Topological Holography

and the

$\mathcal{N}=4$ chiral algebra

(Work in progress with Davide Gaiotto)

Infinite Chiral Symmetry in Four Dimensions

Christopher Beem,[1,2] Madalena Lemos,[3] Pedro Liendo,[3,4] Wolfger Peelaers,[3] Leonardo Rastelli,[1,3] and Balt C. van Rees[3,5]

[1] *Institute for Advanced Study, Einstein Dr., Princeton, NJ 08540, USA*
[2] *Simons Center for Geometry and Physics, Stony Brook University, Stony Brook, NY 11794-3636, USA*
[3] *C. N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, NY 11794-3840, USA*
[4] *IMIP, Humboldt-Universität zu Berlin, IRIS Adlershof, Zum Großen Windkanal 6, 12489 Berlin, Germany*
[5] *Theory Group, Physics Department, CERN, CH-1211 Geneva 23, Switzerland*

Abstract: We describe a new correspondence between four-dimensional conformal field theories with extended supersymmetry and two-dimensional chiral algebras. The meromorphic correlators of the chiral algebra compute correlators in a protected sector of the four-dimensional theory. Infinite chiral symmetry has far-reaching consequences for the spectral data, correlation functions, and central charges of any four-dimensional theory with $\mathcal{N} = 2$ superconformal symmetry.

$\mathcal{N}=2$ superconformal theory

$\Rightarrow$ Chiral Algebra

$\mathcal{N}=4$ with gauge group $U(N) \Rightarrow$ chiral algebra C_N

GOAL: Relate C_N, $N \to \infty$ with Holographic Dual

We will find

Large N chiral algebra $\longleftrightarrow$ Kodaira-Spenser theory on

$SL_2(\mathbb{C}) = AdS_3 \times S^3$

- Setting for holography where both sides can be understood exactly
- Potential to compute beyond the planar limit

$\mathcal{N}=4$ Gauge theory $\longleftrightarrow$ IIB on $AdS_5 \times S^5$
U1 U1

$\boxed{\text{Chiral Algebra} \longleftrightarrow \text{B-model topological string on } AdS_3 \times S^3}$

Supersymmetric localization for type IIB supergravity: C., Si Li (2016)

Alternative derivation:

$\mathcal{N}=4$ Chiral Algebra

= Algebra of operators on a B-brane wrapping

$\mathbb{C} \subseteq \mathbb{C}^3$

$\mathcal{N}=4$ chiral algebra as BRST Reduction:

X_1, X_2 adjoint valued, spin $1/2$, bosonic

$$X_1(0)\cdot X_2(z) \sim 1/z$$

b, c ghosts, fermionic, spins $1, 0$

$$b(0)\cdot c(z) \sim 1/z$$

BRST Current:

$$J_{BRST} = cX_1X_2 + \tfrac{1}{2}bc^2$$

$\mathcal{N}=4$ chiral algebra $=$ cohomology of

$$Q_{BRST} = \oint J_{BRST}\, dz$$

This is the same as the algebra on a B-brane wrapping $\mathbb{C} \subseteq \mathbb{C}^3$

$X_1, X_2 \iff$ fields representing normal motion of the brane

$b, c \iff$ "gauge fields" on the brane

[Complete derivation: • simple argument using open string field theory]

Closed string fields: (X a $CY3$)

- $\alpha \in \Omega^{2,1}(X) = \Omega^{0,1}(X, TX)$

  Beltrami differential: controls deformation of complex structure

- $A \in \Omega^{0,1}(X) \otimes psl(1|1)$

  Holomorphic Chern-Simons gauge field for $psl(1|1)$

Equations of motion:

α defines integrable deformation of complex str

A holomorphic $psl(1|1)$ module

X CY3 $\quad C \subseteq X$ Riemann surface

What is the closed string field sourced by a brane on C?

