Bootstrapping $4d\ \mathcal{N}=2$ conformal theories

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Outline

1. **The Superconformal Bootstrap Program**
2. (A_1, A_2) **Argyres-Douglas Theory**
3. **Landscape of** $4d$ $\mathcal{N} = 2$ **SCFTs**
4. **Summary & Outlook**

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The Superconformal Bootstrap Program

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$\rightarrow$ "Simplest" $\mathcal{N} = 2$ Argyres-Douglas theory?

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Conformal field theory defined by

Set of local operators and their correlation functions

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CFT data

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CFT data strongly constrained

- Unitarity
- Associativity of the operator product algebra

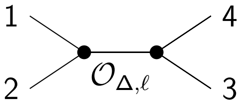
  $(\mathcal{O}_1\mathcal{O}_2)\mathcal{O}_3 = \mathcal{O}_1(\mathcal{O}_2\mathcal{O}_3)$

Conformal Bootstrap

Crossing Symmetry

$\langle(\mathcal{O}_1(x_1)\ \mathcal{O}_2(x_2))\mathcal{O}_3(x_3)\ \mathcal{O}_4(x_4)\rangle =$

$$\sum_{\mathcal{O}_{\Delta,\ell}}$$



Conformal Bootstrap

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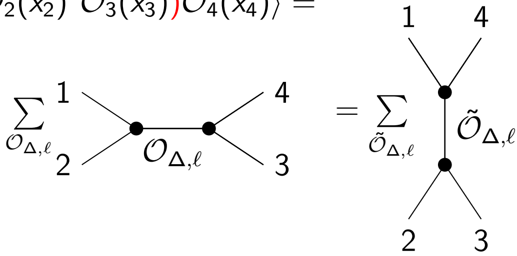
$\langle\, \mathcal{O}_1(x_1)\color{red}{(}\mathcal{O}_2(x_2)\ \mathcal{O}_3(x_3)\color{red}{)}\mathcal{O}_4(x_4)\rangle =$



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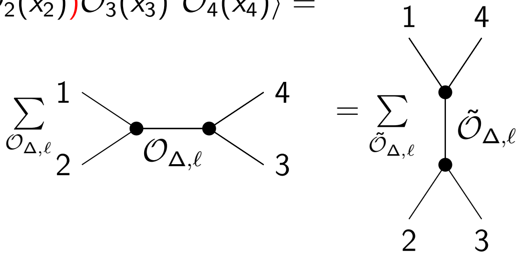

$$\frac{1}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}} \sum_{\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_3\mathcal{O}_4\mathcal{O}_{\Delta,\ell}} g_{\Delta,\ell}(z,\bar{z}) =$$

where $\Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}}$, $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$, $v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$

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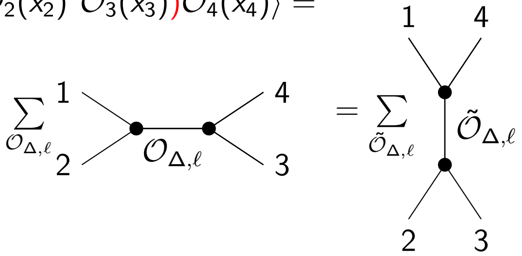

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$$\frac{1}{x_{14}^{2\Delta_{\mathcal{O}}} x_{23}^{2\Delta_{\mathcal{O}}}} \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}} \lambda_{\mathcal{O}_1\mathcal{O}_4\tilde{\mathcal{O}}_{\Delta,\ell}} \lambda_{\mathcal{O}_2\mathcal{O}_3\tilde{\mathcal{O}}_{\Delta,\ell}} g_{\Delta,\ell}(1-z,1-\bar{z})$$

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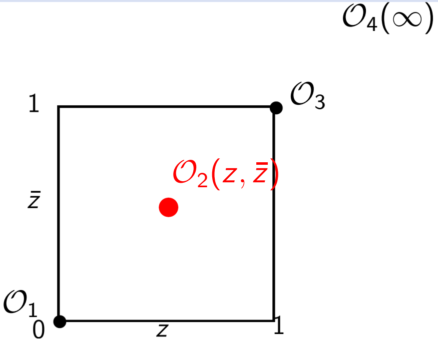
The crossing equation



