Integrability from Emergent Gauge Theory

(Based on work with JiaHui Huang, Minkyoo Kim, Laila Tribelhorn and Jaco Van Zyl)

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Motivation

AdS/CFT offers a non-perturbative definition of quantum gravity in negatively curved spaces (in terms of large N CFT).

Many deep puzzles arise from the study of black holes - can we answer them using AdS/CFT?

Experience with 1/2 BPS sector suggests that operators dual to black holes have dimension $\sim N^2$

For even the strict large N limit we need to learn how to handle non-planar diagrams, a problem of considerable complexity.

Motivation

The usual approach to $1/N$ can't be used. Summing only planar diagrams gives a very poor approximation.

Some natural questions:

1. Is there a systematic $1/N$ expansion for correlation functions of operators of dimension of order N^2?
2. How does the large N limit of these correlators simplify?
3. Can we sum the 't Hooft coupling expansion?

What tools can we develop to answer these questions? We will take a small (ε) step in this direction.

Summary

Consider small deformations of 1/2 BPS sector: simplest non-trivial example.

Use underlying symmetries in a novel way: group representation theory organizes and sums the complete set of ribbon graphs in both free and (sometimes) interacting CFT.

Exhibit a hidden simplicity that implies something of planar integrability survives in these large N but non-planar limits.

Approach: Symmetries

Summing planar diagrams *does not* give an accurate description of large N dynamics. Not known if there is a class of diagrams that dominates: sum everything.

Idea is to use group theory to exploit **permutation symmetries** in the problem in a novel way.

For example, operators constructed from Z and Y complex adjoint scalars are invariant under **permutations of the Z and Y fields** separately.

We can also describe the most general multi-trace operator constructed using Z and Y fields using a permutation

Approach: Permutation Language

$$Y^{i_1}_{i_{\sigma(1)}} \cdots Y^{i_m}_{i_{\sigma(m)}} Z^{i_{m+1}}_{i_{\sigma(m+1)}} \cdots Z^{i_{m+n}}_{i_{\sigma(m+n)}}$$

Two operators, corresponding to the permutations σ_1 and σ_2 are equivalent if

$$\sigma_1 = \gamma\sigma_2\gamma^{-1} \qquad \gamma \in S_n \times S_m$$

$\Rightarrow$ **gauge invariant operators can be identified with elements of the coset**

$$S_{n+m}/S_n \times S_m$$

Each element of the coset is a restricted conjugacy class

This identification provides efficient ways to count the number of gauge invariant operators and this counting comes out correctly.

Approach: Representation Language

Fourier transform on the coset trades restricted conjugacy class for irrep R of S_{n+m} and irrep (r,s) of $S_n \times S_m$ (accounts for symmetry we must mod out).

Modding by $S_n \times S_m \Rightarrow$ restrict S_{n+m} to $S_n \times S_m$ and hence R to (r,s). We need multiplicity labels, α, β.

Restricted Schur basis: dMK, Bhattacharyya, Collins

$$\chi_{R,(r,s)\alpha\beta}(Z,Y)$$

Respects trace relations.

(other possibilities too: see Ramgoolam, Kimura, Brown, Heslop...)

Generalizes for many adjoint scalars, adjoint fermions, covariant derivatives, field strengths.

Approach: General Results

Language of representations provides a **basis for the local gauge invariant operators**.

Computation of correlators reduced to linear algebra: computing free two point function = multiplying two projectors and taking a trace.

$$\langle \chi_{R,(r,s)\alpha\beta} \chi^{\dagger}_{T,(t,u)\gamma\delta} \rangle = \frac{\delta_{RT}\delta_{rt}\delta_{su}\delta_{\alpha\gamma}\delta_{\beta\delta}\mathrm{hooks}_R f_R}{\mathrm{hooks}_r \mathrm{hooks}_s}$$

At one loop level

$$N_{R,(r,s)\mu_1\mu_2;T,(t,u)\nu_1\nu_2} \propto$$

$$\mathrm{Tr}\left(\left[(1,m+1), P_{R,(r,s)\mu_1\mu_2} \right] I_{R'T'} \left[(1,m+1), P_{T,(t,u)\nu_2\nu_1} \right] I_{T'R'} \right)$$

This description of the problem is useful since it exhibits hidden simplicity.

