University of Sussex

On models, physical laws, and cosmology

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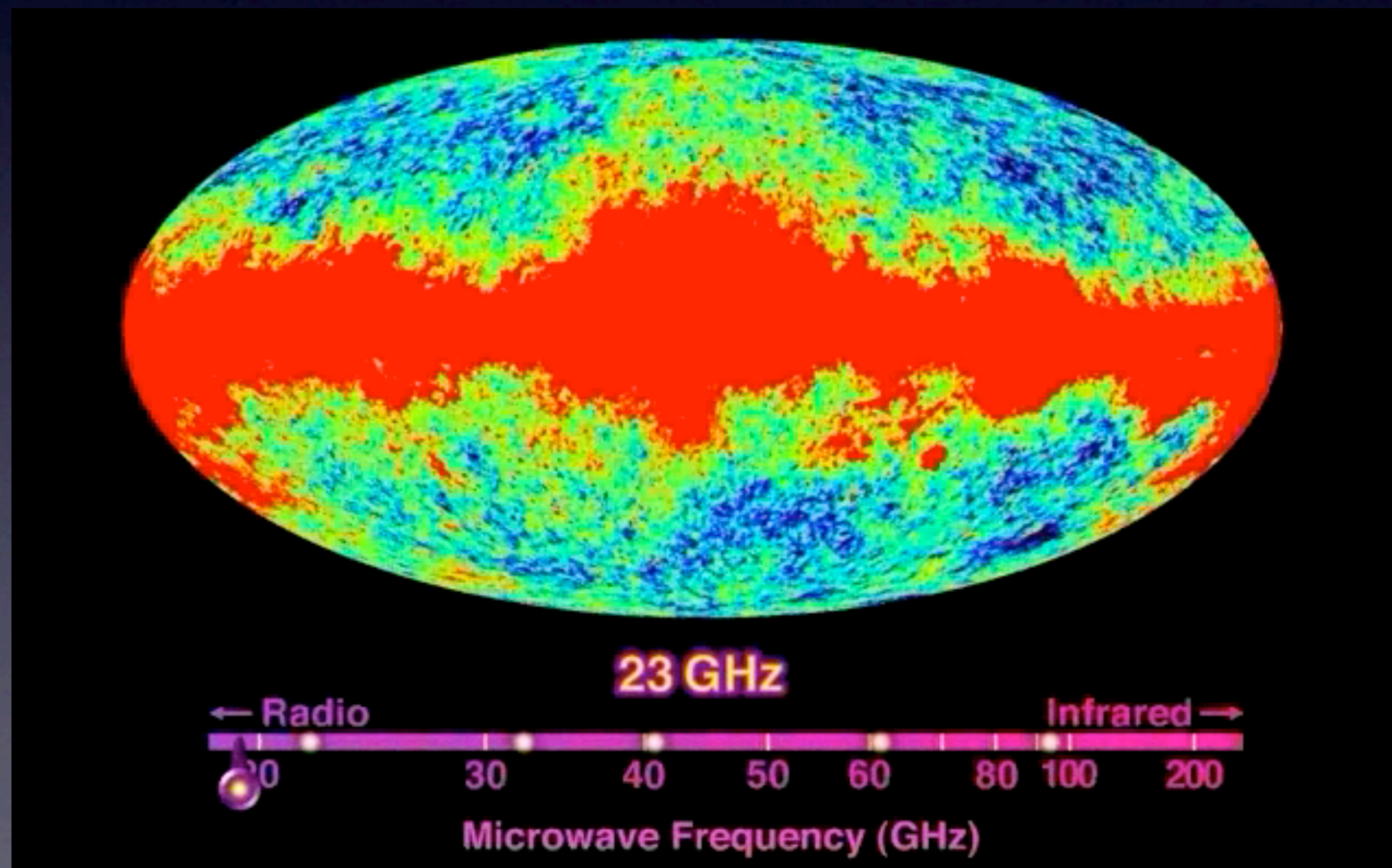
October 2009



Cosmic Microwave Background: WMAP satellite

Microsoft-free presentation

Course aims

My overall aim in these lectures is to explain a framework in which we can carry out science, with particular focus on Bayesian methods of inference and their practical application.

- Lecture 1: Physical laws, models, and cosmological examples
- Lecture 2: Inference
- Lecture 3: Parameter estimation and Monte Carlo methods
- Lecture 4: Model selection and multi-model inference
- Lecture 5: Forecasting and experimental design

Purpose of data analysis

A model is a physical/mathematical construct intended to represent some aspects of the real world. The predictions of the model normally depend on some unknown parameters. Most commonly, the aim of data analysis is to work out what values of those parameters are compatible with the data. More ambitiously, one may ask what choice of parameters is indicated by the data.

Useful models should

- Fit the present data acceptably.
- Have the ability to make predictions for future data.

Useful data should

- Be predictable by the models we aim to test.
- Have modelable intrinsic randomness and experimental error.

If one wants to understand English culture, say the deeper creative aspects from Shakespeare to Pinter, it stands to reason that one really needs to learn English. Analogously, if one really wants to understand the deeper cultural meanings of the sciences, the beautiful relationships that underpin the workings of nature and the physical world - physics in particular, whose language is mathematics - one really does have to learn mathematics.
To those who say that one should be able to explain Science without mathematics, I respond: `Do you think we use mathematics to make life harder for ourselves? No, we use it because it is the only way to understand it. It's not my fault, there really is no other way.'

Harry Kroto
1996 Nobel Prize winner in Chemistry

What is a model, part I?

In the physical sciences, models are usually intended to capture the essence of underlying physical laws, and a principal goal is to uncover the nature of these laws.

Such models usually come in two parts:

Clever Ideas:

e.g. theory of gravity
hot big bang cosmology
quantum mechanics

Parameters:

e.g. strength of gravity
expansion rate of Universe
speed of light

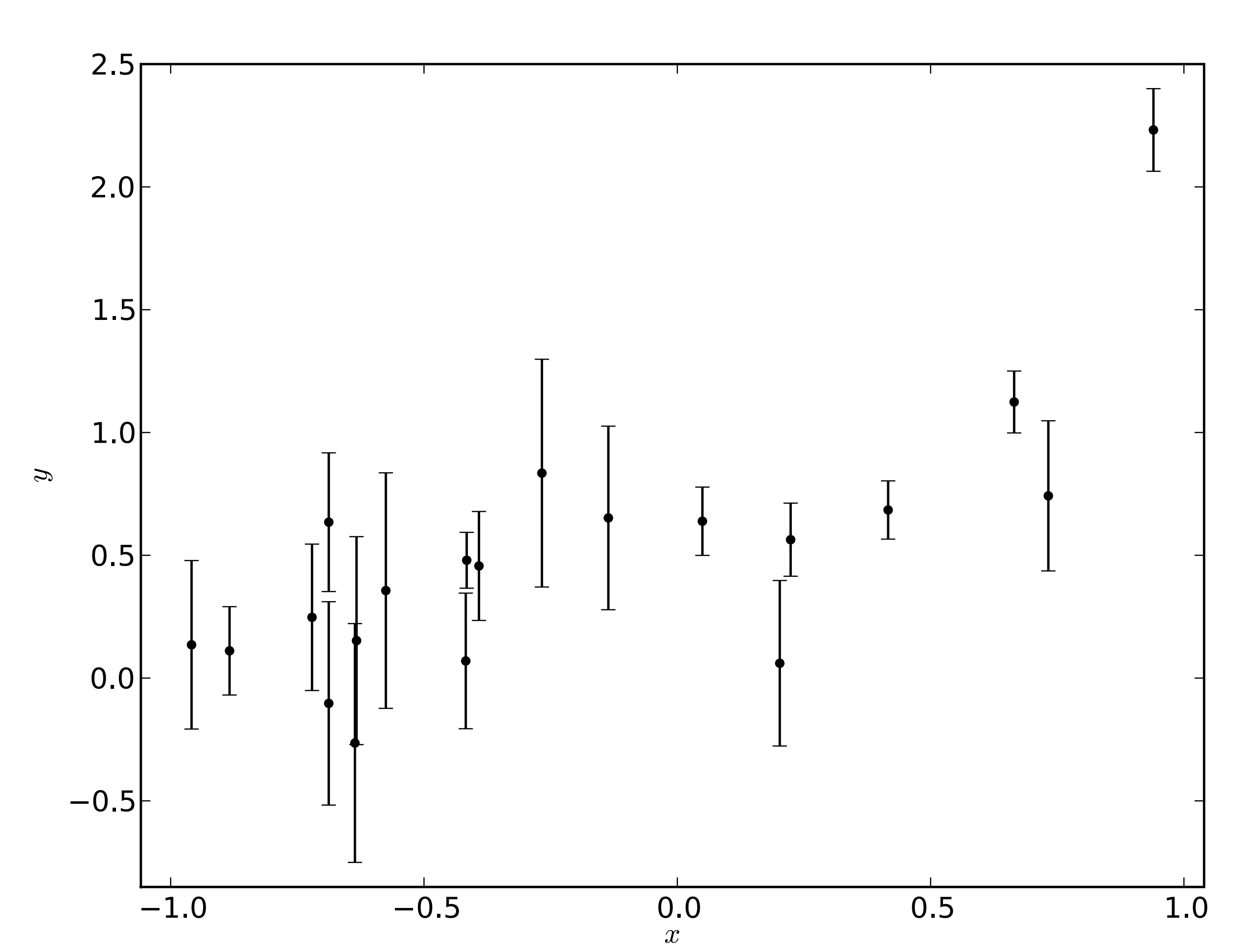
What is a model, part II?

In the biological and geo-sciences, models are usually intended to explain a dataset that has been obtained, to assess causal links, and to develop some predictive power for future data.

