Lecture 1: Neutrinos and Astrophysics

- Early history of the neutrino: probing its properties
- Solar neutrinos and the discovery of neutrino mass
- Inner space/Outer space connections

Wick Haxton Niels Bohr Institute July 2-6, 2018

Multi-Messengers from Compact Sources

Introduction

A great deal of effort and expense has been consumed in recent searches for new physics at the energy frontier

But so far, the specific evidence we have that there is physics beyond the standard model has come primarily from low-energy tests

Neutrino mass and mixing: oscillations of solar and atmospheric neutrinos

Cosmological dark matter: a variety of observations showing that the amount of gravitating mass at various scales is about 7 times the baryonic mass

The former is today's theme, a story with
- exquisitely precise, clean experiments
- persistence, which will continue to be needed …

Story begins in 1914 with Chadwick's studies of β decay, the nuclear process

$$(A, Z) \rightarrow (A, Z+1) + e^-$$

in which he observed that the emitted electrons came out in a continuous spectrum —
which would violate energy conservation

Chadwick speculated that perhaps some unobserved radiation accompanied the decay, accounting for the anomaly — though Rutherford suggested a less radical solution, that the electron lost energy in the target.

This issue was settle in 1927 by Ellis and Wooster, who measured the total energy deposited in a thick target in the decay of ^{210}Bi, finding 0.337 keV per decay, less than the 1.05 MeV nuclear mass difference

So either missing radiation, or QM fails to satisfy energy conservation

Liebe Radioaktive Damen and Herren.....

"... a genius, comparable perhaps only to Einstein himself"   N. Bohr

- In 1930 Pauli hypothesized that an unobserved neutral, spin-1/2 "neutron" accounted for the apparent anomaly -- a new particle with mass < 1% that of the proton, the ν

- At least initially he thought of the neutrino as a stable constituent of the nucleus: this was suggested, for example, by the spin puzzle presented by ^{14}N, with Z=7. A system of seven protons would have half-integer spin, but the addition of a spin-half neutrino constituent would resolve this problem

"I have done a terrible thing. I have postulated a particle that cannot be detected."

The Weak Interaction

- 1932: Chadwick's discovery of the "neutron"

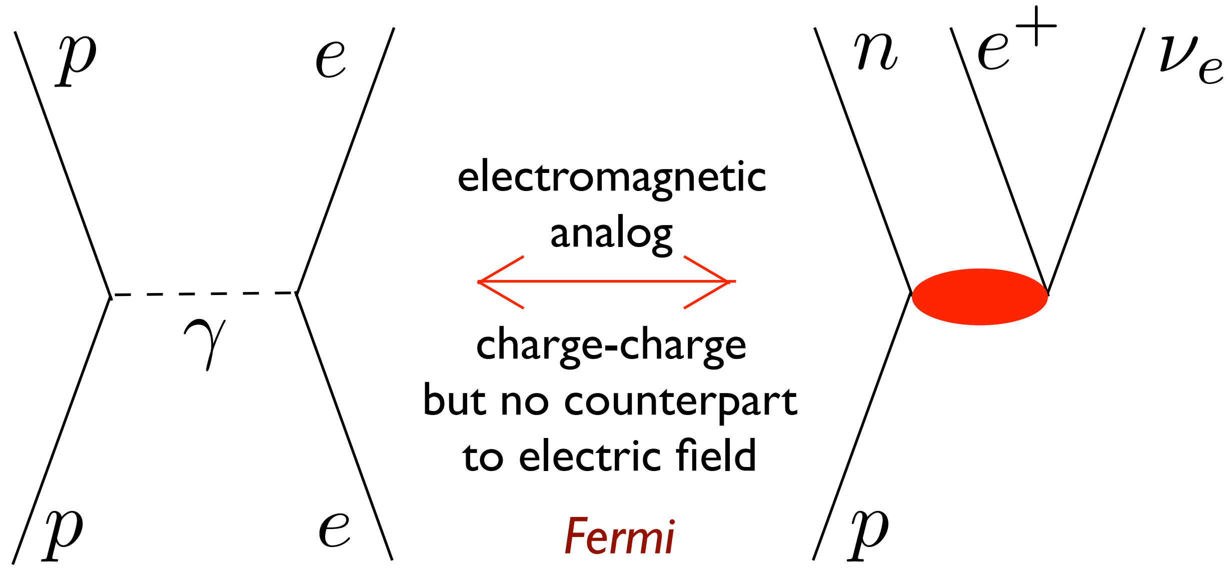
  In the 1933 Solvay conference Pauli finally presented his theory of the "neutrino." Fermi suggested the name "neutrino" to distinguish it from Chadwick's heavy neutral nucleon

- 1934: Fermi's incorporation of both the neutron and the neutrino in his "effective theory" of β decay

$$n_{\text{bound}} \to p_{\text{bound}} + e^- + \bar{\nu}_e$$

  Proposed that the neutrino was produced in the decay, accompanying the outgoing electron

Fermi's treatment of beta decay was based on an analogy with the neutral electromagnetic interaction between static charges, modified to yield a point-like interaction



1933 Solvay Conference

George Gamow attended Solvay 1933

Russian authorities yielded to Gamow's insistence that his physicist wife Lyubov Vokmintseva also be granted visa for the meeting, not knowing the couple had twice previously tried to escape Russia (via kayak!)

Did not return: Curie Institute → Univ. London → Univ. Michigan → George Washington University (Edward Teller)

Viewed from a modern isospin context: $p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \tau_+|n\rangle = |p\rangle \quad \tau_-|p\rangle = |n\rangle$$

the electromagnetic interaction and Fermi's weak analog can be compared

$$[e] \ \frac{1}{r} \ \left[e\frac{1+\tau_3}{2}\right] \quad \Leftrightarrow \quad [e] \ \frac{\delta(\vec{r})}{M^2} \ \left[\mp\frac{1}{\sqrt{2}}e\tau_\pm\right]$$

E&M: $\rho^S + \rho^{V(0)}$ weak $\rho^{V(\pm)}$

makes sense: Fermi used the "missing" components of isovector charge —

but did not consider using the electromagnetic
neutral current itself in the weak interaction

Fermi later recognized that Lorentz invariance meant that this relation must extend to currents (moving charges), $\rho \rightarrow j^{\mu} = (\rho, \vec{j}) = e(1, \vec{p}/M_N)$

$$j^{E\&M} = j_{\mu}^{V;S} + j_{\mu}^{V;V(0)} \qquad \Leftrightarrow \qquad j^{Weak} = j_{\mu}^{V;V(\pm)}$$

Weak current a space-spin vector and an isospin isovector:
E&M and the weak interaction made use of all three isospin components of the vector hadronic current: basic idea of CVC

Then:

THE
PHYSICAL REVIEW
A Journal of Experimental and Theoretical Physics Established by E. L. Nichols in 1893
VOL. 49, NO. 12 JUNE 15, 1936 SECOND SERIES

Selection Rules for the β-Disintegration

G. GAMOW AND E. TELLER, *George Washington University, Washington D. C.*
(Received March 28, 1936)

§1. The selection rules for β-transformations are stated on the basis of the neutrino theory outlined by Fermi. If it is assumed that the spins of the heavy particles have a direct effect on the disintegration these rules are modified. §2. It is shown that whereas the original selection rules of Fermi lead to difficulties if one tries to assign spins to the members of the thorium family the modified selection rules are in agreement with the available experimental evidence.

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Weak current a space-spin vector and an isospin isovector:
E&M and the weak interaction made use of all three isospin components of the vector hadronic current: a step toward unification!

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Fermi's β-decay ↔ electromagnetism analogy ↔ vector weak current ⇒

	$\mu = 0$	$\mu = 1,2,3$
$j_\mu^{weak} = j_\mu^{V;V_\pm}$	$1 \ \tau_\pm$	$\vec{p}/m_N \ \tau_\pm$

Fermi's relativistic correction, noted by G and T

⇒ selection rules for "allowed" decays of

ΔJ = 0   Δπ = 0, e.g., $0^+ \rightarrow 0^+$ decays

with relativistic corrections

ΔJ = 0, ±1 (but no 0→0)   Δπ = 1, e.g., $1^- \rightarrow 0^+$ decays: suppressed by $(v/c)^2$ in transition probabilities

GT added an axial contribution to Fermi's interaction

	$\mu = 0$	$\mu = 1,2,3$
$j_\mu^{weak} = j_\mu^{V;V_\pm}$	$1 \ \tau_\pm$	$\vec{p}/m_N \ \tau_\pm$
$+j_\mu^{A;V_\pm}$	$\vec{\sigma} \cdot \vec{p}/m_N \ \tau_\pm$	$\vec{\sigma} \ \tau_\pm$

ordinary vector

$\sim \vec{r} \times \vec{p}$

carries opposite parity

pseudo- or axial-vector

So that one could obtain in lowest order (allowed)

Fermi: $\Delta J = 0$ $\Delta\pi = 0$, e.g., $0^+ \rightarrow 0^+$ decays and

Gamow-Teller: $\Delta J = 0, \pm 1$ (but no $0 \rightarrow 0$) $\Delta\pi = 0$, e.g., $1^+ \rightarrow 0^+$

"Either the matrix element M_1 or the matrix element M_2 or finally a linear combination of M_1 and M_2 will have to be used to calculate the probabilities of the β-disintegrations. If the third possibility is the correct one, and the two coefficients in the linear combination have the same order of magnitude, then all transitions [satisfying the selection rules] would now [be strong allowed ones]"

- They had deduced the correct rate for beta decay

$$\omega \sim |\langle 1 \rangle|^2 + g_A^2 |\langle \vec{\sigma} \rangle|^2$$

- They obtained this result by generalizing Fermi's interaction into a sum of four-fermion interactions

$$j_\mu^{lep\ V;\mp} j_\mu^{nucl\ V;\pm} \Leftrightarrow j_\mu^{lep\ V;\mp} j_\mu^{nucl\ V;\pm} + j_\mu^{lep\ A;\mp} j_\mu^{nucl\ A;\pm}$$

- But failed to comment on a second possible generalization

$$H_{\rm weak} \sim \frac{G_F}{\sqrt{2}} \left(j_\mu^{lep\ V;\mp} - j_\mu^{lep\ A;\mp} \right) \left(j_\mu^{nucl\ V;\pm} - j_\mu^{nucl\ A;\pm} \right)$$

This alternative gives the same β-decay formula, but implies parity violation, which presumably was so outlandish to GT that it was not worth a comment

More than two decades would pass before it was discovered that the correct low-momentum form of the weak interaction is V-A, and consequently that parity is violated maximally in the weak interaction

Fermi's construction has already introduced the idea implicitly of using all components of an isovector

$$j^{E\&M} = j_{\mu}^{V;S} + j_{\mu}^{V;V(0)} \qquad \Leftrightarrow \qquad j^{Weak} = j_{\mu}^{V;V(\pm)}$$

Had GT considered their current from the same perspective of efficiency, they would have encountered a puzzle

$$j_{\mu}^{A;V(0)} \qquad \Leftrightarrow \qquad j^{Weak} = j_{\mu}^{A;V(\pm)}$$

Where is the neutral axial current - the third component of the isovector?

35 years before the SM & neutral weak currents

Pauli's undetectable neutrino:

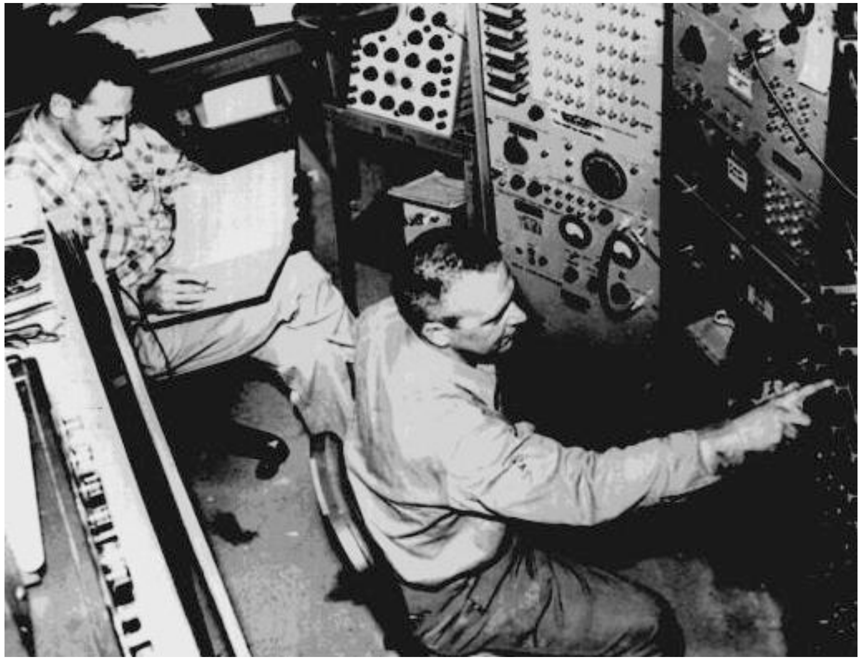
1956 Cowan/Reines experiment

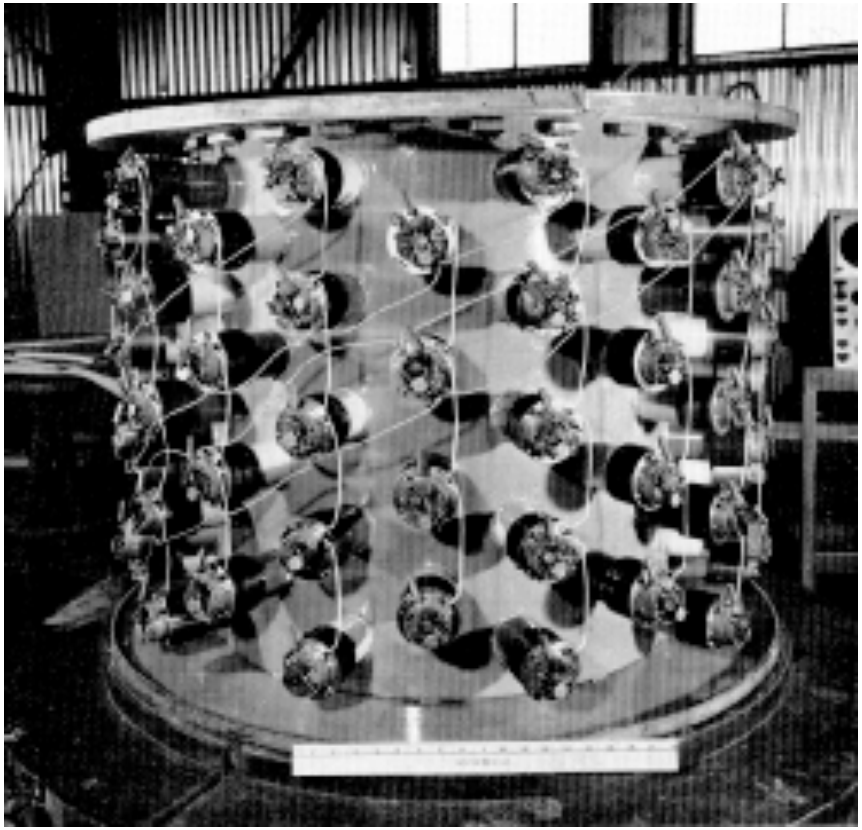
installed a Cd-doped water detector, with a total mass of 200 kg, at the Hanford reactor

$$\bar{\nu}_e + p \rightarrow n + e^+$$

$$n +^{108}\mathrm{Cd} \rightarrow^{109\mathrm{m}}\mathrm{Cd} \rightarrow^{109}\mathrm{Cd} + \gamma$$

with a flux of $5 \times 10^{13} \nu/\mathrm{cm}^2\mathrm{s}$, detected 3 events/hour





Particle-antiparticle conjugation

Another question about the neutrino was raised in 1937 by Majorana

All other fermions in the standard model carry a charge, which changes sign under particle-anti-particle conjugation

Thus $\mathrm{CPT} : e^- \to e^+ \neq e^-$

But a neutrino carries no charge nor any other additively conserved quantum number

Does the neutrino have a distinct anti-particle, or could it be its own anti-particle?