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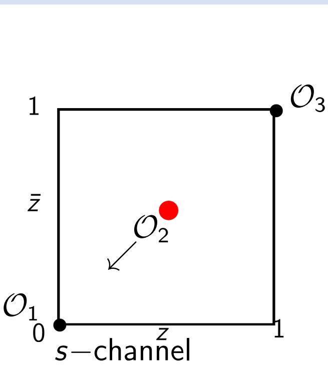

$$\left(\frac{(1-z)(1-\bar{z})}{z\bar{z}}\right)^{\Delta_{\mathcal{O}}} \sum_{\mathcal{O}_{\Delta,\ell}} \lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta,\ell}} g_{\Delta,\ell}(z,\bar{z}) =$$

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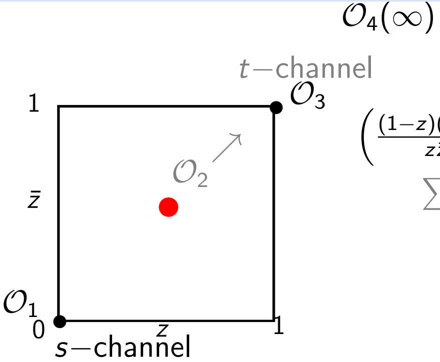

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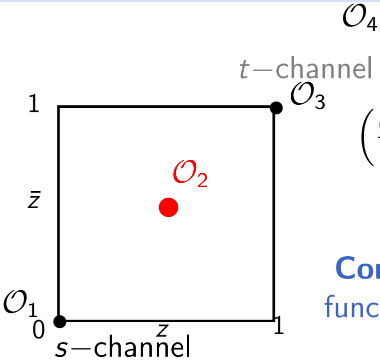

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Consistent CFT $\Rightarrow$ **All** four point functions are **crossing symmetric**

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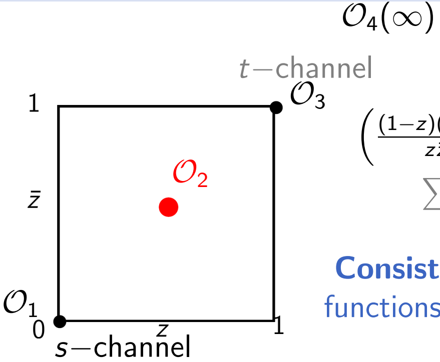

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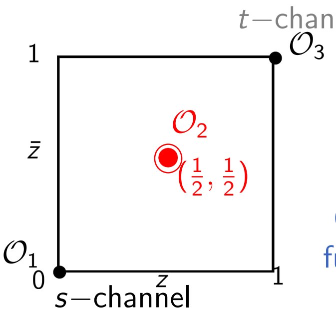
Our tools

The crossing equation

$\mathcal{O}_4(\infty)$



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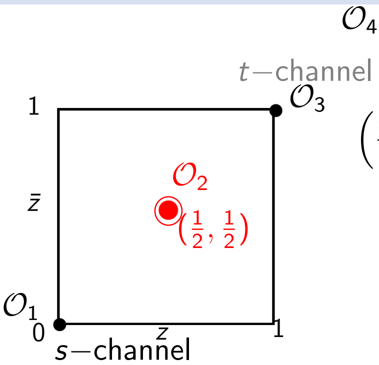

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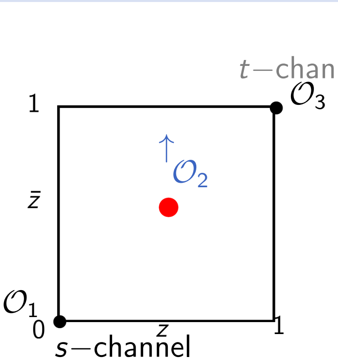

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[Alday Maldacena, Fitzpatrick Kaplan Poland Simmons-Duffin, Komargodski Zhiboedov]