Problem Statement

LLM geometries are 1/2 BPS and result from backreaction of condensate of giant graviton branes. Study excitations of these geometries, using CFT. Entails studying correlators of operators with dimension $\sim N^2$.

Excitations are open string excitations of underlying giant graviton branes. The open strings give rise to an emergent gauge theory. In how much detail can we explore this emergent theory?

Balasubramanian, Berenstein, Feng and Huang, [hep-th/0411205]

The planar limit of the emergent gauge theory is planar $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\text{eff}})$.

Problem Statement: LLM Geometry

$$
\begin{aligned}
ds^2 &= -y(e^G + e^{-G})(dt + V_i dx^i)^2 + \frac{1}{y(e^G + e^{-G})}(dy^2 + dx^i dx^i) \\
&\quad + ye^G d\Omega_3 + ye^{-G} d\tilde{\Omega}_3
\end{aligned}
\tag{1}
$$

$$
z = \frac{1}{2}\tanh(G) \qquad y\partial_y V_i = \epsilon_{ij}\partial_j z \qquad y(\partial_i V_j - \partial_j V_i) = \epsilon_{ij}\partial_y z \tag{2}
$$

$$
\partial_i \partial_i z + y\partial_y \frac{\partial_y z}{y} = 0. \tag{3}
$$

Regularity forces $z = \pm\frac{1}{2}$ on $y = 0$ plane where $(ye^G)(ye^{-G}) = y^2 = 0$.

Lin, Lunin, Maldacena, [hep-th/0409174]

Problem Statement: Boundary Condition

$$\partial_i \partial_i z + y \partial_y \frac{\partial_y z}{y} = 0. \qquad (4)$$

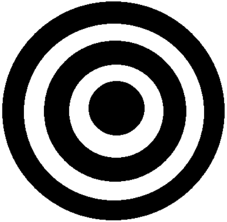


Figure: A coloring of the LLM plane determines z. The black regions are sources of five form flux.

The geometries are 1/2 BPS and are the result of backreaction from a condensate of giant graviton branes.

Problem Statement: Schur Polynomial

For each geometry there is a CFT operator, a Schur polynomials $\chi_B(Z)$.

Corley, Jevicki, Ramgoolam, [hep-th/0111222] Berenstein, [hep-th/0403110]

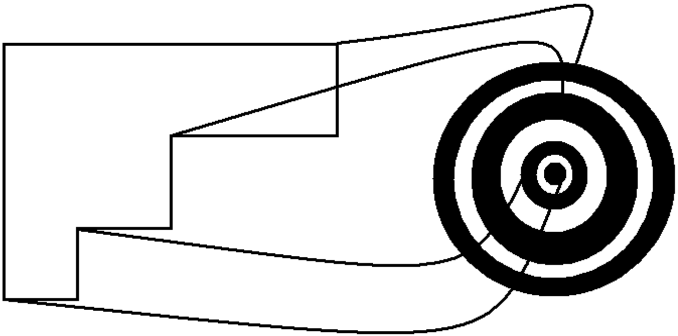
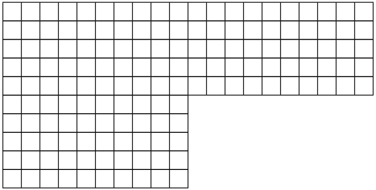
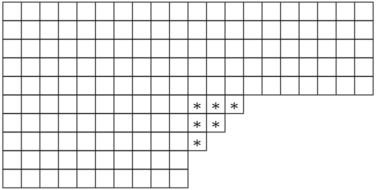


Figure: The coloring of the LLM plane determines both z and B.