Do experiments to find out!

$$p_{\text{bound}} \to n + e^{+} + \nu_e \quad \text{then} \quad \nu_e + n \to p + e^{-}$$

$$\text{but} \quad \nu_e + p \not\to n + e^{+}$$

$$n \to p + e^{-} + \bar{\nu}_e \quad \text{then} \quad \bar{\nu}_e + p \to n + e^{+}$$

$$\text{but} \quad \bar{\nu}_e + n \not\to p + e^{-}$$

well, they seem to do different things

- with these definitions of the ν_e and $\bar{\nu}_e$, they appear operationally distinct, producing different final states

- introduce a lepton "charge" to distinguish the neutrino states and to define the allowed reactions, by the additive conservation law

$$\sum_{\text{in}} \mathbf{l_e} = \sum_{\text{out}} \mathbf{l_e}$$

$lepton$	l_e
e^-	$+1$
e^+	-1
ν_e	$+1$
$\bar{\nu}_e$	-1

Dirac neutrino

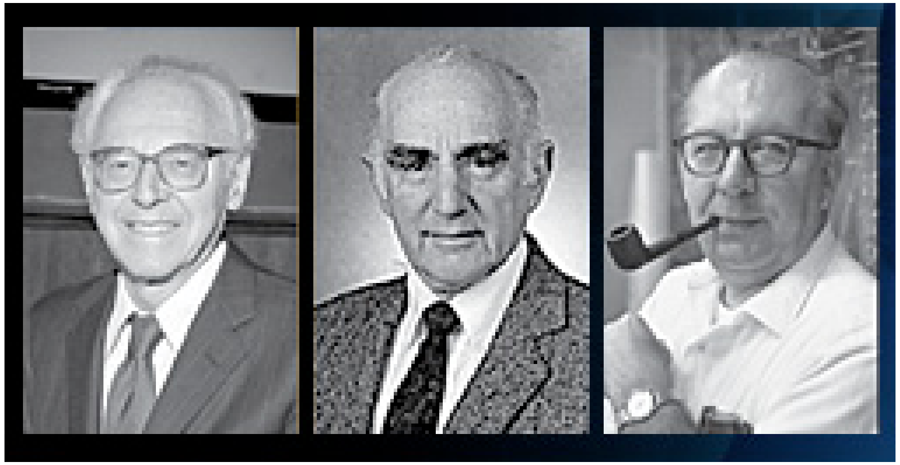
$$\nu_e \perp \bar{\nu}_e \;\Rightarrow\; \text{Dirac neutrino}$$

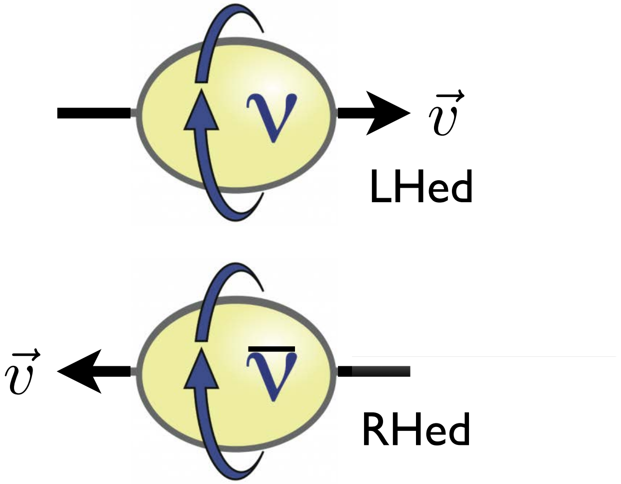
We will see later that this argument is incomplete, and that the question of lepton number is connected in a profound way to the nature of neutrino mass

Neutrinos and the Standard Model

In 1957 the weak interaction was found to violate parity ~ maximally

One of the most dramatic demonstrations of the PNC came from the Goldhaber-Grodzins-Sunyar experiment on the helicity of the neutrino





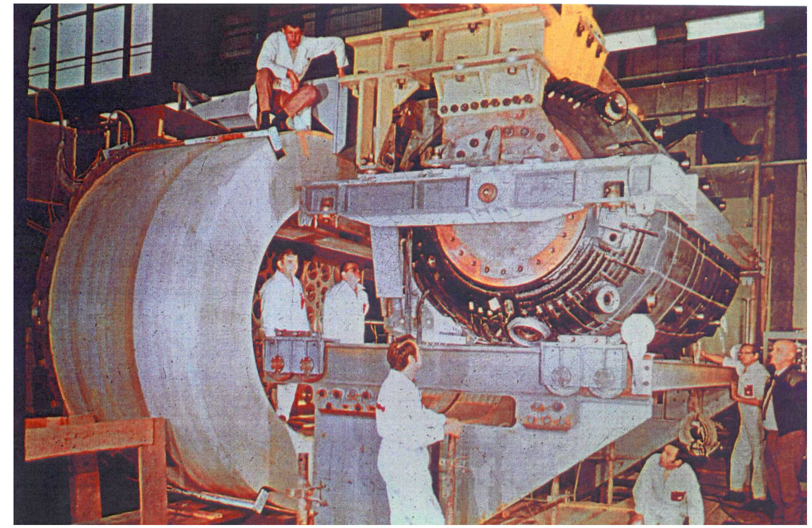
neutrino was found to be left-handed, with its spin opposite its helicity

$$\langle \vec{\sigma} \cdot \hat{p} \rangle = -1 \quad \text{manifestly parity odd, as under P,} \quad \begin{array}{c} \vec{\sigma} \rightarrow \vec{\sigma} \\ \hat{p} \rightarrow -\hat{p} \end{array}$$

In 1962 Lederman, Schwartz, Steinberger discovered the μ neutrino: both charged leptons known at that time (e, μ) have their own neutrinos

(1974-77 the tau charged lepton was discovered, and its distinct neutrino was detected directly in 2000)

From 1972-74 a series of confusing results from the Gargamelle detector, installed in the CERN PS neutrino beam line, resulted in the discovery of neutral current neutrino scattering



In these interactions the third component of the charge-changing axial currents that Goldhaber and Teller had introduced

$$j_\mu{}^{A;V(\pm)}$$

participates (Kate's subject!)

$$j_\mu{}^{A;V(0)}$$

Thus all of the components of the V and A currents play a natural role.

Gargamelle's first neutral current event

In 1958 the V-A theory was formulated:

Feynman and Gell-Mann “Theory of the Fermi Interaction”

Sudarshan and Marshak “Chiral Noninvariance and the Universal Fermi Interaction”

in which only charged currents appeared

1967 Weinberg published his electroweak unification paper in which a neutral partner Z to the W was introduced, so that both charged and neutral currents appeared. This paper’s early citation record:

0 (1967), 0 (1968), 0 (1969), 1 (1970), 4 (1971), 64 (1972), 162 (1973), …

By the end of 1971 both the Higgs mechanism, to generate massive intermediate bosons, and the renormalizability *(Veltman and ’t Hooft)* had been established

In this theory - the standard model - the neutrinos are massless

Solar neutrinos

Another story was evolving in parallel, connect to neutrinos

Eddington had argued that the source of solar energy had to be nuclear (resolving old debates involving Darwin, Lord Kelvin, and others); Gamow developed the theory of QM barrier penetration to explain how fusion could happen at reasonable stellar core temperature. The energy-generating process

$$2e^- + 4\mathrm{p} \rightarrow^4 \mathrm{He} + 2\nu_e + 26.73\ \mathrm{MeV}$$

requires the weak interaction to change protons to neutrons.

The neutrino flux from the sun can be accurately estimated by equating the solar luminosity to the energy-generation rate. This assumes the sun is burning in equilibrium.

While Cowan and Reines succeeded in measuring reactor antineutrinos in 1956, a decade earlier Pontecorvo had suggested detection via

$$\nu_e +^{37}\mathrm{Cl} \to^{37}\mathrm{Ar} + e^- \quad (\nu_e + \mathrm{n}_{\mathrm{bound}} \to \mathrm{p}_{\mathrm{bound}} + e^-)$$

an idea that Louis Alvarez then further developed.

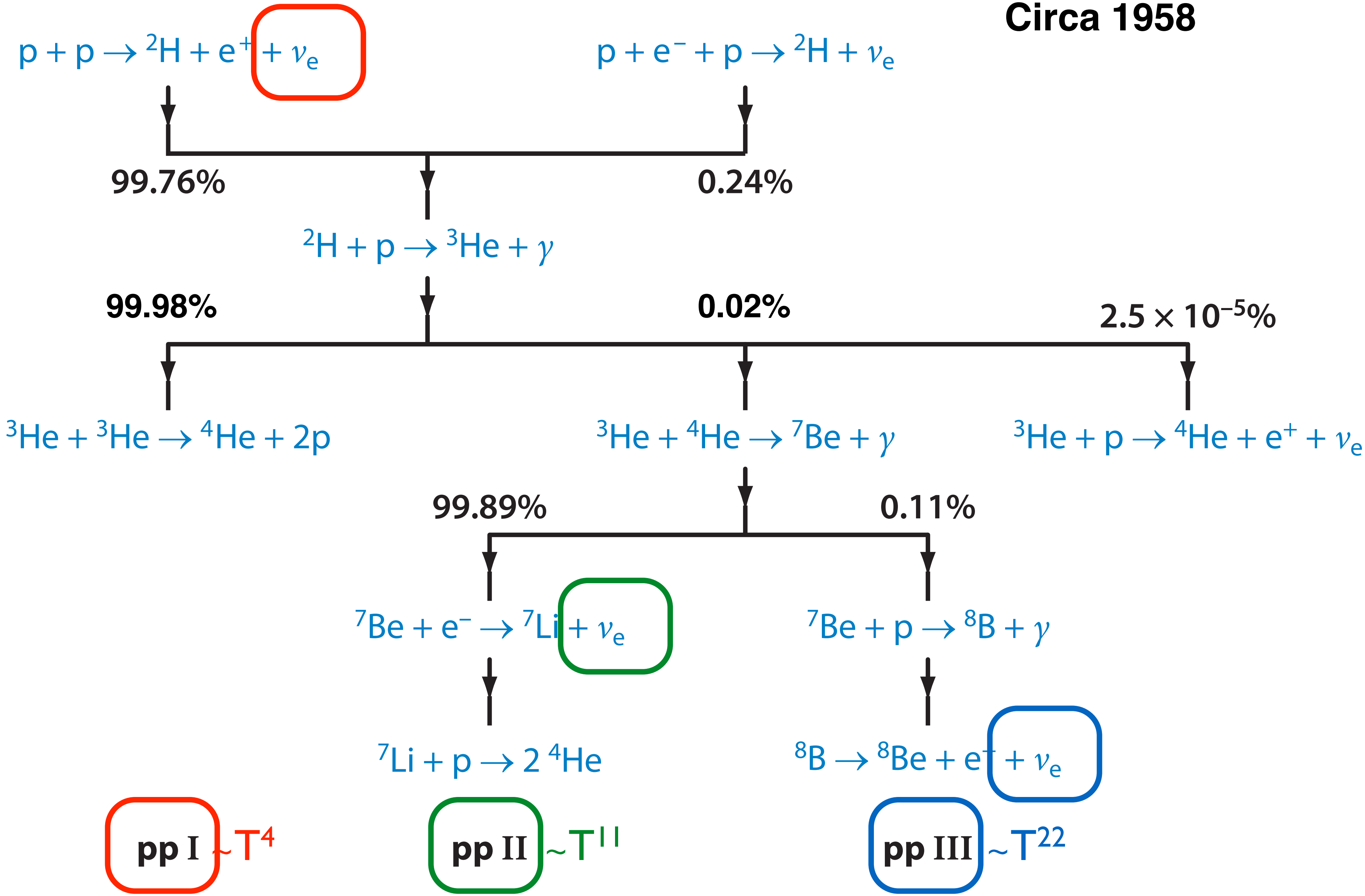
This reaction requires $E_{\nu_e} > 810\ \mathrm{keV}$

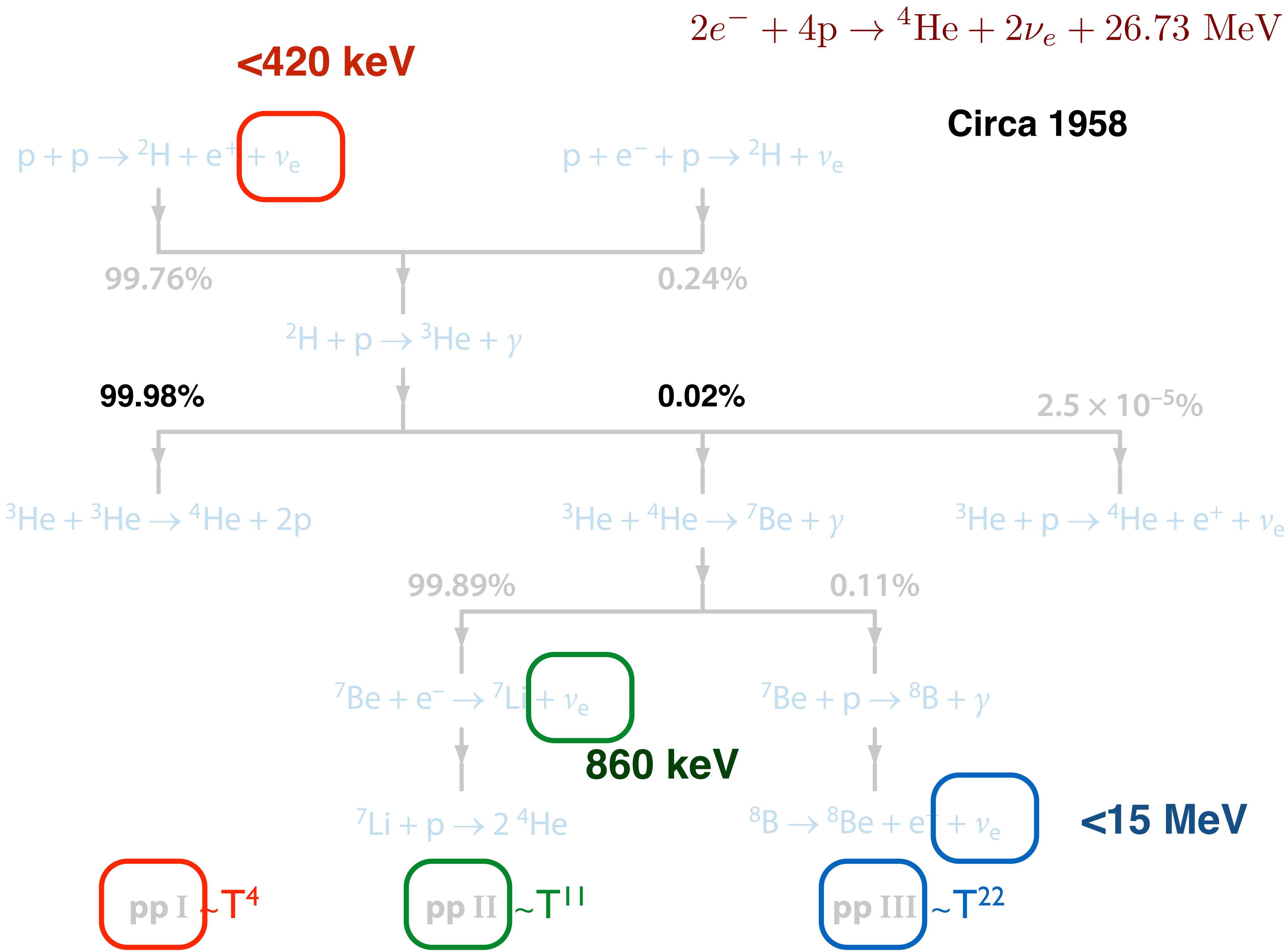
In 1955 Ray Davis Jr. constructed a 1000-gallon $\mathrm{C_2Cl_4}$ detector at BNL, mounted 19 ft underground — setting a limit that was far, far below the expected rate of solar neutrinos

The theory of solar energy generation had been developed in the 1930s by Bethe, Critchfield, and others — with very little thought about the neutrinos… A model of the sun capable of predicting neutrino fluxes with any accuracy would not exist for decades.

$$2e^- + 4\mathrm{p} \rightarrow {}^4\mathrm{He} + 2\nu_e + 26.73\ \mathrm{MeV}$$

Circa 1958

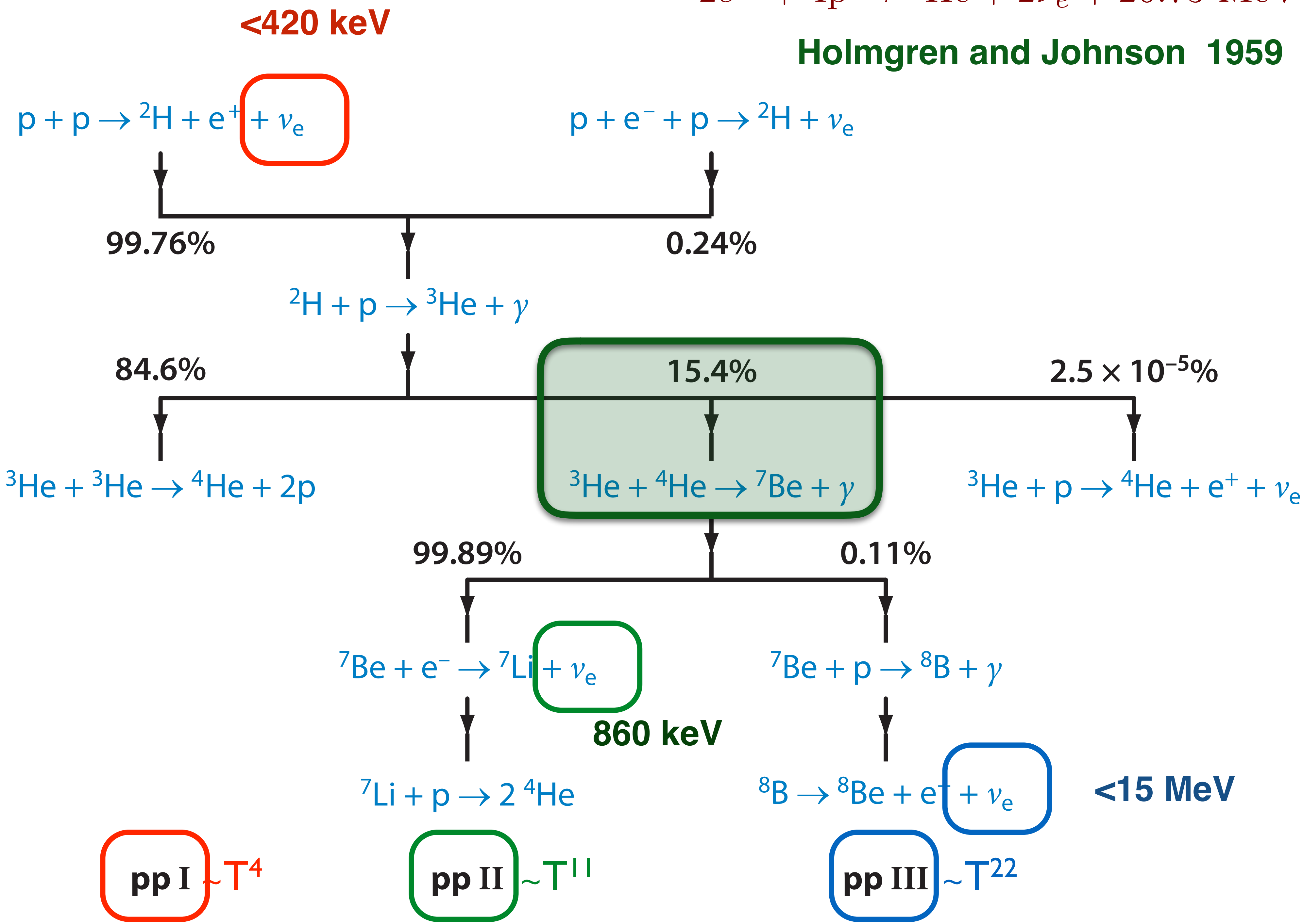




$2e^- + 4p \rightarrow {}^4He + 2\nu_e + 26.73$ MeV
<420 keV
Circa 1958
$p + p \rightarrow {}^2H + e^+ + \nu_e$
$p + e^- + p \rightarrow {}^2H + \nu_e$
99.76%
0.24%
${}^2H + p \rightarrow {}^3He + \gamma$
99.98%
0.02%
2.5×10^{-5}%
${}^3He + {}^3He \rightarrow {}^4He + 2p$
${}^3He + {}^4He \rightarrow {}^7Be + \gamma$
${}^3He + p \rightarrow {}^4He + e^+ + \nu_e$
99.89%
0.11%
${}^7Be + e^- \rightarrow {}^7Li + \nu_e$
${}^7Be + p \rightarrow {}^8B + \gamma$
860 keV
${}^7Li + p \rightarrow 2\,{}^4He$
${}^8B \rightarrow {}^8Be + e^+ + \nu_e$
<15 MeV
pp I $\sim T^4$
pp II $\sim T^{11}$
pp III $\sim T^{22}$

$$2e^- + 4p \rightarrow {}^4\mathrm{He} + 2\nu_e + 26.73\ \mathrm{MeV}$$

Holmgren and Johnson 1959



The implications were immediately recognized

Willy Fowler brought a young John Bahcall, a postdoc expert in weak interactions in stars, to Caltech to join stellar modelers Iben and Sears