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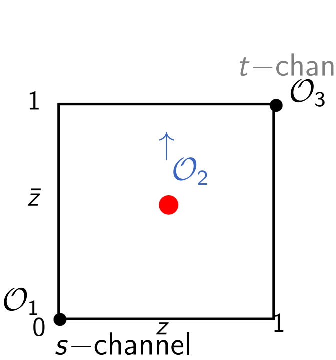

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$\rightarrow$ $t-$channel dominated by low twist

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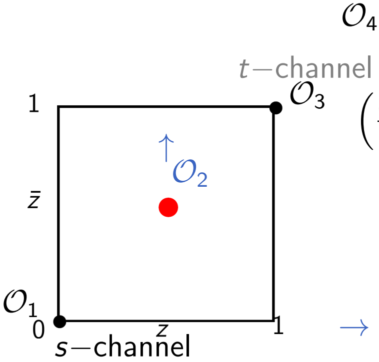

$$\left(\frac{(1-z)(1-\bar{z})}{z\bar{z}}\right)^{\Delta_{\mathcal{O}}} \overbrace{\lim_{\bar{z}\to 1}\sum_{\mathcal{O}_{\Delta,\ell}}}^{\text{large spin}} \lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta,\ell}} g_{\Delta,\ell}(z,\bar{z}) =$$

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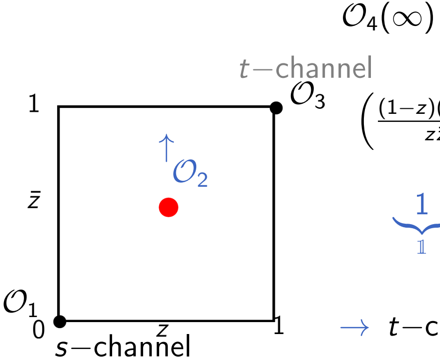

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$\hookrightarrow$ Lorentzian inversion formula of [Caron-Huot, see his talk]

Numerical bootstrap review

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$u = z\bar{z}$, $v = (1-z)(1-\bar{z})$

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$$\sum_{\substack{\mathcal{O}_{\Delta,\ell}\in\mathcal{O}\mathcal{O}\\ \mathcal{O}_{\Delta,\ell}\neq\mathbb{1}}} \lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta,\ell}} \underbrace{\frac{u^{\Delta_{\mathcal{O}}} g_{\Delta,\ell}(v,u) - v^{\Delta_{\mathcal{O}}} g_{\Delta,\ell}(u,v)}{v^{\Delta_{\mathcal{O}}} - u^{\Delta_{\mathcal{O}}}}}_{F_{\Delta,\ell}(u,v)} = 1$$

Numerical bootstrap review

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Numerical bootstrap review

Sum rule

$$\lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta_\star,\ell_\star}} \quad F_{\Delta_\star,\ell_\star} + \sum_{\substack{\mathcal{O}_{\Delta,\ell}\in\mathcal{O}\mathcal{O},\ \mathcal{O}_{\Delta\ell}\neq\mathbb{1} \\ \mathcal{O}_{\Delta,\ell}\neq\mathcal{O}_{\Delta_\star,\ell_\star}}} \lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta,\ell}} \quad F_{\Delta,\ell} = \quad 1$$

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Sum rule

$$\lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta_\star,\ell_\star}} \psi \cdot F_{\Delta_\star,\ell_\star} + \sum_{\substack{\mathcal{O}_{\Delta,\ell}\in\mathcal{O}\mathcal{O},\ \mathcal{O}_{\Delta\ell}\neq\mathbb{1} \\ \mathcal{O}_{\Delta,\ell}\neq\mathcal{O}_{\Delta_\star,\ell_\star}}} \lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta,\ell}} \psi \cdot F_{\Delta,\ell} = \psi \cdot 1$$

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- Find Functional ψ such that
  - $\hookrightarrow$ $\psi \cdot F_{\Delta_\star,\ell_\star}(z,\bar{z}) = 1$
  - $\hookrightarrow$ $\psi \cdot F_{\Delta,\ell}(z,\bar{z}) \geq 0$ for all $\{\Delta, \ell\}$ in spectrum