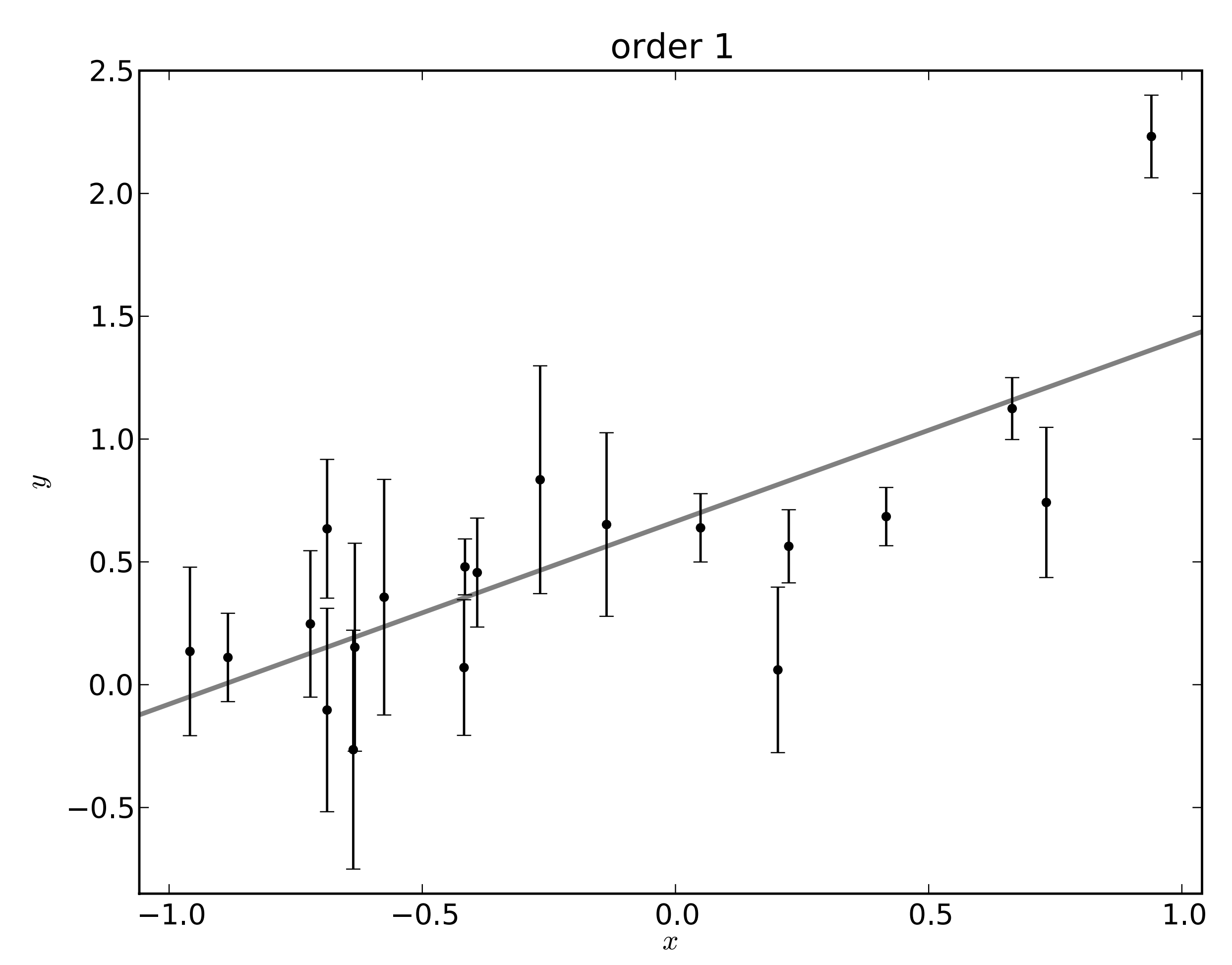
Such models also usually come in two parts:

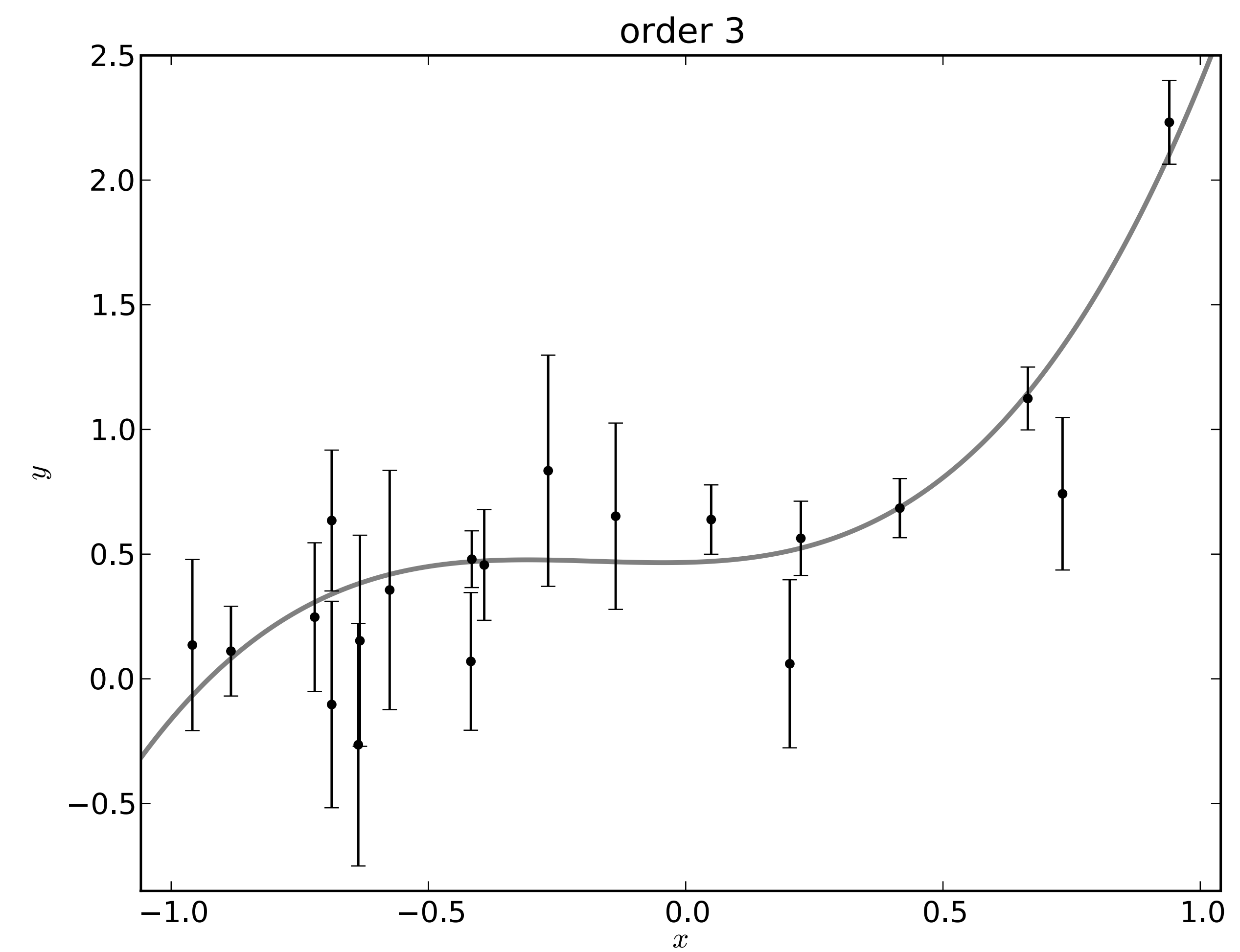
Model structure: e.g. medical outcome related to a linear combination of possible causative factors

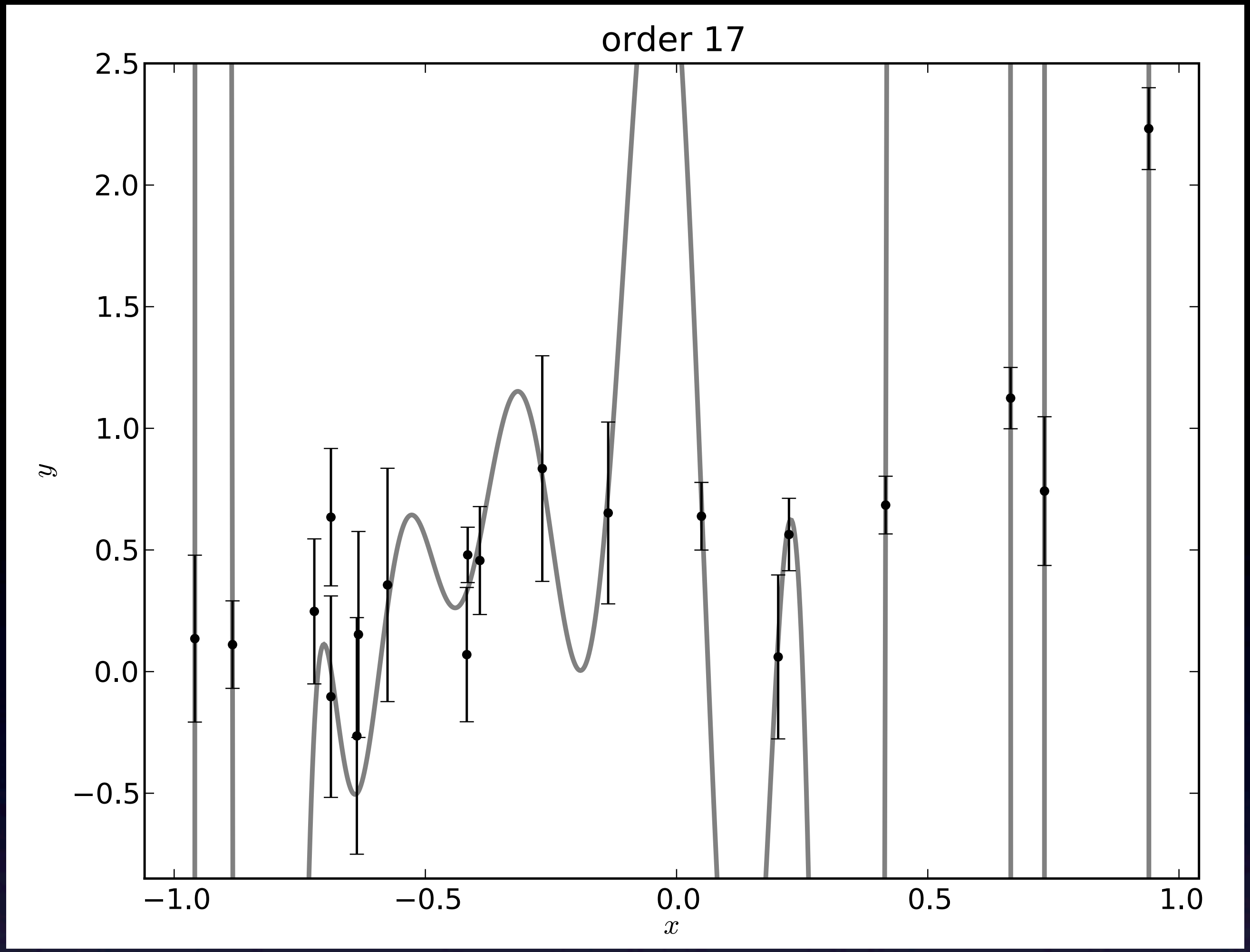
Parameters: e.g. relative weights of different factors in determining an outcome



