Spectrum of a compact jet

Part 1

Imagine a conical jet (fixed opening angle of 10°) originating from a $10M_\odot$ Schwarzschild black hole 1 kpc away, which emits isotropically, and with constant plasma velocity $v = 0.1c$ (ignore special relativistic effects). All internal quantities are conserved.

a) The black hole accretion disk is radiating a power of $0.1L_{\rm Edd}$. Assume the black hole is not spinning, what is the accretion rate $\dot{M}$ (gm/s)?

b) Assume 10% of the accretion power ($\dot{M}c^2$) enters the jets (just think about one jet) at the base with the plasma velocity given above. The base of the jets have a radius of $5r_g$, where $r_g = \frac{GM}{c^2}$. Assume the energy density of the plasma is split equally between the magnetic field and electrons which have a distribution $n_e(E_e) = CE_e^{-p}$ erg^{-1} cm^{-3} where p=2. What are the values of B and N_e (the integrated, total number of electrons per volume) at the base of the jets?

c) Assume the magnetic field is mainly torroidal. How do B and n_e evolve along the jets as a function of r?

Part 2

According to R&L (Rybicki & Lightman), for a powerlaw of electrons the synchrotron self-absorption coefficient is given by:

$$\alpha_\nu \propto CB^{(p+2)/2}\nu^{-(p+4)/2}, \tag{1}$$

where C is the same electron distribution normalization mentioned above.

a) What is the expression for the optical depth τ in terms of lengthscale r and ν? (*hint: think units*).

b) At the photosphere (defined where $\tau = 1$), what is the relationship between r and ν? (*hint: reduce all quantities to their dependencies on either r or ν*).

Part 3

According to R&L, for a power law of electrons the total radiated power per volume per frequency is:

$$P_\nu \approx \frac{10^{-22} C B}{(p+1)} \left(\frac{10^{-7} \nu}{B} \right)^{-(p-1)/2} \quad \text{erg cm}^{-3}\text{s}^{-1}\ \text{Hz}^{-1} \tag{2}$$

a) Express the isotropic flux density at Earth in terms of dependence on r and ν.

b) Use your answer to find the flux density of the photosphere as a function of frequency, what is the dependence on ν?

c) Give the value of the flux density from the jet base observed from Earth?