They began the task to develop a “standard model” of main-sequence stellar evolution that, if applied to the Sun, could predict the fluxes of solar neutrinos — to motivate a large Cl experiment

Required enormous work in theory and nuclear/atomic experiment — 50 years of effort that still continues

A truly daunting challenge: T^{11}, T^{22} power dependence ⇒ predict the central temperature of the Sun to 1%

The Standard Solar Model: Davis to SNO

- a model of low-mass, main-sequence stellar evolution
  - local hydrostatic equilibrium: gas pressure gradient counteracting gravitational force
  - hydrogen burning: pp chain, CN cycle
  - energy transport by radiation (interior) and convection (envelope)
  - boundary conditions: today's mass, radius, luminosity

- The implementation of this physics requires
  - electron gas EOS - close to an ideal gas
  - low-energy nuclear cross sections
  - radiative opacity
  - some means of fixing the composition at ZAMS, including the ratios X:Y:Z

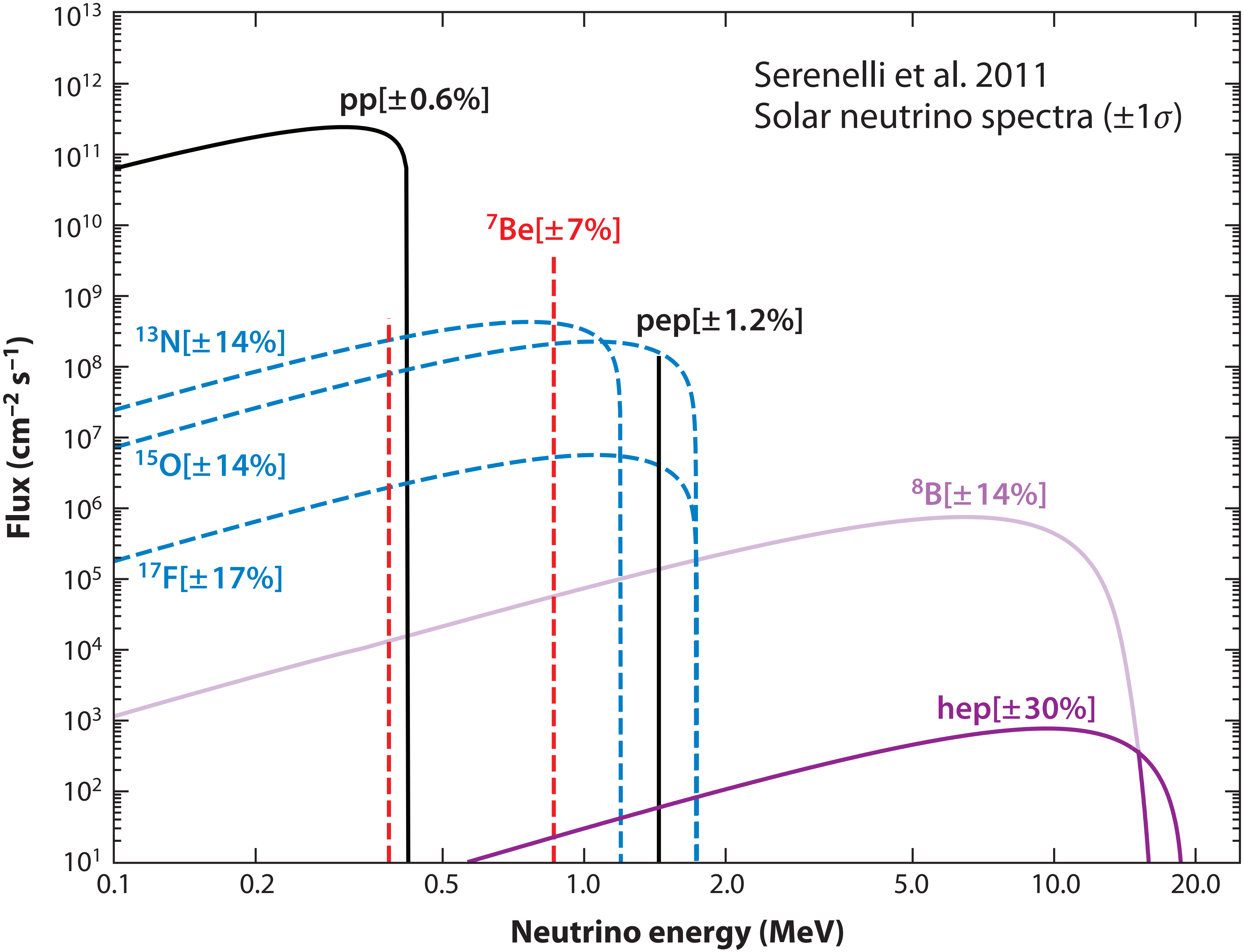
Model tests:

- **Solar neutrinos:** direct measure of core temperature to ~ 0.5%

- **Helioseismology:** inversions map out the local sound speed, properties of the convective zone

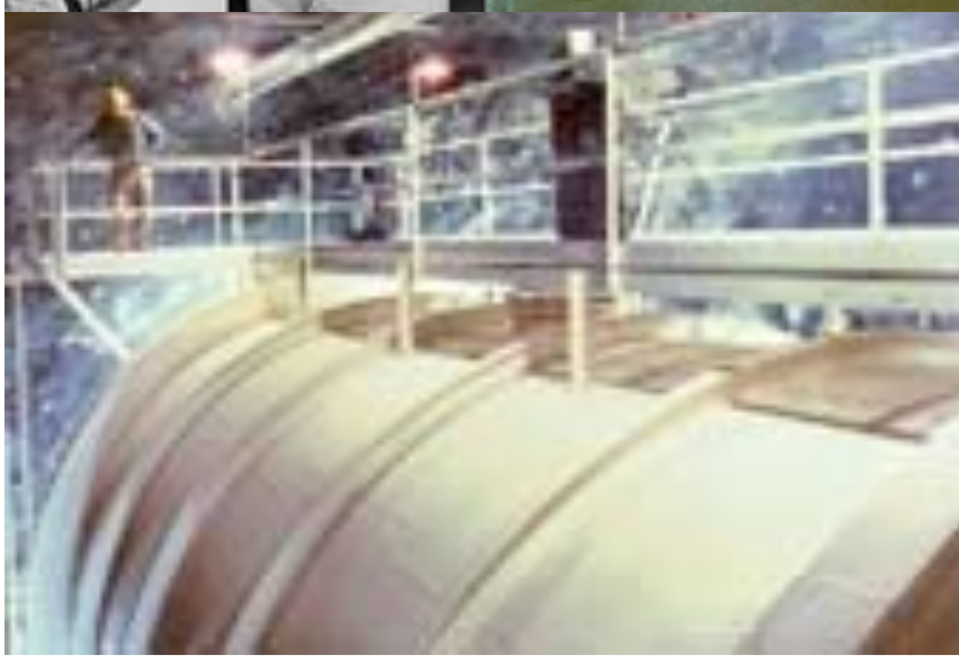
As sound speed measurements reached 1% in the 1990s, it became apparent that the SSM was marvelously predictive …

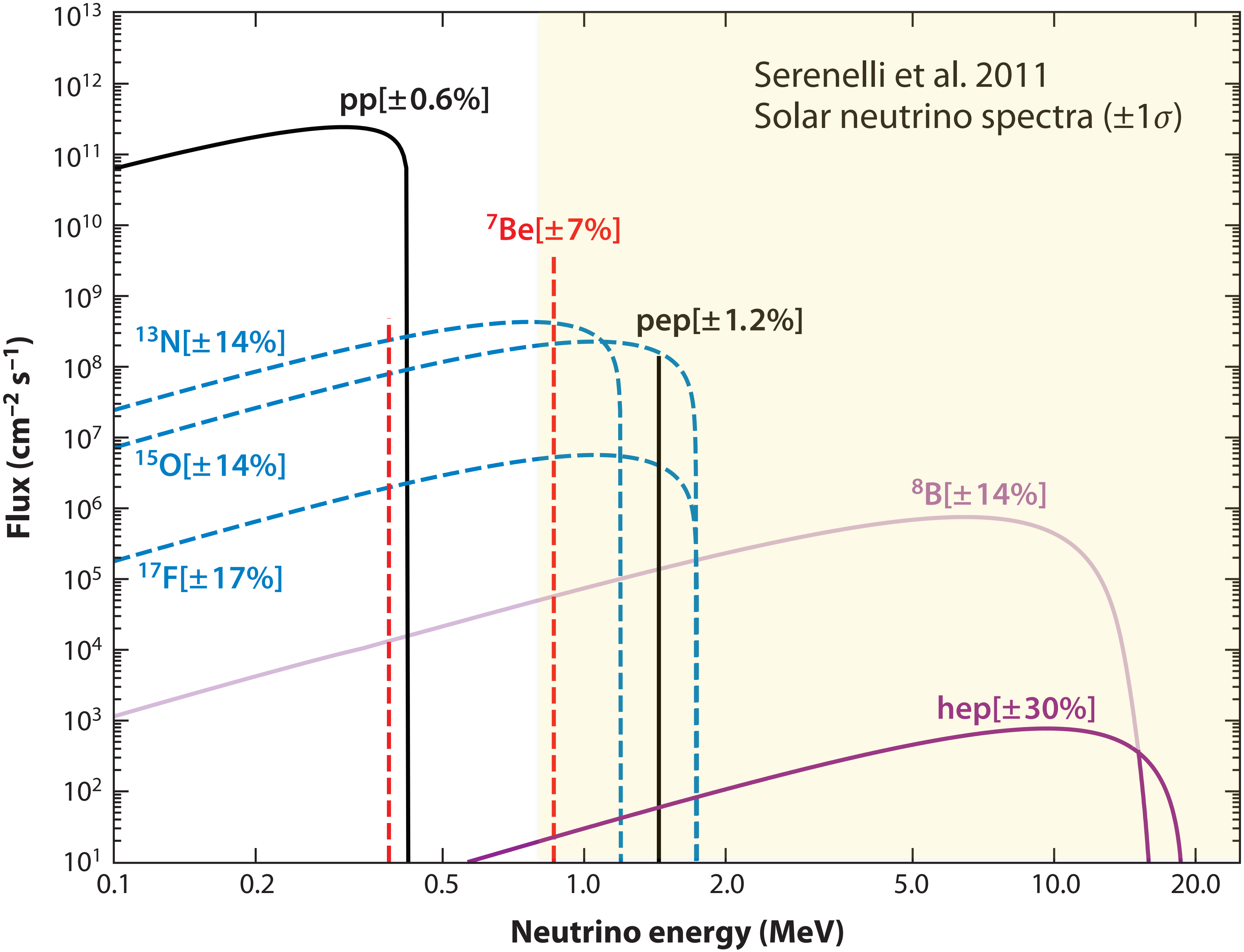
But the story with neutrinos was complicated



Serenelli et al. 2011
Solar neutrino spectra (±1σ)
pp[±0.6%]
7Be[±7%]
pep[±1.2%]
13N[±14%]
15O[±14%]
17F[±17%]
8B[±14%]
hep[±30%]
Flux (cm−2 s−1)
Neutrino energy (MeV)
0.1
0.2
0.5
1.0
2.0
5.0
10.0
20.0



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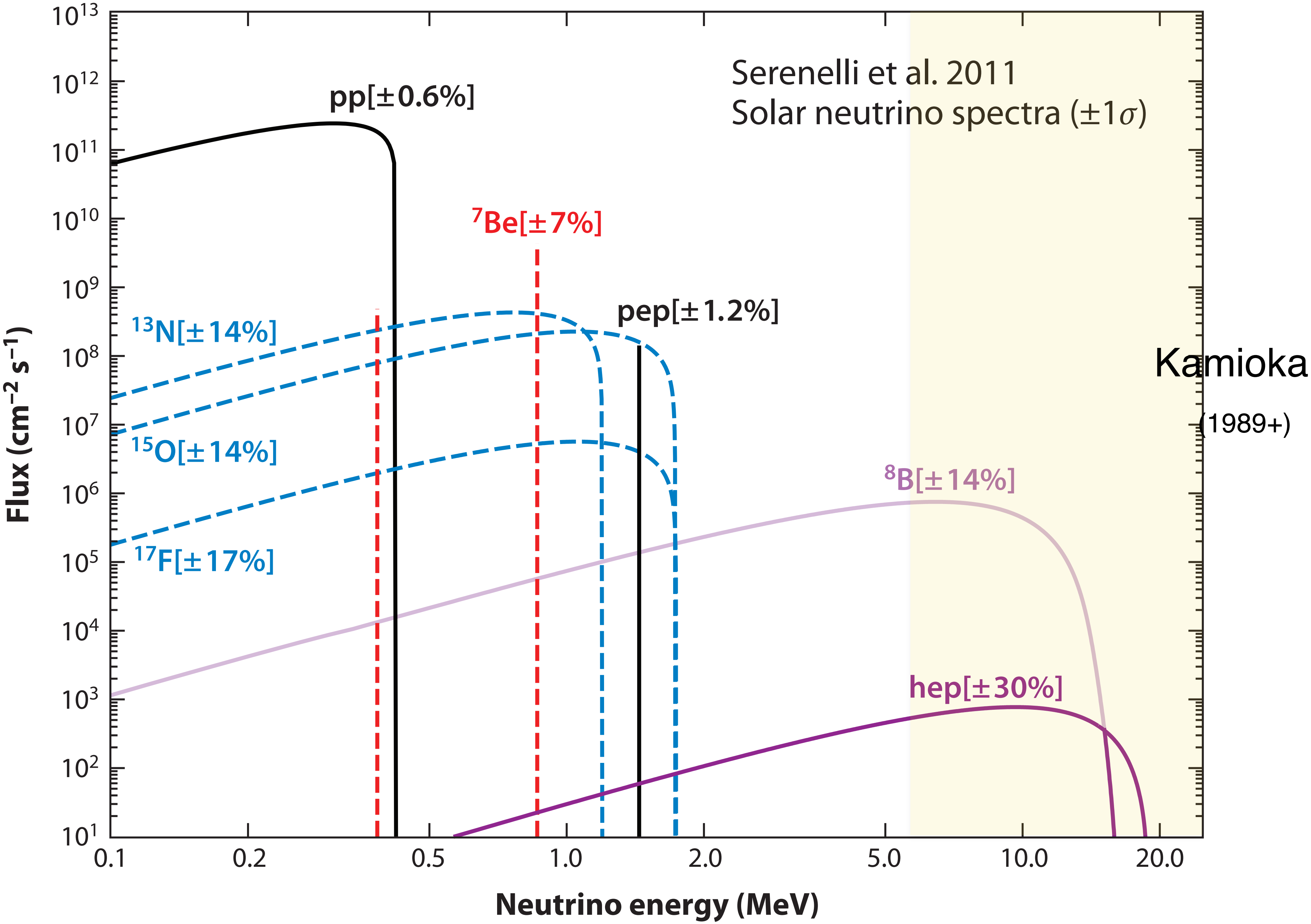
Flux (cm−2 s−1)
Neutrino energy (MeV)
10^13
10^12
10^11
10^10
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
0.1
0.2
0.5
1.0
2.0
5.0
10.0
20.0

^{37}Cl

(1968+)