[Data images from a lecture by David Hogg]







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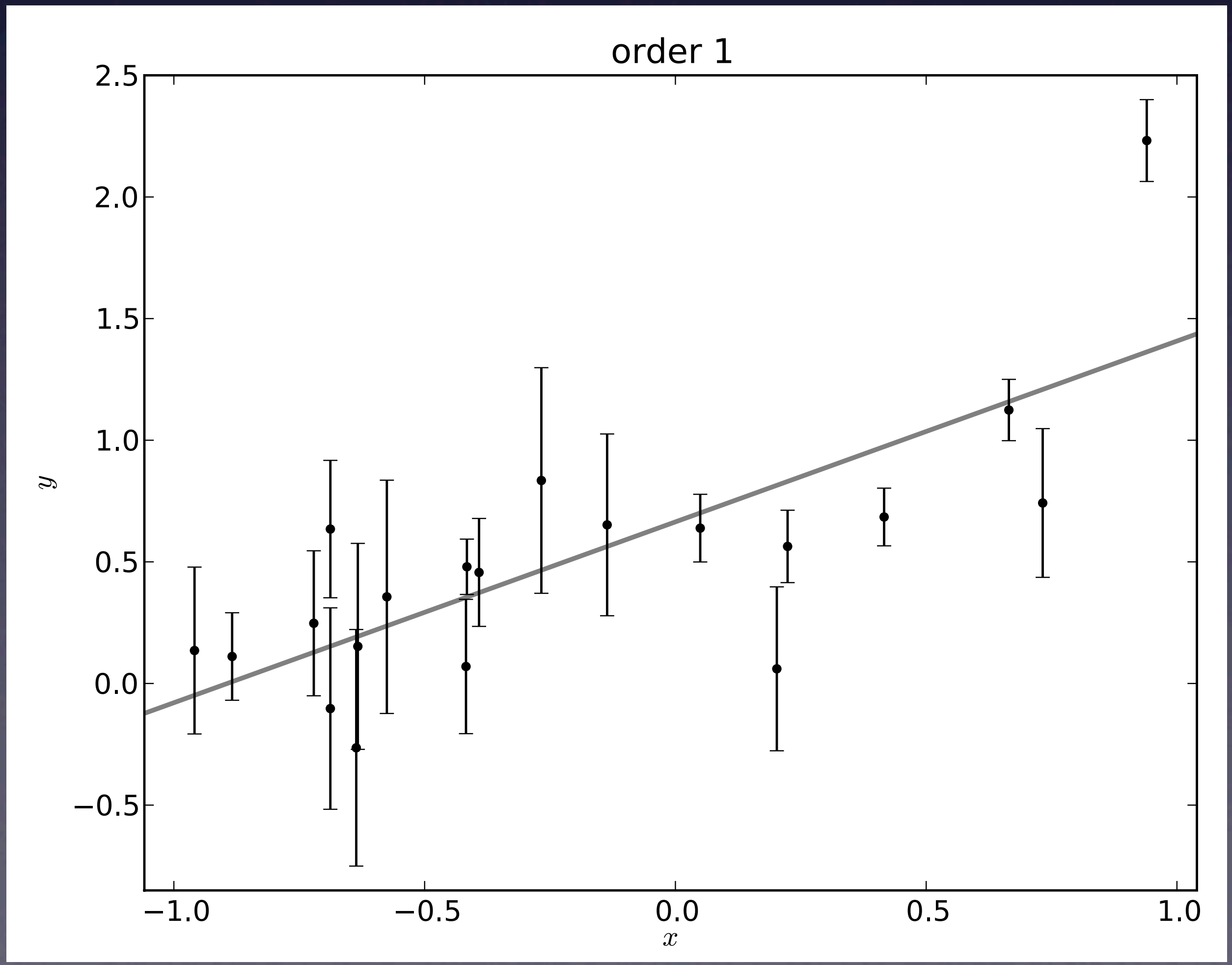
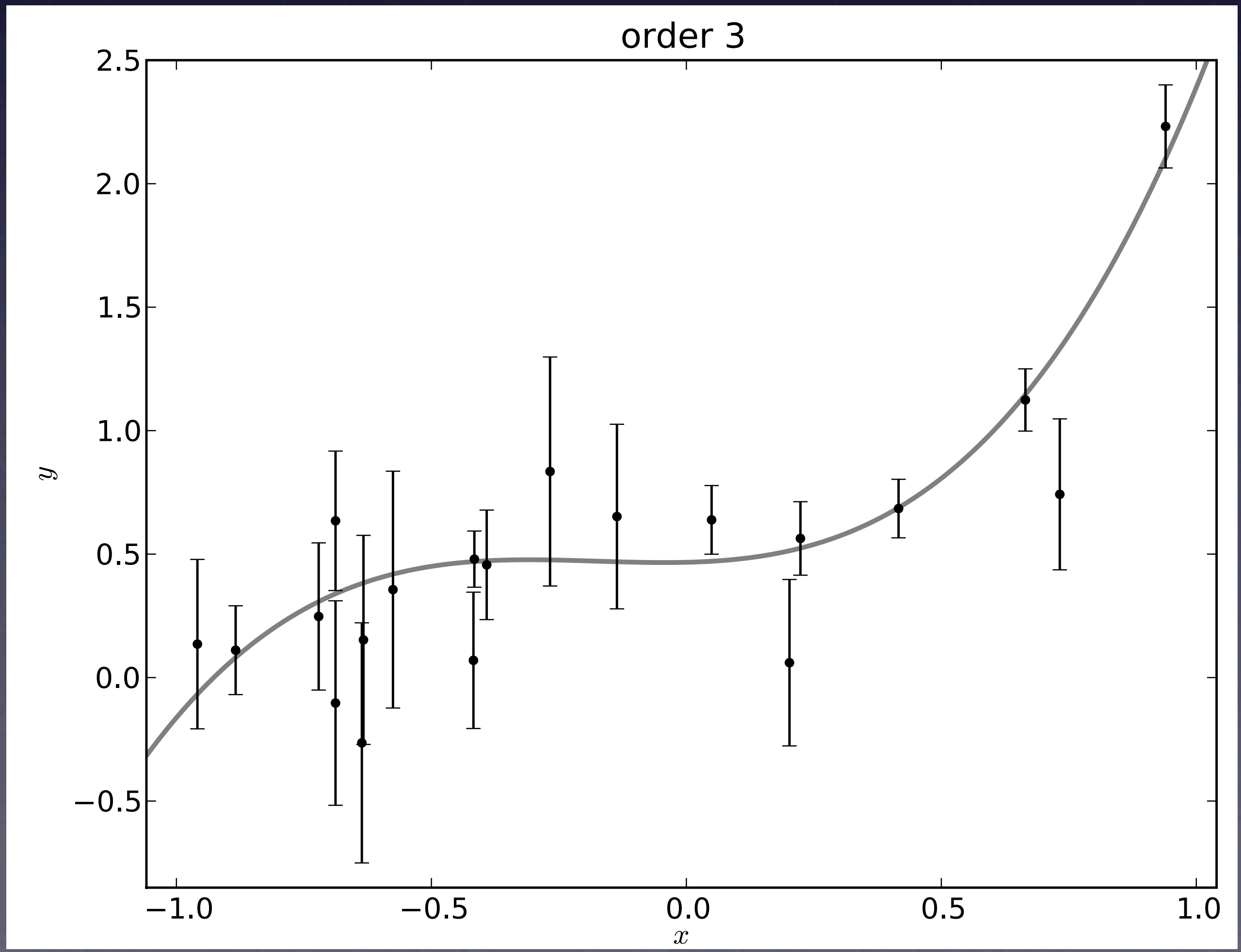
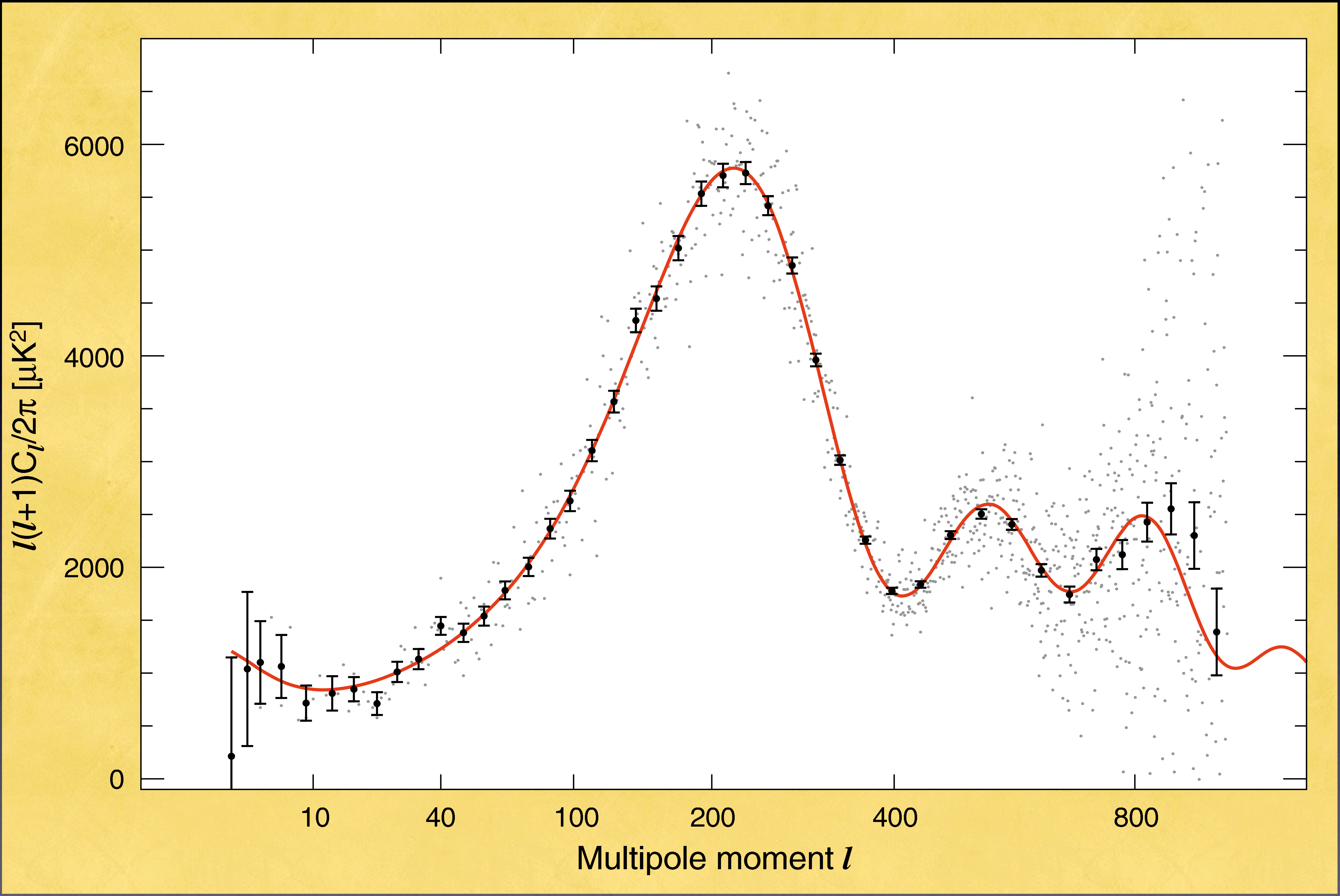