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$$\lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta_\star,\ell_\star}}\psi\cdot F_{\Delta_\star,\ell_\star}+\sum_{\substack{\mathcal{O}_{\Delta,\ell}\in\mathcal{O}\mathcal{O},\ \mathcal{O}_{\Delta\ell}\neq\mathbb{1}\\ \mathcal{O}_{\Delta,\ell}\neq\mathcal{O}_{\Delta_\star,\ell_\star}}}\lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta,\ell}}\,\psi\cdot F_{\Delta,\ell}=\psi\cdot 1$$

- Find Functional ψ such that
  - $\hookrightarrow$ $\psi\cdot F_{\Delta_\star,\ell_\star}(z,\bar{z})=1$
  - $\hookrightarrow$ $\psi\cdot F_{\Delta,\ell}(z,\bar{z})\geq 0$ for all $\{\Delta,\ell\}$ in spectrum
  - $\hookrightarrow$ Minimize $\psi\cdot 1$

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  - $\hookrightarrow$ Minimize $\psi \cdot 1$
- $\boxed{\lambda^2_{\phi\phi\mathcal{O}_{\Delta_\star,\ell_\star}} \leqslant \psi \cdot 1}$

Numerical bootstrap review

Sum rule

$$\lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta_\star,\ell_\star}}\psi\cdot F_{\Delta_\star,\ell_\star}+\sum_{\substack{\mathcal{O}_{\Delta,\ell}\in\mathcal{O}\mathcal{O},\ \mathcal{O}_{\Delta\ell}\neq\mathbb{1}\\ \mathcal{O}_{\Delta,\ell}\neq\mathcal{O}_{\Delta_\star,\ell_\star}}}\lambda^2_{\mathcal{O}\mathcal{O}\mathcal{O}_{\Delta,\ell}}\ \psi\cdot F_{\Delta,\ell}=\psi\cdot 1$$

- Find Functional ψ such that
  - $\hookrightarrow$ $\psi\cdot F_{\Delta_\star,\ell_\star}(z,\bar z)=1$
  - $\hookrightarrow$ $\psi\cdot F_{\Delta,\ell}(z,\bar z)\geq 0$ for all $\{\Delta,\ell\}$ in spectrum
  - $\hookrightarrow$ Minimize $\psi\cdot 1$
- $\boxed{\lambda^2_{\phi\phi\mathcal{O}_{\Delta_\star,\ell_\star}}\leqslant\psi\cdot 1}$
- Truncate $\psi=\sum_{m,n}^{m,n\leqslant\Lambda}a_{mn}\partial_z^m\partial_{\bar z}^n|_{z=\bar z=\frac{1}{2}}$

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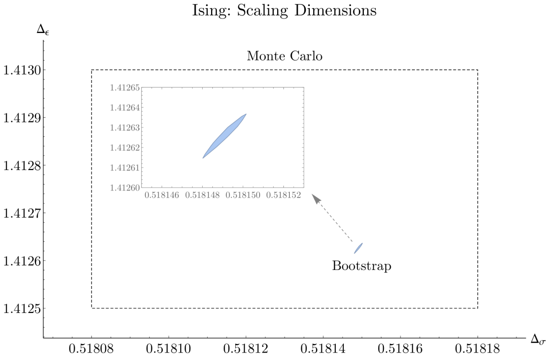
  - $\hookrightarrow$ Always true bounds

$3d$ **Ising Model**

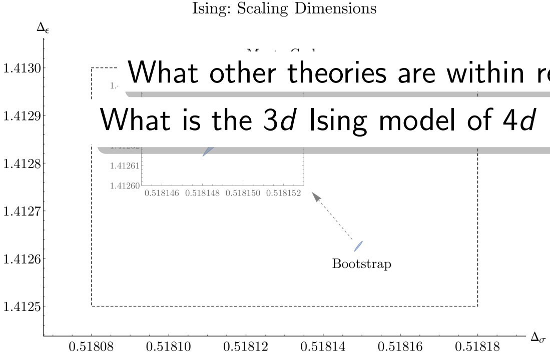
[Poland Simmons-Duffin Kos, Simmons-Duffin, Poland Simmons-Duffin Kos Vichi]



