Traversable Wormholes

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Two parts

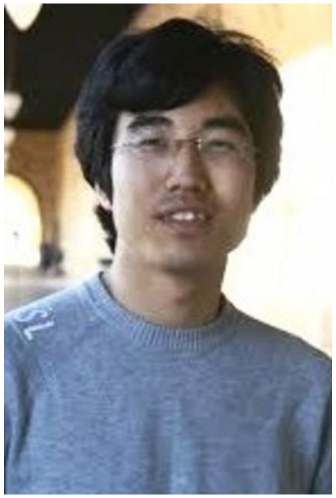
- Making Nearly AdS_2 with global time translation symmetry.

- Traversable wormhole solution in 4d inspired by the above.

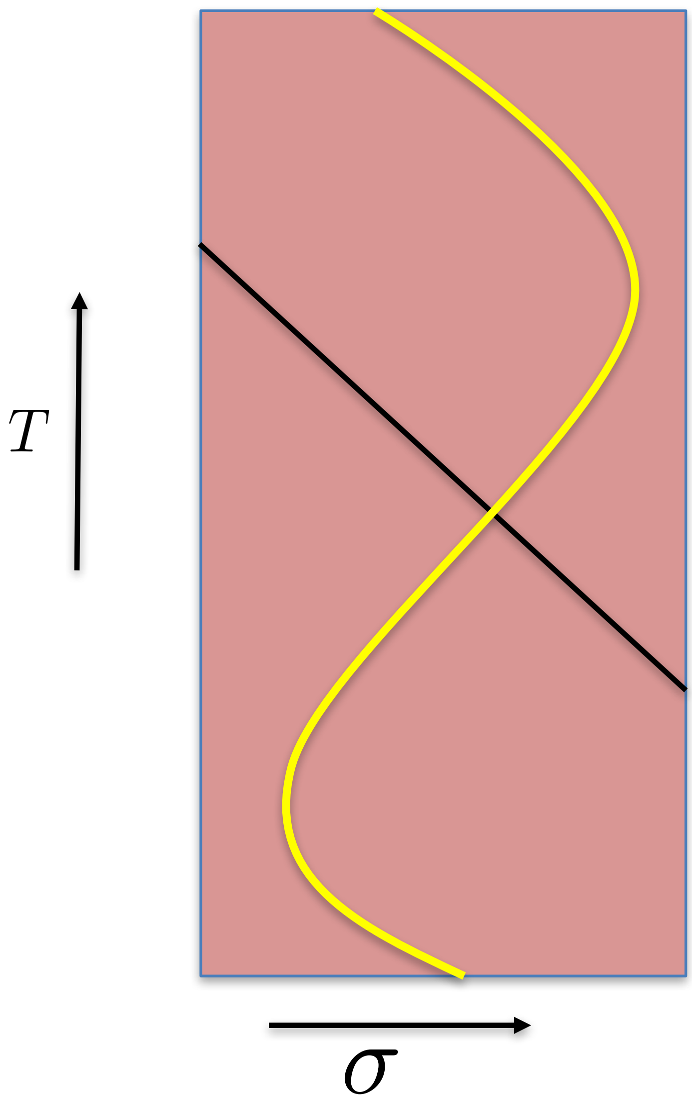
How to make a wormhole

Understanding global AdS_2

Work with Xiaoliang Qi



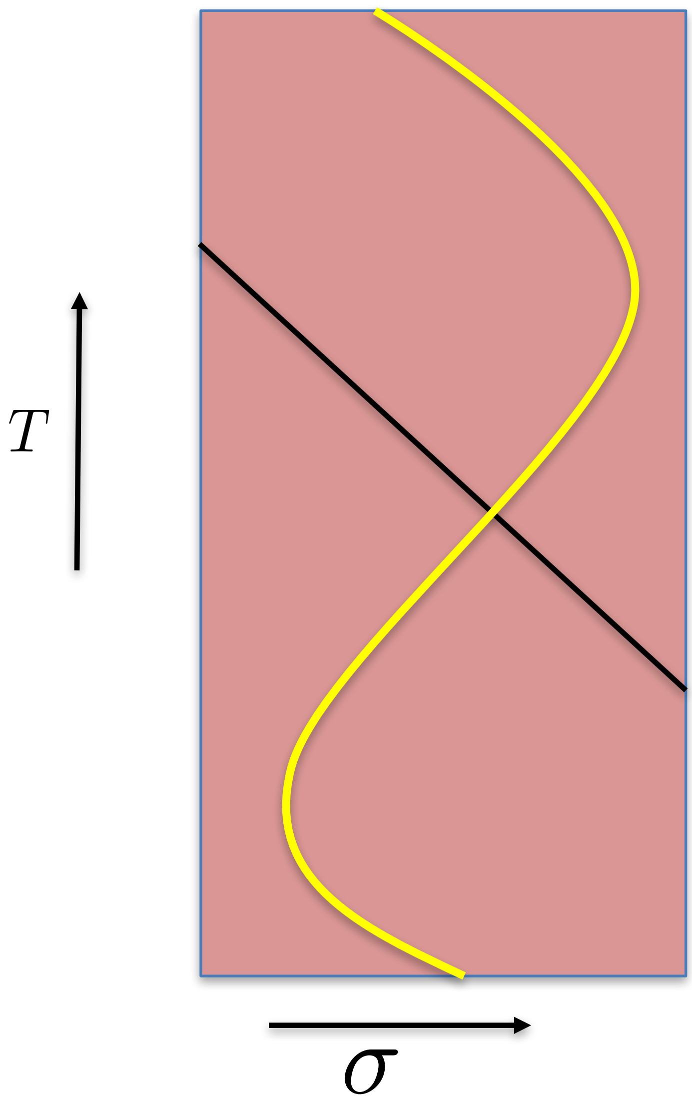
AdS_2 - Global coordinates



$$ds^2 = \frac{-dT^2 + d\sigma^2}{(\sin\sigma)^2}$$

- SL(2,R) isometries
- Two boundaries
- Causally connected
- Particle dynamics → oscillatory behavior → gapped spectrum
- Global coordinates

AdS_2 : A traversable wormhole



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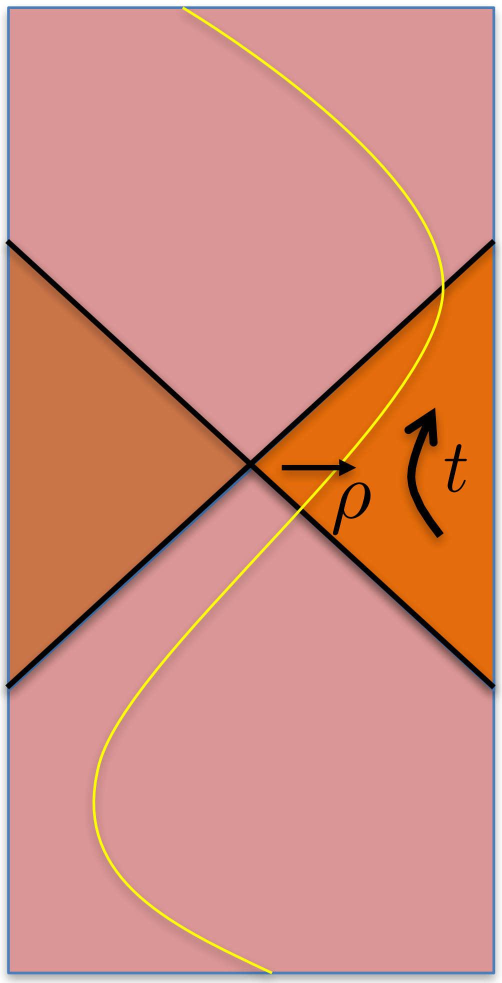
AdS$_2$ - thermal (Rindler) coordinates



$$ds^2 = -dt^2 \sinh^2 \rho + d\rho^2$$

- Two boundaries
- Cover only a portion of AdS$_2$
- Causally disconnected

$T\,,\ t$ are conjugate to two different elements of the SL(2,R) isometries of AdS$_2$

Non-travesable wormhole

AdS_2 vs $NAdS_2$ asymptotic boundary conditions

- Exact AdS_2 boundary conditions do not make sense

JM, Michelson, Strominger

- Need to break some of the AdS_2 isometries slightly
- We should think about nearly-AdS_2

- Nearly AdS_2 with t-isometry → TFD of Nearly CFT_1
- Nearly AdS_2 with T-isometry → ?