6000
4000
2000
0
$l(l+1)C_l/2\pi$ [μK²]
10
40
100
200
400
800
Multipole moment l

Basic model construction

Model inputs:

- Model structure
- Known parameters
- Unknown parameters

Predictions:

Predictions for observables

Observations:

Measurements of observables, with uncertainties

Model inputs:

Model structure

Known parameters

Unknown parameters

Predictions:

Predictions for observables

Observations:

Measurements of observables, with uncertainties

Typical objectives:
a) To use the observations to measure or constrain the unknown parameters.
b) To test whether the model structure is capable of explaining the data obtained.

What do we do with the outcome of our analysis?

Suppose you have satisfactorily explained the data with our model and extracted its preferred parameter values. Then you might ...

- Sit back happily, satisfied that you have figured out a little piece of how the world works.

  E.g. an astronomer might be content to have measured the expansion rate of the Universe to the best accuracy to date.

- Use the validated model/parameters to make predictions for future outcomes.

  E.g. a medical statistician might use the model to make predictions of disease susceptibility in future patients.

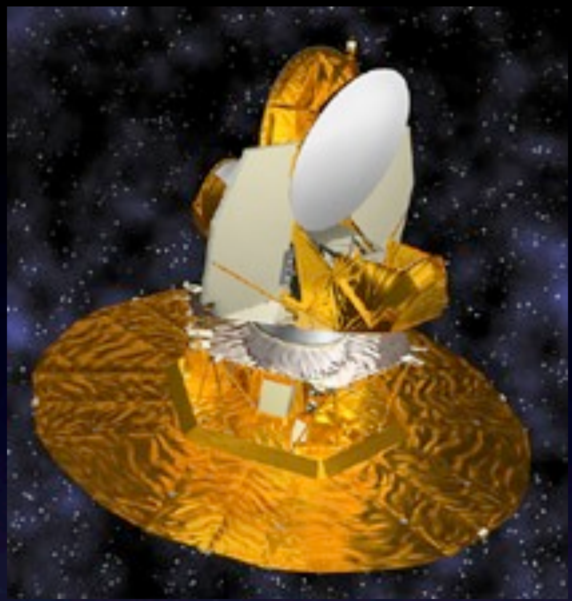
- Decide you can think of a new model that might do even better.

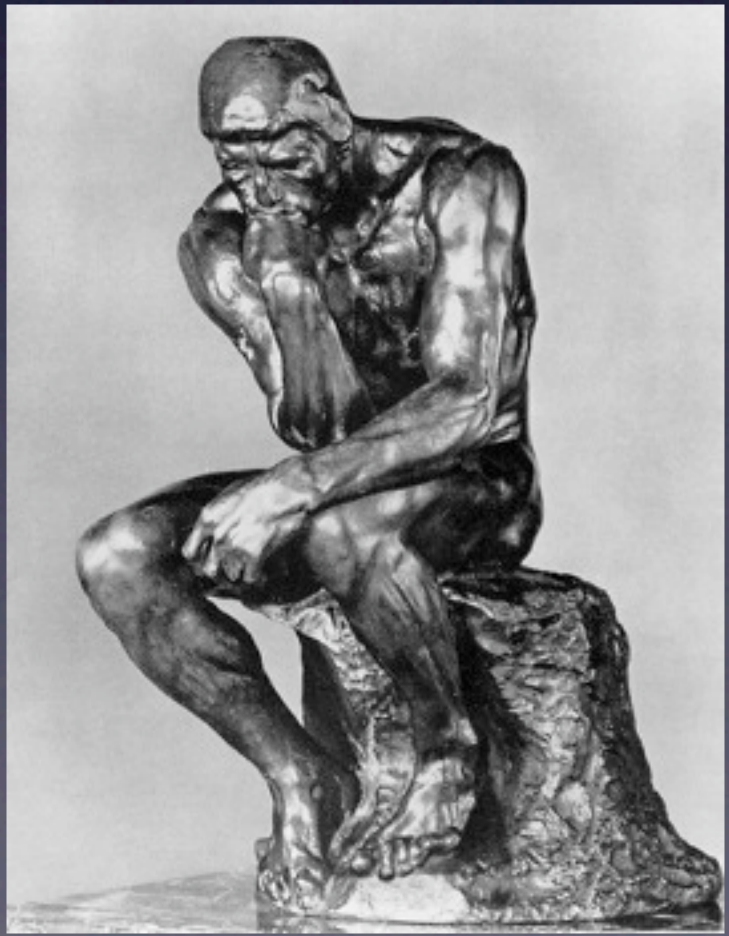
Some cosmology

The era of precision cosmology

Precision observations

Precision theory





The cosmological quest

Modern cosmology is a global and collective effort.

Our aim? To put in place a precision description of our Universe that can stand the test of time, and perhaps outlive us all.

This ambition is possible because

- The superb quality of new observational data, from many sources.
- The development of theoretical models containing enough ingredients to explain observed phenomena.
- The power of modern high-performance computers for data acquisition and analysis, and for determining theoretical predictions from models.

What cosmological model?

These are the principles and physical laws underpinning the Universe.

- Hot big bang cosmology

  Describes the global properties of the Universe, its expansion, and its material content.

- Galaxy formation by gravitational instability

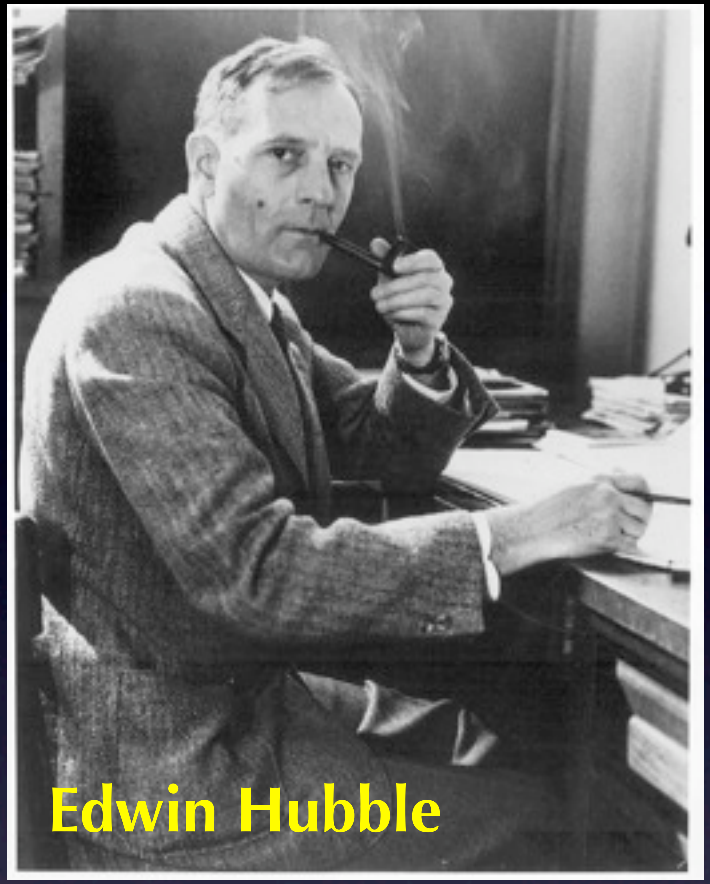
  Describes the growth of structure from initially-small irregularities. Gravity for the initial collapse, lots of other physics in the details.

- Inflationary cosmology

  The leading candidate theory for explaining where those initial irregularities came from: quantum fluctuations during rapid expansion of the young Universe.

The Hot Big Bang

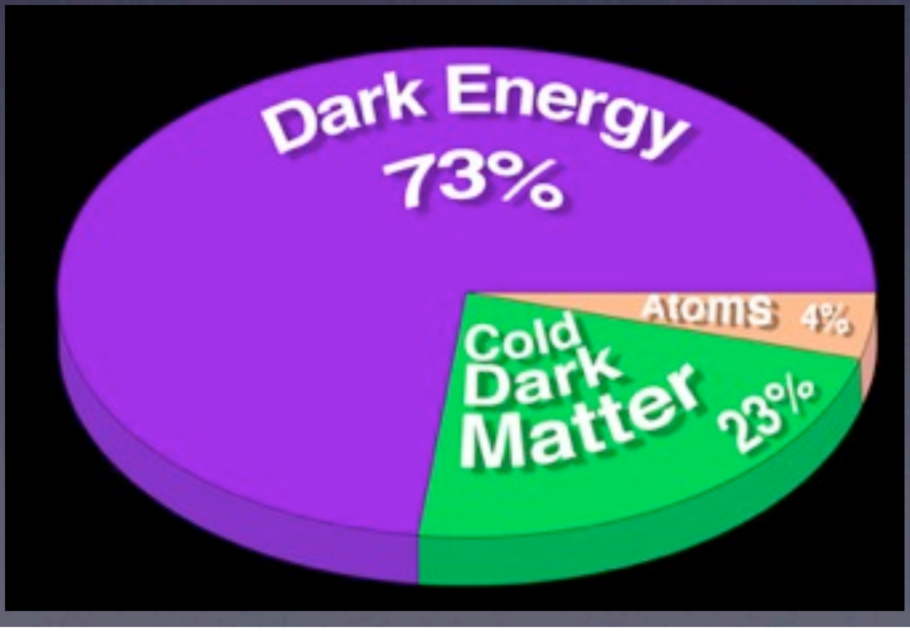
It has been known for about 80 years that the Universe is expanding. Einstein's theory of general relativity is used to explain this.