One $\mathbb{Z}_2$−even, one $\mathbb{Z}_2$−odd relevant scalar operator

3*d* Ising Model

[Poland Simmons-Duffin Kos, Simmons-Duffin, Poland Simmons-Duffin Kos Vichi]



What other theories are within reach?

What is the 3*d* Ising model of 4*d* (S)CFTs?

One $\mathbb{Z}_2-$even, one $\mathbb{Z}_2-$odd relevant scalar operator

The Superconformal Bootstrap

Conformal field theory defined by

$\{\mathcal{O}_{\Delta,\ell,\ldots}(x)\}$ and $\{\lambda_{\mathcal{O}_i\mathcal{O}_j\mathcal{O}_k}\}$

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Q: Is there a solvable truncation of the crossing equations? [see Rastelli's talk]

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$\rightarrow$ Yes, for $4d$ $\mathcal{N} \geqslant 2$ [Beem ML Liendo Peelaers Rastelli van Rees]

(and also $6d$ $\mathcal{N} = (2,0)$ and $2d$ $\mathcal{N} = (0,4)$ [Beem Rastelli van Rees])

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$\rightarrow$ Subsector $\mathcal{N} \geqslant 2$ SCFTs captured by $2d$ chiral algebra

A solvable subsector

$4d$ $\mathcal{N} = 2$ **SCFTs** $\to$ $2d$ **chiral algebra** **[see Rastelli's talk]**

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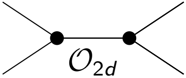
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 

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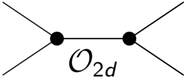
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 

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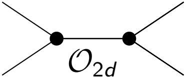

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assumptions: interacting theory, unique stress tensor

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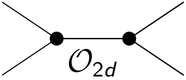

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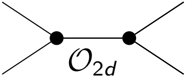
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 

$\rightarrow$ $\lambda^2_{\mathcal{O}_{2d}} \rightsquigarrow \lambda^2_{\mathcal{O}_{4d}} \underbrace{\geqslant}_{4d \text{ unitarity}} 0$ $\Rightarrow$ **New unitarity bounds**

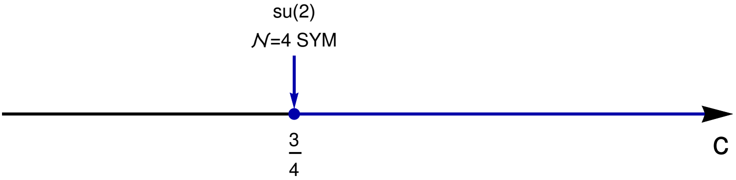
assumptions: interacting theory, unique stress tensor

Landscape of $4d$ $\mathcal{N} \geqslant 2$ SCFTs

From $2d$ (super-)stress tensor four-point function

(assumptions: interacting theory, unique stress tensor)

$\rightarrow$ $4d$ $\mathcal{N} = 4$ SCFTs $c = a \geqslant \frac{3}{4}$ [Beem Rastelli van Rees]



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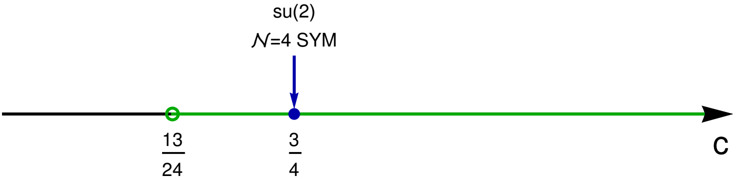

