1 Inverse Compton Monte Carlo Code

Here, we will construct a simple numerical model of an accretion disk corona.

Consider an ionized, hot slab of gas (the corona) of temperature T_{corona} with some finite thickness L, located above an optically thick source of blackbody photons (the disk) with temperature $T_{\text{BB}} \ll T_{\text{corona}}$. The slab contains free electrons at density n that can scatter the radiation.

Following the sketch in Fig. 1, set up your coordinate system with the corona above the disk in the z-direction and measure angles from the z-axis as $\mu = \cos(\theta)$.

Photons enter the slab from below with an isotropic distribution of angles $0 < \mu \leq 1$ and can exit the corona at the top surface. Downgoing photons are re-absorbed by the disk.

Simplifying assumptions: We will make the simplifying assumption that the disk and corona are extended to infinity in the x and y-directions so that our problem is symmetric about the z-axis. We measure optical depth as $d\tau = n\sigma_{\text{T}}ds$ along the photon trajectory.

Neglect the non-linear effects of Compton cooling on the temperature of the corona or the disk (that is, assume that both maintain a fixed temperature).

Assume the electrons are non-relativistic ($v \ll c$), such that we can neglect Klein-Nishina effects and use the Thomson cross section σ_{T}. You may choose to neglect the small recoil loss on the photon in the frame of the electron.

Make the simplifying assumption that scattering is *isotropic* in the frame of the electron (given that the electron distrution is isotropic, this is permissible). Further, neglect polarization.

For the first part of the problem, further simplify the problem by assuming a single energy E_{e} and velocity v_{e} for the electrons and a single input energy $h\nu_{\text{source}}$ for the source photons.

A) Write a function to Lorentz transform a four-vector P^{μ} into a frame moving with velocity $\vec{\beta} = \vec{v}/c$. Use the general matrix form of the Lorentz transform,

$$\Lambda^{\mu}{}_{\nu}(\beta) = \begin{pmatrix} \gamma & -\gamma\beta_1 & -\gamma\beta_2 & -\gamma\beta_3 \\ -\gamma\beta_1 & 1 + (\gamma - 1)\beta_1^2 & (\gamma - 1)\beta_1\beta_2 & (\gamma - 1)\beta_1\beta_3 \\ -\gamma\beta_2 & (\gamma - 1)\beta_1\beta_2 & 1 + (\gamma - 1)\beta_2^2 & (\gamma - 1)\beta_2\beta_3 \\ -\gamma\beta_3 & (\gamma - 1)\beta_1\beta_3 & (\gamma - 1)\beta_2\beta_3 & 1 + (\gamma - 1)\beta_3^2 \end{pmatrix} \tag{1.1}$$

B) Consider an electron moving in direction $\vec{v} = (v_x, v_y, v_z)$ scattering a photon of energy $h\nu_0$ moving in direction $\hat{\vec{k}}_0 = (k_{x,0}, k_{y,0}, k_{z,0})/k_0$. The photon four-momentum in the observer frame is $P_0^{\mu} = h\nu/c(1, \hat{\vec{k}}_0)$. Using your result from (A), write a function that performs an inverse Compton scattering by Lorentz transforming the photon into the electron frame, Thomson-scattering, and transfoming back to the observer's frame. Your function should take $\hat{\vec{k}}_0$, $h\nu_0$, and v as input and return scattered $\hat{\vec{k}}_1$ and $h\nu_1$. To do this, assume scattering is *isotropic* and *elastic* in the electron frame. You will have to use a random number generator to generate random directions $\hat{\vec{k}}$ for the scattered photon in the electron frame and the electron velocity vector $\hat{\vec{v}}$.

C) From the solution to the radiative transfer equation, write down the probability of a photon traveling to an optical depth τ before being scattered by an electron (that is, out of N phtons, what fraction will travel to an optical depth τ or less). Invert your answer to find an expression of optical depth τ (or distance $s = \tau/(n\sigma_{\rm T})$) as a function of probability P.

D) Write a function that follows the trajectory of a single photon injected in some initial direction $\hat{\vec{k}}_{\rm source}$ through multiple scatterings until it either escapes the corona ($z > L$) or is re-absorbed by the disk ($z < 0$).

For each scattering step i, draw a random probability P (i.e., random number) between 0 and 1 and use your expression from (C) to convert that to a travel distance $s = \tau/(n\sigma_{\rm T})$. From s and $\hat{\vec{k}}$, determine the location of the scattering event.

If the scattering occurs inside the corona, evaluate your scattering function from (B) to determine the new direction $\hat{\vec{k}}_{\rm i}$ and energy $h\nu_{\rm i}$ of the scattered photon. If scattering occurs outside the corona, your photon has escaped or been re-absorbed.

E) Write a wrapper that executes the function from (D) for N photons injected in random directions $\hat{\vec{k}}_{\rm source}$ and tabulate all out-going photons. Your function should take as input N, L, n, $v_{\rm e}$, and $h\nu_{\rm source}$ and return a list of outgoing photon energies $h\nu_{\rm out}$.

F) Holding $N \gg 1$, $v_{\rm e} = 0.1c$, and $h\nu_{\rm source} \ll m_{\rm e}c^2$ fixed, log-log-plot the histograms of $h\nu_{\rm out}$—that is, the scattered spectra—for $\tau_{\rm z} \equiv nL\sigma_{\rm T} = 0.1$, $\tau_{\rm z} = 1$, $\tau_{\rm z} = 3$, and $\tau_{\rm z} = 10$. Evaluate the fraction of transmitted and re-absorbed photons in each case.

G) **Extra credit 1**: Generalize to the case of a Maxwellian electron distribution of temperature $T_{\rm e}$. Plot the output spectra for your choice of T. You can use `numpy.random.randn` for this, for example.

H) **Extra credit 2**: Generalize to the case of a Planck source photon distribution of temperature $T_{\rm BB}$. Plot the input and output spectrum for your choice of $T_{\rm BB}$. You will have to work out how to randomly inject photons from a Planck distribution.

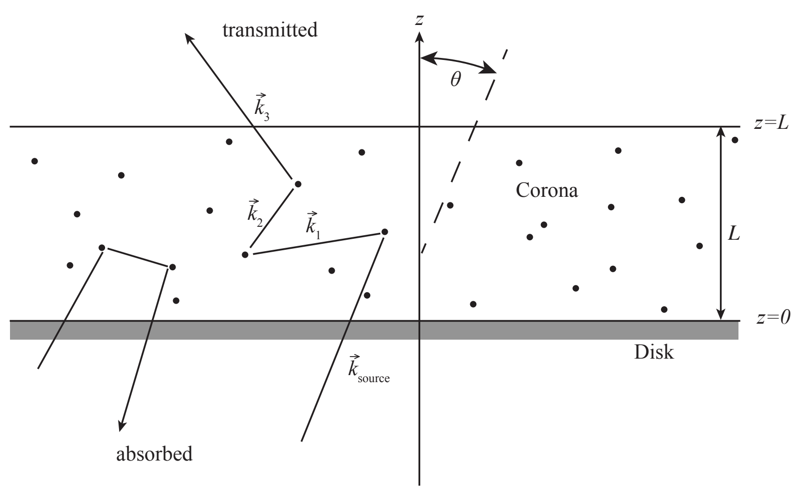

Figure 1: Illustration of different possible photon paths that lead to reflection and transmission.