Excitations



Excitations are described by adding boxes to B.

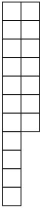


Add only at one inward pointing corner.

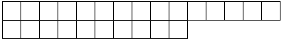
How to read a Young diagram

Each box is a field (adjoint scalar, fermion, field strength or covariant derivative)

Columns are giant gravitons (expanded into S^5; constrained by stringy exclusion principle)



Rows are dual giants (expanded into AdS_5; number constrained $\leq N$)



Strategy

Argue that planar limit of emergent gauge theories agrees with planar limit of $\mathcal{N} = 4$ super Yang-Mills theory in three steps:

1. Construct a bijection between operators in planar $\mathcal{N} = 4$ super Yang-Mills and planar limit of emergent gauge theory.
2. Argue that three point functions in planar emergent gauge theory vanish, so OPE coefficients match.
3. Argue that spectrum of anomalous dimensions matches spectrum of $\mathcal{N} = 4$ super Yang-Mills.

Useful ingredient: background independence

Classify ingredients of the computation as background independent / dependent.

Background independent: something that **takes the same value on any inward pointing corner** or even in the absence of a background, i.e. **planar limit of original CFT**.

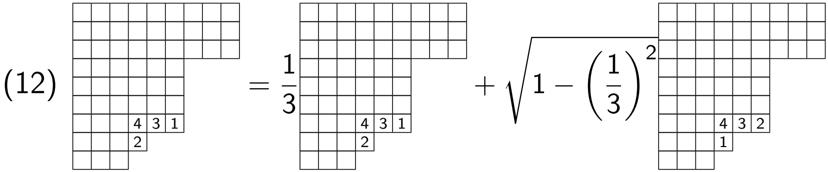
Take the same value regardless of which collection of branes are excited, hence the name “background independent”.

Background dependent: does depend on the collection of branes we excite.

Example: background independent

$$R = \begin{array}{|c|c|c|}\hline & & \\ \hline & \multicolumn{2}{c}{} \\ \cline{1-1} \end{array}$$

$$(12)\ \begin{array}{|c|c|c|}\hline 4 & 3 & 1 \\ \hline 2 & \multicolumn{2}{c}{} \\ \cline{1-1} \end{array} = \frac{1}{3} \begin{array}{|c|c|c|}\hline 4 & 3 & 1 \\ \hline 2 & \multicolumn{2}{c}{} \\ \cline{1-1} \end{array} + \sqrt{1 - \left(\frac{1}{3}\right)^2} \begin{array}{|c|c|c|}\hline 4 & 3 & 2 \\ \hline 1 & \multicolumn{2}{c}{} \\ \cline{1-1} \end{array}$$

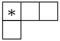
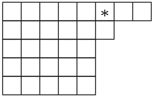


Action of the permutation subgroup; projection operators; intertwining operators,.... = any symmetric group data.

Example: background dependent

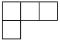
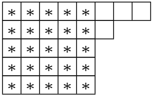
There are two background dependent quantities that will plays a role:

The factor of a box in row i and column j: $(= N - i + j)$

 $\rightarrow \quad N$  $\rightarrow \quad N + 5$

All N dependence comes from factors of the excitation.

Ratios of hooks (note definition of R and $+R$)

$R =$  $\qquad + R =$ 

$$\frac{\text{hooks}_{+R}}{\text{hooks}_{+r}} = \alpha^4 \frac{\text{hooks}_R}{\text{hooks}_r}$$

Can absorb α into field normalization - **ignore this factor.**

Bijection

Operators in planar $\mathcal{N}=4$ super Yang-Mills (R has at most order $\sqrt{N}$ boxes)

$$O_A = \sum_{R,r,s,\alpha,\beta} a^{(A)}_{R,(r,s),\alpha,\beta} \chi_{R,(r,s)\alpha\beta}(Z,Y,X,\cdots)$$