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13N[±14%]
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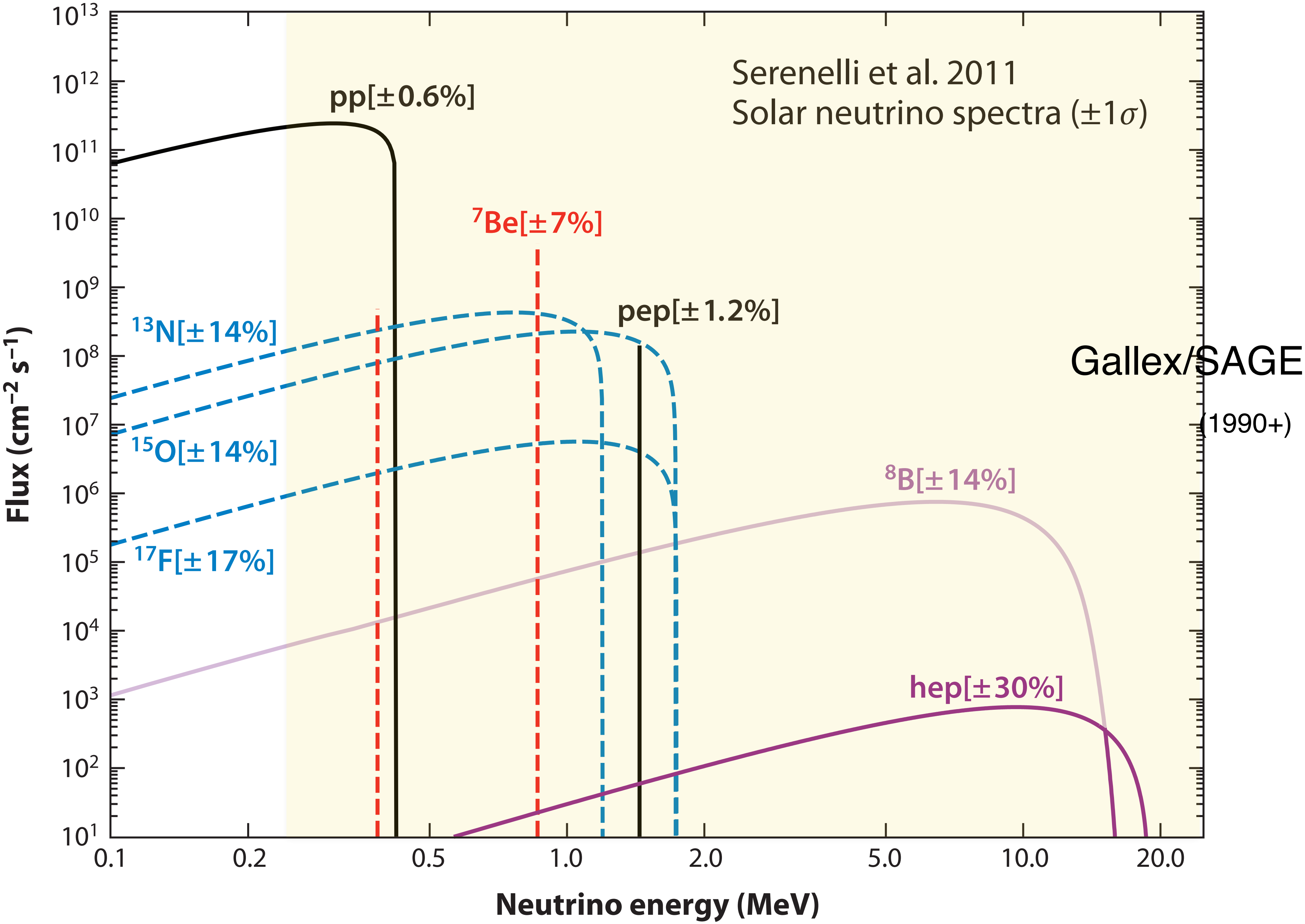
17F[±17%]
8B[±14%]
hep[±30%]
Kamioka
(1989+)
Flux (cm−2 s−1)
Neutrino energy (MeV)
10^13
10^12
10^11
10^10
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
0.1
0.2
0.5
1.0
2.0
5.0
10.0
20.0



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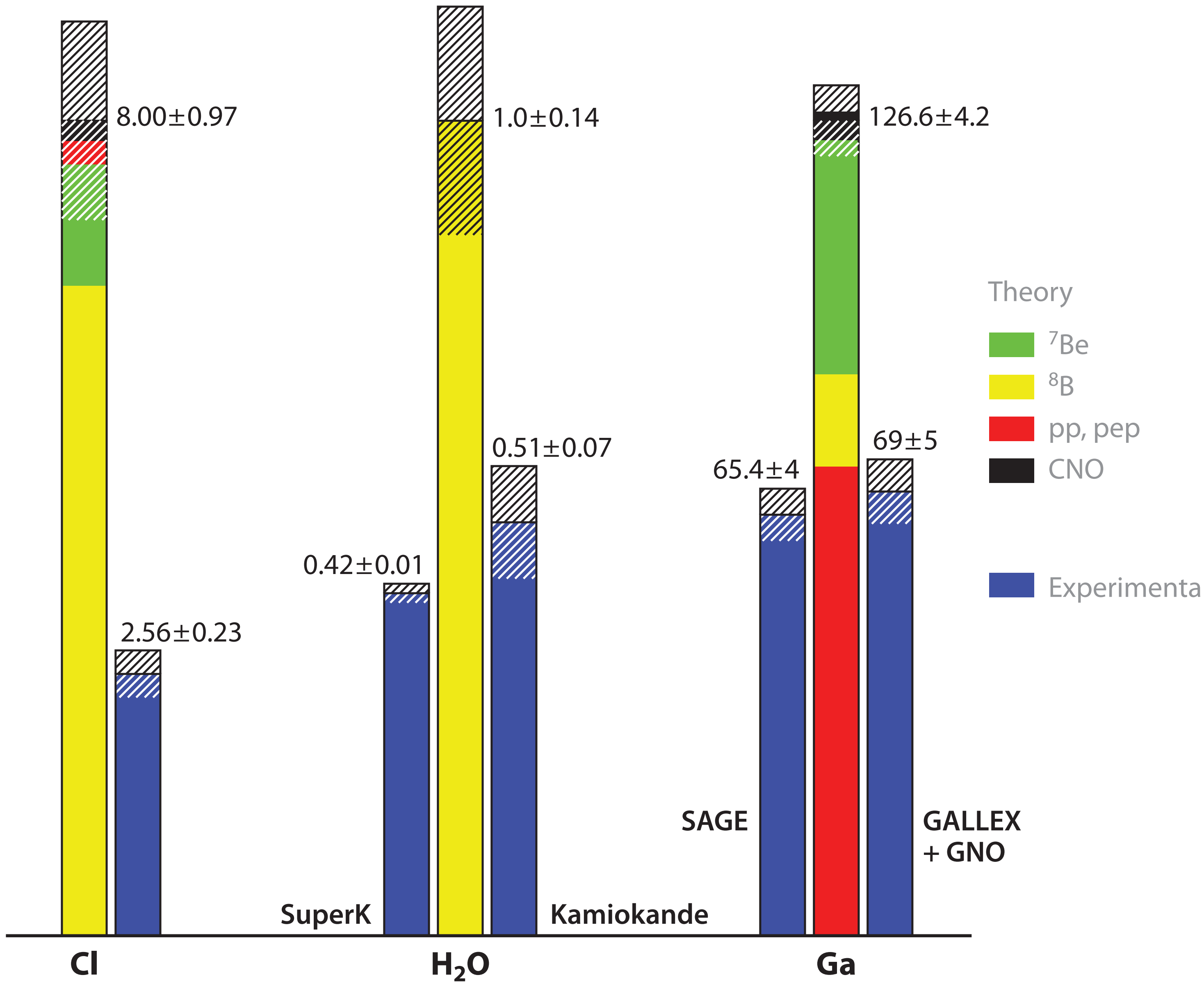
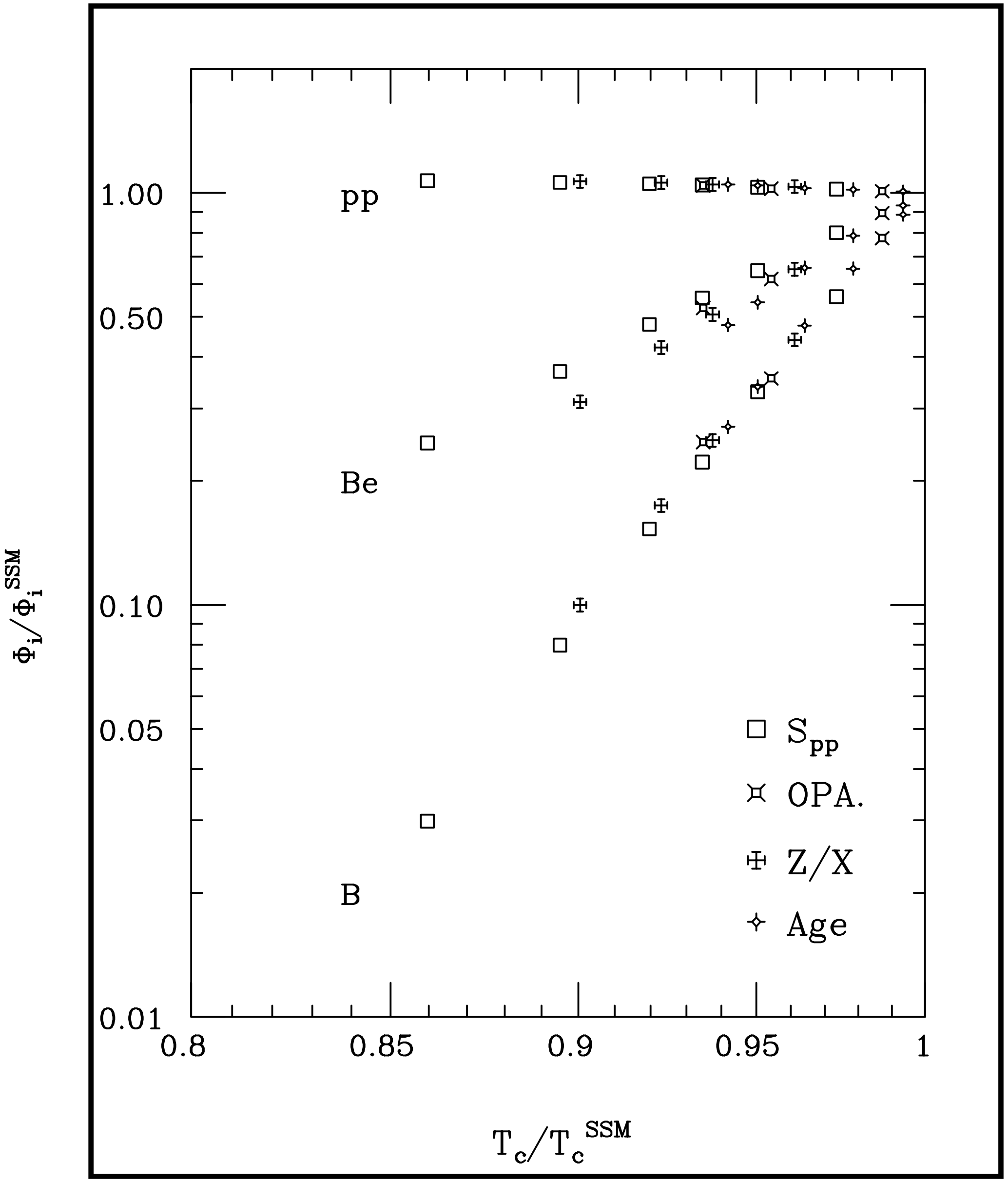
17F[±17%]
8B[±14%]
hep[±30%]
Gallex/SAGE
(1990+)
Flux (cm−2 s−1)
Neutrino energy (MeV)
0.1
0.2
0.5
1.0
2.0
5.0
10.0
20.0

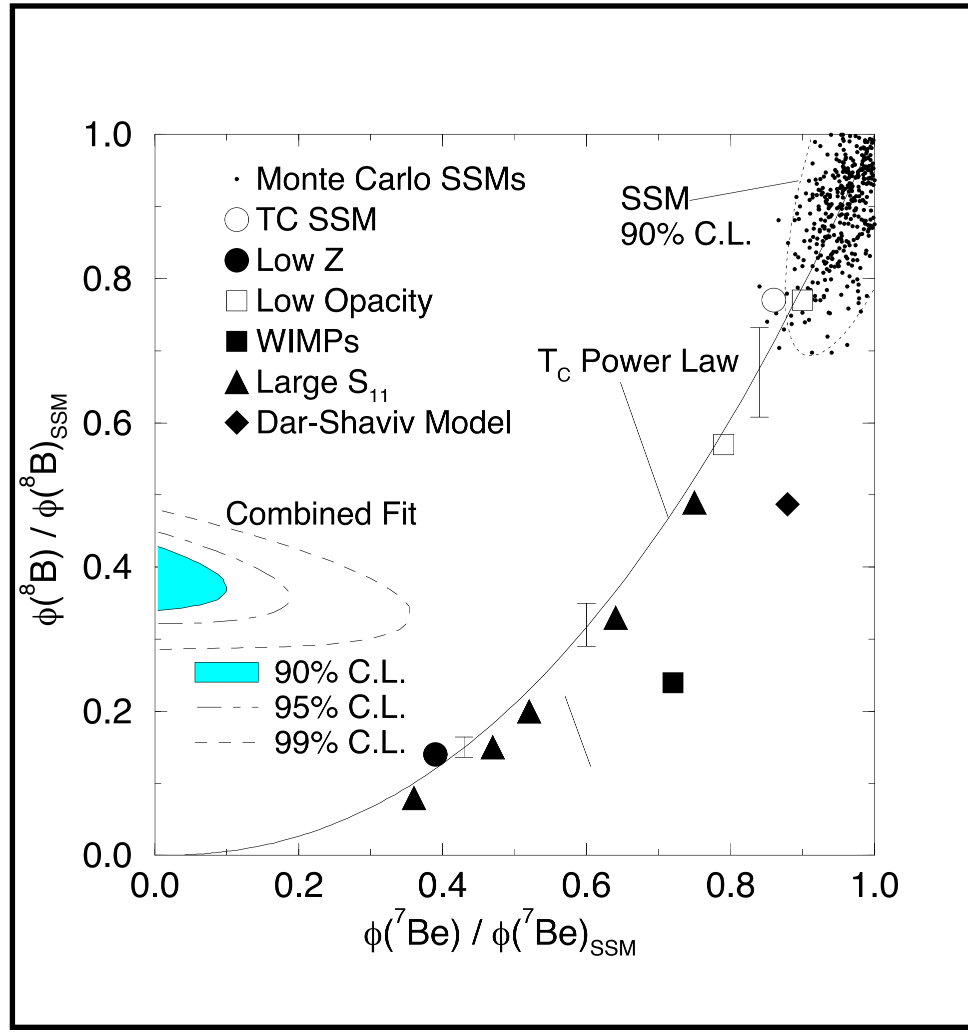


FROM J N Bahcall

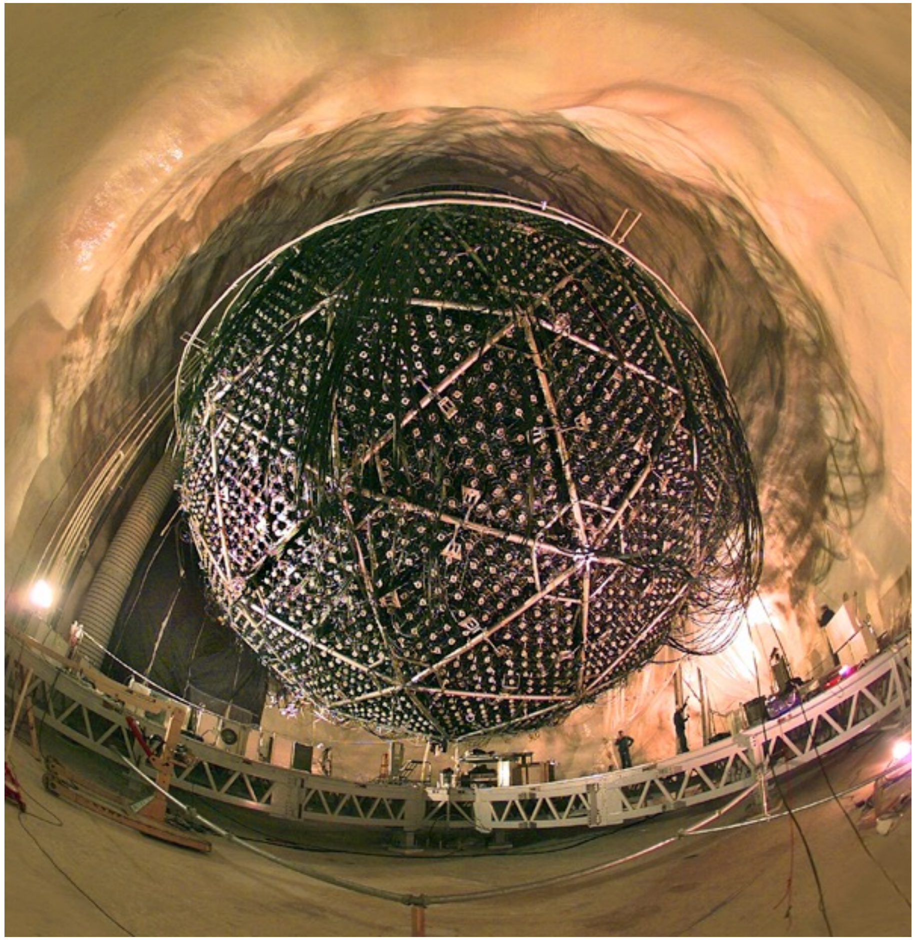
By mid-1990s model-independent arguments showed that the results not only differed from the SSM, but did so in a way that could not be fixed