First recall some facts about nearly AdS_2 boundary conditions…

Nearly AdS$_2$ gravity

Keep the leading effects that perturb away from AdS$_2$

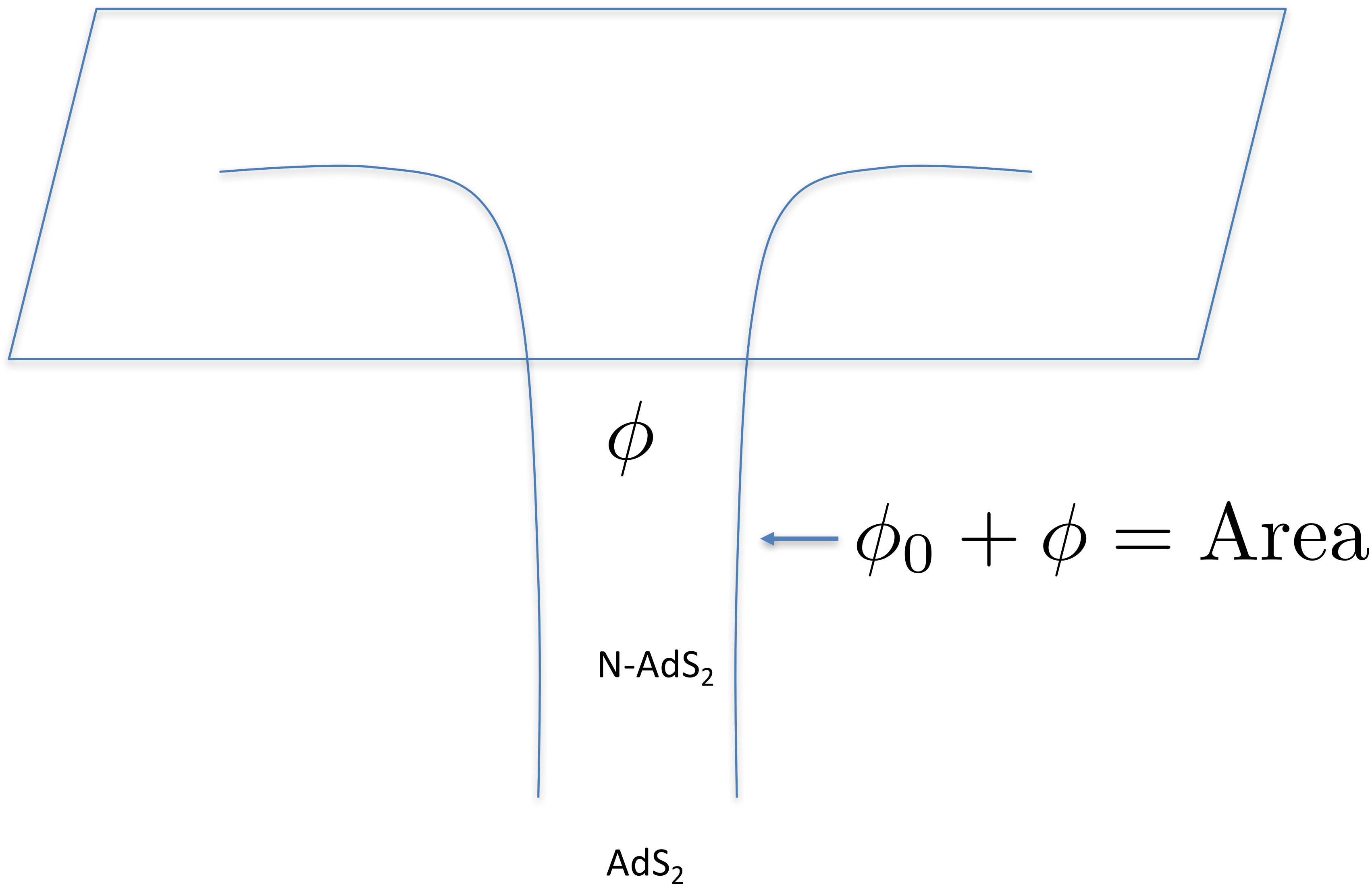
Jackiw Teitelboim
Almheiri Polchinski

$$\int d^2x\sqrt{g}\phi(R+2) + \phi_0 \int d^2x\sqrt{g}R$$

Topological term.
Ground state entropy

Gives leading gravitational dynamics.

Universal description for near extremal black holes.


ϕ
$\phi_0 + \phi = \mathrm{Area}$
N-AdS$_2$
AdS$_2$

Nearly AdS_2 gravity

Keep the leading effects that perturb away from AdS_2

Teitelboim Jackiw
Almheiri Polchinski

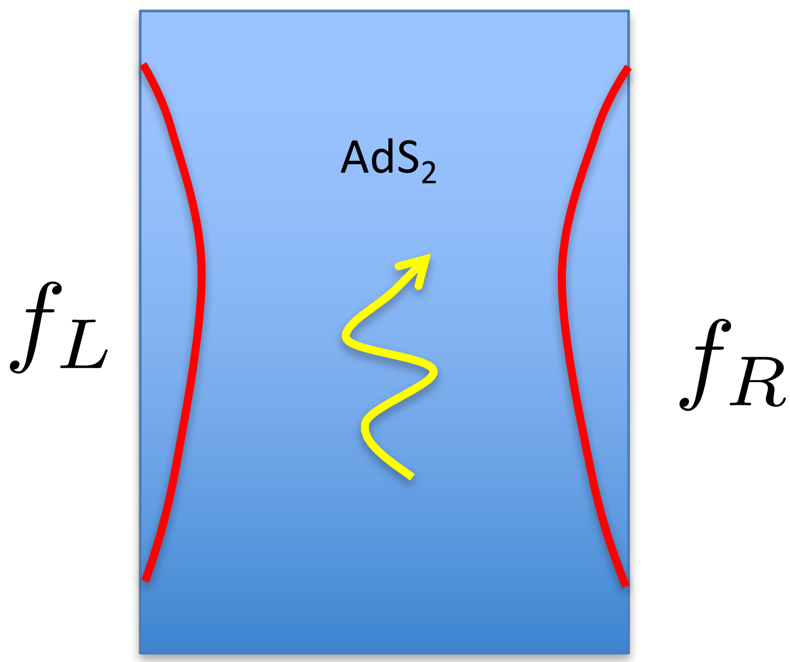
$$\int d^2x \sqrt{g}\phi(R+2) + \phi_0 \int d^2x \sqrt{g} R$$

Comes from the area of the additional dimensions.

Ground state entropy

No bulk excitations → only "boundary gravitons" → location of the physical boundary in AdS_2

Gravitational dynamics



$$\int \phi(R+2)$$

Rigid AdS$_2$

Physical boundary given by dilaton

Dynamics is in the position of the boundary.

Boundary graviton: encodes the motion of the boundary.

$$(H_{f_L} \times H_{\text{bulk}} \times H_{f_R})/SL(2,R)$$

- With pure gravity, the only solution with ϕ growing towards both boundaries is the thermal AdS_2 , with t- isometry.

- We need some sort of matter.

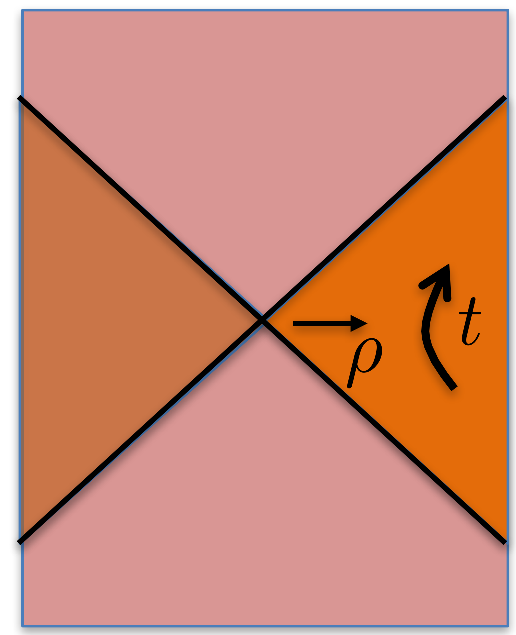
- No ordinary matter →

$$-\, e^{-2\omega}\partial_+\phi\Big|_{-\infty}^{+\infty} = \int_{-\infty}^{\infty} dx^+ T_{++} < 0$$

Integrate

$$-\partial_+(e^{-2\omega}\partial_+\phi) = e^{2\omega}T_{++}\ , \qquad ds^2 = -e^{2\omega}dx^+dx^-$$

(Raychaudhuri eqn)



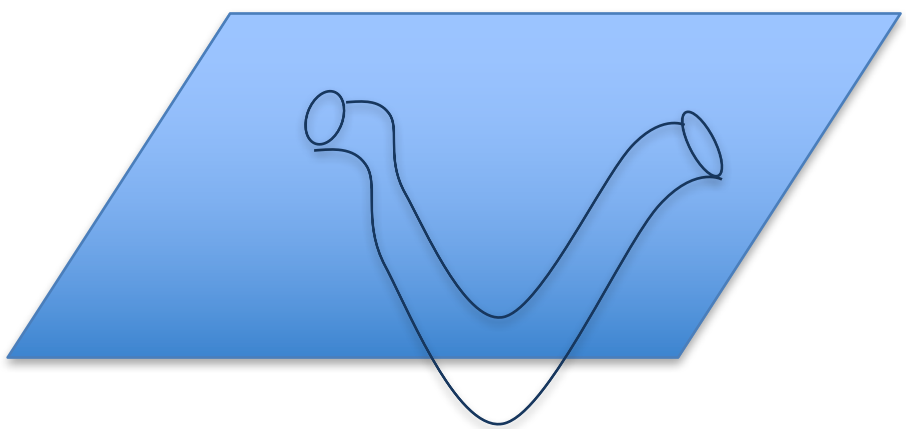
A small digression..

Topological Censorship in GR

- We cannot have disconnected boundaries that are causally connected through the bulk.
- We cannot have non-trivial topology in asymptotically flat space (traversable wormhole).
  Even if the length of the wormhole is larger than the distance between its two mouths.

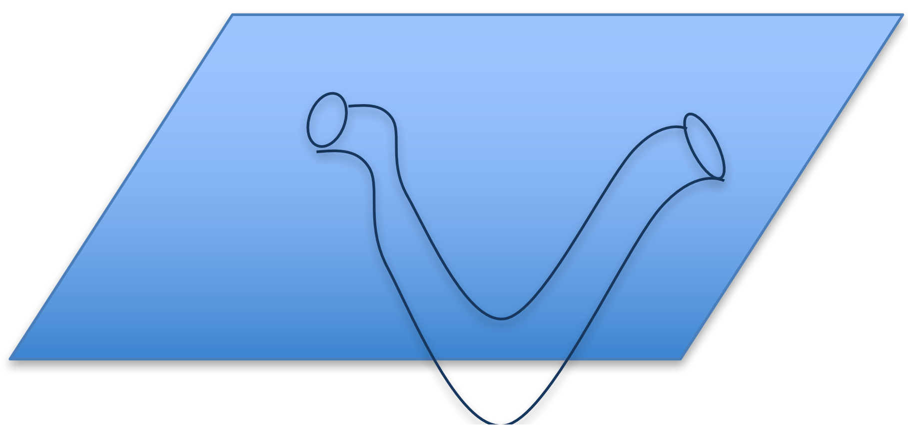
Friedman, Schleich, Witt
Galloway, Schleich, Witt, Woolgar

- If we obey the positive average null energy condition.



Not true when we include quantum effects

- Quantum effects should connect the boundaries.
- Do not obey the positive null energy condition.
- In principle, we could have non-trivial topology in asymptotically flat space (traversable wormhole) (mouths should be closer than length of the wormhole)



Back to AdS_2 …

We will look at a simple example

- Nearly-AdS_2 gravity
- Plus matter
- Plus boundary conditions connecting the two sides (as in Gao-Jafferis-Wall)

$$S_{int} = \mu \int du \chi_L(u) \chi_R(u)$$

u is proper length along the boundary, or boundary time.

- This generates negative null energy and allows for a solution with the global time isometry, where ϕ grows towards both boundaries

In parallel we will look at a similar problem in the SYK model.

