One-point functions in defect CFT, Integrable Matrix Product States, and boundary integrability

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and

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Work with Lorenzo Piroli (SISSA) and Eric Vernier (Oxford)

Outline

- One point functions in defect CFT

$$\langle \mathcal{O}(x) \rangle = \frac{C_{\mathcal{O}}}{(x_\perp)^\Delta}, \qquad C_{\mathcal{O}} = ?$$

($\mathcal{O}$ normalized by the two-point function)

- In $\mathcal{N} = 4$ SYM, planar limit: scaling operators are found by diagonalization of an integrable spin chain

- The one-point functions are given by a (normalized) overlap

$$C_{\mathcal{O}} \sim \frac{\langle \Psi | MPS \rangle}{\langle \Psi | \Psi \rangle^{1/2}}$$

- The state $|MPS\rangle$ and its integrability

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The Field Theory Setting

Details in:

M. de Leeuw, C. Kristjansen, K. Zarembo, JHEP 8 98 (2015)

I. Buhl-Mortensen, M. de Leeuw, C. Kristjansen, K. Zarembo, JHEP 02 052 (2016)

M. de Leeuw, C. Kristjansen, S. Mori, Phys. Lett. B 763 197 (2016)

M. de Leeuw, C. Kristjansen, G. Linardopoulos, J.Phys. A50 (2017) 254001

M. de Leeuw, C. Kristjansen, G. Linardopoulos, Phys.Lett. B781 (2018) 238

The Field Theory Setting

- A 2+1 D defect in 3+1 D SYM, separating regions with different gauge groups.
- Corresponding to the D3-D5 brane setting on the AdS side, with the D5 brane carrying k units of magnetic flux.
- For $z > 0$ gauge group $SU(N)$ broken down to $SU(N-k)$ by VEVs
- For $z < 0$ ubroken $SU(N-k)$ with vanishing VEVs
- On the broken side the scalar fields Φ_j have (classical) expectation values

$$\Phi_j^{cl} = \frac{1}{z}\begin{pmatrix} (t_j)_{k\times k} & 0_{k\times(N-k)} \\ 0_{(N-k)\times k} & 0_{(N-k)\times(N-k)} \end{pmatrix}, \qquad j = 1,2,3$$

$$\Phi_j^{cl} = 0, \qquad j = 4,5,6$$

Here t_j are the k-dimensional generators of $SU(2)$.

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- Gauge invariant scaling operators built from the scalars:

$$\mathcal{O} = \Psi^{j_1 j_2 \ldots j_L} \mathrm{Tr}\left(\Phi_{j_1} \Phi_{j_2} \ldots \Phi_{j_L}\right), \qquad j_1, \ldots, j_L = 1 \ldots 6$$

- The "wave function" Ψ from diagonalizing the dilatation operator. At one loop: Solving the $SO(6)$-symmetric spin chain with

$$H = \sum_{l=1}^{L} (2 - 2P_{l,l+1} + K_{l,l+1})$$

- One-point functions $\langle \mathcal{O}(z) \rangle$

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- As a first approximation take the classical values and compute

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- Correcting the normalization we get

$$\langle\mathcal{O}(z)\rangle = \frac{C}{z^L}, \qquad C = \left(\frac{8\pi^2}{\lambda}\right)^{L/2}\frac{1}{\sqrt{L}} \times \frac{\langle\Psi|MPS\rangle}{\langle\Psi|\Psi\rangle^{1/2}}$$

- MPS generally: $\omega^{(i)}$, $i = 1, \dots, N$

$$|MPS\rangle = \sum_{i_1,\dots,i_L=1}^{N} \mathrm{tr}_0\left[\omega^{(i_1)}\omega^{(i_2)}\dots\omega^{(i_L)}\right]|i_1, i_2, \dots, i_L\rangle$$

- Now specifically:

$$\omega^{(j)} = \begin{cases} t^{(j)} & \text{for } j = 1,2,3 \\ 0 & \text{for } j = 4,5,6 \end{cases}$$

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Overlaps

- Eigenstates are characterized using Bethe rapidities

$$|\Psi\rangle = |\{\lambda\}_N\rangle$$

  Can be nested, etc.