Answer: $\alpha \in \Omega^{2,1}(X)$ such that

$$\bar\partial \alpha = N \delta_C \in \Omega^{2,2}(X)$$

Example: $\mathbb{C} \subseteq \mathbb{C}^3$

$z \qquad z, w_1, w_2$

$$\alpha = \frac{dw_1\, dw_2 (\bar w_1 d\bar w_2 - \bar w_2 d\bar w_1)}{|w|^4} \sim \frac{(\bar w_1 d\bar w_2 - \bar w_2 d\bar w_1)}{|w|^4} \partial_z$$

(Using $\partial_z \vee (dw_1 dw_2 dz) = dw_1 dw_2$)

Holographic Dual to theory on $\mathbb{C} \subseteq \mathbb{C}^3$:

Complex manifold $\mathbb{C}^3 \setminus \mathbb{C}$ where holomorphic functions are $F(z, \bar{z}, \omega_i, \bar{\omega}_i)$ satisfying

$$\boxed{\bar{\partial} F + N \frac{\varepsilon_{ij} \bar{\omega}_i d\bar{\omega}_j}{|w|^4} \partial_z F = 0}$$

Solutions to this equation: ω_1, ω_2 and

$$u_1 = \omega_1 z - N \frac{\bar{\omega}_2}{|w|^2}$$

$$u_2 = \omega_2 z + N \frac{\bar{\omega}_1}{|w|^2}$$

$$\boxed{u_2 \omega_1 - u_1 \omega_2 = N}$$

Conclusion: Holographic dual to the chiral algebra is topological B-model on

$$SL_2(\mathbb{C}) = \mathbb{H}^3 \times S^3$$

How does this relate to ordinary holography?

C., Si Li (2016): conjecture that localization of type IIB on $AdS_5 \times S^5$ is top'l B-model on

$$\underset{w_i}{(\mathbb{C}^3} \setminus 0) \times \underset{z_j}{\mathbb{C}^2}$$

with background field

$$N \, \partial_{z_1} \partial_{z_2} \frac{\varepsilon_{ijk} \bar{w}_i \, d\bar{w}_j \, d\bar{w}_k}{|w|^6}$$

This top' string has $psl(3|3)$ symmetry (residue of $psl(4|4)$ on $AdS_5 \times S^5$)

Rastelli et al: Localize using $Q \in psl(3|3)$

Corresponding super-ghost is

$$\omega_1 z_1 + N \frac{\bar{\omega}_2 d\bar{\omega}_3 - \bar{\omega}_3 d\bar{\omega}_2}{|w|^4} \partial_{z_2} + \cdots$$

Localizes top' string to $\omega_1 = 0,\ z_1 = 0$

Conclusion: Holographic dual to the chiral algebra is topological B-model on

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How to recover the large N chiral algebra?

Effective theory on $\mathbb{H}^3$ is "higher spin gravity"

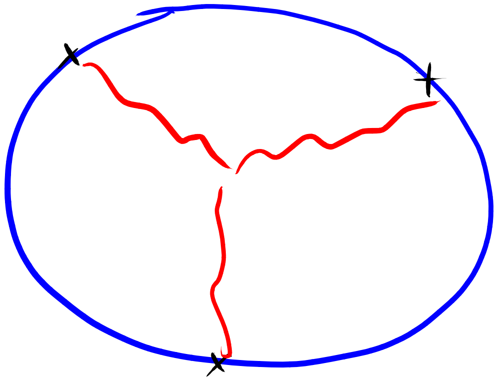
Usual holographic dictionary:

