Lecture 1

Aim of this class: To discuss BBH mergers (in particular LIGO results)

⇒ (1) how to retrieve BH params

(2) testing GR (or 'inconsistency tests' of GR)

(3) astrophysical implications.

Before continuing:

recap 100 years of theory and experimental advances + breakthroughs.

(do not forget funding was critical for LIGO).

From Jon's lectures:

$$L = \frac{G}{5c^5} \langle \dddot{I}_{ij} \dddot{I}_{ij} \rangle \qquad h \sim \frac{\ddot{I}_{ij}}{D}$$

rate of energy loss

$$I_{ij} = \int \rho x_i x_j \, dV.$$

⇒ use these as o. of mag. and for freq spectrum

- For $\mathcal{M} \sim 30M_\odot$, must have $M \geq 70M_\odot$.
- BHs have size $R_{BH} = 2M_{BH}$ and last stable cirular orbit at $D = 6M_{BH}$ (note $M_\odot \sim 1.5$km).
- Suppose Kepler formula holds until peak GW frequency ~ 150Hz; what is the most massive BH consistent with this frequency?
- $M_{BH} = 1/(6^{3/2}\Omega) \sim 130M_\odot$
- Could it be something besides a BH?
- No: Stars have $R \sim 10^6$km and are too large; WD and NS have maximum masses $M \sim M_\odot$ are too light to explain $\mathcal{M}$.
- Must be BBH with $m_2/m_1 \in [\sim 0.1, 1]$ with masses in range $(130M_\odot, 10M_\odot)$–$(35M_\odot, 35M_\odot)$ satisfying chirp-mass constraint $\mathcal{M} \sim 30M_\odot$

② Blanchet 2015,

Post Newtonian Source Modelling

* No analytic soln. to relativistic 2 body problem

(Newtonian gravity: $\nabla^2 \Phi_N = 4\pi G \rho$)

$$\underbrace{\Phi_N (t, \vec{x})}_{\text{Newtonian potential.}} = - G \int \frac{\rho(t, \vec{x}\,')}{|\vec{x} - \vec{x}\,'|} \, d^3\vec{x}\,'$$

changes in mass density cause inst. changes in grav. field

→ does not

* Physical params : mass ratio
  spin ang momentum $\vec{S}_{1,2}$
  eccentricity
  tidal params

} morphology of GW waveform

* Challenge : general formalism for relativistic 2 body problem

Notably, local dynamics ⟷ Far-zone formalism GW waveform

Relating asymptotic GW form h_{ij} @ some detector to in sources wave zone stress energy $T^{\alpha\beta}$ of matter field.

②

Blanchet 2015,

Post Newtonian Source Modelling

* No analytic soln. to relativistic 2 body problem

(Newtonian gravity : $\nabla^2 \Phi_N = 4\pi G \rho$)

$$\underbrace{\Phi_N (t, \vec{x})}_{\text{Newtonian potential.}} = - G \int \frac{\rho(t, \vec{x}\,')}{|\vec{x} - \vec{x}\,'|} \, d^3 \vec{x}\,'$$

Changes in mass density cause inst. changes in grav. field

→ does not

* Physical params : mass ratio

spin ang momentum $\vec{S}_{1,2}$

eccentricity

tidal params

} morphology of GW aveform

* Challenge : general formalism for relativistic 2 body problem

Notably, local dynamics ⟷ Far - zone formalism GW waveform

Relating asymptotic GW form h_{ij} @ some detector to in sources wave zone stress energy $T^{\alpha\beta}$ of matter field.

To describe PN inspiral:

1) Find a soln to E.F.E :

$$G^{\alpha\beta} = \frac{8\pi G}{c^4} T^{\alpha\beta} \Rightarrow \nabla T^{\alpha\beta} = 0$$

valid for a general compact support and reg. matter

matter density in formal PN.