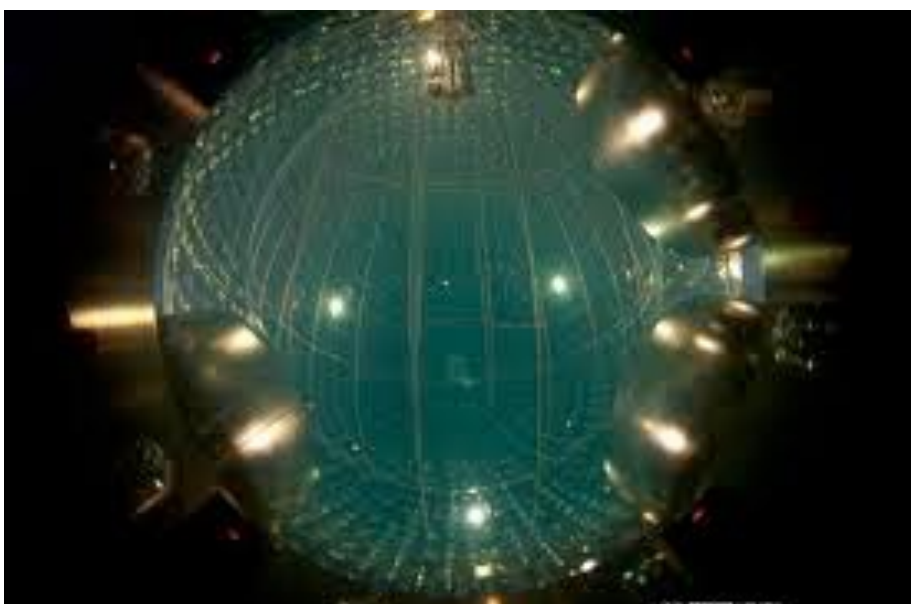


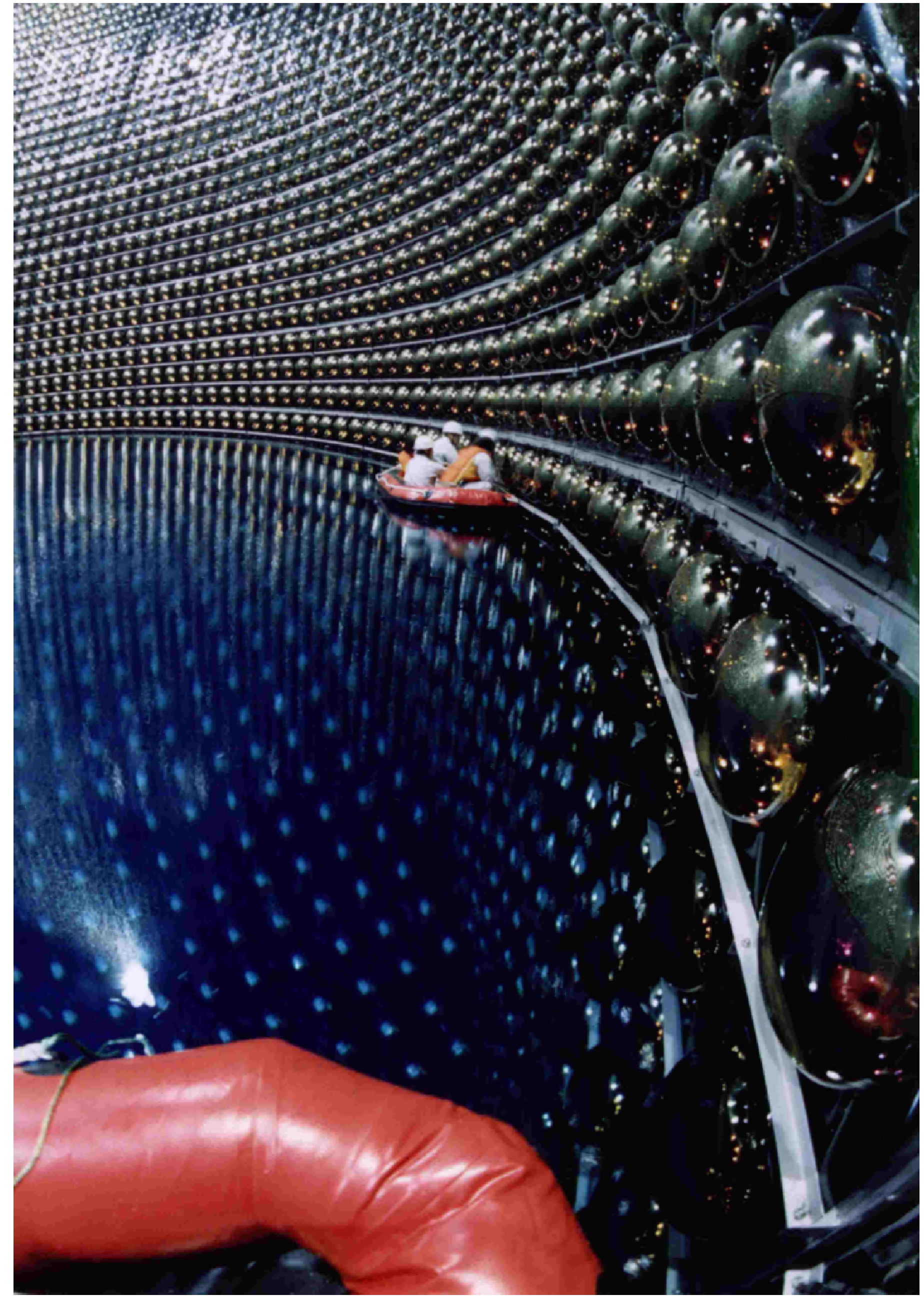
Castellani et al.



Hata et al.







possibiity of new neutrino physics:
SNO, Super-Kamiokande, Borexino

One attractive non-solar solution had been suggested by Pontecorvo in 1957: neutrino oscillations with require a mass (massless particles travel at the speed of light and thus have no “clock”)

And they require mixing

$$|\nu_{m_1}\rangle,\ |\nu_{m_2}\rangle,\ |\nu_{m_3}\rangle\ \neq\ |\nu_e\rangle,\ |\nu_\mu\rangle\ ,|\nu_\tau\rangle$$

$$\text{mass eigenstate}\ \neq\ \text{flavor eigenstates}$$

(eigenstates of free propagation) (production eigenstates)

$$|\nu_e\rangle = \sum_i U_{ei}|\nu_i\rangle$$

e.g., for the mixing of just two flavors ,

$$\begin{aligned} |\nu_e\rangle &= \quad \cos\theta_{12}|\nu_1\rangle + \sin\theta_{12}|\nu_2\rangle \\ |\nu_\mu\rangle &= -\sin\theta_{12}|\nu_1\rangle + \cos\theta_{12}|\nu_2\rangle \end{aligned}$$

Then it is straightforward to show, for a coherent localized neutrino wave packet

$$|\nu(t=0) = |\nu_e\rangle \quad \Rightarrow \quad P_{\nu_\mu}(t) = |\langle \nu(t)|\nu_\mu\rangle|^2 \sim \sin^2 2\theta_{12} \;\; \sin^2 \frac{\pi c t}{L_0}$$

$$L_0 = \frac{4\pi \;\hbar c \; E_\nu}{\delta m_{21}^2 c^4} \qquad \delta m_{21}^2 = m_2^2 - m_1^2$$

So this can reduce the number of surviving $\nu_e s$ that Davis was measuring. But

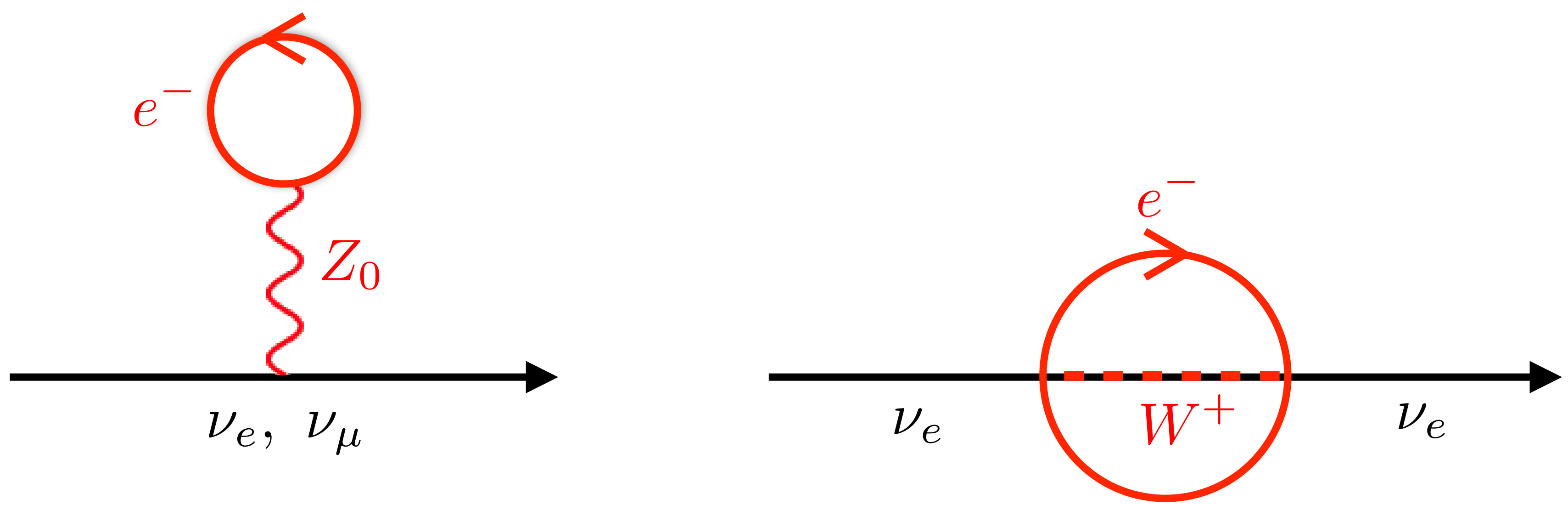
$$P_{\nu_e} \rightarrow 1 - \frac{1}{2} \sin^2 2\theta_{12}$$

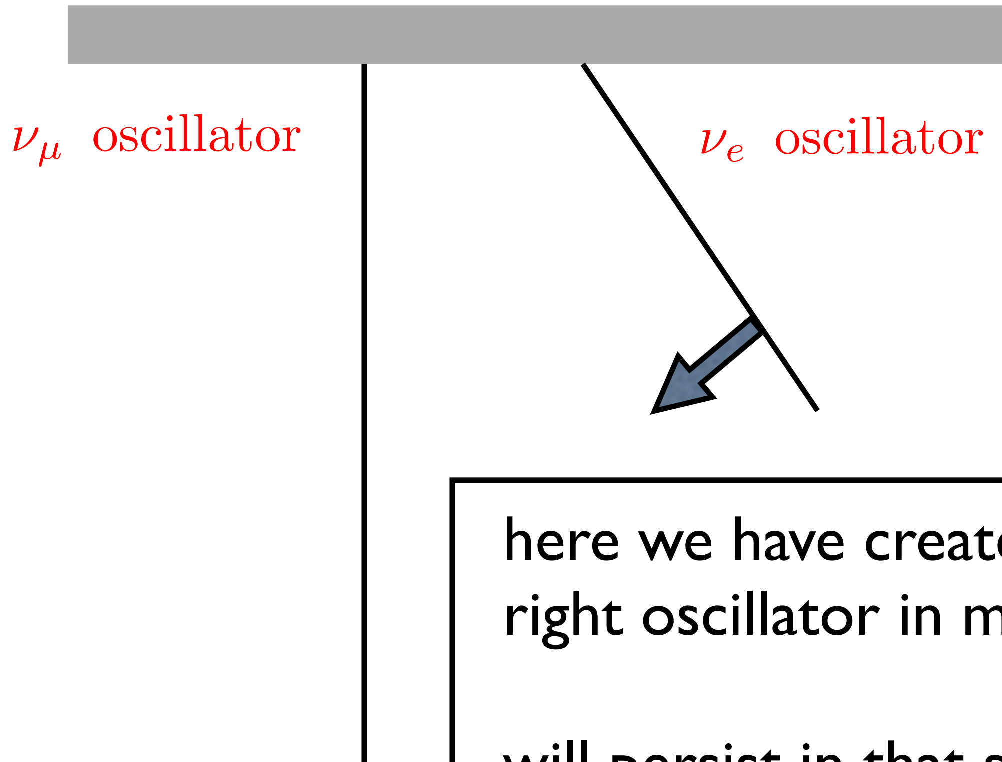
Getting sufficient reduction in the flux difficult…

It was know, just as photons acquire a mass when they travel through and react with water, the neutrinos will acquire a mass when traveling through solar matter

These masses (and mass differences) evolve with ρ_{solar}

$$\delta m^2_{21} \;\Rightarrow\; \delta m^2_{21}(\rho)$$





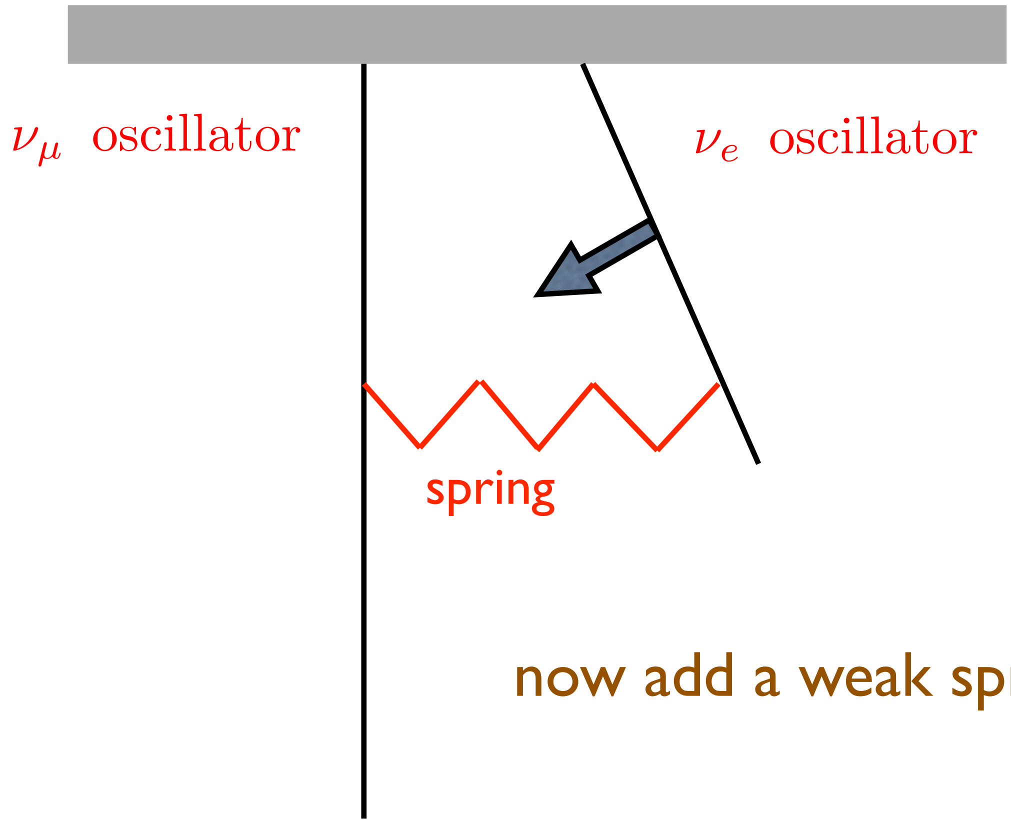
<u>Example #1</u>
masses? yes
mixing? no
matter? no

here we have created a ν_e by putting right oscillator in motion at t=0

will persist in that state forever:
flavor and mass eigenstates coincident

analogs: left(right) oscillator $\leftrightarrow \nu_\mu \; (\nu_e)$
normal modes $\leftrightarrow$ mass eigenstates $m_H \, (m_L)$

here the pendula are uncoupled:
each pendulum is both a mass eigenstate and a flavor eigenstate



Example #2
masses? yes
mixing? yes
matter? no

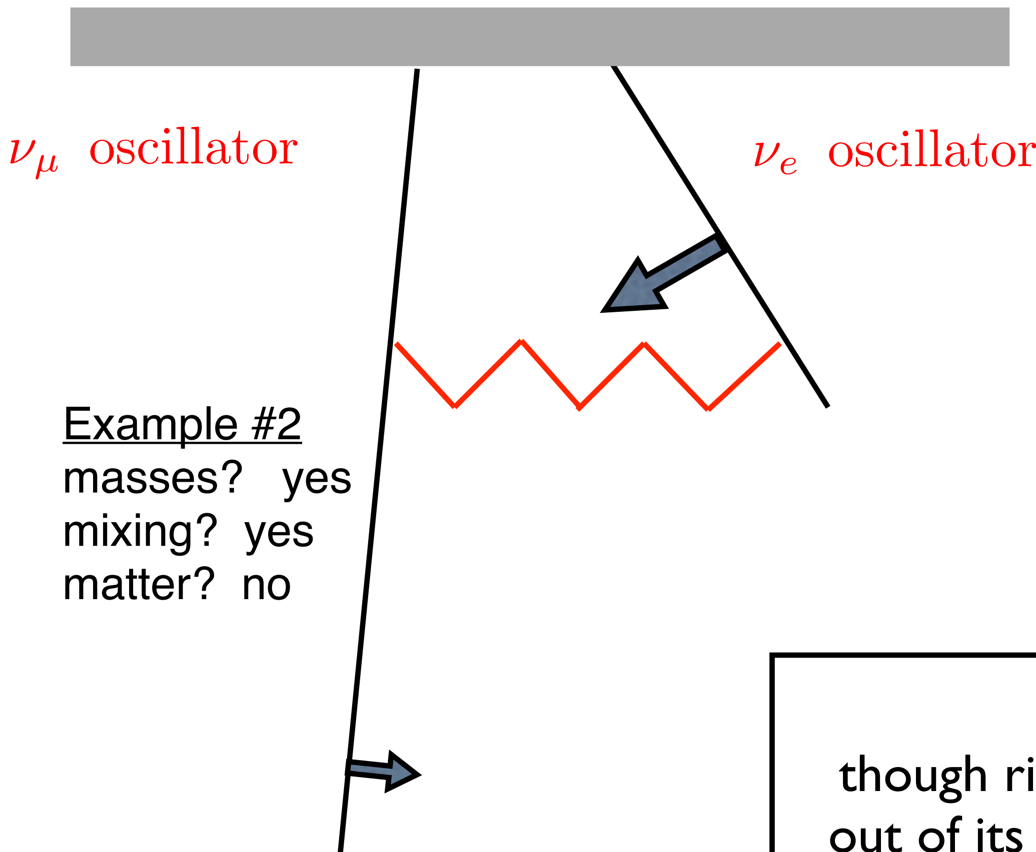
now add a weak spring ⇔ flavor mixing

⇔ flavor, mass eigenstates distinct

(pendulum analog: oscillators not coincident with normal mode states)

create a solar ν_e at t=0:

right oscillator put in motion, subsequent evolution now interesting

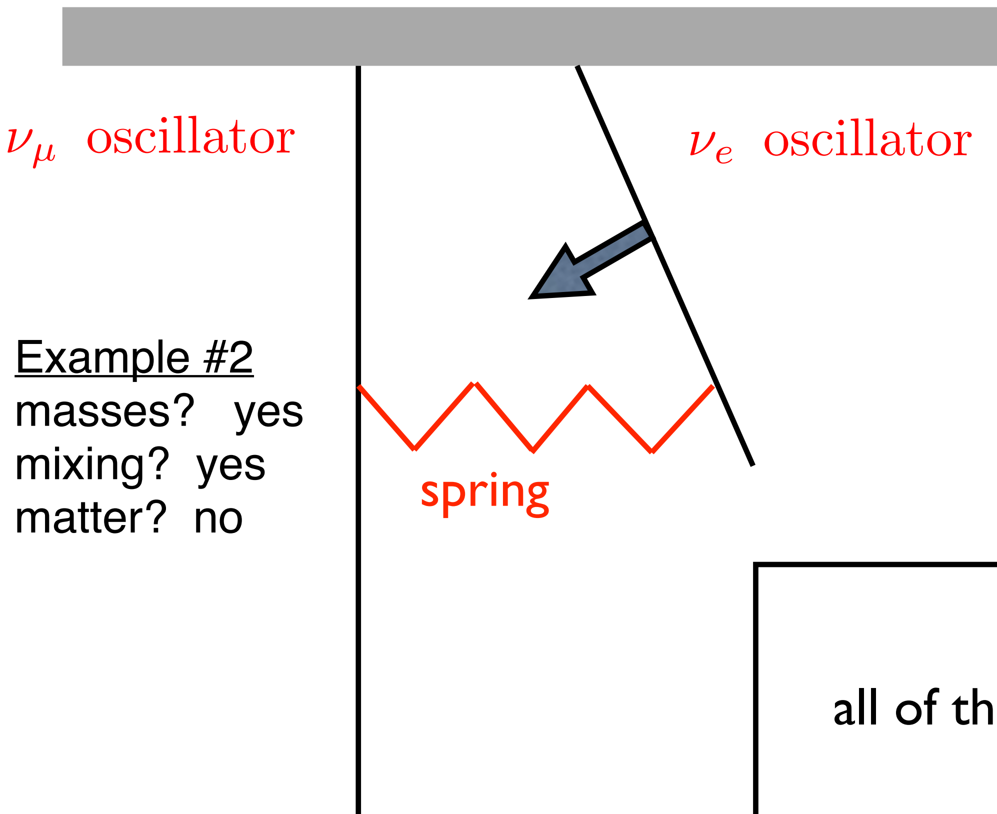


<u>Example #2</u>
masses? yes
mixing? yes
matter? no

system a bit later, t>0:
though right oscillator is pushing the left oscillator out of its resonant frequency, there is some motion induced in the left pendula:

we have a state that is mostly "muon neutrino" but also has some "electron neutrino"

vacuum oscillations: effect of the flavor mixing



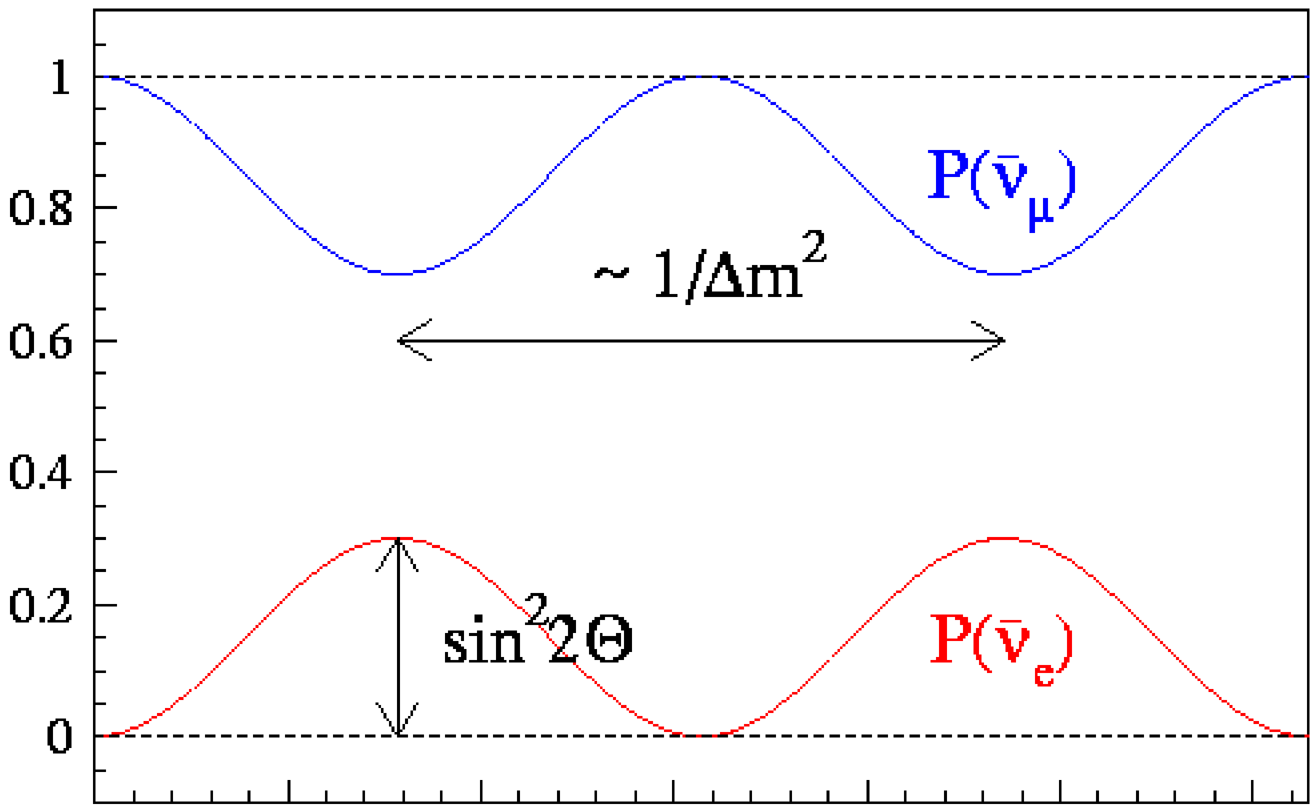
<u>Example #2</u>
masses? yes
mixing? yes
matter? no

and still later, t>0:
all of the pushing has cancelled out -- the right pendulum is back at rest

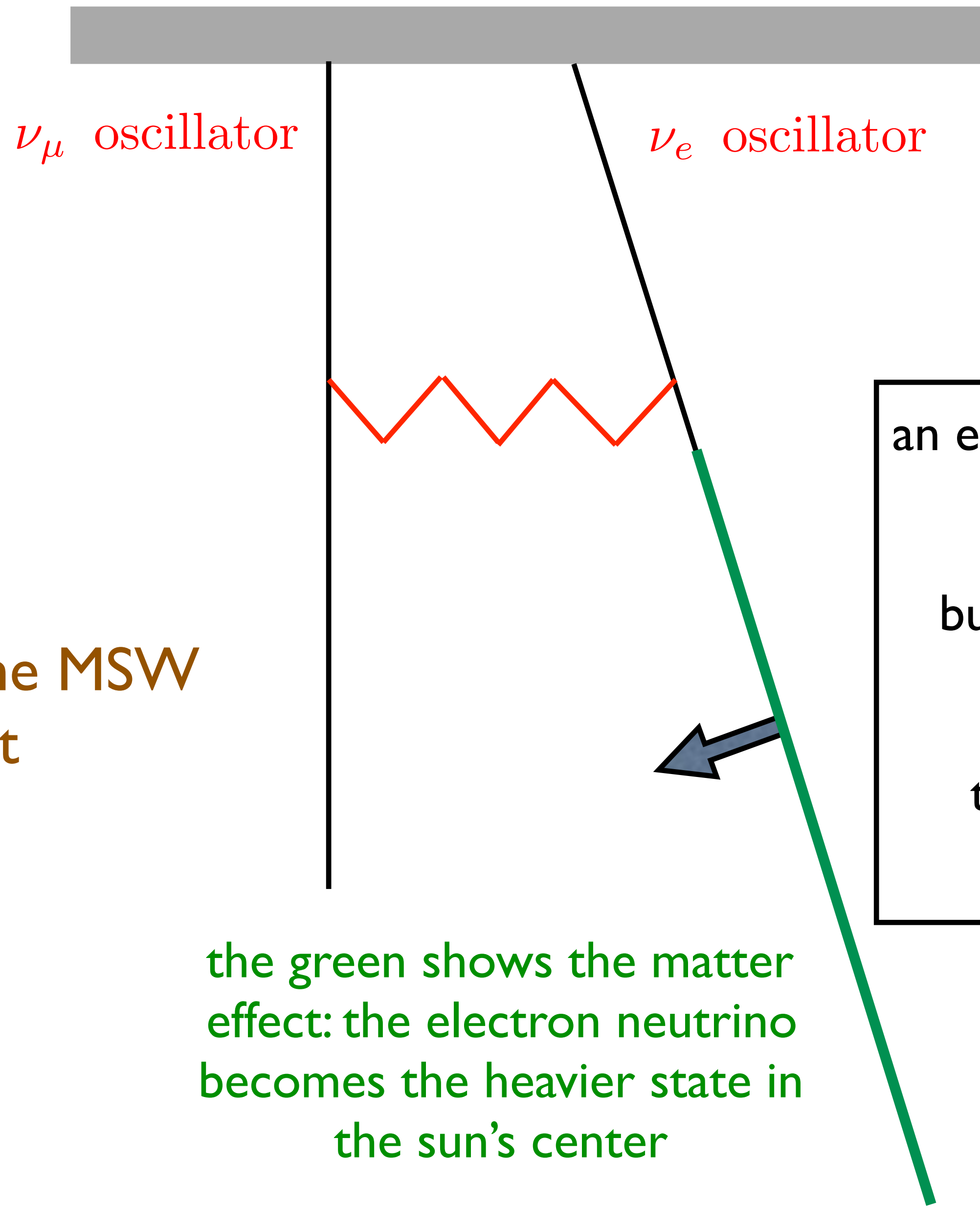
$$\text{pure } \nu_e \leftrightarrow \tau = 2\pi/(\omega_1 - \omega_2)$$

and the cycle repeats

Coupled oscillator analog of vacuum oscillations



Now add solar matter, large at t=0 (core) then exponentially decreasing to 0 (at solar surface)



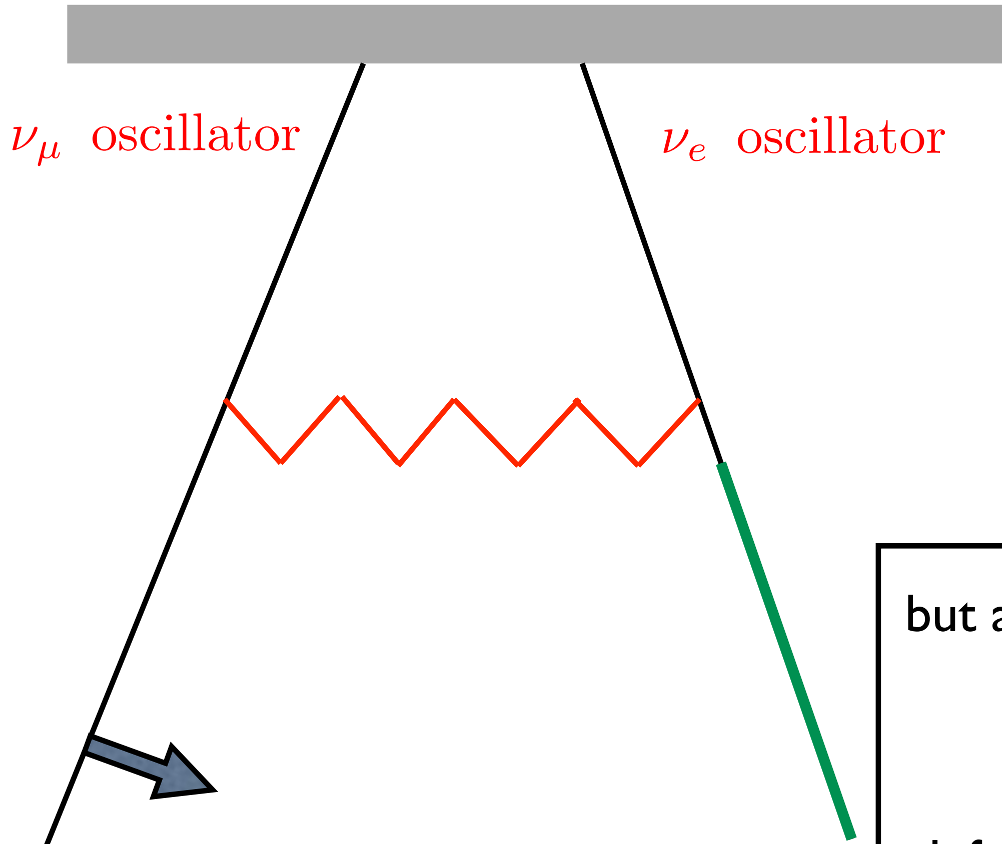
an electron is produced at t=0 at solar center

but it is locally the heavier state: matter effect

transfers a bit to muon neutrino side

known as the MSW effect

the green shows the matter effect: the electron neutrino becomes the heavier state in the sun's center

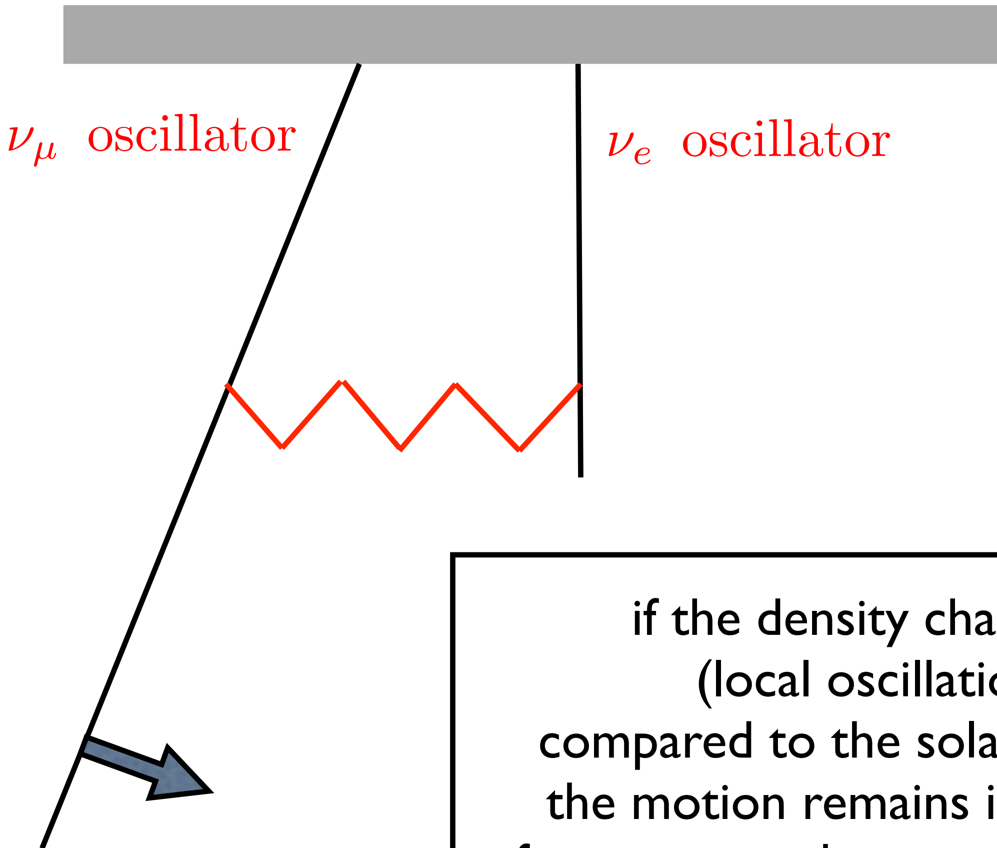


but a critical density is encountered on the way out of sun:

matter effect is "just so"
left and right oscillators momentarily in resonance

two normal modes split just by effects of the spring

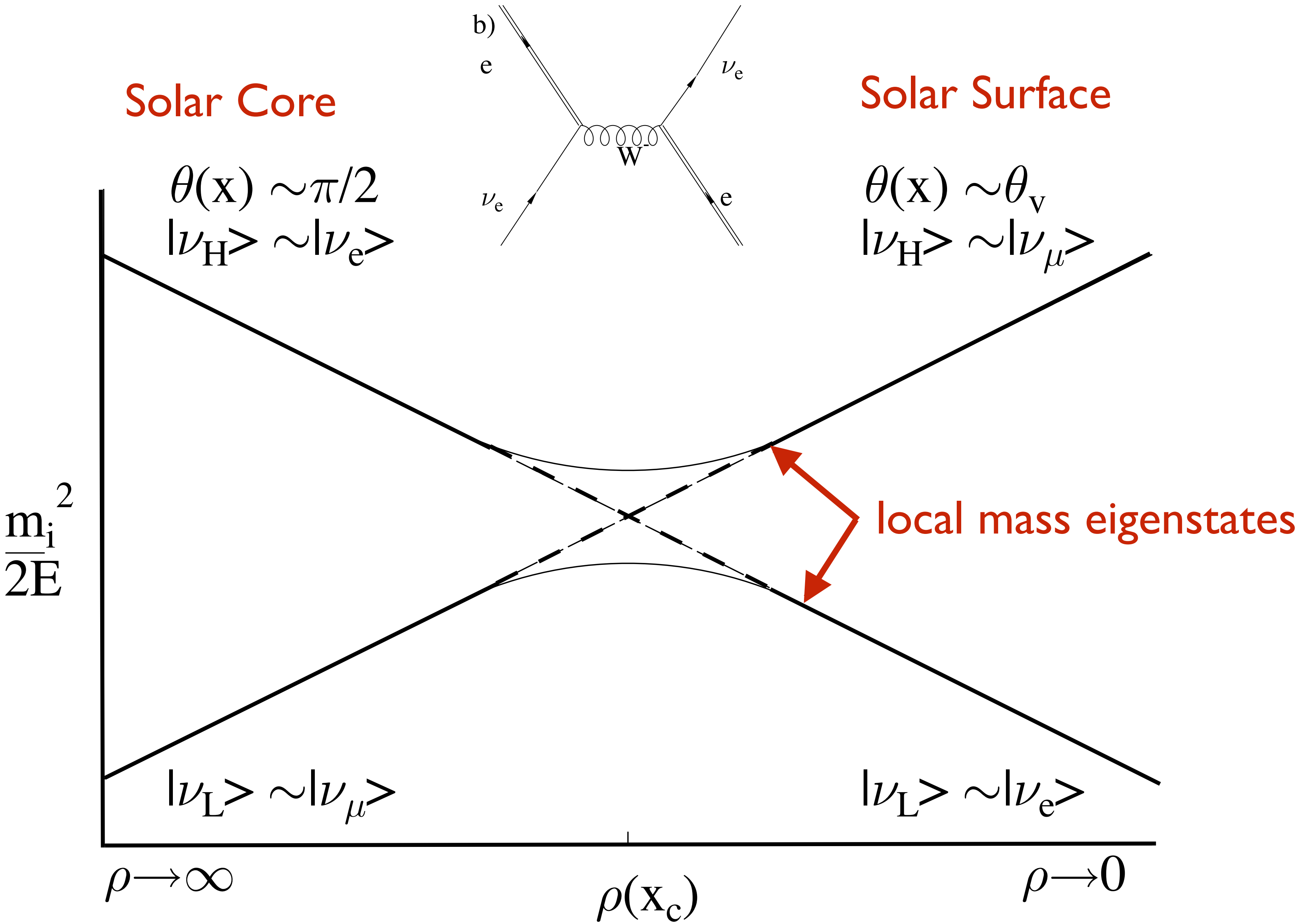
what then happens as the neutrino continues out of the sun?