Edwin Hubble


Vesto Slipher

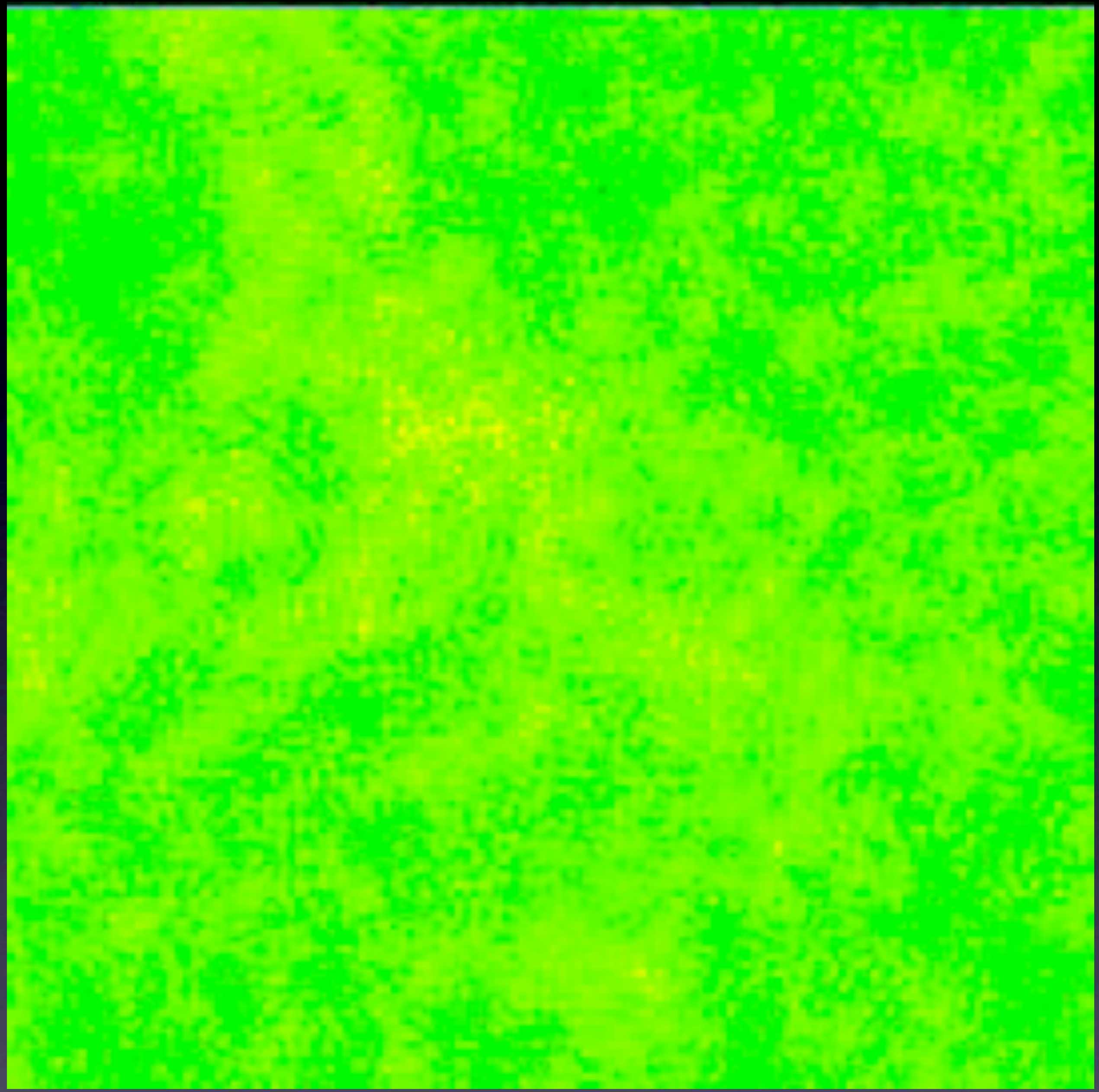
Our challenge is to figure out the material constitution of the Universe, currently believed to have five parts.

- Baryons (ie protons, neutrons and electrons)
- Radiation (photons)
- Neutrinos (a very weakly-interacting particle)
- Dark matter
- Dark energy (eg cosmological constant)



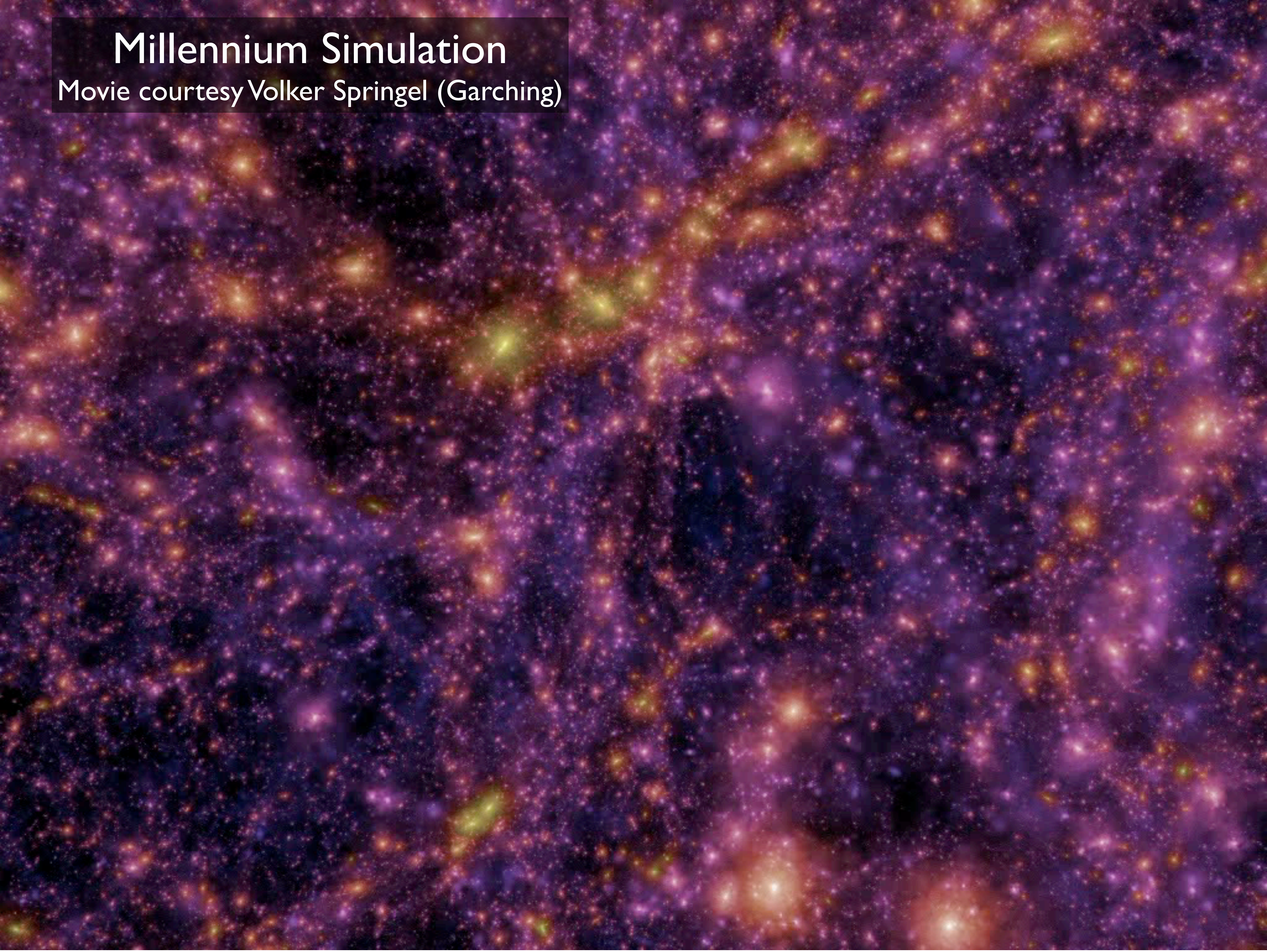
Structure formation

The Universe evolves, and will continue to do so. Galaxies form and are presently assembling into galaxy clusters and superclusters.



Gravity is the main player: initially overdense regions exert greater gravitational attraction on their neighbouring regions and accumulate material.

The details of the gravitational instability mechanism depend on the properties of the Universe, such as its material composition. Study structure formation, and you learn about the Universe.