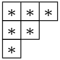
Operator in the planar emergent gauge theory

$$O_{+A} = \sum_{R,r,s,\alpha,\beta} a^{(A)}_{R,(r,s),\alpha,\beta} \chi_{+R,(+r,s)\alpha\beta}(Z,Y,X,\cdots)$$

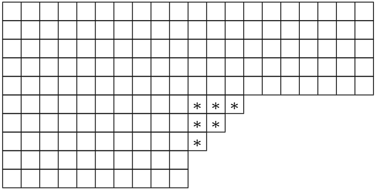
The coefficients appearing in the two sums are the same. Only change $R \leftrightarrow +R$, $r \leftrightarrow +r$

Bijection

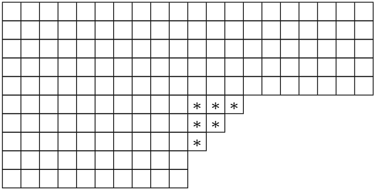
$N = 4 \quad SYM\ (R)$ $\qquad$ $Emergent \quad YM\ (+R)$



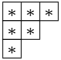
$\longrightarrow$



$Emergent \quad YM\ (+R)$ $\qquad$ $N = 4 \quad SYM\ (R)$



$\longrightarrow$



Reminder: Correlator

$$\langle \chi_{R,(r,s)\alpha\beta} \chi^{\dagger}_{T,(t,u)\gamma\delta} \rangle = \frac{\delta_{RT}\delta_{rt}\delta_{su}\delta_{\alpha\gamma}\delta_{\beta\delta}\mathrm{hooks}_R f_R}{\mathrm{hooks}_r \mathrm{hooks}_s}$$

Bhattacharyya, Collins, dMK, [arXiv:0801.2061 [hep-th]]

Free field theory result; all ribbon graphs were summed, so valid for operators of any dimension.

All N dependence is inside the product of factors f_R.

Correlators: Free Field Theory

Planar correlator:

$$\langle O_A(x_1) O_B(x_2)^\dagger \rangle = \sum_{R,r,s,\alpha} \frac{a^{(A)}_{R,(r,s),\alpha,\beta} a^{(B)*}_{R,(r,s),\alpha,\beta} \mathrm{hooks}_R f_R}{\mathrm{hooks}_r \mathrm{hooks}_s} \frac{1}{|x_1 - x_2|^{2J}}$$

Emergent gauge theory correlator:

$$\langle \cdots \rangle_B = \frac{\langle \cdots \rangle}{f_B} |x_1 - x_2|^{2|B|}$$

$$\langle O_{+A}(x_1) O_{+B}(x_2)^\dagger \rangle_B = \sum_{R,r,s,\alpha} \frac{a^{(A)}_{R,(r,s),\alpha,\beta}\, a^{(B)*}_{R,(r,s),\alpha,\beta}\, \mathrm{hooks}_{+R}\, f_{+R}}{f_B\, \mathrm{hooks}_{+r} \mathrm{hooks}_s} \frac{1}{|x_1 - x_2|^{2J}}$$

Correlators: Free Field Theory

From planar correlator:

$$\frac{\text{hooks}_R}{\text{hooks}_r} \times f_R$$

From emergent gauge theory correlator:

$$\frac{\text{hooks}_{+R}}{\text{hooks}_{+r}} \times \frac{f_{+R}}{f_B}$$

Explicit computation shows that

$$\frac{f_{+R}}{f_B} = f_R\big|_{N \to N_{\text{eff}}}$$

$$\frac{\text{hooks}_{+R}}{\text{hooks}_{+r}} = \frac{\text{hooks}_R}{\text{hooks}_r}$$

Correlators

$$\langle O_A(x_1) O_B(x_2)^\dagger \rangle = F_{AB}(N) \frac{1}{|x_1 - x_2|^{2J}}$$

$$\langle O_A(x_1) O_B(x_2)^\dagger \rangle_B = F_{AB}(N_{\text{eff}}) \frac{1}{|x_1 - x_2|^{2J}} \left(1 + O\left(\frac{1}{N} \right) \right)$$

Planar three point functions of CFT single traces vanish. Planar three point functions of emergent gauge theory single traces vanish. OPE coefficients of both agree in free field theory.