Sachdev, Ye, Kitaev model (SYK)

Quantum mechanical model, only time.

$$\{\psi_i, \psi_j\} = \delta_{ij}$$

N Majorana fermions

$$H = \sum_{i_1,\cdots,i_4} J_{i_1 i_2 i_3 i_4} \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4}$$

$$\left(H_q = J_{i_1 \cdots i_q} \psi^{i_1} \cdots \psi^{i_q}\right)$$

random couplings

$$\langle J^2_{i_1 i_2 i_3 i_4} \rangle = J^2 / N^3$$

J = single dimension one coupling.

N large, strong coupling 1 << (time) J << N (still exponentially many energy levels)

- The SYK model has some properties in common with nearly AdS_2 gravity.
- It has the same gravitational dynamics.
- This dynamics is expected to be universal for any system with an almost conformal symmetry in the IR (which is not integrable).

SYK model

Low energies

Conformal invariant part + reparametrizations

Nearly AdS_2 gravity

QFT on AdS_2 + boundary dynamics

Not the same

same

$$S = -C \int du \{ f(u), u \}$$

Schwarzian action

Boundary gravitons

Kitaev

JM, Stanford

Zhang, Suh

- Low temperature entropy
- Gravitational backreaction
- Chaos exponent
- Wormhole traversability (location of horizon)

Emergent reparametrization symmetry which is spontaneously and explicitly broken

Two copies of SYK + Interaction

$$H = H_L + H_R + \mu \int du \sum_i \psi_L^i(u)\psi_R^i(u)$$

Two copies of SYK + Interaction

$$H = H_L + H_R + \mu \int du \sum_i \psi_L^i(u)\psi_R^i(u)$$

$$\Big\downarrow \text{ Low energies}$$

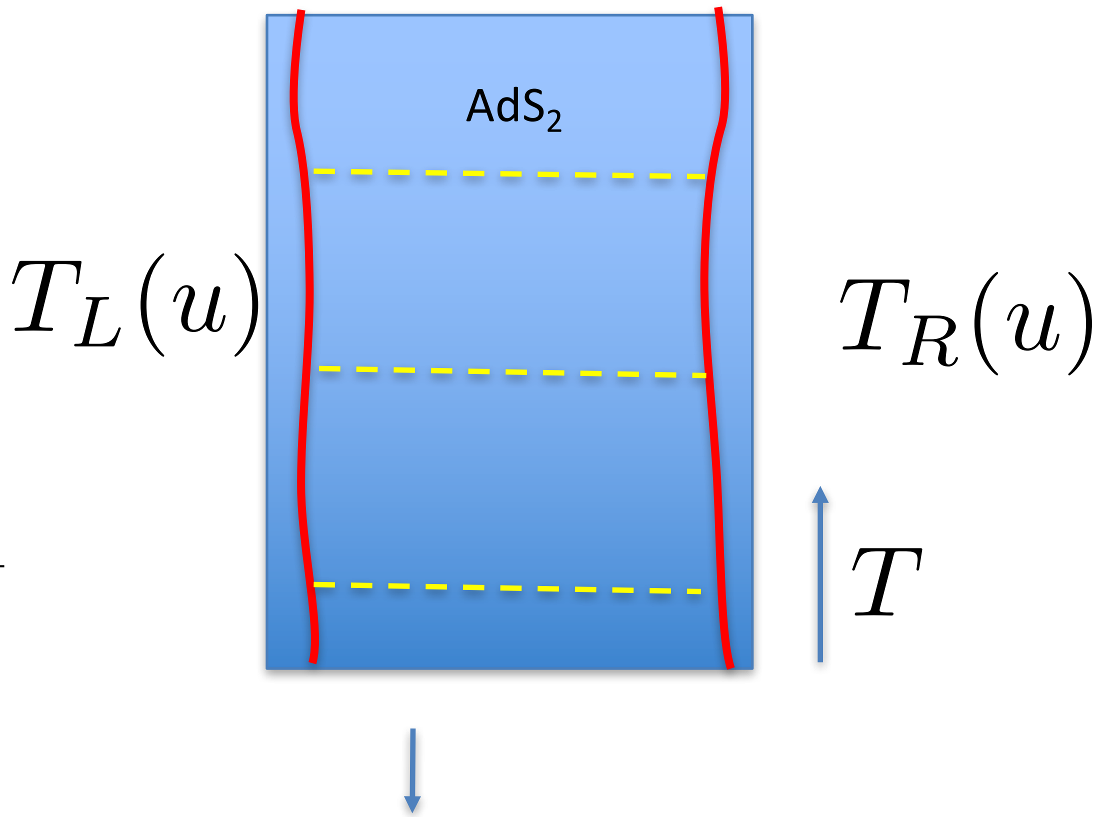
$$S = \frac{N\alpha_S}{J} \int du \{f_L(u), u\} + \{f_R(u), u\} + N\mu \int du \left[\frac{f_L'(u) f_R'(u)}{|f_L(u) - f_R(u)|^2} \right]^{\Delta}$$

+ Global SL(2,R) gauge symmetry → set total SL(2,R) charge to zero.

$$f(u) = \tan(T(u)/2)$$

AdS$_2$ gravity +

Interaction



$$ds^2 = \frac{-dT^2 + d\sigma^2}{(\sin\sigma)^2}$$

$$S = \frac{N\alpha_S}{J}\int du\{\tan\frac{T_L(u)}{2}, u\} + \{\tan\frac{T_R(u)}{2}, u\} + N\mu\int du\left[\frac{T_L'(u)T_R'(u)}{\cos^2\left(\frac{T_L(u)-T_R(u)}{2}\right)}\right]^\Delta$$

+ Global SL(2,R) gauge symmetry → set total SL(2,R) charge to zero.   u is proper time.

$$f(u) = \tan(T(u)/2)$$

$$T_L(u) = T_R(u) = (\text{constant})u = T'u$$

is a solution of the equations of motion.

$$\Big\downarrow \; \text{zero SL(2,R) charges}$$

$$N \frac{1}{J} (T')^2 \propto N\mu \left(\frac{T'}{J} \right)^{2\Delta}$$

$$\Big\downarrow$$

$$\left(\frac{T'}{J} \right)^{2(1-\Delta)} \propto \frac{\mu}{J}$$

A solution always exists for small $\frac{\mu}{J} \ll 1$

$$\text{for} \quad 0 < \Delta < \frac{1}{2} \,, \qquad \mu \ll T' \ll J$$

$$T_L(u) = T_R(u) = (\text{constant})u = T'u$$
$$\text{for} \quad 0 < \Delta < \frac{1}{2}\,, \qquad \mu \ll T' \ll J$$

sets the scale of the energy gap. Relation between AdS_2 time and boundary time, u.

Field in AdS_2 corresponding to a boundary operator of dimension Δ →

$$E = E_u = T'(\Delta + n)$$

Spectrum governed by conformal symmetry
(like in higher dimensional global AdS)

Schwarzian, or dynamical boundary degree of freedom (boundary graviton) →
For small perturbations: one harmonic oscillator with energy

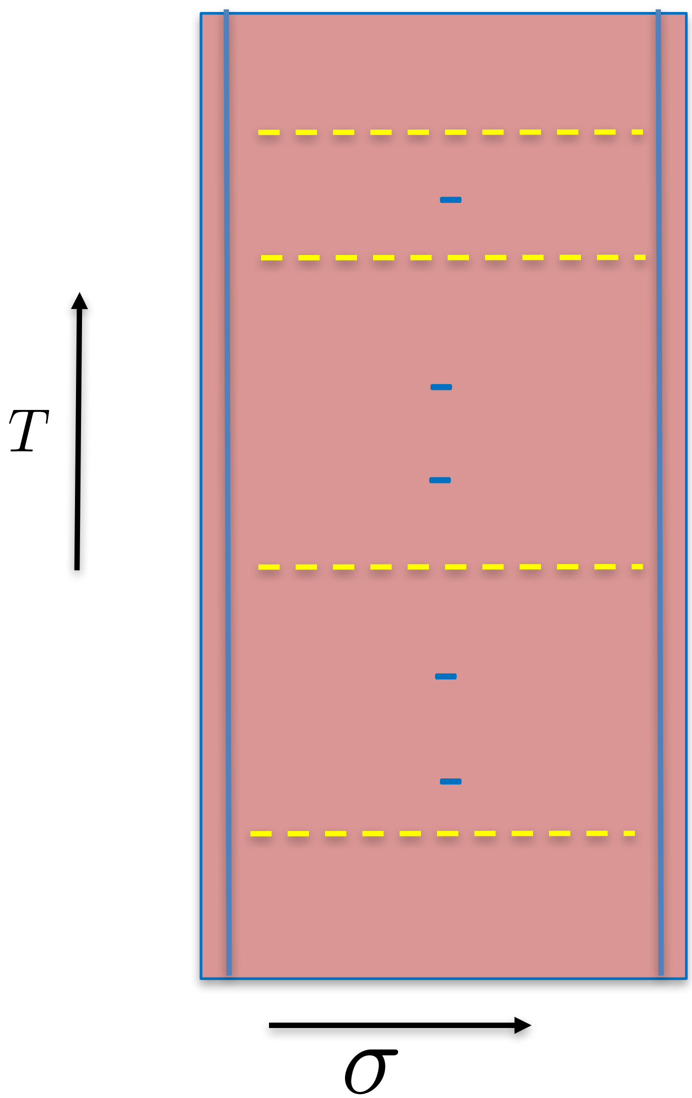
$$E = T'\sqrt{2(1-\Delta)}(n + \frac{1}{2})$$

Stable equilibrium

Same energy scale as the particles inside

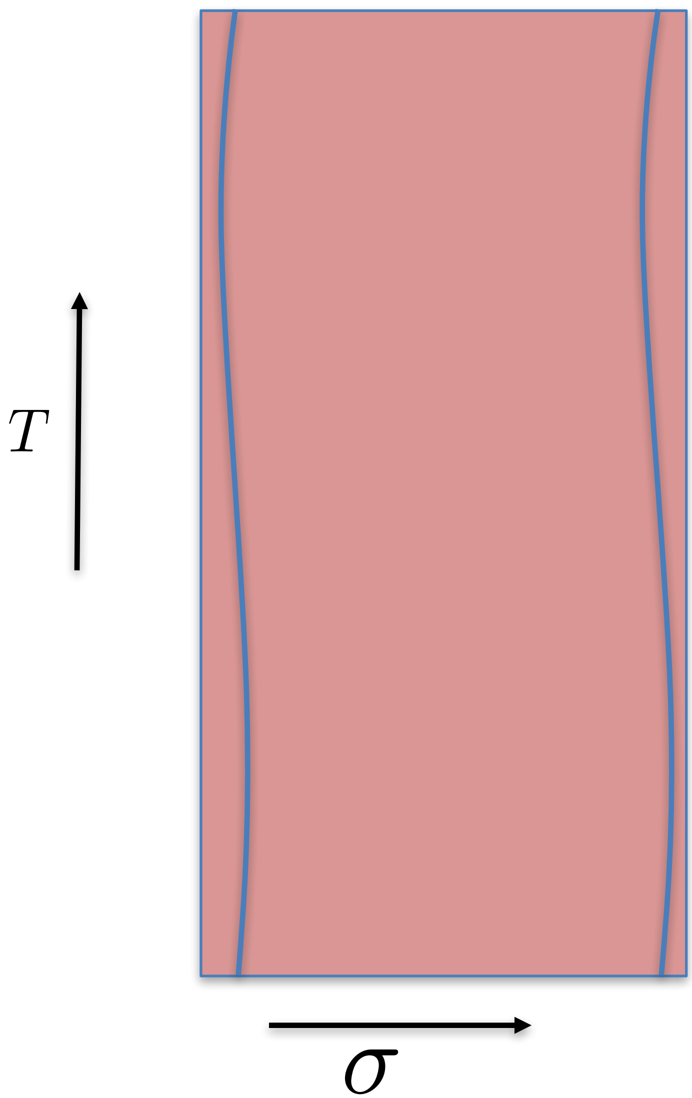
This part is not conformal invariant.