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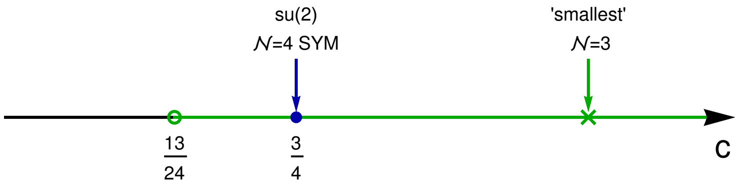

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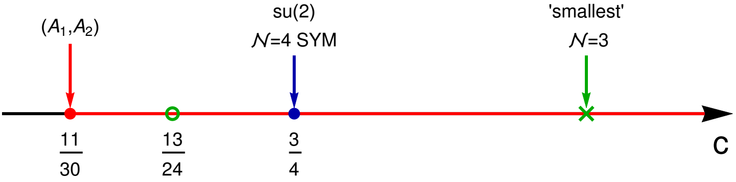

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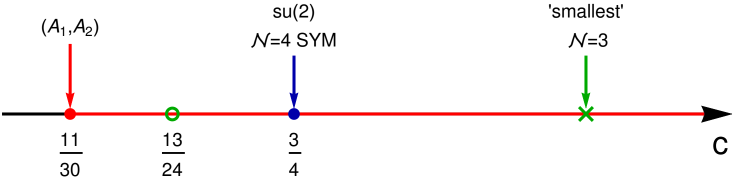
$\hookrightarrow$ Saturated by the (A_1, A_2) Argyres-Douglas theory



Outline

The “simplest” Argyres-Douglas theory

→ Originally obtained on the Coulomb branch of a $4d$ $\mathcal{N} = 2$ susy gauge theory with gauge group $SU(3)$

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Our tools beyond protected subsector

- Numerical bootstrap
  [Rattazzi Rychkov Tonni Vichi]

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- Numerical bootstrap [Rattazzi Rychkov Tonni Vichi]
- Lightcone bootstrap

[Alday Maldacena, Fitzpatrick Kaplan Poland Simmons-Duffin, Komargodski Zhiboedov]

↪ Lorentzian inversion formula of [Caron-Huot]

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(only in $\phi\bar{\phi}$ channel) [Fitzpatrick Kaplan Khandker Li Poland Simmons-Duffin]

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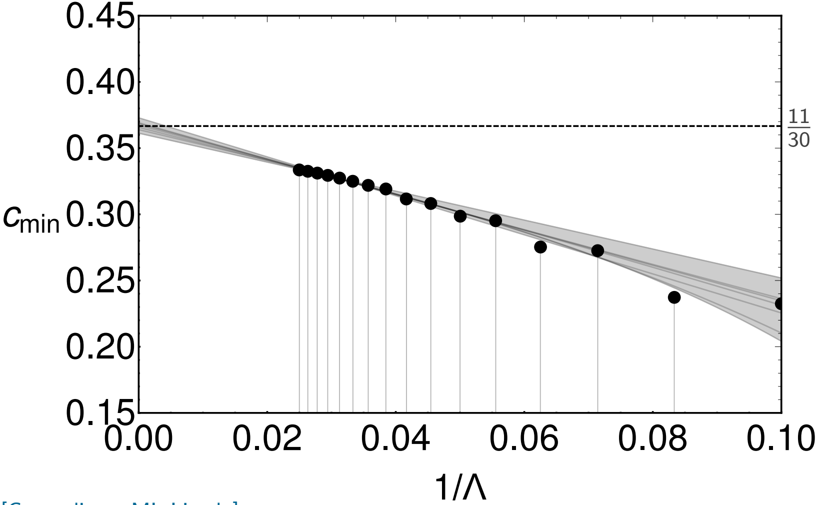

[Cornagliotto ML Liendo]