We **CONJECTURE** that this is a property of the full interacting theory.

Value of N_{eff}

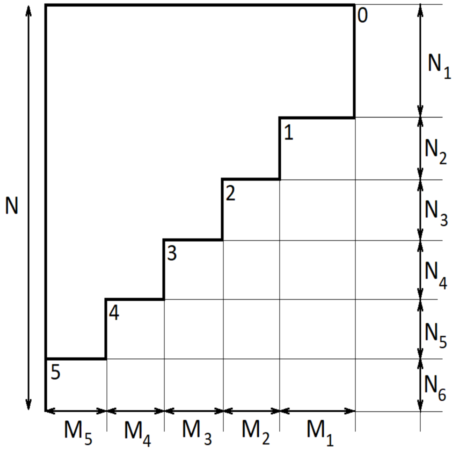


Figure: At corner 2, $N_{\text{eff}} = N - N_1 - N_2 + M_3 + M_4 + M_5$.

Correlators

$$\langle O_A(x_1) O_B(x_2)^\dagger \rangle = F_{AB}(N) \frac{1}{|x_1 - x_2|^{2J}}$$

$$\langle O_A(x_1) O_B(x_2)^\dagger \rangle_B = F_{AB}(N_{\rm eff}) \frac{1}{|x_1 - x_2|^{2J}} \left(1 + O\left(\frac{1}{N}\right)\right)$$

Planar three point functions of CFT single traces vanish. Planar three point functions of emergent gauge theory single traces vanish. OPE coefficients of both agree in free field theory.

We **CONJECTURE** that this is a property of the full interacting theory.

Anomalous dimensions

$$DO_{+R,(+r,s)\mu_1\mu_2}(Z,Y) =$$

$$\sum_{T,(t,u)\nu_1\nu_2} N_{+R,(+r,s)\mu_1\mu_2;+T,(+t,u)\nu_1\nu_2} O_{+T,(+t,u)\nu_1\nu_2}(Z,Y)$$

where

$$N_{+R,(+r,s)\mu_1\mu_2;+T,(+t,u)\nu_1\nu_2} = -\frac{g_{YM}^2}{8\pi^2}\sum_{+R'}\frac{c_{+R,+R'}d_{+T}nm}{d_{+R'}d_{+t}d_u(n+m)}$$

$$\times\sqrt{\frac{f_{+T}\mathrm{hooks}_{+T}\mathrm{hooks}_{+r}\mathrm{hooks}_s}{f_{+R}\mathrm{hooks}_{+R}\mathrm{hooks}_{+t}\mathrm{hooks}_u}}$$

$$\times\mathrm{Tr}\left(\left[(1,m+1),P_{+R,(+r,s)\mu_1\mu_2}\right]I_{+R'+T'}\left[(1,m+1),P_{+T,(+t,u)\nu_2\nu_1}\right]I_{+T'+R'}\right)$$

De Comarmond, dMK, Jefferies, [arXiv:1012.3884 [hep-th]]

Anomalous dimensions

$$c_{+R,+R'}\sqrt{\frac{f_{+T}}{f_{+R}}} = c_{R,R'}\sqrt{\frac{f_T}{f_R}}\Big|_{N\to N_{\mathrm{eff}}}$$

$$\frac{d_{+T}\,nm}{d_{+R'}\,d_{+t}d_u(n+m)}\sqrt{\frac{\mathrm{hooks}_{+T}\mathrm{hooks}_{+r}\mathrm{hooks}_s}{\mathrm{hooks}_{+R}\mathrm{hooks}_{+t}\mathrm{hooks}_u}}d_{+r'}$$
$$= \frac{d_T\,nm}{d_{R'}\,d_t d_u(n+m)}\sqrt{\frac{\mathrm{hooks}_T\mathrm{hooks}_r\mathrm{hooks}_s}{\mathrm{hooks}_R\mathrm{hooks}_t\mathrm{hooks}_u}}d_{r'}$$