Nearly AdS_2



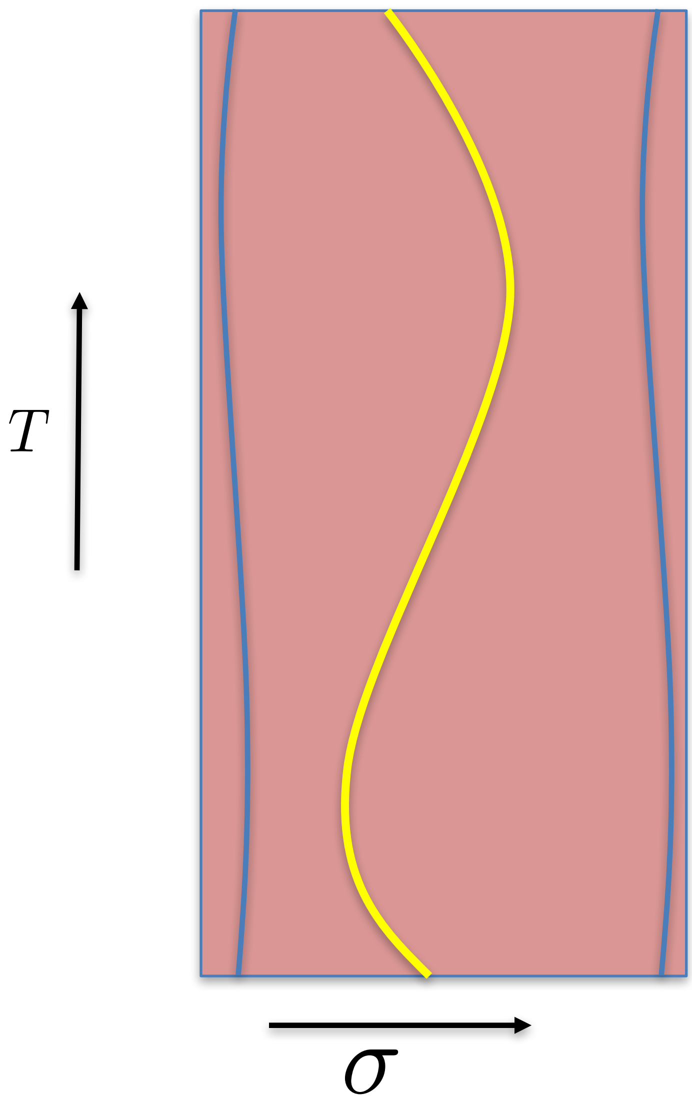
Casimir force due to the boundary conditions connecting the left and right sides → attractive force between the two boundaries.

Nearly AdS_2



Small oscillations around equilibrium position.

Matter oscillations



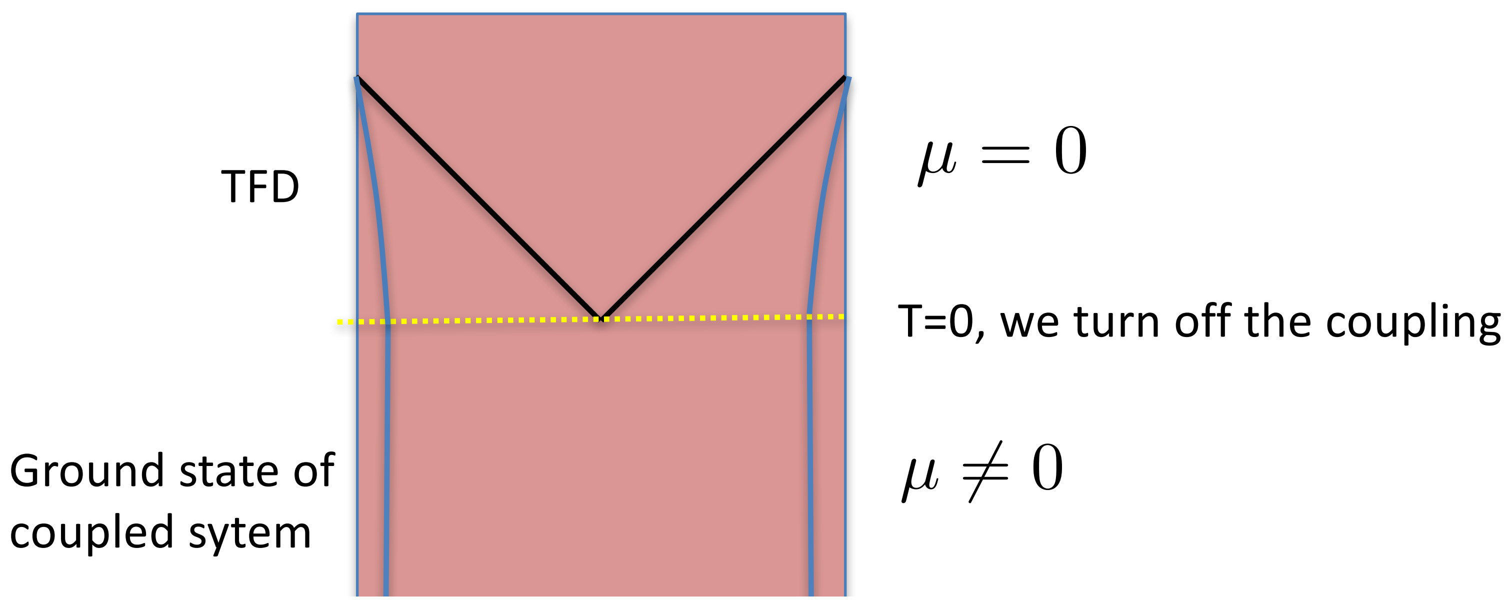
Bulk matter particle → Oscillations → excitation goes from mostly from the left SYK to mostly on the right SYK.

Governed by conformal symmetry.

Making the TFD

- Create two SYK systems.
- Couple term. $\mu \neq 0$
- Couple them further to a heat sink and let them cool down to find its ground state.
- At t=0, turn off the left-right coupling. $\mu = 0$

- → Get a state that is close to the TFD.



In SYK we can also solve the theory beyond the low energy limit.

There is an interesting phase diagram when we go to finite temperature

Conclusions

- As a variant of the Gao-Jafferis-Wall teleportation idea, we can generate states that lead to traversable wormholes, similar to AdS_2 in global coordinates.
- Can be analyzed in the SYK context.
- The TFD is close to ground state of the coupled system.
- Displays universal properties based on the symmetry patterns of the problem. E.g. a spectrum that is mostly in SL(2) representations.

This reasoning inspired the following solution

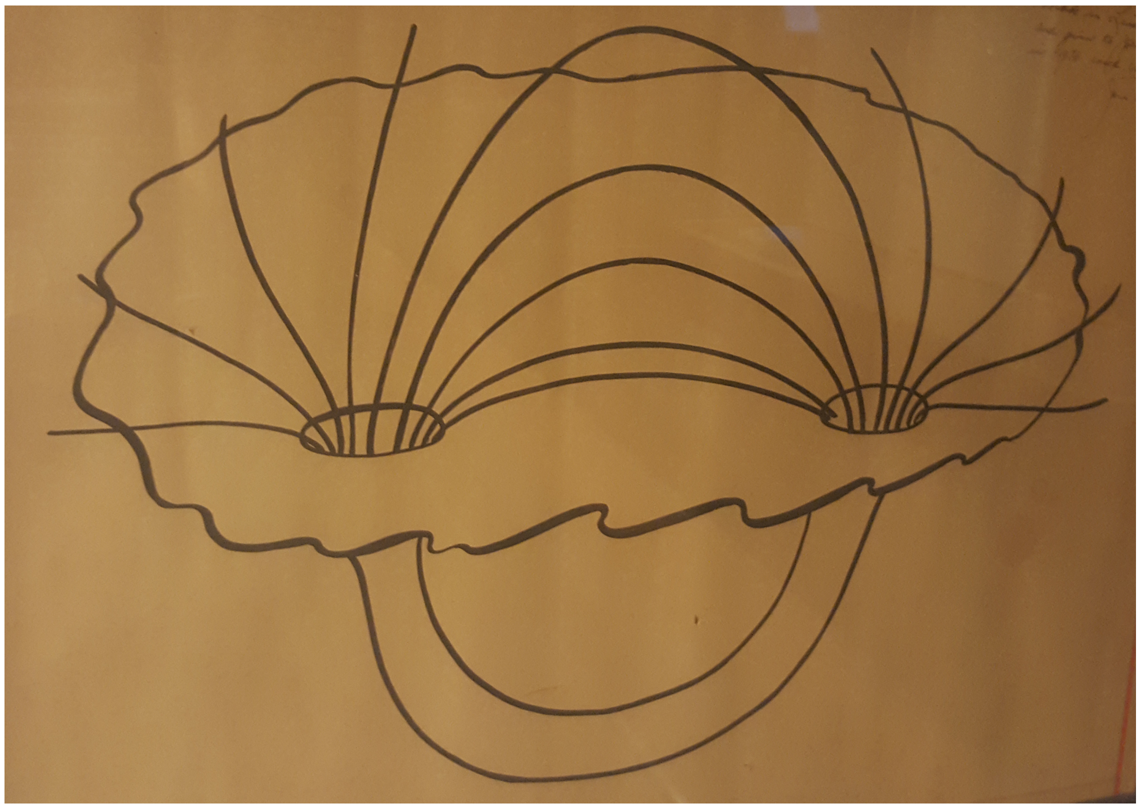
Wormholes in 4 dimensions

Based on work with:

Alexey Milekhin

Fedor Popov

Drawing by John Wheeler, 1966



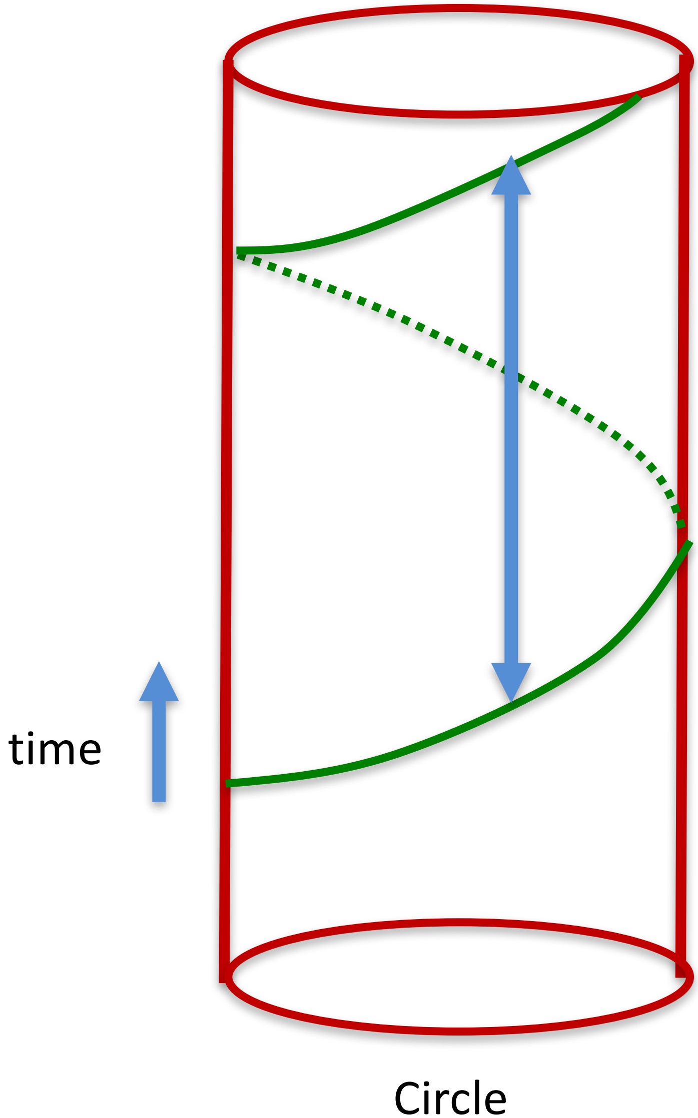
Charge without charge.
Mass without mass

Spatial geometry. Traversable wormhole

Recall that we need negative null energy

Negative null energy in QFT

Eg. Two spacetime dimensions



$$T_{--} < 0$$

$$E \propto -\frac{c}{L}$$

Negative Casimir energy

Quantum effect

The null energy condition does not hold for null lines that are not achronal!

Some necessary elements

- We need something looking like a circle to have negative Casimir energy.
- Large number of bulk fields to enhance the size of quantum effects.
- We will show how to assemble these elements in a few steps.

The theory

$$S = \int d^4x \left[R - F^2 + i\bar{\psi}\ \not{D}\psi \right]$$

Einstein + U(1) gauge field + massless charged fermion

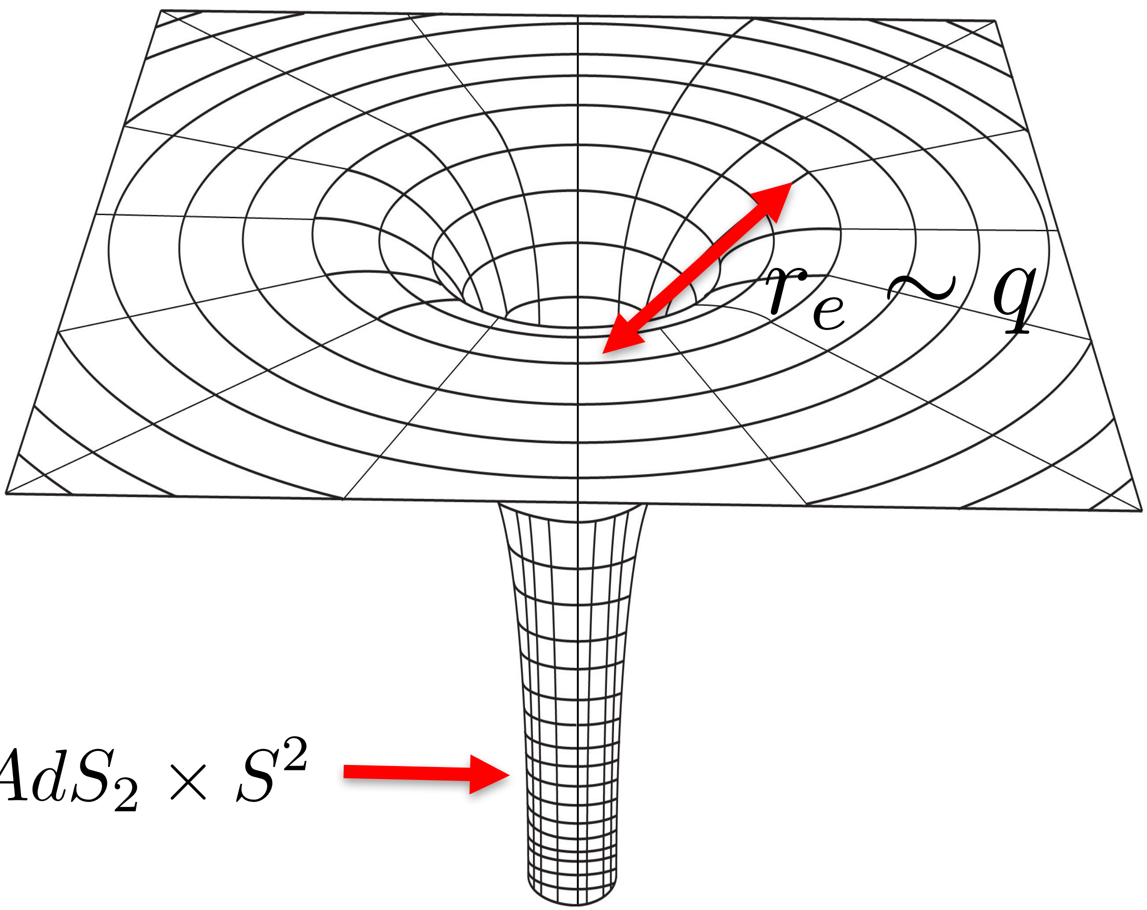
Could be the Standard Model at very small distances, with the fermions effectively massless. The U(1) is the hypercharge. SU(3) x SU(2) x U(1).

$l_{\text{Planck}} = 1$

The first solution: Extremal black hole

Magnetic charge q

$$\int_{S^2} F = q = \text{integer}$$

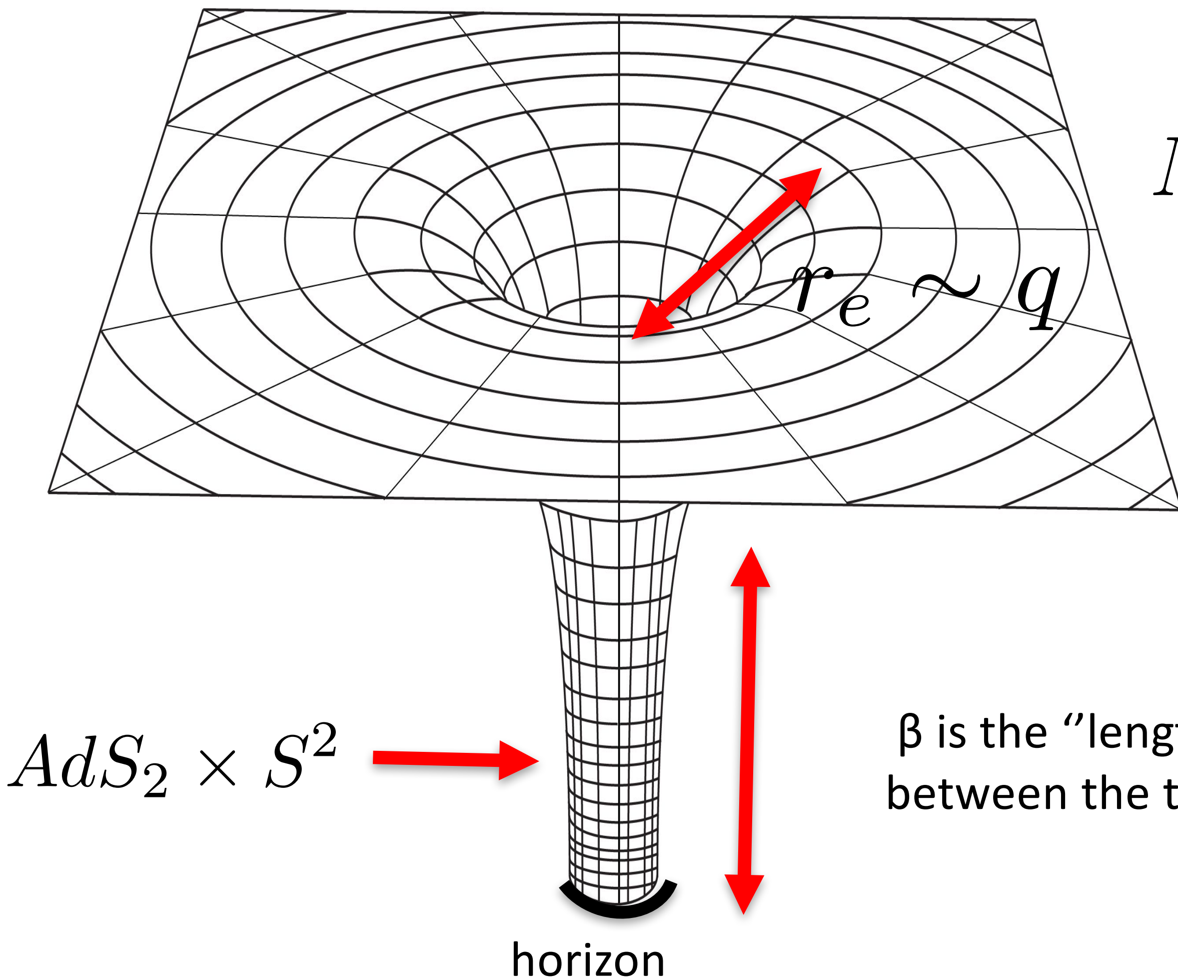


$$M = q$$

$AdS_2 \times S^2$

$$l_{\text{Planck}} = 1$$

The next solution: Near Extremal black hole



$$M - q = q^3 \underbrace{T^2}_{\text{Very small}} = \frac{q^3}{\beta^2}$$

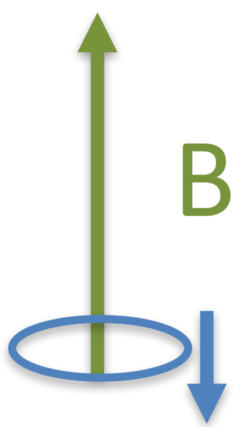
β is the "length" of the throat. Redshift factor between the top and the bottom