- Overlaps with some state $|\Psi_0\rangle$:

$$\langle\Psi_0|\{\lambda\}_N\rangle = f(\{\lambda\})$$

  In principle all known from coordinate Bethe Ansatz

- Difficulty: Difference between off-shell and on-shell formulas
- Studied independently in the (cond-mat) integrability community

$$|\Psi_0\rangle \quad \to \quad |\Psi(t)\rangle = e^{-iHt}|\Psi_0\rangle \quad \to \quad \langle\mathcal{O}(t)\rangle = ?$$

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Integrable initial states

A subset of states $|\Psi_0\rangle$ with special properties:

- Overlaps display the pair structure, are non-zero only if

$$\{\lambda\}_N = \quad \{\lambda^+, -\lambda^+\}_{N/2} \quad \text{or} \quad \{\lambda^+, -\lambda^+\}_{(N-1)/2} \cup \{0\}$$

- They take a factorized form. In simple cases

$$\frac{|\langle\Psi_0|\{\lambda\}_N\rangle|^2}{\langle\{\lambda\}_N|\{\lambda\}_N\rangle} = \prod_{j=1}^{N/2} v(\lambda_j^+) \times \frac{\det G^+}{\det G^-}$$

More generally some linear combination of these, with different v-functions

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Common properties of integrable states

Common property:

$$Q_{2j+1}|\Psi_0\rangle = 0$$

$$Q_{2j+1}|\Psi_0\rangle = Q_{2j+1}\left(\sum_{\{\lambda\}} |\{\lambda\}\rangle\langle\{\lambda\}|\Psi_0\rangle\right) =$$
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$$= 0$$

- Easy to check directly
- Implies the pair structure
- Does NOT help with the overlaps

L. Piroli, BP, E. Vernier (2017)

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Examples

- In XXZ: Néel, Dimer states

$$|N\rangle = \otimes_{j=1}^{L/2} |\uparrow\downarrow\rangle \qquad |D\rangle = \otimes_{j=1}^{L/2} \frac{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle}{\sqrt{2}}$$

- Arbitrary two-site states in spin-1/2 XXZ
- A subset of two-site states in higher spin XXZ, and in higher rank models ($SU(N)$-symmetric chains)
- All MPS that arise in $\mathcal{N} = 4$ SYM with a defect

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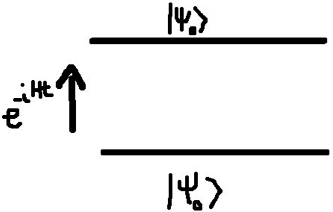

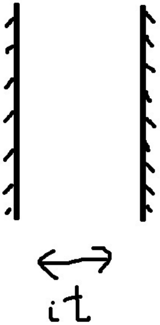

S. Ghoshal and A. B. Zamolodchikov, Int. J. Mod. Phys., A9 3841, `hep-th/9306002`

$$Q_s|\Psi_0\rangle = 0 \quad \text{for some } s \in S \subset \{1, 2, \dots\}$$

$$|\Psi_0\rangle = \exp\left(\int du\; K(u)Z(u)Z(-u)\right)|0\rangle \qquad K(u) = R(i\pi/2 - u)$$

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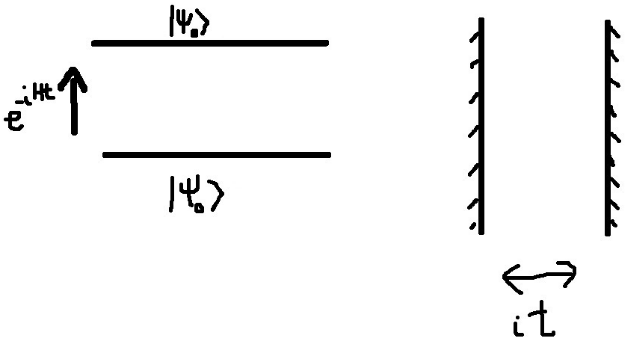