$$\frac{\mathrm{Tr}\left(\left[(1,m+1),P_{+R,(+r,s)\mu_1\mu_2}\right]I_{+R'+T'}\left[(1,m+1),P_{+T,(+t,u)\nu_2\nu_1}\right]I_{+T'+R'}\right)}{d_{+r'}}$$
$$= \frac{\mathrm{Tr}\left(\left[(1,m+1),P_{R,(r,s)\mu_1\mu_2}\right]I_{R'T'}\left[(1,m+1),P_{T,(t,u)\nu_2\nu_1}\right]I_{T'R'}\right)}{d_{r'}}$$

In the end: $N\to N_{\mathrm{eff}}$ in planar anomalous dimensions.

Summary

We argued for three things:

1. A bijection between operators in planar $\mathcal{N} = 4$ super Yang-Mills and the planar emergent gauge theory.
2. Argued planar three point functions in FREE emergent gauge theory vanish, so OPE coefficients vanish. Conjectured this is true when interactions are included.
3. Argued that planar spectrum of anomalous dimensions matches planar spectrum of $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\rm eff})$.

Conclusion: Planar limit of emergent gauge theory is planar limit of $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\rm eff})$. It is an integrable system.

dMK, Huang, Tribelhorn [arXiv:1806.06586 [hep-th]]

Weak coupling tests

Evidence supporting the above result:

1. Explicit computations of anomalous dimensions of the emergent gauge theory, to two loops, agree with $N \to N_{\text{eff}}$ rule.
2. Using $su(2|2)$ symmetry two magnon S-matrix is determined. Agrees up to two loops with a computation performed in the emergent gauge theory, using a coordinate space Bethe ansatz.

Beisert, [hep-th/0511082].

dMK, Mathwin, van Zyl, [arXiv:1601.06914 [hep-th]]

dMK, Kim, Van Zyl, arXiv:1802.01367 [hep-th]

Magnon $SU(2|2)$ representation

$$Q^\alpha_a|\phi^b\rangle = a\delta^b_a|\psi^\alpha\rangle\,, \qquad Q^\alpha_a|\psi^\beta\rangle = b\epsilon^{\alpha\beta}\epsilon_{ab}|\phi^b\rangle\,,$$
$$S^a_\alpha|\phi^b\rangle = c\epsilon_{\alpha\beta}\epsilon^{ab}|\psi^\beta\rangle\,, \qquad S^a_\alpha|\psi^\beta\rangle = d\delta^\beta_\alpha|\phi^a\rangle\,,$$

where

$$a = \sqrt{g}\eta\,, \qquad b = \frac{\sqrt{g}}{\eta}f\left(1 - r_0\frac{x^+}{x^-}\right)\,,$$
$$c = \frac{\sqrt{g}i\eta}{fx^+}\,, \qquad d = \frac{\sqrt{g}x^+}{i\eta}\left(1 - r_0\frac{x^-}{x^+}\right)\,,$$

with

$$x^+ + \frac{1}{x^+} - r_0x^- - \frac{r_0}{x^-} = \frac{i}{g} \qquad r_0 = \sqrt{\frac{N_{\rm eff}}{N}}$$

For magnon in $\mathrm{AdS}_5\times\mathrm{S}_5$, $N_{\rm eff} = N \Rightarrow r_0 = 1$. For boundary magnon of $Z = 0$ giant graviton $N_{\rm eff} = 1 \Rightarrow r_0 = 0$.