$l_{\mathrm{Planck}} = 1$

Motion of charged fermions

- Magnetic field on the sphere.
- There is a Landau level with precisely zero energy.
- Orbital and magnetic dipole energies precisely cancel.
- Explained by an anomaly argument

Ambjorn, Olesen



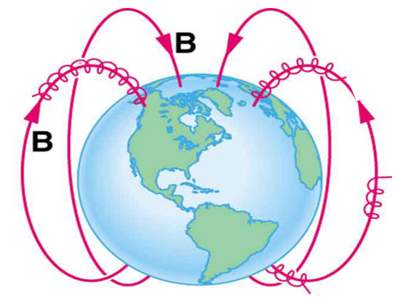
Massless fermions → U(1) chiral symmetry

4d anomaly → 2d anomaly → there should be massless fermions in 2d.

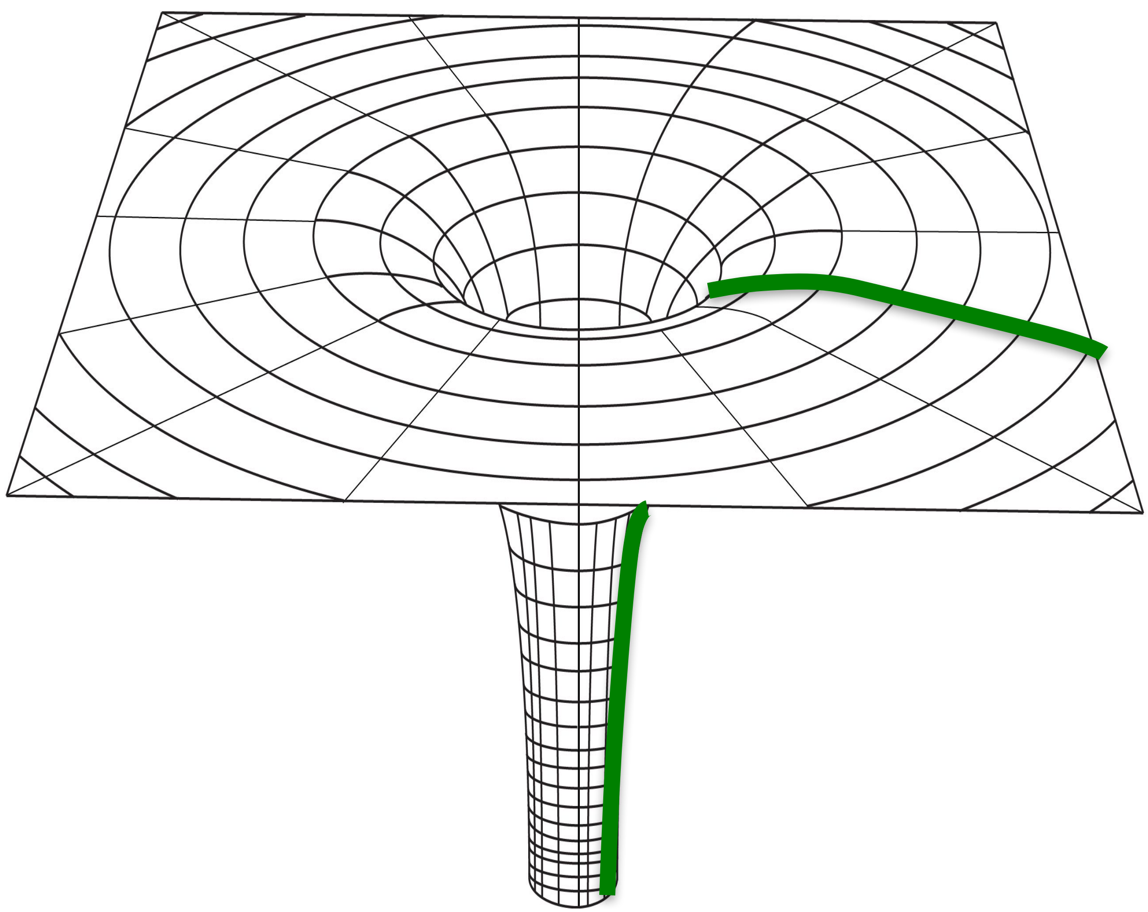
(Here we view F as non-dynamical).

Motion of charged fermions

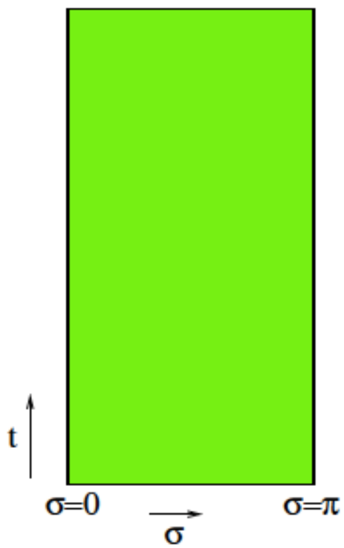
- Degeneracy = q = flux of the magnetic field on the sphere. Form a spin j, representation of SU(2), 2j +1 =q.
- We effectively get q massless two dimensional fermions along the time and radial direction.
- We can think of each of them as following a magnetic field line.

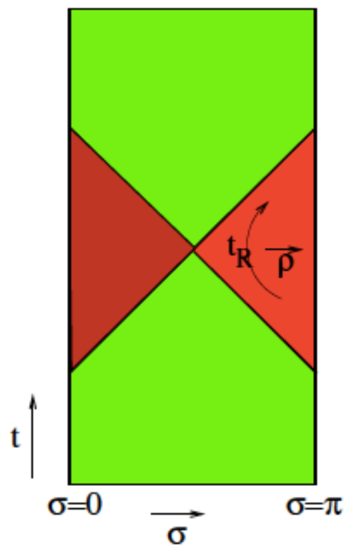


q massless two dimensional fields, along field lines.



AdS_2





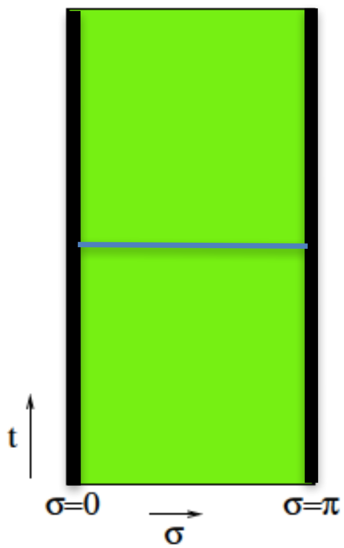
$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma}$$