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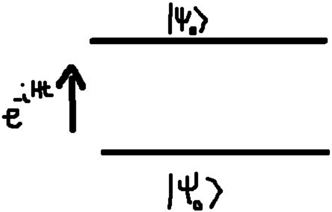

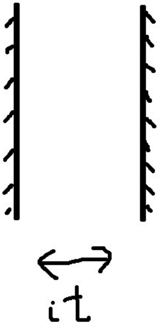

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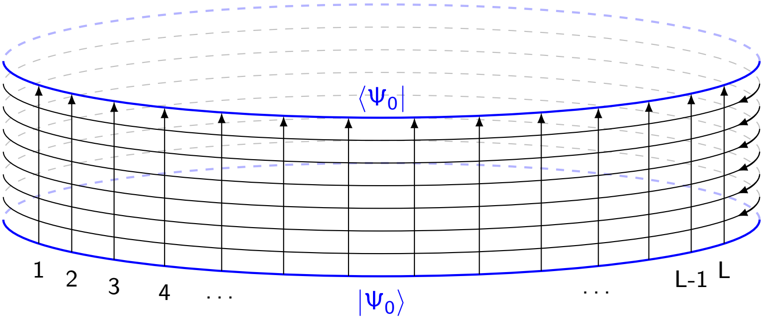
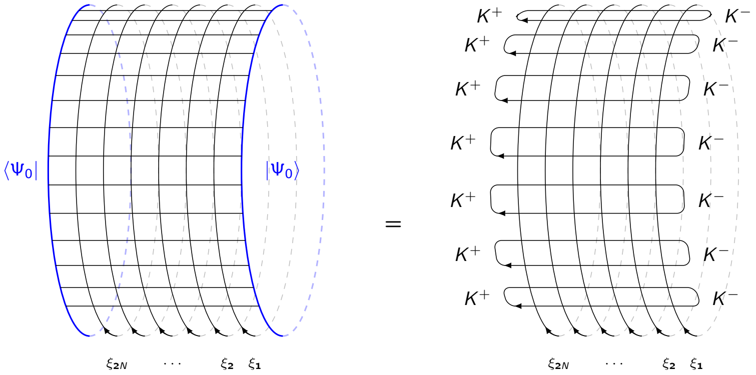


The Loschmidt amplitude



For two site product state: boundary transfer matrix

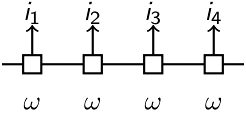
$$\tau(u) = \mathrm{Tr}_0\{K_+(u)T(u)K_-(u)T^{-1}(-u)\}$$

BP, J. Stat. Mech. (2013) P10028

Integrable MPS

- Constructing MPS from $\omega^{(j)}$, $j = 1 \dots N$

$$|MPS\rangle = \sum_{i_1,\dots,i_L=1}^{N} \mathrm{tr}_0 \left[\omega^{(i_1)}\omega^{(i_2)} \dots \omega^{(i_L)}\right] |i_1, i_2, \dots, i_L\rangle$$



- Let's baxterize them! Looking for matrices $\psi_{ab}(u)$ with $a, b = 1 \dots N$

- Initial condition:

$$\psi_{ab}(0) = \omega^{(a)}\omega^{(b)}$$

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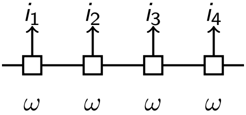

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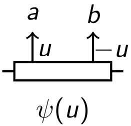

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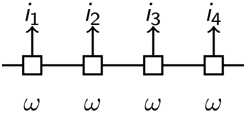

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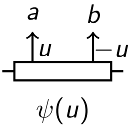