Strong coupling tests

Finite size giant magnon soln to NG in LLM: $\phi = \sigma - \tau$, $t = \kappa\tau$, $r = r(\sigma)$; ϕ and r coords of bubbling plane. Eqn of motion can be integrated once

$$r'(\sigma) = \frac{\kappa r\sqrt{1-\frac{r^2}{C^2}}}{\sqrt{(1-\kappa)^2 h^4(r) r^2 - (\kappa - (1-\kappa)V_\phi(r))^2}} \qquad h^{-2} = 2y\cosh(G)$$

C is an integration constant.

$$E = \frac{\sqrt{\lambda}}{2\pi}\int_{\sigma_{\rm min}}^{\sigma_{max}} d\sigma \frac{\partial L_{NG}}{\partial \dot{t}} \qquad J = \frac{\sqrt{\lambda}}{2\pi}\int_{\sigma_{\rm min}}^{\sigma_{max}} d\sigma \frac{\partial L_{NG}}{\partial \dot{\phi}},$$

$$E - J = \frac{\sqrt{\lambda}}{\pi} r_0 \sin\left(\frac{p}{2}\right) - 4\frac{\sqrt{\lambda}}{\pi} r_0 \sin^3\left(\frac{p}{2}\right) e^{-2\left(\frac{\pi}{\sqrt{\lambda} r_0 \sin\left(\frac{p}{2}\right)}+1\right)} + \cdots$$

Net effect $\sqrt{\lambda} \to \sqrt{\lambda} r_0 = \sqrt{g_{YM}^2 N}\sqrt{\frac{N_{\rm eff}}{N}}$ which is $N \to N_{\rm eff}$.

dMK, Kim, Van Zyl, arXiv:1802.01367 [hep-th]

Further strong coupling test

In [Kim, Van Zyl, arXiv:1805.12460 [hep-th]] semiclassical string solutions that live on white regions of the bubbling plane were constructed and studied.

Solutions are labeled by conserved charges E, J and S.

Holographically dual to operators in $SL(2)$ sector of $\mathcal{N} = 4$ super-Yang Mills (made of covariant derivatives acting on complex scalar fields Z).

In an appropriate short string limit the $N \to N_{\text{eff}}$ rule is again confirmed.

Beyond the planar limit

The planar limit of the emergent gauge theory matches the planar limit of $\mathcal{N}=4$ super Yang-Mills theory with gauge group $U(N_{\text{eff}})$.

Beyond the planar limit, these two theories differ: if the emergent gauge theory really was $\mathcal{N}=4$ with $U(N_{\text{eff}})$ we would expect

1. Giant gravitons with momenta $\leq N_{\text{eff}}$.
2. Dual giant gravitons with an arbitrarily high momentum.

The momenta of giant gravitons is cut off well below N_{eff}. It is cut off by "the size of the corner". The momenta of dual giant gravitons is also cut off by "the size of the corner".

Dual giant gravitons and giant gravitons that have a momentum close to the cut off start to interact with excitations localized at adjacent corners.

Summary and a Future Direction

We have studied the planar limit of the emergent gauge theory that arises by exciting the branes that backreact to produce the LLM geometry.

Using group representation theory techniques we have managed to learn enough about the dynamics to suggest its matches the planar limit of $\mathcal{N}=4$ super Yang-Mills with gauge group $U(N_{\rm eff})$.

Passes weak and strong coupling tests. (two loop anomalous dimensions, two loop S-matrix, finite size correction to energy using Nambu-Goto description)

Beyond the planar limit the emergent gauge theory differs from $\mathcal{N}=4$ super Yang-Mills with gauge group $U(N_{\rm eff})$.

Next: Emergent gauge theory living on the intersection of giant graviton branes.

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Poster

Stanley Mandelstam was an eminent South African-born American Theoretical Physicist and a Wits graduate. He made seminal contributions to Particle Physics and String Theory. This series of schools, named in Stanley Mandelstam's honor, aims to expose local South African Theoretical Physicists, both post-graduate students and academics, to exciting recent developments in Theoretical High Energy Physics. The School is followed by a high level workshop that aims to stimulate discussion and interaction which might lead to new research directions and collaborations. The meeting brings leading experts together to accomplish these goals.