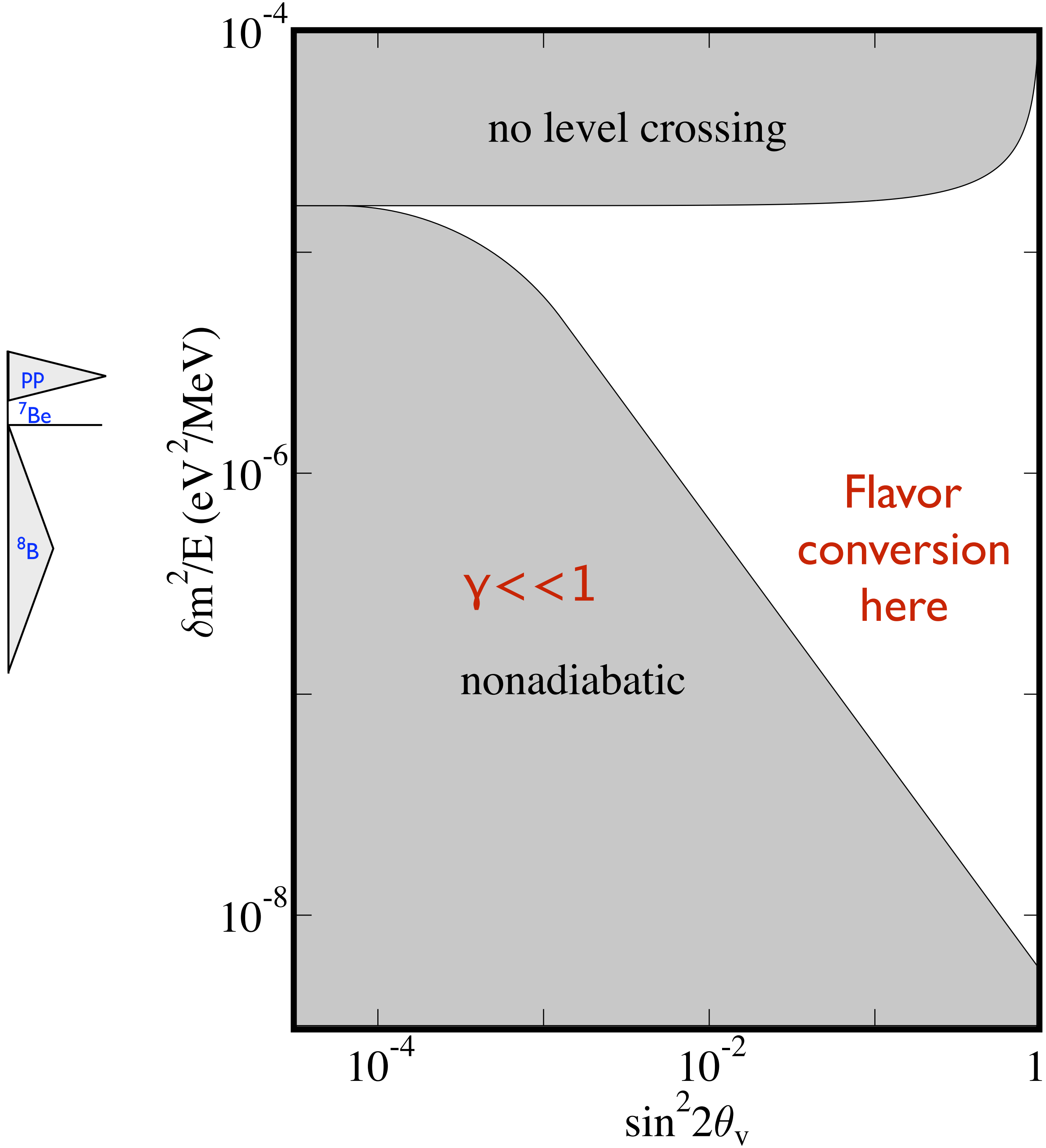
if the density changes are adiabatic
(local oscillation length small
compared to the solar density scale height)
the motion remains in the mode closest in
frequency to the resonance mode -- the right
oscillator (muon neutrino) mode

effectively a complete conversion
of electron neutrinos to muon neutrinos

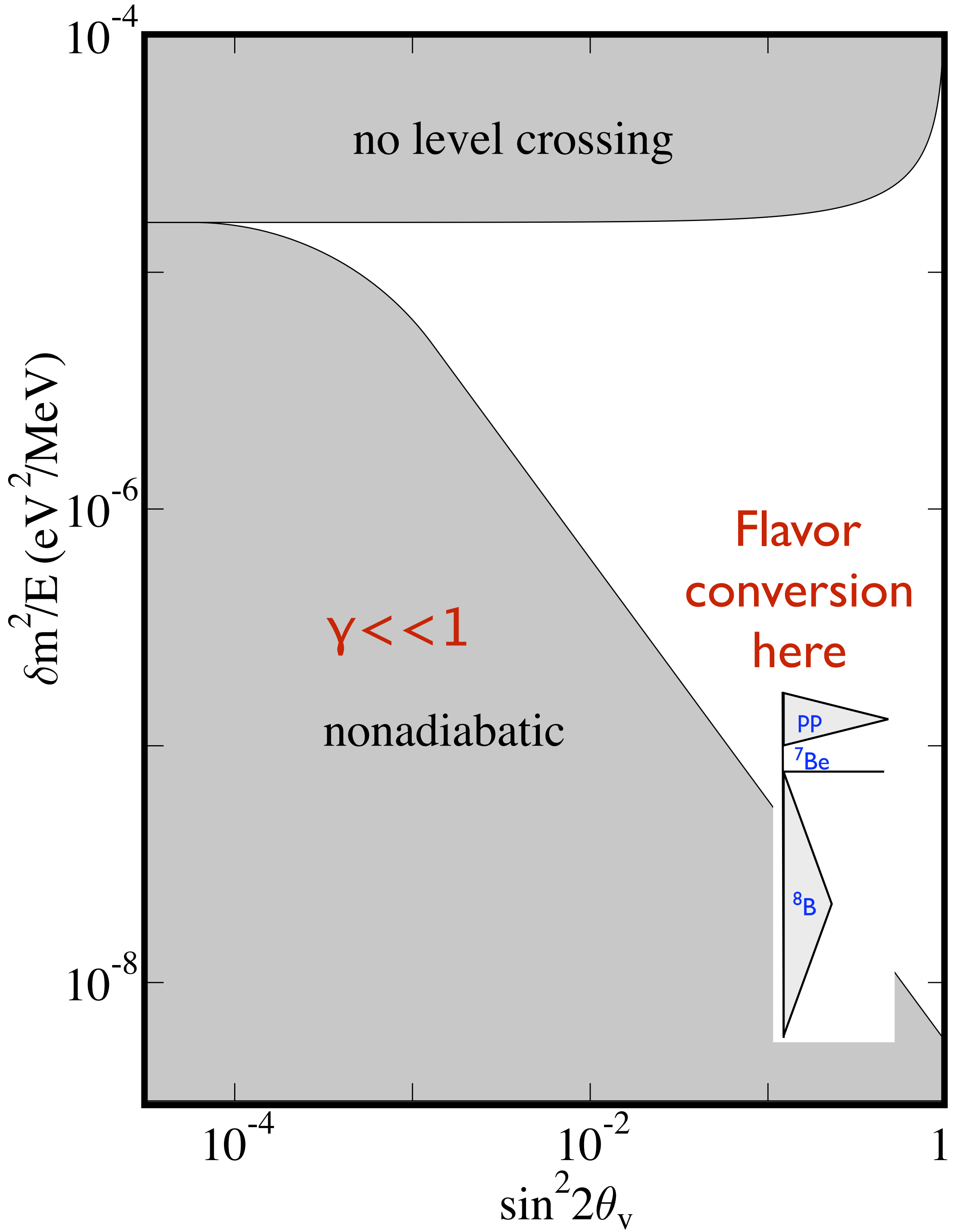
an important effect in the solar neutrino
problem, and in supernovae



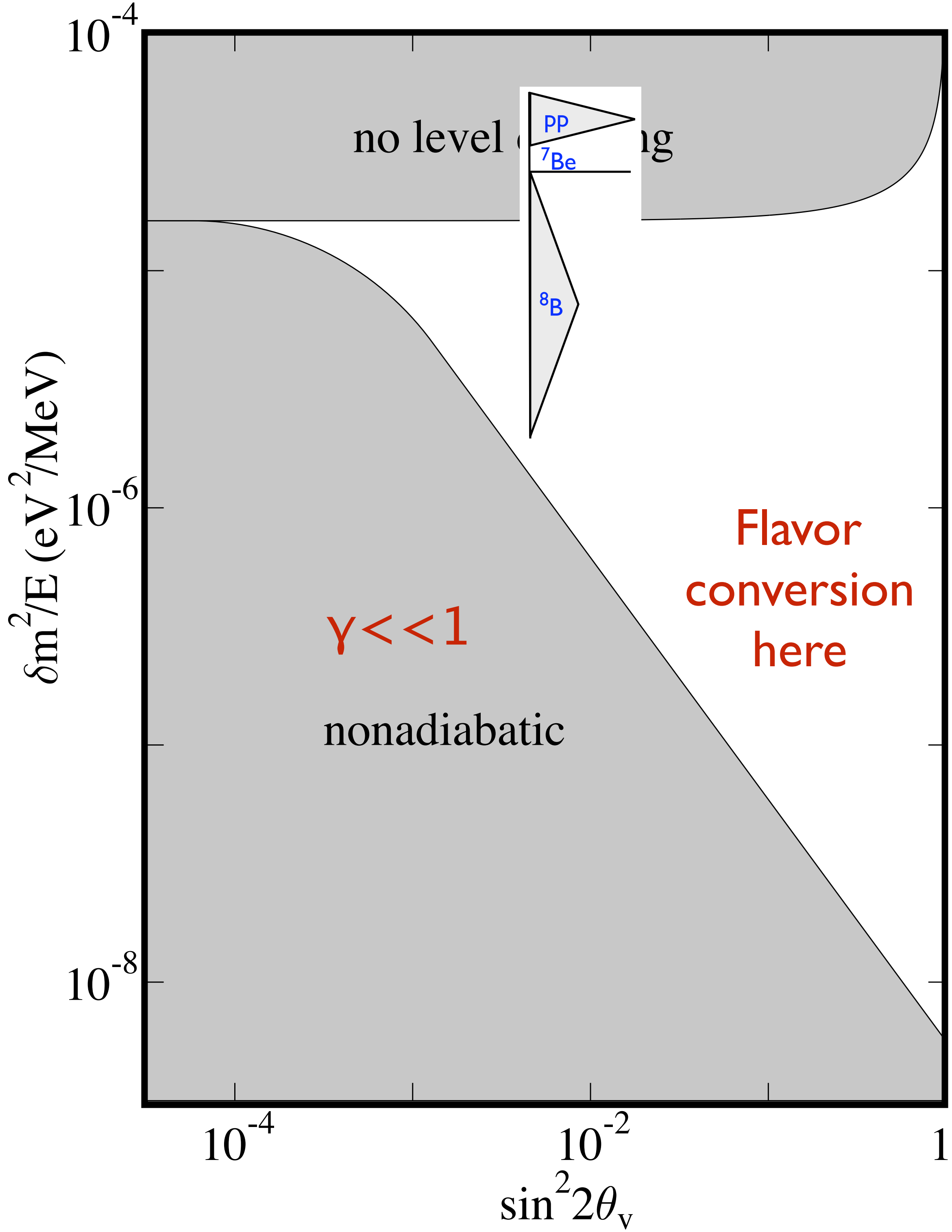
adiabatic passage of an avoided level crossing:
"the MSW mechanism"



pp
^{7}Be
^{8}B
10^{-4}
10^{-6}
10^{-8}
$\delta m^2/E$ (eV^2/MeV)
no level crossing
$\gamma << 1$
nonadiabatic
Flavor conversion here
10^{-4}
10^{-2}
1
$\sin^2 2\theta_v$

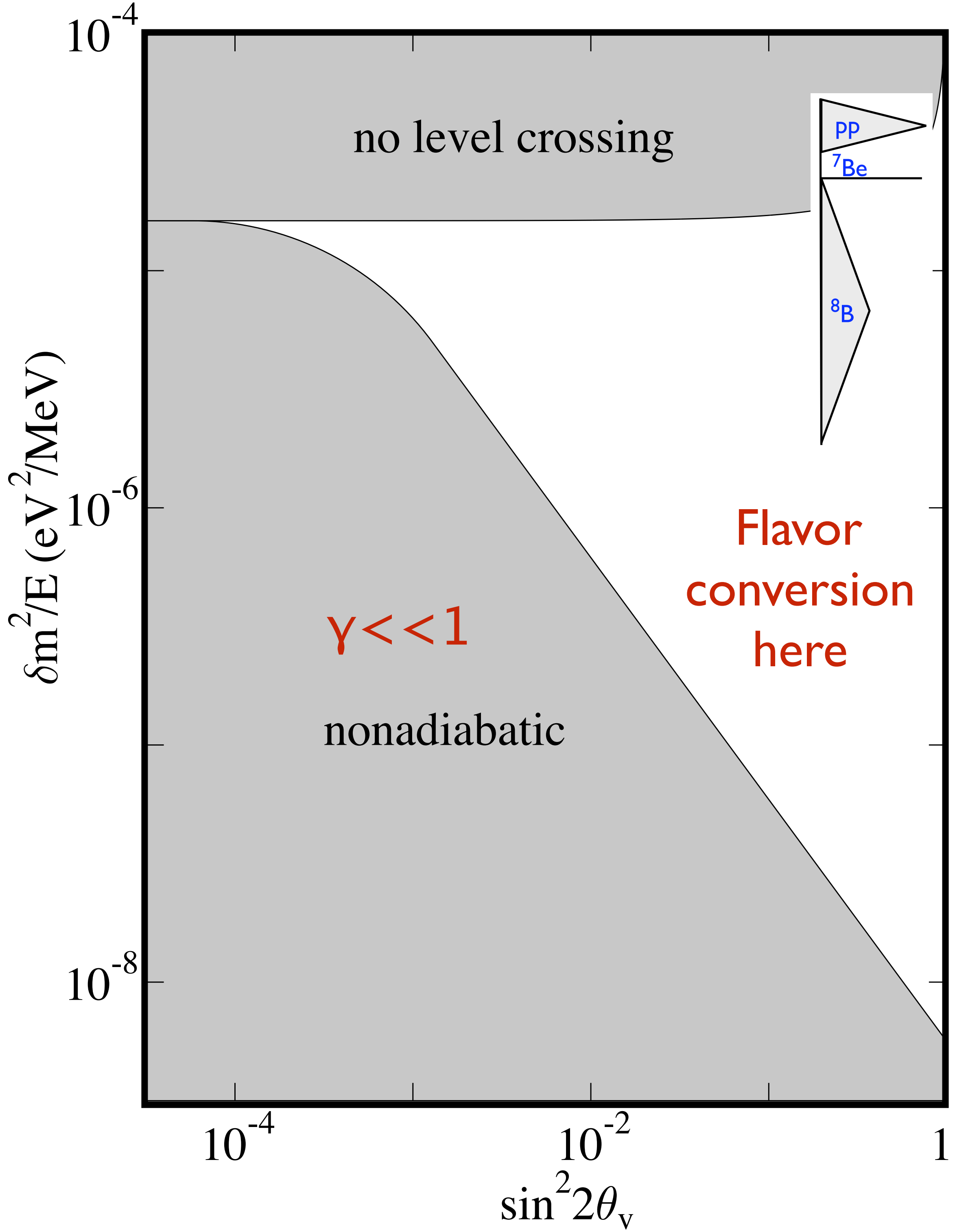


Low Solution
pp nus convert
^{8}B nus remain:
no



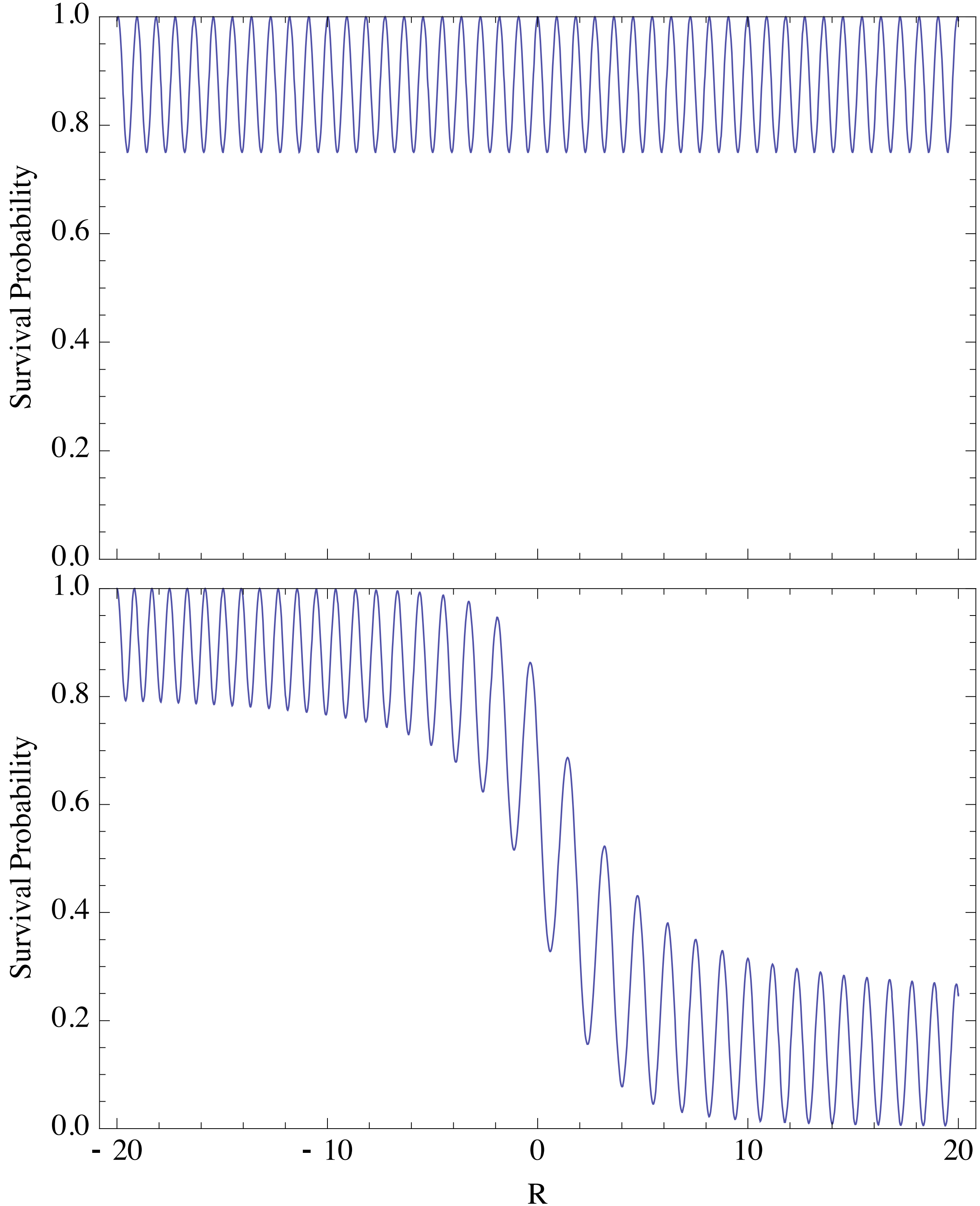
Small angle solution
pp nus remain
^{8}B nus oscillate

better: but no



Large angle solution
pp nus remain
^{8}B nus oscillate

yes!