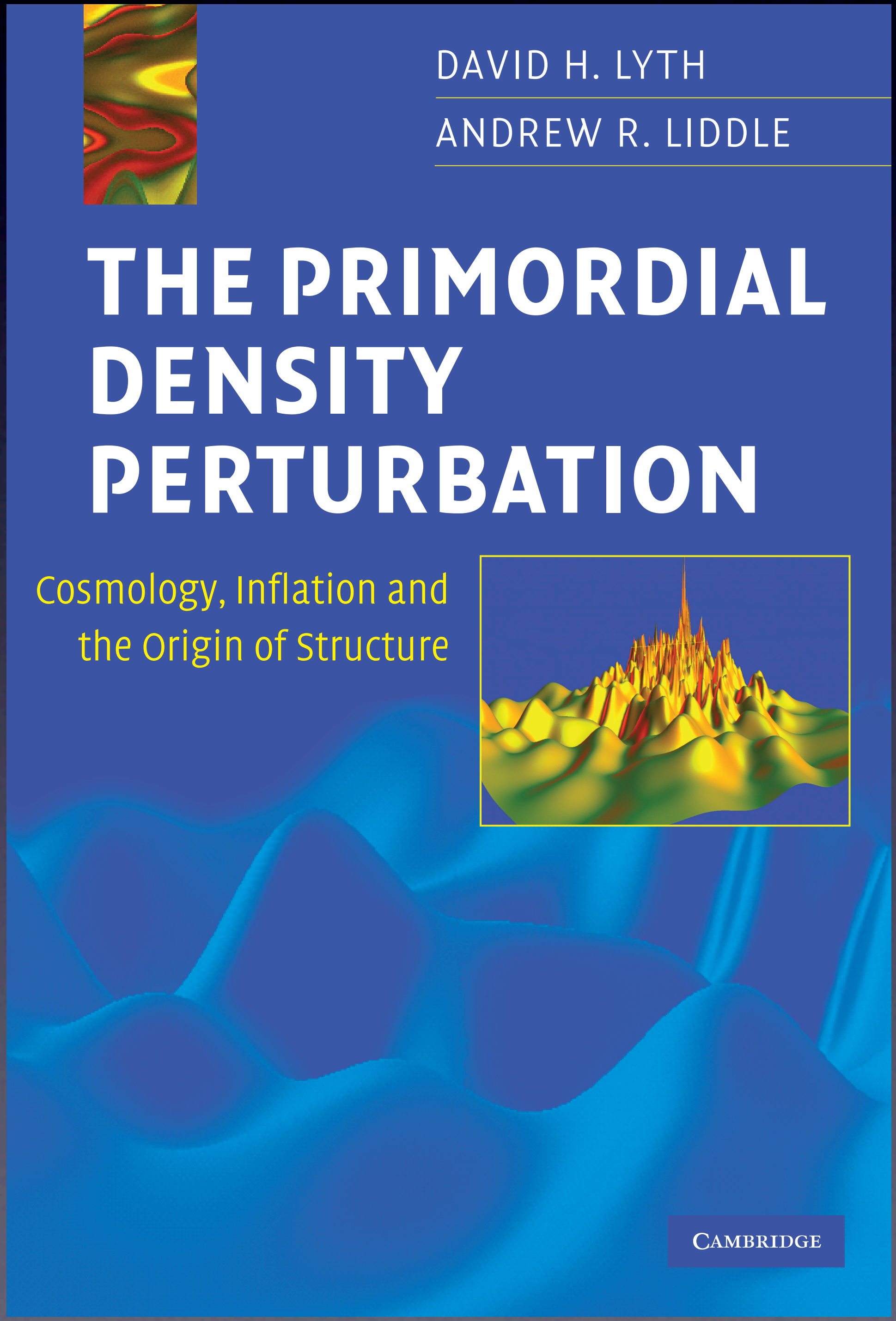

Millennium Simulation
Movie courtesy Volker Springel (Garching)

Inflationary cosmology

What created the seeds that gravitational instability amplified?

According to the leading paradigm, cosmological inflation, the seeds were created by quantum uncertainty in the very young Universe.

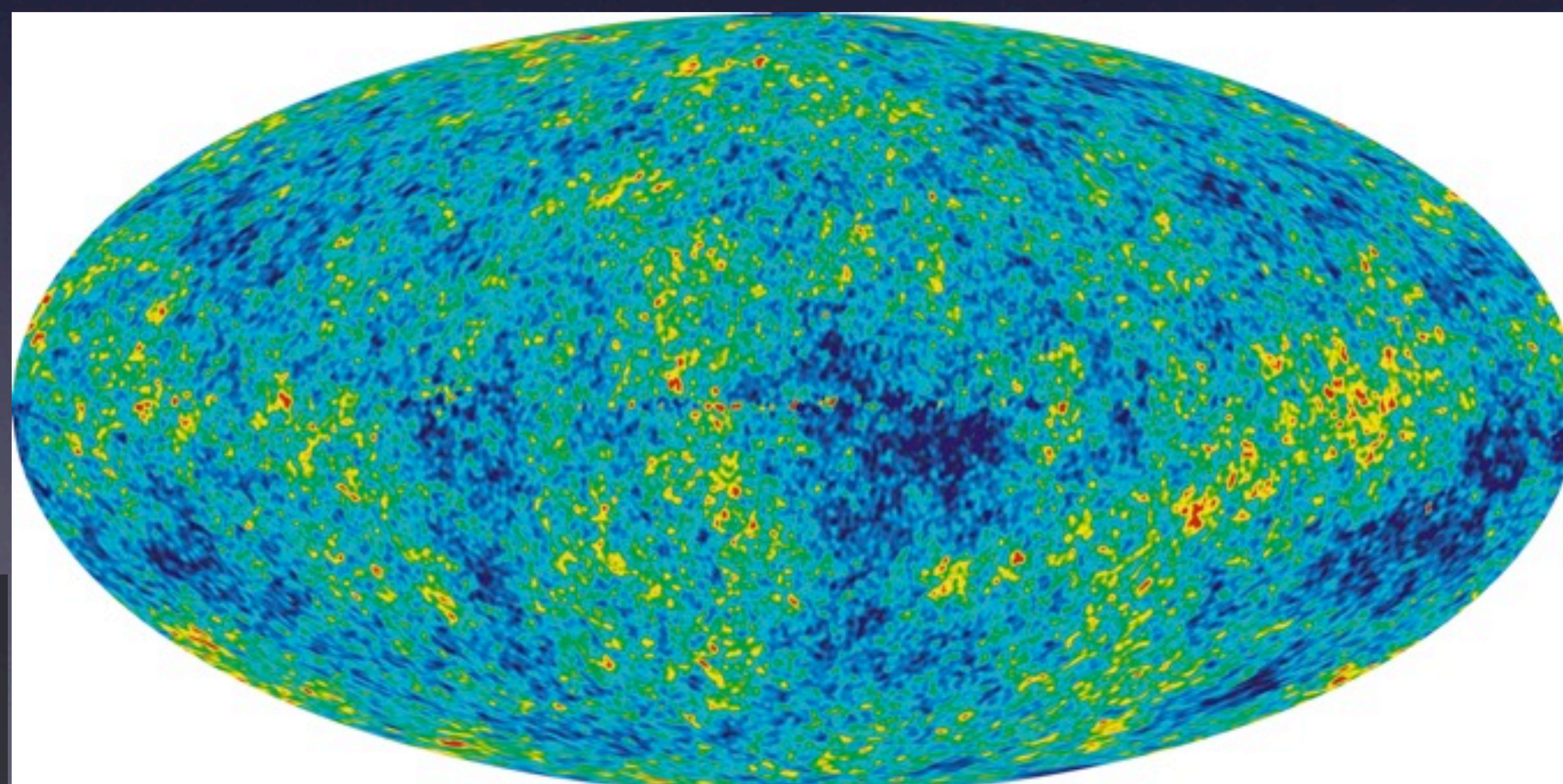
By studying the seeds from which galaxies grew, we learn about processes in the very young Universe. These processes determine how the Universe looks today.



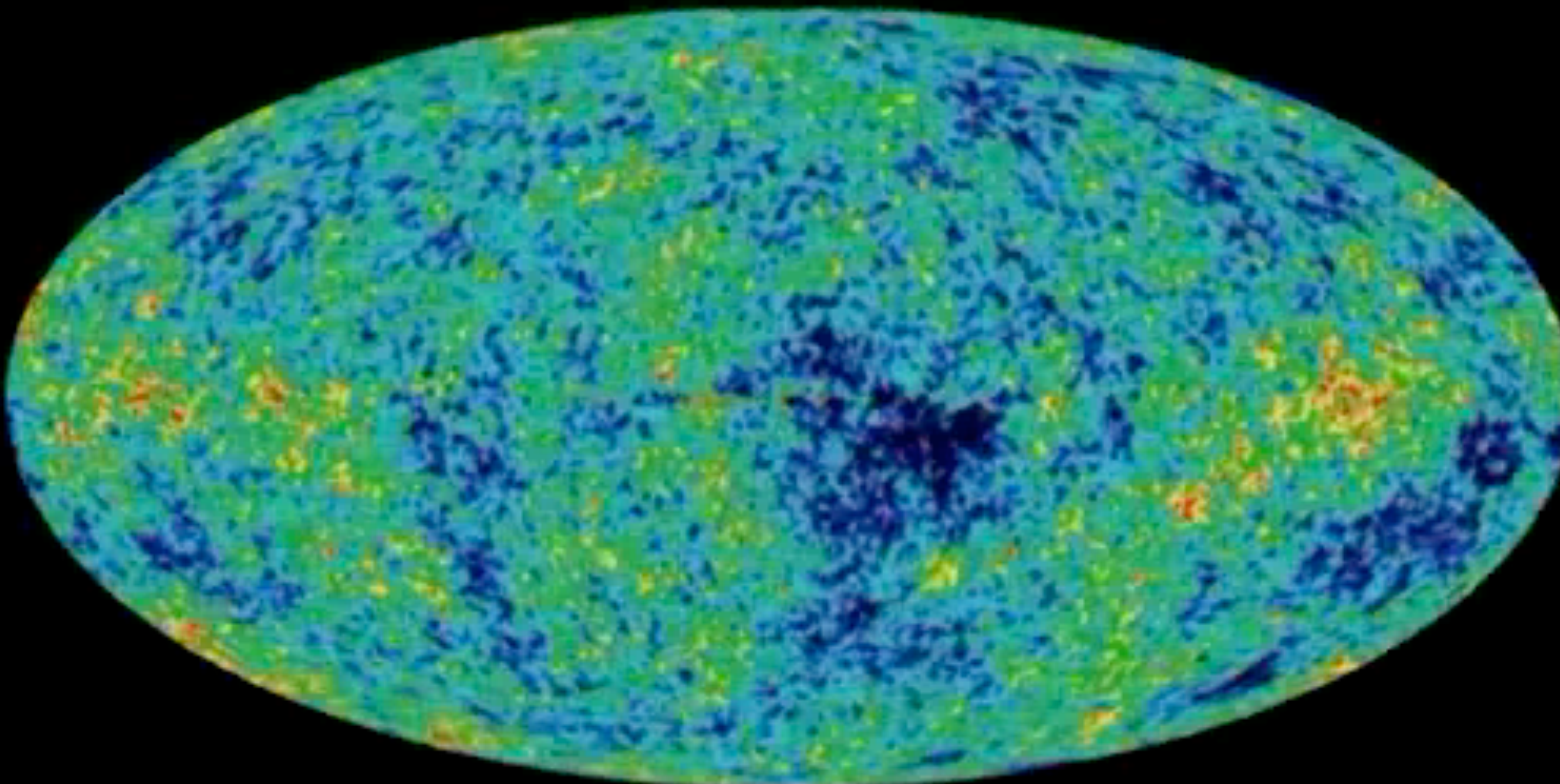
The cosmic microwave background

This is relic radiation left over from the hot early stages of the Big Bang. It tells us about physical conditions in the young Universe.

Discovery of `seed' irregularities in the CMB led to the award of the 2006 Nobel Prize to John Mather and George Smoot.