- Modifications of boundary conditions on

$$\partial \mathbb{H}^3 = \mathbb{CP}^1$$

$\longleftrightarrow$ Single trace operators at large N

CFT correlators $\longleftrightarrow$ Witten diagram computations

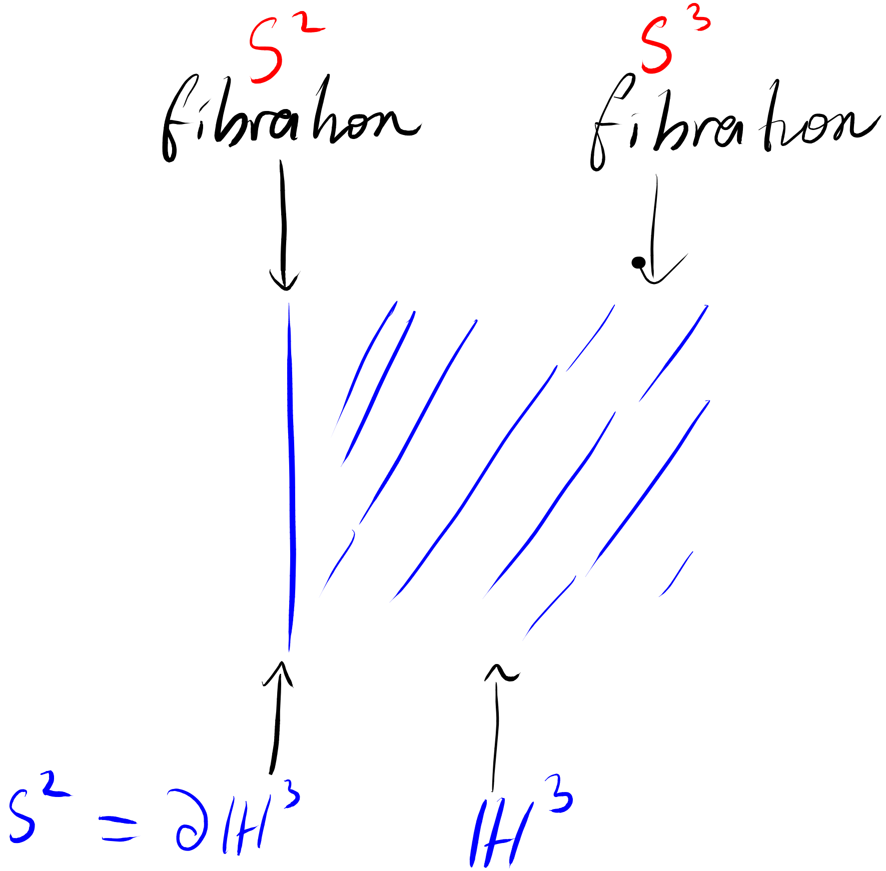


Holomorphic Picture:

$SL_2(\mathbb{C})$ compactifies to

$$U_1 W_2 - W_1 U_2 = N Y^2 \subseteq \mathbb{CP}^4$$

$Y=0$: $\mathbb{CP}^1 \times \mathbb{CP}^1$; holomorphic boundary



Holomorphic boundary is $\mathbb{CP}^1 \times \mathbb{CP}^1$, fibred over $\partial \mathbb{H}^3 = \mathbb{CP}^1$

Boundary conditions in holomorphic language:

$\alpha \in \Omega^{2,1}(SL_2(\mathbb{C}))$ has order 1 pole at $Y=0$

$A \in \Omega^{0,1}(SL_2(\mathbb{C})) \otimes psl(1|1)$ has order 1 zero at $Y=0$

Near boundary of $SL_2(\mathbb{C})$ have coordinates

w on $\mathbb{CP}^1_R$ (rotated by $SU(2)_R$)

z on $\mathbb{CP}^1_C$ (chiral algebra plane)

r normal direction (pole at $z=\infty, w=\infty$)

Modifications of boundary conditions at

$z=0, \; r=0$

$\longleftrightarrow$ Local operators of dual gauge theory

Possible Modifications: allow

$$A \in \Omega^{0,1}(SL_2(\mathbb{C})) \otimes psl(1/1)$$

to have a pole of order $k \geq 0$ at $x=0$:

$$A \sim x^{-k} \omega^l \delta_{z=0}, \quad l \leq k$$

Choices transform in representation

$[k+1, k/2+1]$ of

$$SU(2)_R \times SO(2)$$

(Dim^n $k+1$, spin $k/2$ under $SO(2)$ Lorentz transformations)

$\alpha \in \Omega^{2,1}(SL_2(\mathbb{C}))$ modified boundary conditions:

$$\alpha \sim r^{-k} dr\, \delta_{z=0} \quad \in \left[k, \frac{k-1}{2}\right]$$

Or:

$$\alpha \sim r^{-k} dw\, \delta'_{z=0} + r^{-1-k} dr\, dw\, \iota_{\partial_z} \delta_{z=0}$$

$$\in [k-1, k/2+1]$$

Possible deformations of boundary conditions:

Bosonic: $\left[k+1, \frac{k}{2}\right]_{k\geq 1}$ and $\left[k+1, \frac{k}{2}+2\right]_{k\geq 0}$