Venue: The Protea Hotel Edward Durban.

Format: We have number of international plenary lecturers who will each deliver a pedagogical talk (aimed at graduate students) at the School. Each lecture will last for one hour. The workshop will be used for cutting edge research talks, both by the School lecturers and by international and local participants. The loose format of the school leaves plenty of time for discussion and collaboration.

Funding: Funding is provided by the National Research Foundation (NRF) and the Mandelstam Institute for Theoretical Physics (MITP).

Plenary Speakers:

- Fernando Alday (Oxford U.)
- Nicolas Boulanger (Université de Mons)
- Simone Giombi (Princeton U.)
- Jeff Murugan (Cape Town U.)
- Hugh Osborn (Cambridge U.)
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Organizers:

Thanks for your attention!

Decoupling

Conclusion: The planar limit of the emergent gauge theory is the planar limit of $\mathcal{N} = 4$ super Yang-Mills with gauge group $U(N_{\rm eff})$. It is an integrable system.

The planar emergent gauge theory is decoupled from the rest of the theory in the sense that starting in the Hilbert space of the emergent gauge theory will not evolve you out of this space.

Interactions between excitations of the planar limit of different emergent gauge theories are $1/N$ suppressed.

Interactions between delocalized excitations and the planar limit of an emergent gauge theory are $1/N$ suppressed.

Weak coupling tests

Perform a coordinate space Bethe ansatz: M. Staudacher [hep-th/0412188]

$$\psi_{AB} \equiv \sum_{l_2>l_1} \psi_{AB}(l_1,l_2)\mathrm{Tr}(Z^{l_1}AZ^{l_2-l_1}BZ^{J-l_1-l_2})$$

Solve

$$D\Psi_{AB} = E\Psi_{AB}$$

with D the two loop dilatation operator. Ansatz for wave function

$$\psi_{YX} = e^{i(p_1 l_1+p_2 l_2)} + Ae^{i(p_2 l_1+p_1 l_2)} + \delta_{l_2,l_1+1}\phi_{YX}(l_1)$$

The energy E agrees with the planar two loop anomalous dimension after replacing $N \to N_{\rm eff}$. The S-matrix element A agrees with the planar result after replacing $N \to N_{\rm eff}$.

Fourier transform

Schur polynomial: Functions on $\mathrm{Diag}(S_n) \setminus S_n \times S_n / \mathrm{Diag}(S_n)$

$$\chi_R(Z) = \frac{1}{n!} \sum_{\sigma \in S_n} \chi_R(\sigma) Z^{i_1}_{i_{\sigma(1)}} \cdots Z^{i_n}_{i_{\sigma(n)}}$$

$$\chi_R(\sigma) = \mathrm{Tr}(\Gamma_R(\sigma)) = \sum_i \langle R, i | \Gamma_R(\sigma) | R, i \rangle$$

Restricted Schurs: Functions on $\mathrm{Diag}(S_n \times S_m) \setminus S_{n+m} \times S_{n+m} / \mathrm{Diag}(S_n \times S_m)$

$$\chi_{R,(r,s)\alpha\beta}(Z, Y) = \frac{1}{n!m!} \sum_{\sigma \in S_{n+m}} \chi_{R,(r,s)\alpha\beta}(\sigma) Y^{i_1}_{i_{\sigma(1)}} \cdots Y^{i_m}_{i_{\sigma(m)}} Z^{i_{m+1}}_{i_{\sigma(m+1)}} \cdots Z^{i_{n+m}}_{i_{\sigma(n+m)}}$$

$$\chi_{R,(r,s)\alpha\beta}(\sigma) = \sum_i \langle (r,s)\alpha, i | \Gamma_R(\sigma) | (r,s)\beta, i \rangle$$