All-sky CMB map by the WMAP satellite (2003-2008)



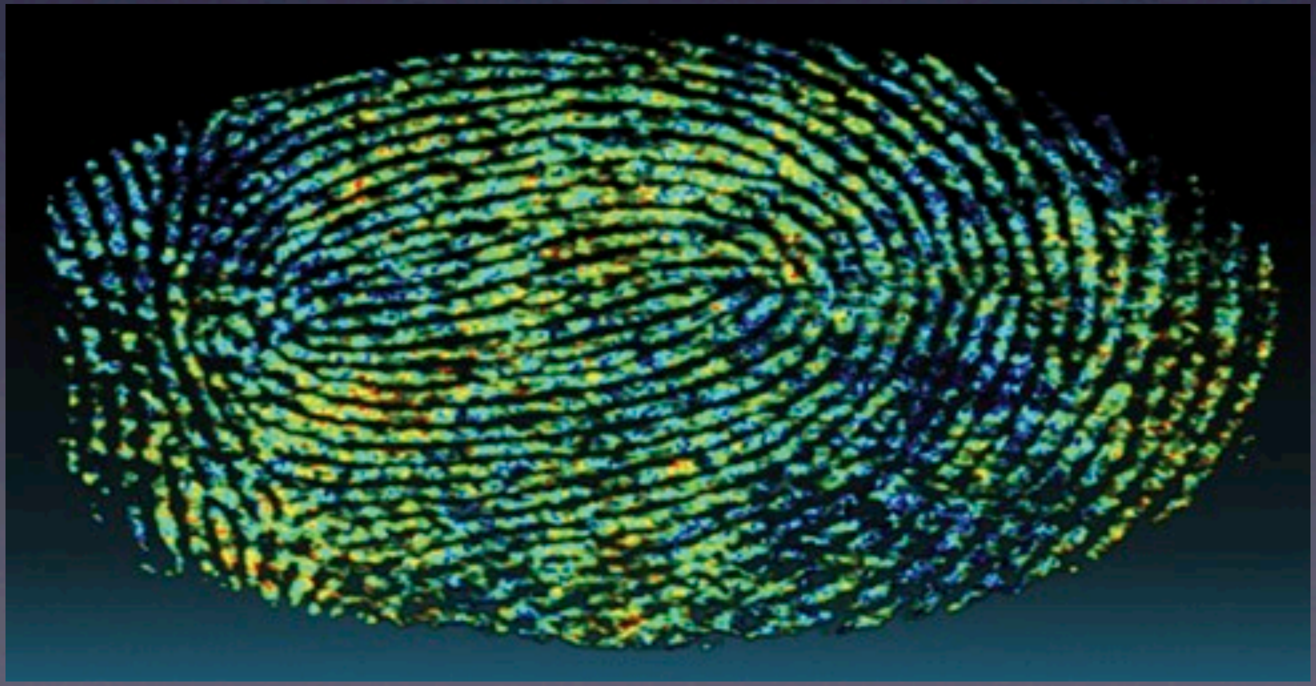
Movie credit: NASA/WMAP Team

The cosmic fingerprint

Questions:

- How fast is the Universe expanding?
- What are the amounts of the different kinds of materials in it?
  Atoms versus **radiation** versus **dark matter** versus **dark energy**
- How old is the Universe?
- What form do the initial seed irregularities take?
- ....

Each of the different possible Universes predicts a distinctive pattern in the structures seen in the cosmic microwave background. Measure them, and we find out which Universe is ours!



What cosmological parameters?

Cosmological parameters can be broken up into two classes: those describing the homogeneous Universe and those describing the irregularities.

- Homogeneous Universe
  - Hubble parameter
  - Spatial curvature
  - Baryon density
  - Dark matter density
  - Reionization optical depth
  - Galaxy bias parameter
  - ...
- Perturbations
  - Initial amplitude of perturbations
  - Scale dependence of perturbations
  - Primordial gravitational waves
  - ...

Old Universe – ***New*** Numbers

$\Omega_{tot} = 1.02^{+0.02}_{-0.02}$

$w < -0.78$ (95% CL)

$\Omega_\Lambda = 0.73^{+0.04}_{-0.04}$

$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$

$\Omega_b = 0.044^{+0.004}_{-0.004}$

$n_b = 2.5 \times 10^{-7}{}^{+0.1\times10^{-7}}_{-0.1\times10^{-7}}$ cm^{-3}

$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$

$\Omega_m = 0.27^{+0.04}_{-0.04}$

$\Omega_\nu h^2 < 0.0076$ (95%CL)

$m_\nu < 0.23$ eV (95% CL)

$T_{cmb} = 2.725^{+0.002}_{-0.002}$ K

$n_\gamma = 410.4^{+0.9}_{-0.9}$ cm^{-3}

$\eta = 6.1 \times 10^{-10}{}^{+0.3\times10^{-10}}_{-0.2\times10^{-10}}$

$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$

$\sigma_8 = 0.84^{+0.04}_{-0.04}$ Mpc

$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$

$A = 0.833^{+0.086}_{-0.083}$

$n_s = 0.93^{+0.03}_{-0.03}$

$dn_s/d\ln k = -0.031^{+0.016}_{-0.018}$

$r < 0.71$ (95% CL)

$z_{dec} = 1089^{+1}_{-1}$

$\Delta z_{dec} = 195^{+2}_{-2}$

$h = 0.71^{+0.04}_{-0.03}$

$t_0 = 13.7^{+0.2}_{-0.2}$ Gyr

$t_{dec} = 379^{+8}_{-7}$ kyr

$t_r = 180^{+220}_{-80}$ Myr (95% CL)

$\Delta t_{dec} = 118^{+3}_{-2}$ kyr

$z_{eq} = 3233^{+194}_{-210}$

$\tau = 0.17^{+0.04}_{-0.04}$

$z_r = 20^{+10}_{-9}$ (95% CL)

$\theta_A = 0.^\circ598^{+0.002}_{-0.002}$

$d_A = 14.0^{+0.2}_{-0.3}$ Gpc

$l_A = 301^{+1}_{-1}$

$r_s = 147^{+2}_{-2}$ Mpc

WMAP Science Team

To the future ...

"... Cry as you
will, take what you
need, the night is young
and limitless our greed."
J.H. Prynne, The White Stones

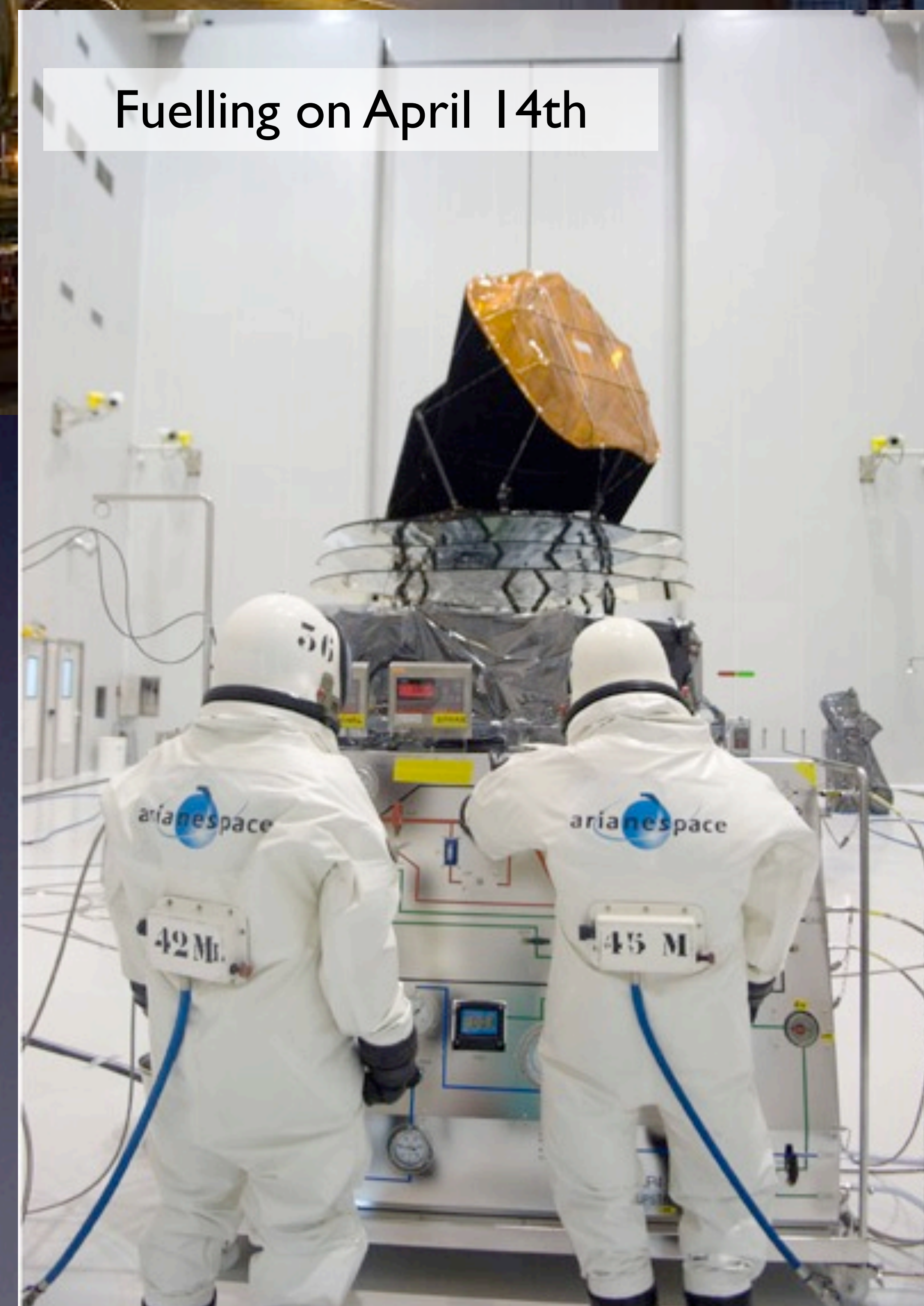
The standard cosmological models answers many questions about our Universe - what is its material composition, how old is it, how fast does it expand - but leaves some unanswered and raises some new ones.

- What is the dark matter?
- Why is the Universe filled with matter, not anti-matter?
- What is the dark energy? Does it evolve? How will it affect the future evolution of the Universe?