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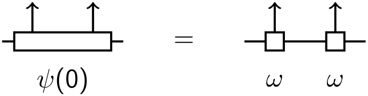
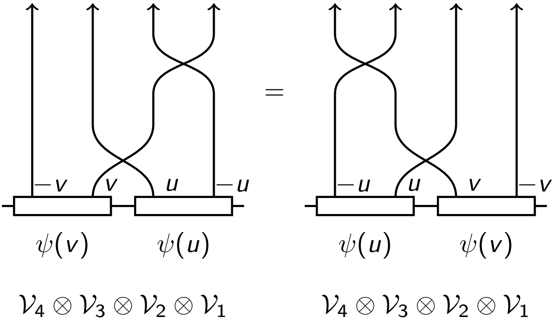


Boundary Yang-Baxter relation in $\mathcal{V}_4 \otimes \mathcal{V}_3 \otimes \mathcal{V}_2 \otimes \mathcal{V}_1$:

$$\check{R}_{12}(u-v)\check{R}_{23}(-u-v)(\psi(v)\otimes\psi(u)) = \check{R}_{34}(u-v)\check{R}_{23}(-u-v)(\psi(u)\otimes\psi(v))$$

with $\check{R}(u) = PR(u)$



Integrability condition can be proven

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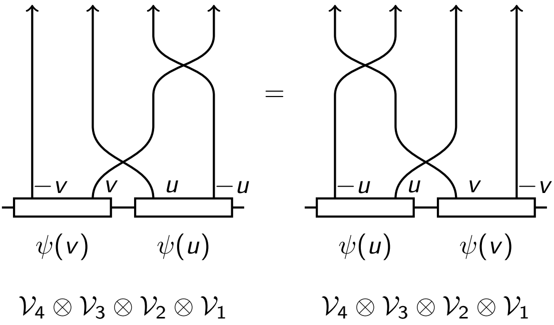

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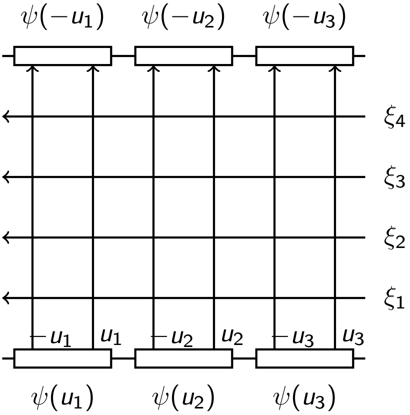
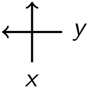
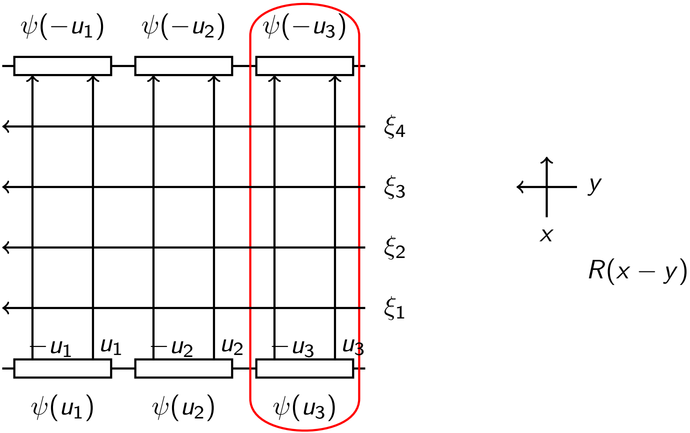
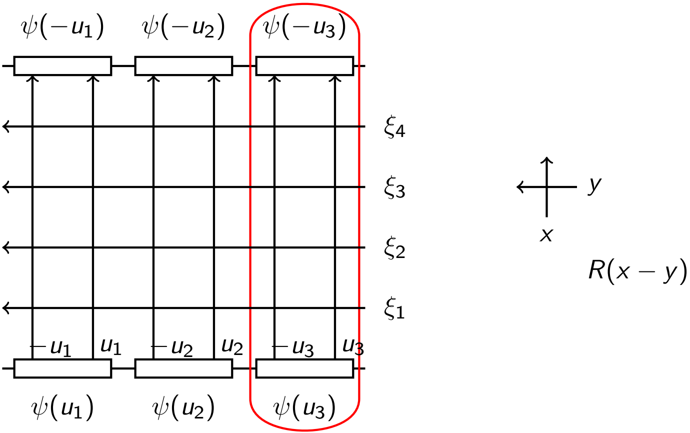