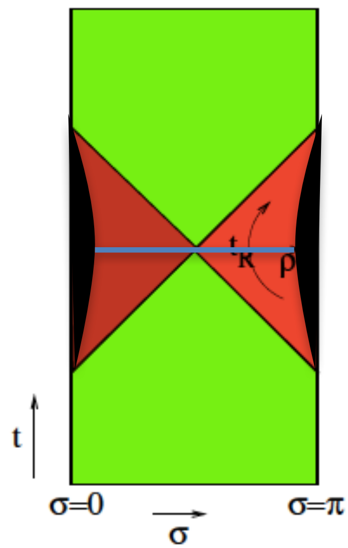
Global

$$ds^2 = -(r^2 - 1)d\tau^2 + \frac{dr^2}{(r^2 - 1)}$$

Thermal/Rindler

Nearly AdS_2





$$ds^2 = \frac{-dt^2 + d\sigma^2}{\sin^2 \sigma}$$

Global

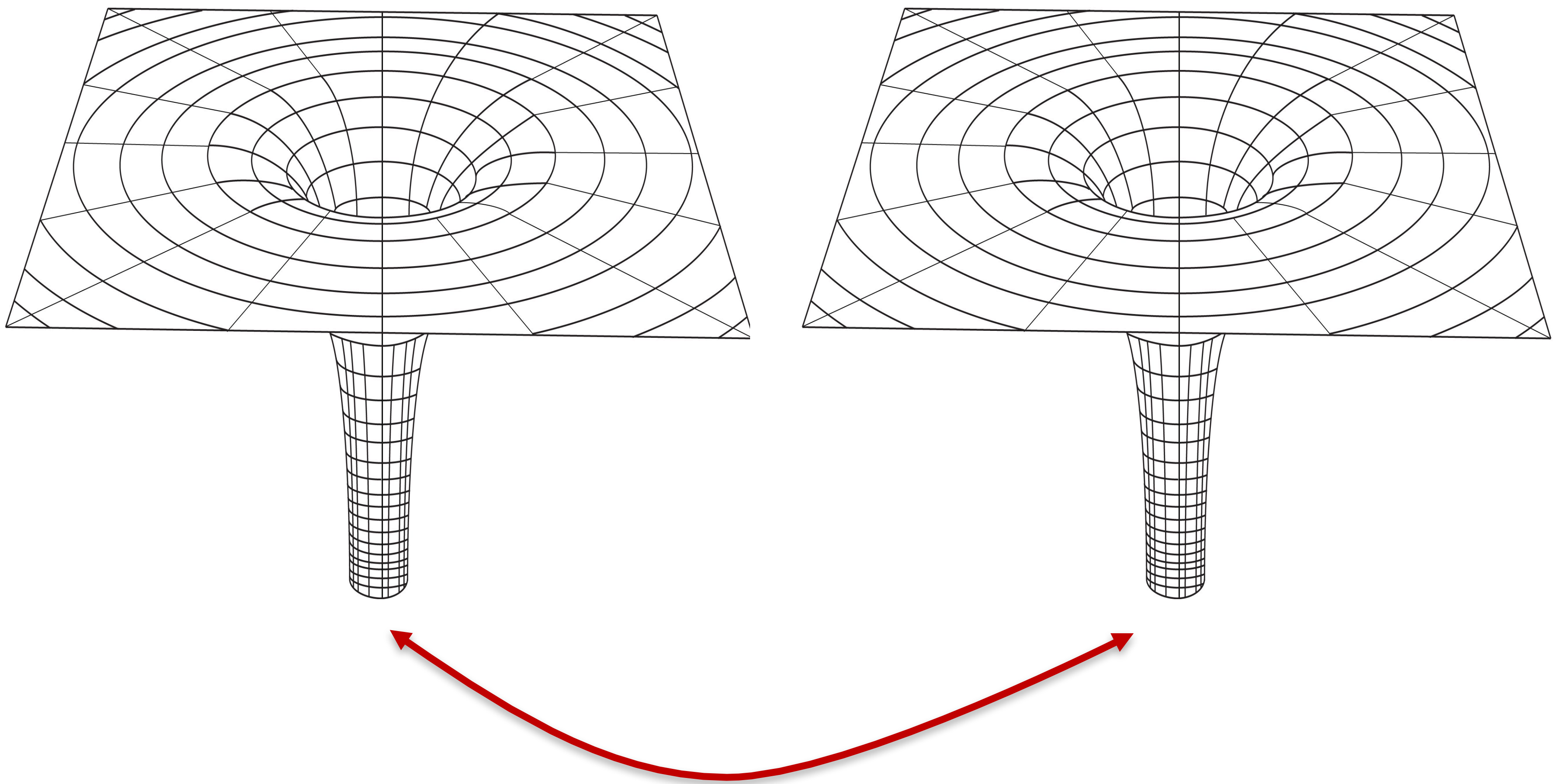
$$ds^2 = -(r^2 - 1)d\tau^2 + \frac{dr^2}{(r^2 - 1)}$$

Thermal/Rindler

Connect them to flat space, so that t is an isometry.
The acquire non-zero energy when the throat has finite length

$$M - q = q^3 T^2 = \frac{q^3}{\beta^2}$$

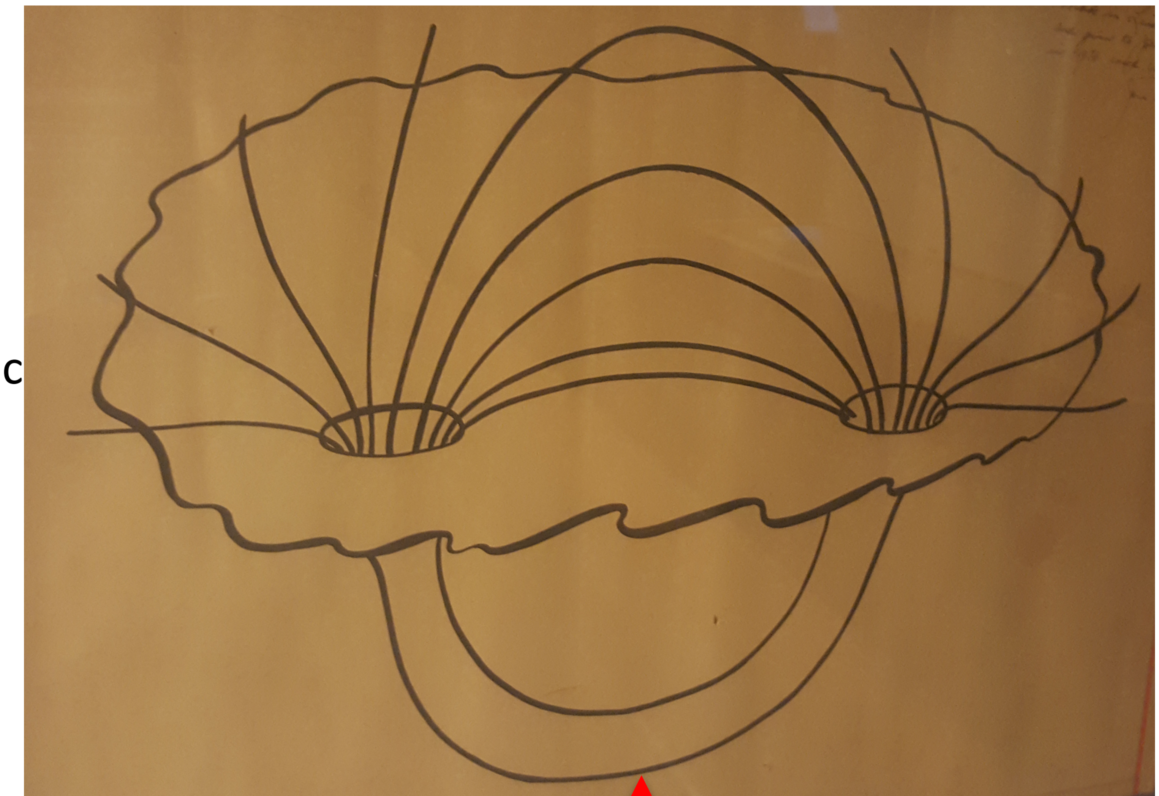
Connect a pair black holes



Connect, and set to global AdS_2

Connect a pair black holes

Positive magnetic charge



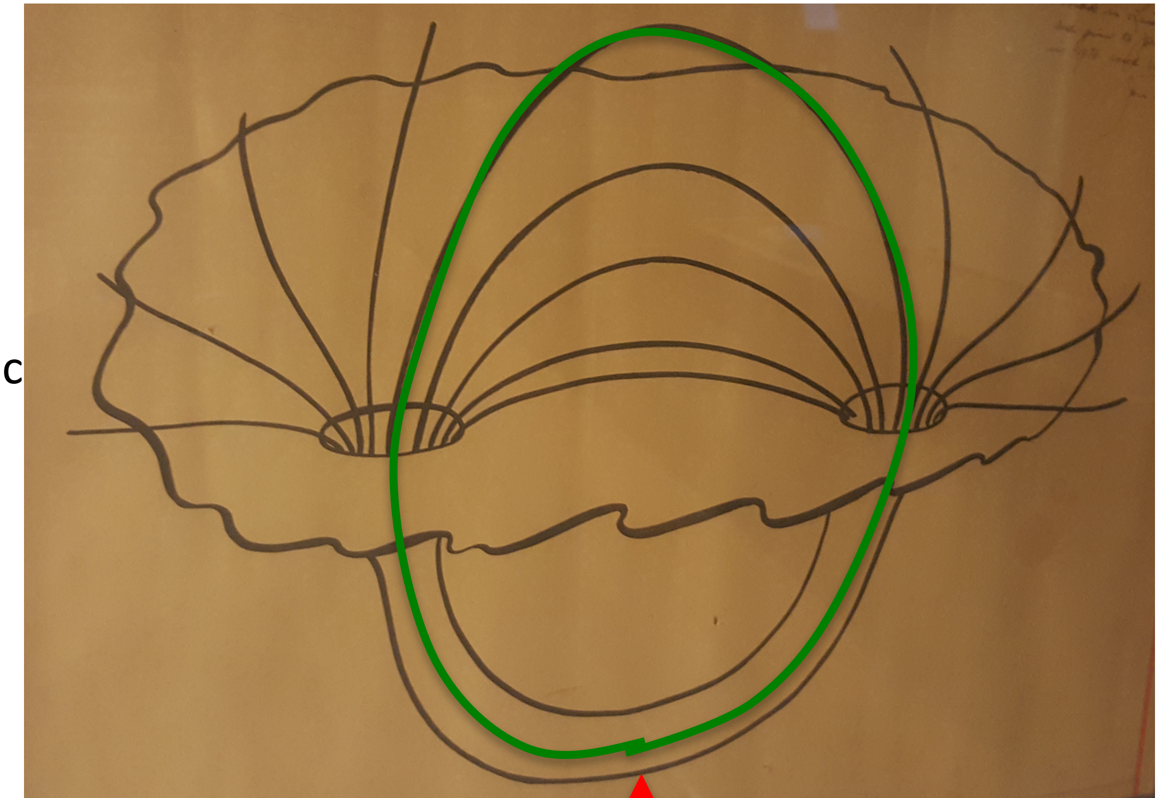
Negative magnetic charge

Not a solution yet.
Not a black hole.

Nearly $AdS_2 \times S^2$ wormhole of finite length

Fermion trajectories

Positive magnetic charge



Negative magnetic charge

Charged fermion moves along this closed circle.

Casimir energy

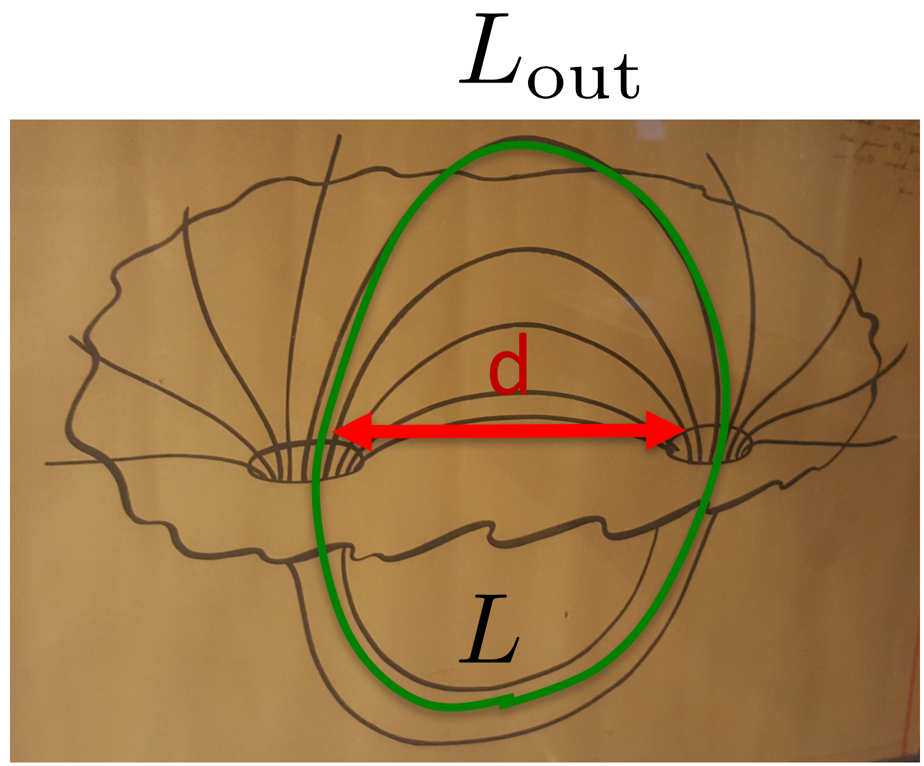
Assume: “Length of the throat” is larger than the distance.