Fermionic: $\left[k+1, \frac{k}{2}+1\right]_{k\geq 0}$ (two copies)

(+ z derivatives)

Single Trace operators of large N chiral algebra:

Bosonic: $\mathrm{Tr}\, X_1^k \in [k+1, k/2] \quad k \geq 1$

$\mathrm{Tr}\, X_1^k (X_i \partial_z X_j)\varepsilon_{ij} \in [k+1, k/2 + 2] \quad k \geq 0$

Fermionic: $\mathrm{Tr}\, X_1^k b \in [k+1, k/2 + 1] \quad k \geq 0$

$\mathrm{Tr}\, X_1^k \partial c \in [k+1, k/2 + 1] \quad k \geq 0$

Perfect match! ($\Rightarrow$ indices are the same but also no cancellation)

Conjecture: $\mathcal{N}=4$ chiral algebra correlation functions

$$\langle \mathcal{O}_1(z_1) \dots, \mathcal{O}_k(z_k)\rangle = \left(\frac{\partial}{\partial b_1} \cdots \frac{\partial}{\partial b_k} Z_{KS}[b]\right)_{b=0}$$

b = Boundary condition

b_i = Variations of b.c. corresponding to $\mathcal{O}_i$

Can make explicit computations of 2 and 3 point functions from the "gravity" side.

These match the chiral algebra computations

[No complete proof yet, verified for certain simple operators]

Example computation

$$\langle \mathrm{Tr}(b\, X_1^k), \mathrm{Tr}(\partial c\, X_2^k) \rangle \sim N^{k+1} z^{-k-2}$$

How to compute from gravity side?

$\mathrm{Tr}(\partial c\, X_2^k) \longleftrightarrow$ boundary operator

$$\int_{\mathbb{CP}^1 \times \mathbb{Z}} \omega^k \, d\omega \, \partial_n^{k+2} A^{0,1}$$

$\mathrm{Tr}(b\, X_1^k)$ corresponds to a boundary field with leading singularity $n^{-k} \delta_{z=0}$

Want to find $A^{0,1}$ with this leading singularity satisfying $\bar\partial A^{0,1} = 0$ (EOM)

$\bar\partial$ operator deformed by

$$N \frac{n^2 d\bar{w}}{(1+|w|^2)^2} \partial_z$$

Explicit Solution:

$$A = n^{-k}\delta_{z=0} + N n^{2-k} \frac{\bar{w}}{(1+|w|^2)} \delta'_{z=0} + \frac{N^2 n^{4-k}}{2} \left(\frac{\bar{w}}{(1+|w|^2)}\right)^2 \delta^{(2)}_{z=0}$$

$$+ \cdots + \frac{1}{k!} n^k N^k \left(\frac{\bar{w}}{1+|w|^2}\right)^k \delta^{(k)}_{z=0}$$

$$+ \frac{1}{k!} N^{k+1} n^{k+2} \frac{\bar{w}^k d\bar{w}}{(1+|w|^2)^{k+2}} \quad \frac{1}{z^{k+2}}$$

Only term present at $z \neq 0$ is

$$\frac{1}{k!} N^{k+1} n^{k+2} \frac{\bar{w}^k d\bar{w}}{(1+|w|^2)^{k+2}} \frac{1}{z^{k+2}}$$

Apply operator

$$\int_{\mathbb{CP}^1 \times z} w^k dw\, \partial_n^{k+2} A$$ corresponding to $\mathrm{Tr}\, \partial c\, X_2^k$

find gravitational two point function

$$N^{k+1} \frac{1}{z^{k+2}} \int_w \frac{|w|^{2k} dw\, d\bar{w}}{(1+|w|^2)^{k+2}}$$

Outlook

- Further Examples:

  D3 probing ADE singularity $\longleftrightarrow$ $SL_2(\mathbb{C})/\Gamma$

- "Giant gravitons"

  Local operators are sourced by B-branes in $SL_2(\mathbb{C})$ living on complex curves

- Other classes of examples:

  M2 branes / matrix vector models (C., 2017)

  Line defects in 2d YM (Ishtiaque, Moosavian, Zhou: 2018)