$R(x - y)$



$$\mathcal{T}(u) = \sum_{a_1,a_2,b_1,b_2=1}^{N} \psi^{(1)}_{a_2 a_1}(u) \otimes \left[T_{a_2 b_2}(-u)\, T_{a_1 b_1}(u) \right] \otimes \psi^{(2)}_{b_2 b_1}(-u)$$



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$$[\mathcal{T}(u), \mathcal{T}(v)] = 0$$

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- Relation to (operator valued) K-matrices $K(u) \in End(\mathcal{V})$

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- Twisted BYB relation in $End(\mathcal{V} \otimes \mathcal{V})$

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Solutions

Generally solutions characterized by the pair $(\mathfrak{g}, \mathfrak{g}')$

$\mathfrak{g}$ decribes the symmetry of the model, and $\mathfrak{g}' \subset \mathfrak{g}$ the symmetry of $\psi(u)$

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Looking for solutions with factorizability condition

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For $\mathfrak{g} = \mathfrak{gl}(N)$ we have $R(u) = P + uI$

- For $N = 3$, $(\mathfrak{gl}(3), \mathfrak{so}(3))$:

$$\psi_{ab}(u) = \sigma_a \sigma_b - 2u\delta_{ab}$$

  Describes $\omega^{(j)} = \sigma_j$, the 1pt functions with $k = 2$ in the $SU(3)$ sector

- Higher spin representations

$$\psi_{ab}(u) = (1 + u)S_a S_b - uS_b S_a - u^2 \delta_{ab}$$

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For $\mathfrak{g} = \mathfrak{so}(N)$ we have $R(u) = (u+c)P + u(u+c)I - uK, \qquad c = \frac{N-2}{2}$

This is crossing symmetric: $R^{t_1}(u) = R(-u-2)$

- Completely $SO(N)$ symmetric solution:

$$\psi_{ab}(u) = (-2u + c)\gamma_a\gamma_b + 2u(1+u)\delta_{ab}$$

- Solution for $(\mathfrak{so}(6), \mathfrak{so}(3) \oplus \mathfrak{so}(3))$: (indices $a, b = 1..3$, $I, J = 4..6$)

$$\psi_{ab}(u) = 2(1-u)\sigma_a\sigma_b + u(2u-1)\delta_{ab}$$
$$\psi_{aI}(u) = \psi_{Ia} = 0$$
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For $u = 0$ describes the AdS/CFT MPS in the $SO(6)$ sector with $k = 2$

Change of variables $u \to -u + 1$: (DeWolfe and Mann, 2004)

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Change of variables $u \to -u + 1$: (DeWolfe and Mann, 2004)

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For $\mathfrak{g} = \mathfrak{so}(N)$ we have $R(u) = (u+c)P + u(u+c)I - uK, \qquad c = \frac{N-2}{2}$

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Describes the D3-D7 brane setting, with $N = 6$, $D = 5$.

M. de Leeuw, C. Kristjansen, G. Linardopoulos, J.Phys. A50 (2017) 254001

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- Proof of integrability for the MPS
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Overlaps

Interpretation of the Gaudin-like determinants:

Non-trivial density of states due to the pair structure
In QFT: M. Kormos, BP, 2010
In the XXZ chain in the TDL: BP, 2018

Thank you for the attention!!!