2) Apply binary model of 'pt.' particles

$$T^{\alpha\beta}_{p.p} = \frac{1}{\sqrt{-g}} \sum_A \frac{m_A v_A{}^{\alpha} v_A{}^{\beta}}{\sqrt{(g_{\mu\nu})_A v_A{}^{\mu} v_B{}^{\nu}}} \underbrace{\delta(x - y_A)}_{\delta \text{ fn.}}$$

infinite self field

Locally confined spatial support → regularisation

3) Assumptions

near zone validity

- Weak grav field : $h^{\alpha\beta} \equiv \sqrt{-g}\, g^{\alpha\beta} - \eta^{\alpha\beta}$
- weakly self-gravit : slow motion of source $\quad \epsilon \sim \frac{v}{c} \sim \frac{a}{c} \ll 1$

$r \ll \lambda$

characteristic wavelength of emit. radn

$$\epsilon \equiv \max \left\{ \left| \frac{T^{0i}}{T^{00}} \right|, \left| \frac{T^{ij}}{T^{00}} \right|^{1/2}, \left| \frac{U}{c^2} \right|^{1/2} \right.$$

harmonic gauge

PN expansion → direct iterative method; non linear

retarded potentials,

$$\bar{h}^{\mu\nu}(x,t,c) = \sum \frac{(\ln c)^q}{c^p} h^{\mu\nu}_{p,q}(\vec{x},t).$$

⇒ determine E. of motion → Express in terms of source multiples.

Relate to "wave zone" $\boxed{\text{distance} \gg \text{characteristic wavelength}}$

EXPAND IN POWERS of 'G',

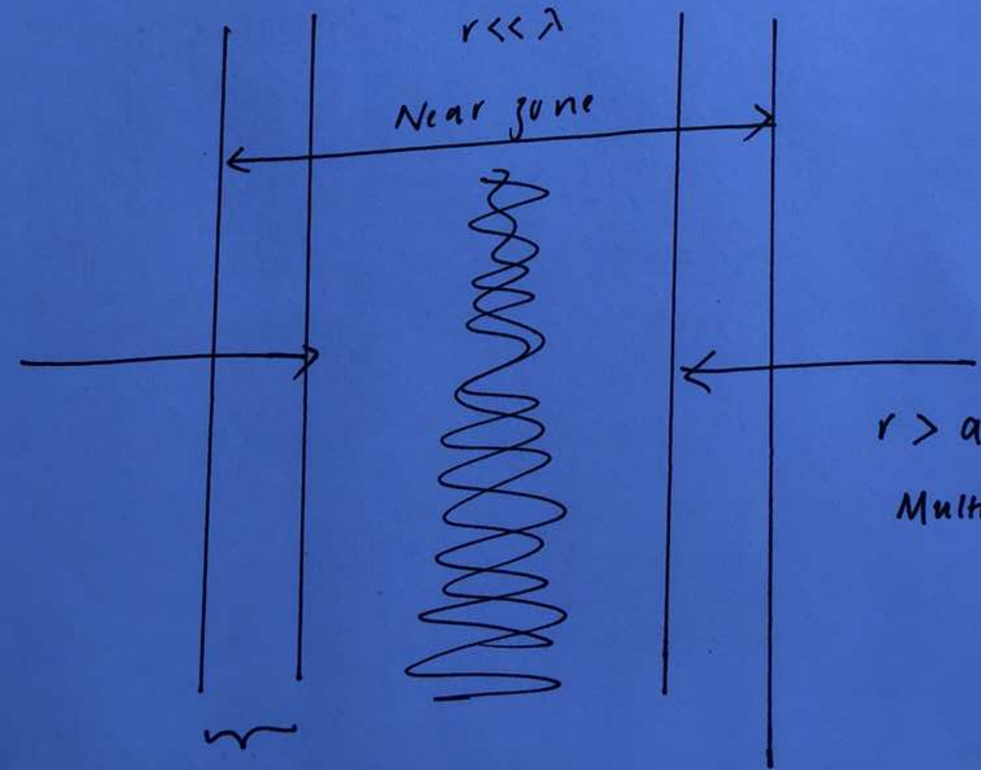
→ find solutions interms of multipoles

$$h_n^{\alpha\beta} = \sum_{k=1}^{\infty} \sum_{p=0}^{n-1} G^{\alpha\beta}_{L(k,p,n)}(t_{ret}) \times \hat{N}_L \frac{(\log r)^p}{r^k}$$

ASYMPTOTIC MATCHING

(choose radiative coordinates.
→ radiative multipoles)

$$h_{ij} \text{ (PN near zone)} \longleftrightarrow \bar{h}_{ij} \text{ (exterior multiple zone)}.$$



$$\bar{h}^{\mu\nu} = \frac{16\pi G}{c^4}\left[\tilde{\Box}_R^{-1}[\bar{\tau}^{\mu\nu}] + \sum_{l=\infty}^{\infty} \frac{(-1)^l}{l} \partial_L \left(\frac{R_L^{\mu\nu}(t-r/c) - R_L^{\mu\nu}(t+r/c)}{2r}\right)\right.$$

Numerical Relativity

Darmois
Lichnerowicz
Foures - Bruhat
Arnowitt, Deser + Misner
York

Spacetime is foliated Σ_t $\bar{c}$ $t = \text{const.}$ slices.