$L \gg L_{out} > d$

Casimir energy is of the order of

$$E \propto -\frac{q}{L}$$

Full energy also need to take into
Account the conformal anomaly because
AdS_2 has a warp factor.
That just changes the numerical factor.



Finding the solution

Solve Einstein equations in the throat region with the negative quantum stress tensor

=

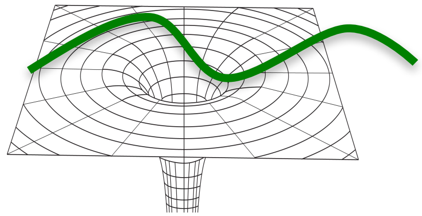
Balance the classical curvature + gauge field energy vs the Casimir energy.

$$M - q = \frac{q^3}{L^2} - \frac{q}{L}, \qquad \frac{\partial M}{\partial L} = 0 \longrightarrow L \sim q^2$$

Now the throat is stabilized. Negative binding energy.

$$E_{\rm binding} = M - q = -\frac{1}{q} = -\frac{1}{r_s}$$

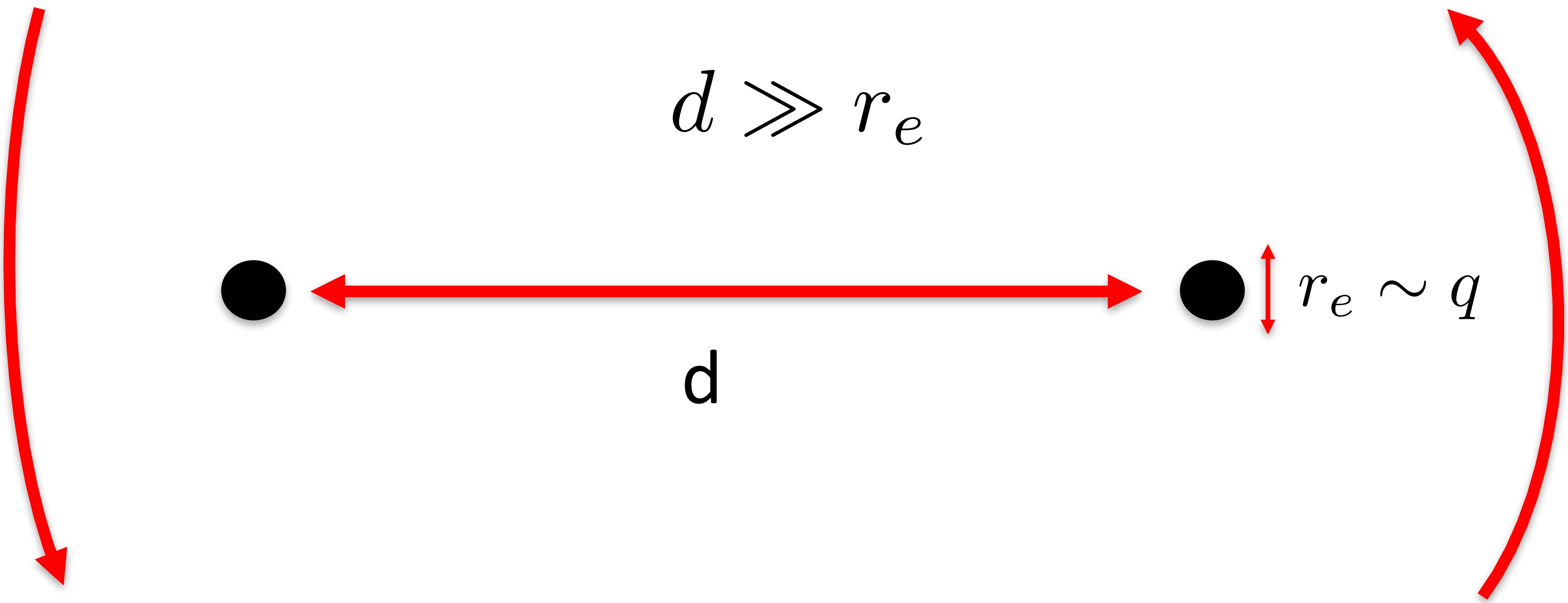
Very small. Only low energy waves can explore it



This is not yet a solution in the outside region:

The two objects attract and would fall on to each other

Adding rotation



$$\Omega = \sqrt{\frac{r_e}{d^3}}$$ Kepler rotation frequency

Throat is fragile

- We must make sure not to start sending matter into the throat that can accumulate there and produce a black hole.
- Rotation → radiation → effective temperature: $T \sim \Omega$
- We need that Ω is smaller than the energy gap of the throat

$$\Omega \ll \frac{1}{L}$$

- The configuration will only live for some time, until the black holes get closer.. (they emit electromagnetic and gravitational radiation)
- These issues could be avoided by going to AdS_4 ...

Some necessary inequalities

$$L \sim q^2$$

From stabilized throat solution

$$d \ll L \longrightarrow d \ll q^2$$

Black holes close enough to that Casmir energy computation was correct.

$$\sqrt{\frac{q}{d^3}} = \Omega \ll \frac{1}{L} \longrightarrow q^{\frac{5}{3}} \ll d$$

Kepler
rotation frequency

Black holes far enough so that they rotate slowly compared to the energy gap.

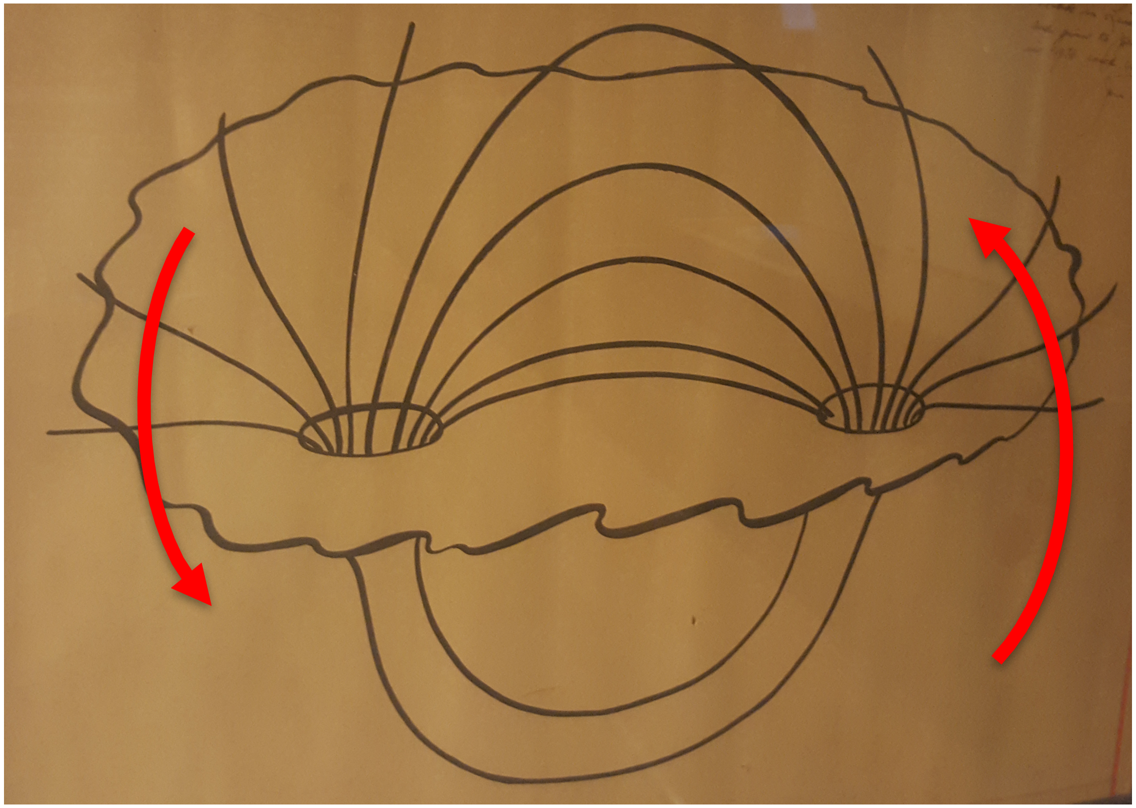
Unruh-like temperature less than energy gap

They are compatible

$$q^{\frac{5}{3}} \ll d \ll q^2$$

Other effects we could think off are also small :
can allow small eccentricity, add electromagnetic and gravitational radiation, etc. Has a finite lifetime.

Final solution



Looks like the exterior of two near extremal black holes. But they connected.
But there is no horizon!. Zero entropy solution.
It has a small binding energy.

Wormholes in the Standard Model

If nature is described by the Standard Model at short distances and d is smaller than the electroweak scale,

$$1 \ll q \ll 10^8$$

Distance d smaller than electroweak scale.

If the standard model is not valid → similar ingredients might be present in the true theory.

That it <u>can</u> exist, does not mean that it is <u>easily</u> produced by some natural or artificial process.

Entropy and entanglement

- Total spacetime has no entropy and no horizon.
- If we only look at one object → entanglement entropy = extremal black hole entropy
- Wormhole = two entangled black holes
- Total Hamiltonian $$H = H_L + H_R + H_{\rm int}$$

Generated by fermions in exterior

Conclusions

- We displayed a solution of an Einstein Maxwell theory with charged fermions.
- It is a traversable wormhole in four dimensions and with no exotic matter.
- It balances classical and quantum effects.
- It has a non-trivial spacetime topology, which is forbidden in the classical theory.
- It has no horizon and no entropy.
- Can be viewed as a pair of entangled black holes.

Questions

- If we start from disconnected near extremal black holes: Can they be connected quickly enough ? → topology change.
- Could we turn it into a prediction from quantum gravity ?