vacuum
oscillations

tells one $\sin^2 2\theta_{12}$
as well as both the
magnitude and *sign*
of δm^2_{12}

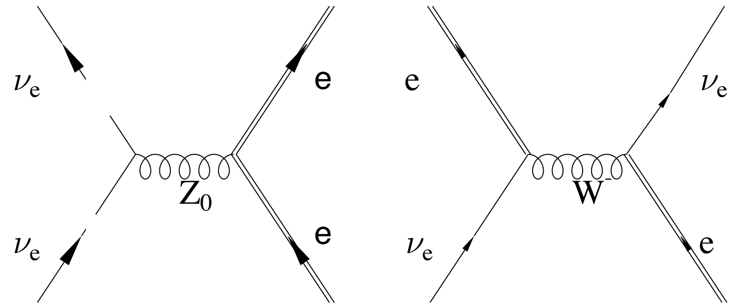
solar MSW
oscillations

Sudbury Neutrino Observatory (SNO)

Central acrylic vessel contained one ton of heavy water

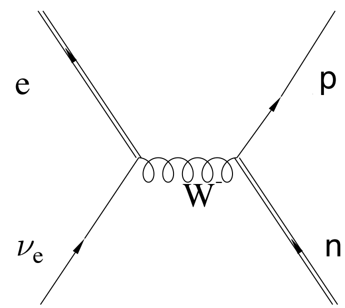
1) $\nu_x + e^- \rightarrow \nu_x + e^-$

$$\text{sensitivity} : \nu_e/\nu_\mu/\nu_\tau \sim 6/1/1$$



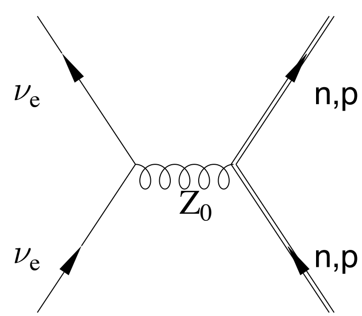
2) $\nu_e + D \rightarrow e^- + p + p$

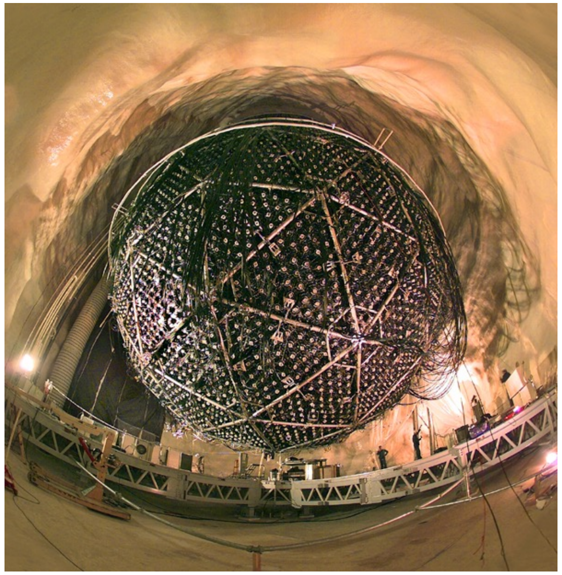
$$\text{sensitivity} : \nu_e$$



3) $\nu_x + D \rightarrow \nu_x + p + n$

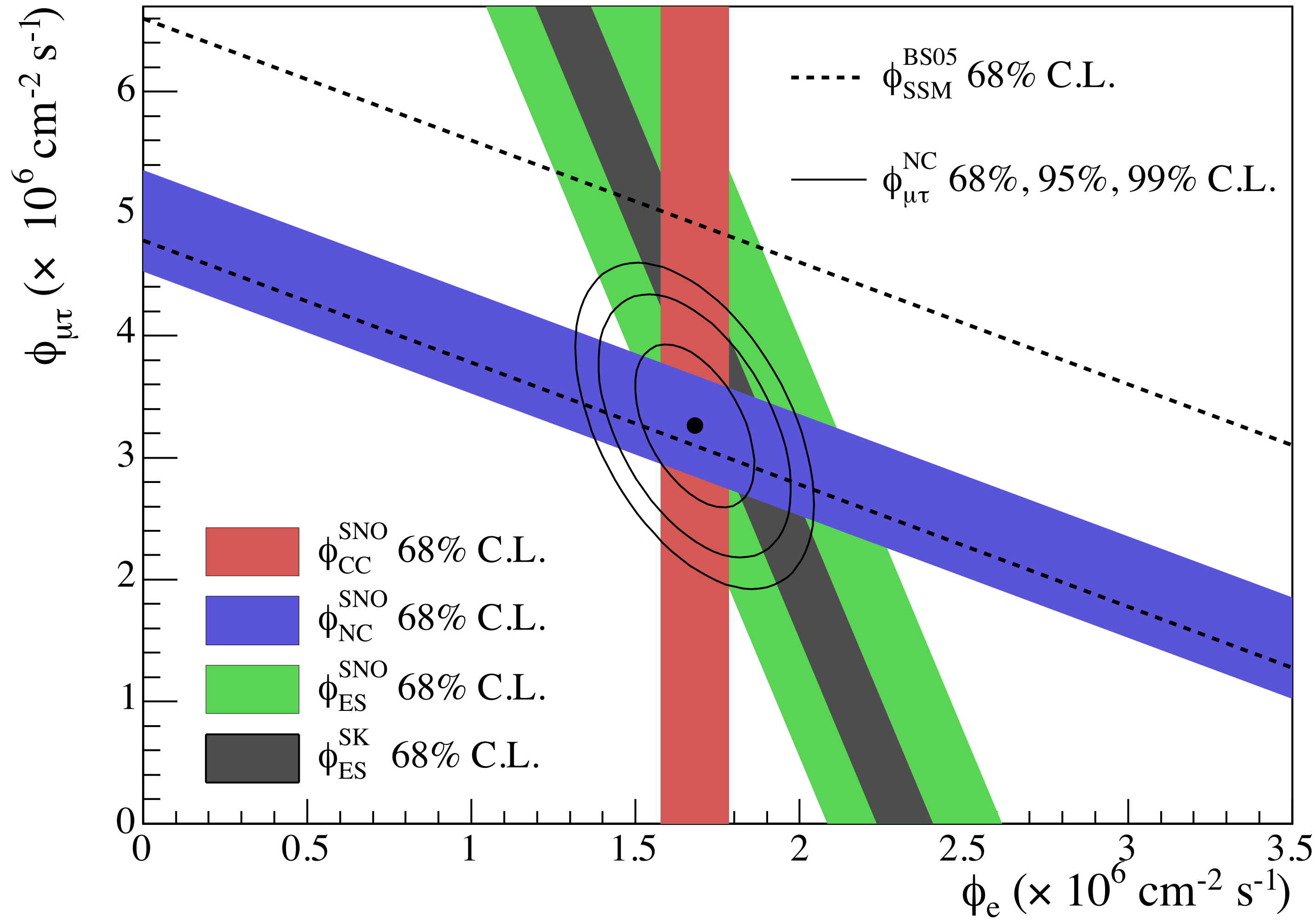
$$\text{sensitivity} : \nu_e/\nu_\mu/\nu_\tau \sim 1/1/1$$





the “solar ν problem” was definitively traced to new physics by SNO

flavor conversion $\nu_e \rightarrow \nu_{heavy}$

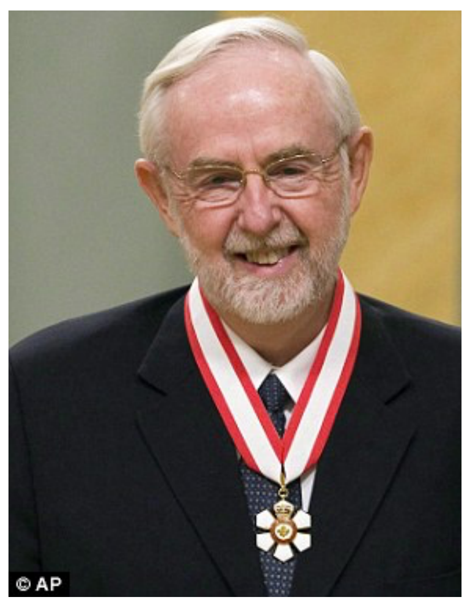


requires an extension of the SM -- Majorana masses or ν_R

Fifty years on experimental effort, initiated by the desire to use the neutrino to probe astrophysics, yielded two separate Nobel prizes

Davis (Cl),
Koshiba (Kamioka)
awarded
Nobel Prize, 2002

McDonald (SNO),
Kajita (SuperK)
in 2015



A more algebraic treatment, so we can introduce the mass^2 matrix

Slightly generalize our earlier vacuum case for arbitrary initial state

$$|\nu(0)\rangle \;\rightarrow\; a_e(0)|\nu_e\rangle + a_\mu(0)|\nu_\mu\rangle$$

yielding

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

vacuum m_ν^2 matrix

which we noted was modified in matter in its e-e component

$$m^2_{\nu_e} = 4E\sqrt{2}G_F\;\rho_e(x)$$

inserting this into mass matrix generates the 2-flavor MSW equation

$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta + 4E\sqrt{2}G_F\rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

or equivalently (by taking an average phase away)

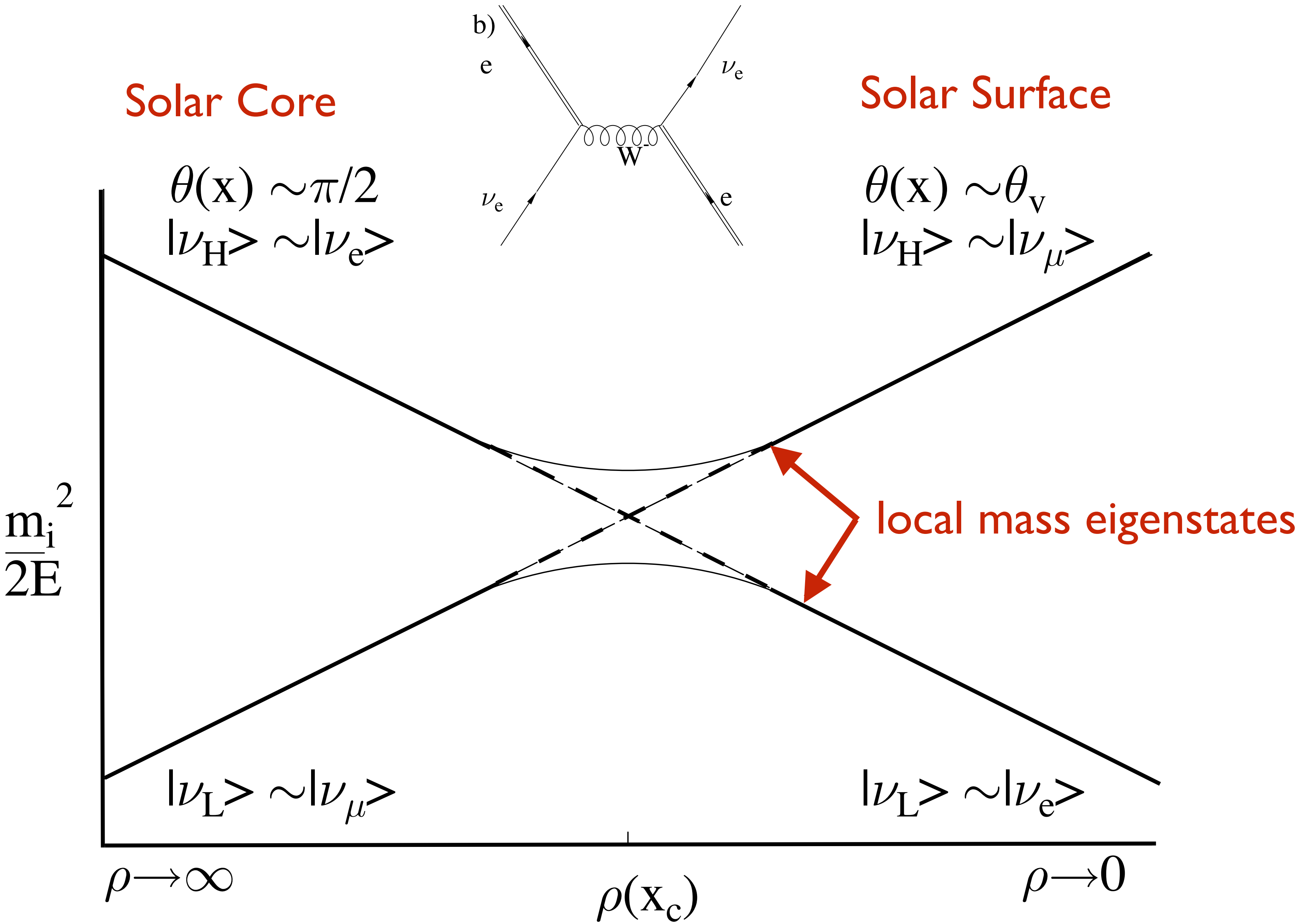
$$i\frac{d}{dx}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E}\begin{pmatrix} -\delta m^2 \cos 2\theta + 2E\sqrt{2}G_F\rho_e(x) & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & -2E\sqrt{2}G_F\rho_e(x) + \delta m^2 \cos 2\theta \end{pmatrix}\begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}$$

the m_ν^2 matrix's diagonal elements vanish at a critical density

$$\rho_c : \quad \delta m^2 \cos 2\theta \equiv 2E\sqrt{2}G_F\rho_c$$

Now freeze the density at some arbitrary value and diagonalize the RHS, finding the “local” mass eigenstates appropriate to that density (see HW)

In terms of this (changing) mass eigenstate basis



adiabatic passage of an avoided level crossing:
"the MSW mechanism"

In terms of local mass eigenstates,

$$|\nu(x)\rangle = a_H(x)|\nu_H(x)\rangle + a_L(x)|\nu_L(x)\rangle$$

$$i\frac{d}{dx}\begin{pmatrix} a_H(x) \\ a_L(x) \end{pmatrix} = \frac{1}{4E}\begin{bmatrix} m_H^2(x) & i\alpha(x) \\ -i\alpha(x) & m_L^2(x) \end{bmatrix}\begin{pmatrix} a_H(x) \\ a_L(x) \end{pmatrix}$$

- mass splittings small at ρc: avoided level crossing
- $\nu_H(x) \sim \nu_e$ at high density
- if vacuum θ small, $\nu_H(0) \sim \nu_\mu$ in vacuum

thus there is a local mixing angle θ(x) that rotates from $\sim \pi/2 \rightarrow \theta_v$ as $\rho_e(x)$ goes from $\infty \rightarrow 0$

The HW will allow you to verify these results

- it must be that $\alpha(x) \sim \frac{d\rho}{dx}$ (the coupling between local eigenstates)
- if derivative gentle (change in density small over one local oscillation length) we can ignore: matrix then diagonal, easy to integrate

$$\Rightarrow \; P_{\nu_e}^{adiabatic} = \frac{1}{2} + \frac{1}{2}\cos 2\theta_v \cos 2\theta_i \rightarrow 0 \text{ if } \theta_v \sim 0, \theta_i \sim \pi/2$$

- most adiabatic behavior is near the crossing point: small splitting ⇒ large local oscillation length ⇒ can "see" density gradient

- derivative at ρ_c governs nonadiabatic behavior (Landau Zener)

$$P_{\nu_e}^{LZ} = \frac{1}{2} + \frac{1}{2}\cos 2\theta_v \cos 2\theta_i (1 - 2P_{hop})$$

so $\rightarrow 1 \text{ if } \theta_v \sim 0, \theta_i \sim \pi/2, P_{hop} \sim 1$

(jumping to the other mass eigenstate at the crossing ⇒ no oscillations)

$$P_{hop}^{linear} = e^{-\pi \gamma_c/2} \qquad \gamma_c = \frac{\sin^2 2\theta}{\cos 2\theta} \frac{\delta m^2}{2E} \frac{1}{\left| \frac{1}{\rho_c} \frac{d\rho}{dx} \right|}$$

$\Upsilon_c >> 1 \Leftrightarrow$ adiabatic, no hopping, so strong flavor conversion

$\Upsilon_c << 1 \Leftrightarrow$ nonadiabatic, unit probability hopping, little flavor conversion

(the sudden approximation you learned in QM)

so two conditions for strong flavor conversion:

sufficient density to create a level crossing

adiabatic crossing of that critical density

MSW mechanism is about passing through a level crossing

(I can show you how to derive the LZ result in discussions)