'time' coordinate

On each Σ_t, we have $\{x^i\}_{i \in \{1,2,3\}}$

$$\Rightarrow g_{\mu\nu} dx^\mu dx^\nu = ds^2$$

$$= -N^2 (dt)^2 + \gamma_{ij} \left(dx^i + \beta^i dt\right)\left(dx^j + \beta^j dt\right)$$

where N : lapse fn.

γ_{ij} : 3-metric (spatial)

β^μ : shift vector

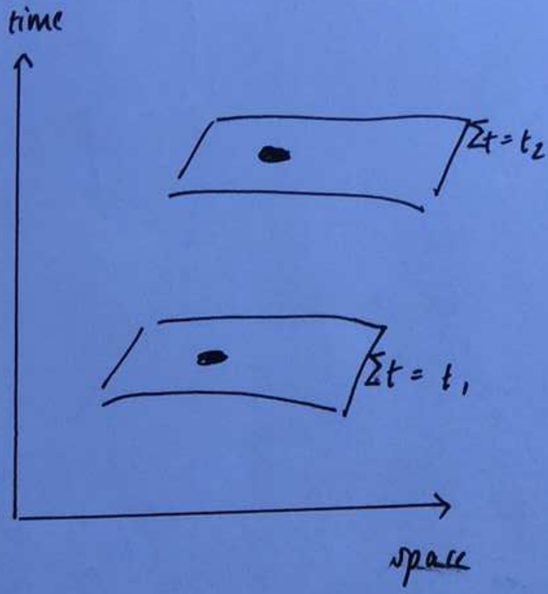
$$\gamma_{\mu\nu} = g_{\mu\nu} + \underbrace{n_\mu n_\nu}_{\text{unit normal to } \Sigma_t}$$

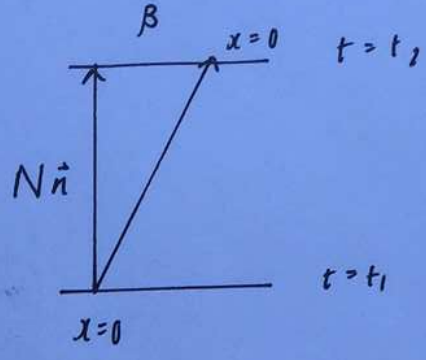
$$t^\mu = N n^\mu + \beta^\mu.$$

K^{ij} : extrinsic curvature

$$= -\frac{1}{2} \mathcal{L}_n \gamma_{ij}$$

specifies fully how each hypersurface is contained in the spacetime.





Relationship between coords. on neighbouring slices is given by lapse N and shift β_μ.

EFE in 3+1 :

Cauchy Surface

Hamiltonian Constraint $\bar{R} + K^2 - K_{ij}K^{ij} = 0$

Momentum Constraint $\bar{\nabla}_j (K^{ij} - \gamma^{ij} K) = 0$

Dynamical equations :

$$\partial_t K_{ij} = N[\bar{R}_{ij} - 2K_{il}K^l{}_j + K K_{ij} - 8\pi G S_{ij} + 4\pi G \gamma_{ij} (S - \rho)] - \bar{\nabla}_i \bar{\nabla}_j N + \beta^l \bar{\nabla}_l K_{ij} + K_{il} \bar{\nabla}_j \beta^l + K_{jl} \bar{\nabla}_i \beta^l$$

Kinematic relation between γ_{ij} and K_{ij} :

$$\partial_t \gamma_{ij} = -2 N K_{ij} + \bar{\nabla}_i \beta_j + \bar{\nabla}_j \beta_i$$

Non trivial ? :

How does one specify (γ_{ij}, K_{ij}) and solve constraints for BBH mergers ?

Constraints must be solved for spacetime itself + specify dynamical, constrained variables & gauge freedom in (γ_{ij}, K_{ij}).