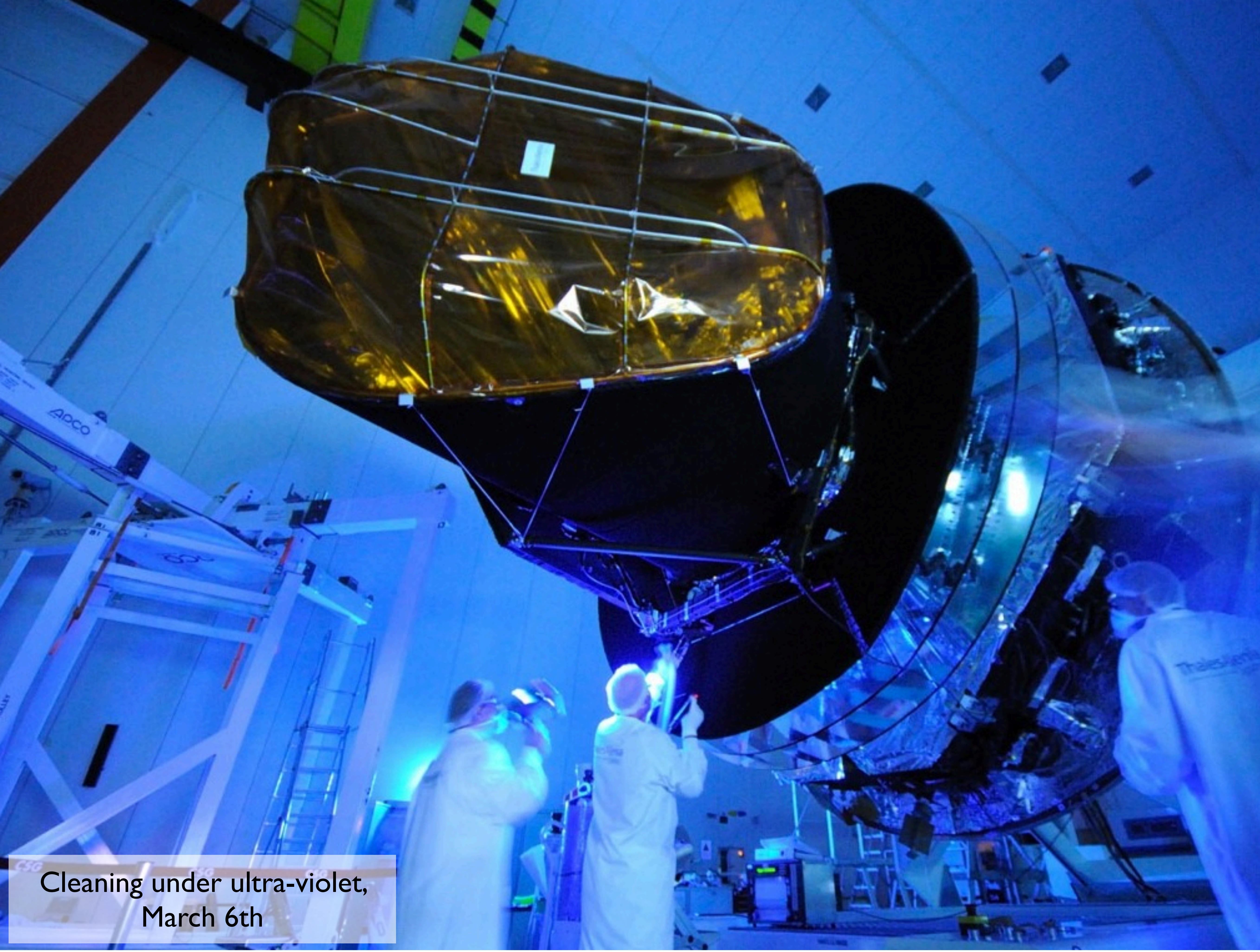
The answer, as always, is "get more data"!

Planck Satellite arrives in French Guiana, February 18th

Planck satellite integration March 2nd

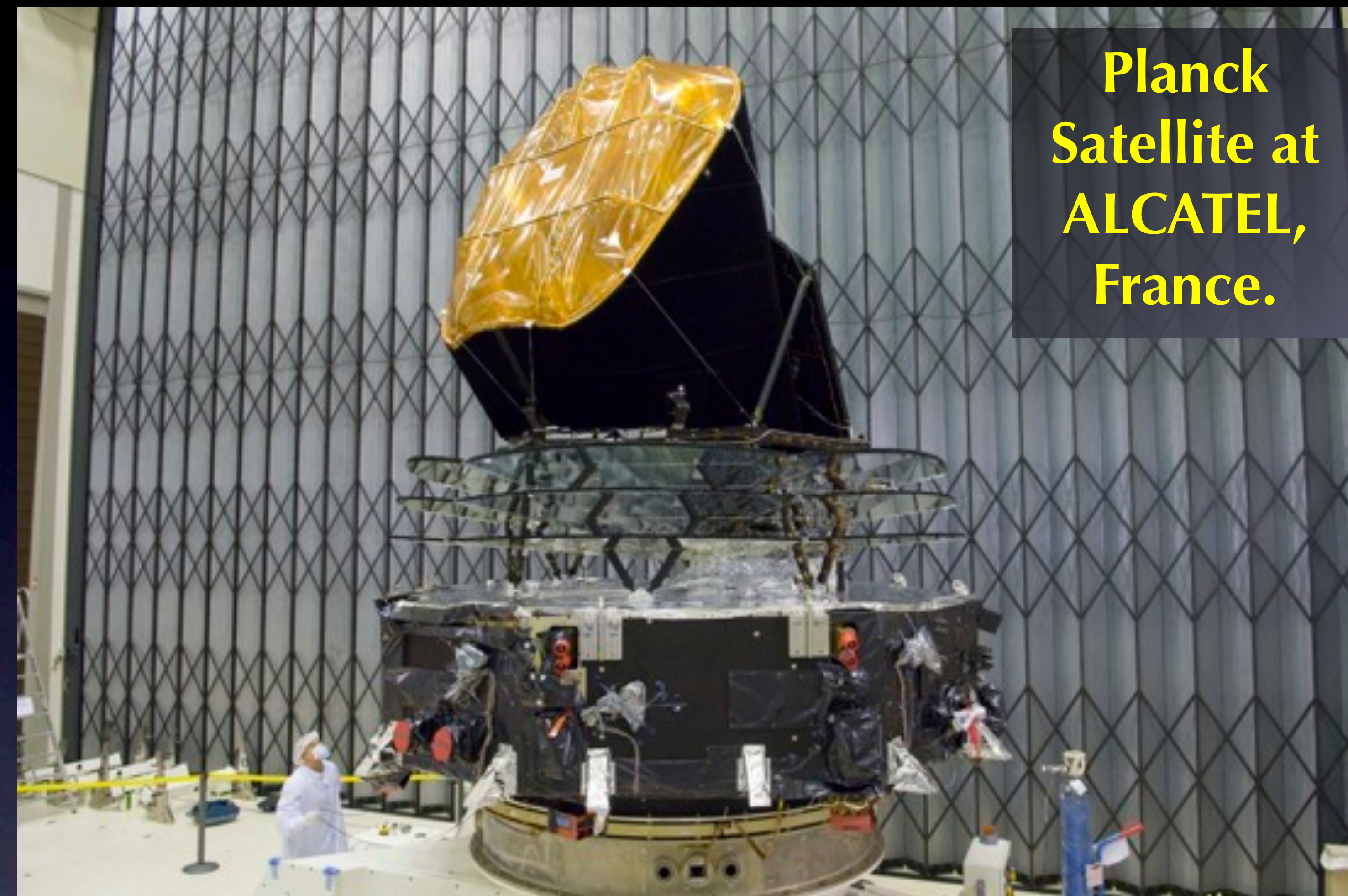


Fuelling on April 14th

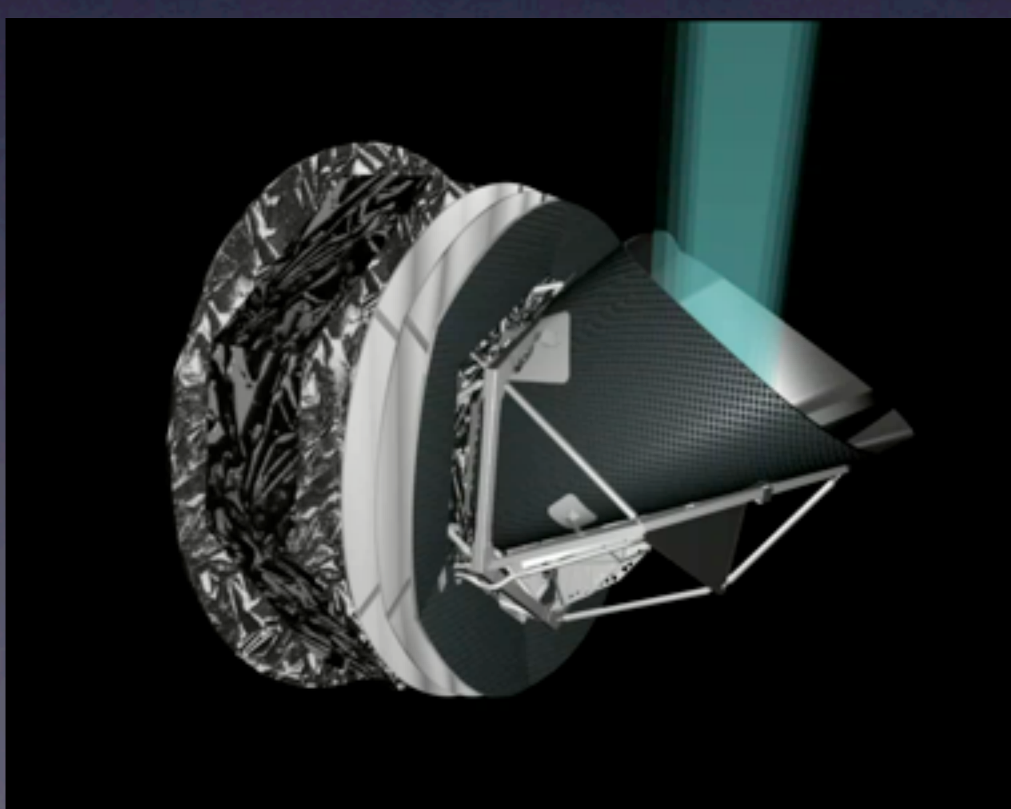


Cleaning under ultra-violet,
March 6th

Planck Satellite



Planck Satellite at ALCATEL, France.



Operations:

Successfully launched on May 12th 2009.
First all-sky survey started in late August.

Publication schedule for main results:

Late 2012: suite of science papers and first data release.

Statistical challenges for cosmology

Within cosmology, we face statistical challenges of various types, many of which I will use as concrete illustrations during this course.

- Estimation of cosmological parameters

  How do we best extract information about our cosmological models from the complex datasets at hand?

- Cosmological model selection

  How can we use the data to inform us about the possible choices of cosmological model?

- Survey forecasting and design

  How can we assess how well future experiments will be able to address the questions we wish to answer? And what can we do to optimize their chances of doing so?