NBIA & DARK Summer School: Multi-Messengers from Compact Sources

WARMUP #1

Solution: Interaction rate is given by $R_{\rm int} = \sigma\phi N_t$, where σ is the cross section, ϕ is the flux and N_t is the number of targets. For this back-of-the-envelope estimate we will take the σ and ϕ as constant, although a more accurate calculation would fold both σ and ϕ as a function of energy.
The number of targets is $N_t \sim \frac{MtN_A}{A}$, where M is the mass, N_A is Avogadro's number, A is the molar mass and t is targets per atom or molecule. Plugging in $M = 80$ kg, $t = 10$ electrons per water molecule, $A = 18$ g/mol for water, and $N_A = 6\times10^{23}$, and then plugging in (eyeballing from the plots in lecture) $\sigma \sim 5\times10^{-44}$ cm^2 (electron scattering cross-section above a few MeV) and $\phi \sim 2\times10^6$ cm^{-2}s^{-1} (flux above a few MeV, mostly from ^{8}B neutrinos), and then multiplying by 80 years[1], we get about 7 solar neutrino interactions per lifetime.

WARMUP #2

Here we employ the same relation $R_{\rm int} = \sigma\phi N_t$. For the cross section we take $\sigma \sim 2\times10^{-41}$ cm^2, and for $N_t = 1$ we plug in $M = 4\times10^{10}$ g, $A = 40$ g/mol for argon. To find the total fluence (flux integrated over time) at the center of the Milky Way ($\sim$ 8.5 kpc), we assume the neutron star binding energy is about $E_b \sim 2.5\times10^{53}$ ergs, and that this energy goes entirely into neutrinos, equally distributed among flavors. If each neutrino has about $E_\nu \sim 15$ MeV, then the fluence per flavor is $F = \frac{E_b}{6\cdot4\pi d^2 E_\nu} \sim 2\times10^{11}$ cm^{-2} at $d = 8.5$ kpc.

Plugging this in, we get an expected number of neutrino events of 2600. (Note that a high-energy tail in the spectrum will increase this, as cross section goes as E_ν^2.)

WARMUP #3

[1]Useful fact: one year is close to $\pi\times10^7$ seconds.

The Poisson probability of detecting n events if the mean is μ is given by $\frac{\mu^n}{n!}e^{-\mu}$. The probability of detecting $n = 0$ is $e^{-\mu}$, so the probability of detecting 1 or more is $1 - e^{-\mu}$. The mean number of events expected scales as the inverse square of the distance, and as detector mass.