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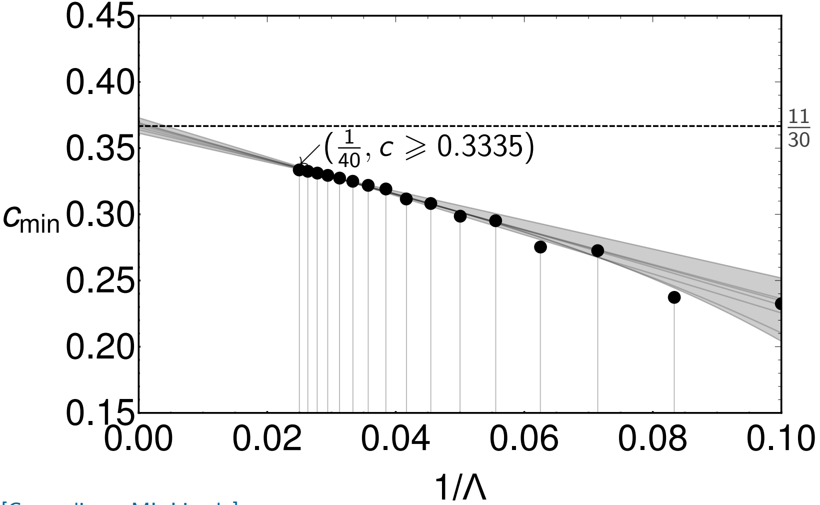

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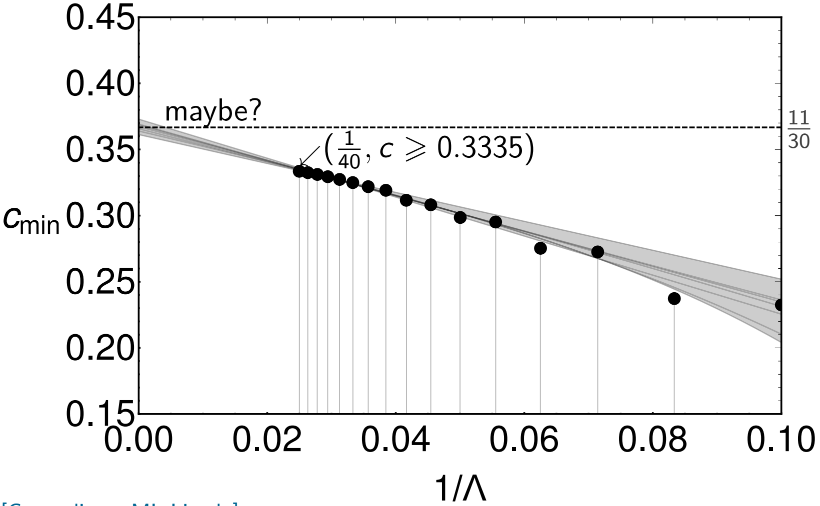
Minimum allowed central charge

Does $\langle \phi\phi\bar{\phi}\bar{\phi} \rangle$ know about $c \geqslant \frac{11}{30}$?



[Cornagliotto ML Liendo]

Bounding OPE coefficients

$$\phi\phi \sim \underbrace{\lambda^2_{\phi^2}}_{\text{unknown}} \underbrace{\phi^2}_{\Delta=2\Delta_\phi} + \cdots$$

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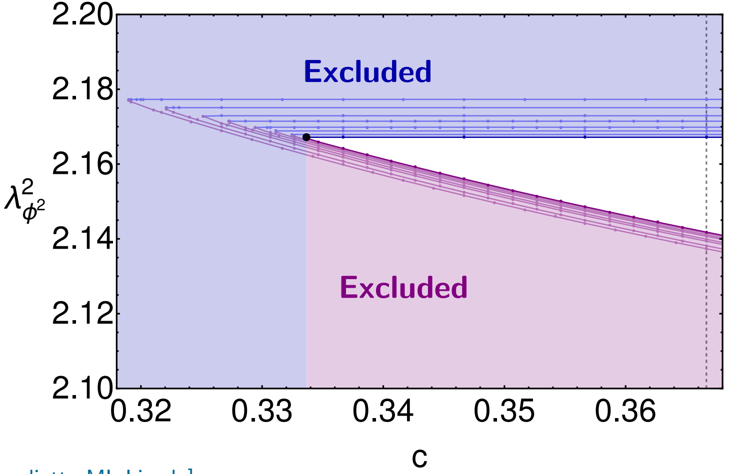

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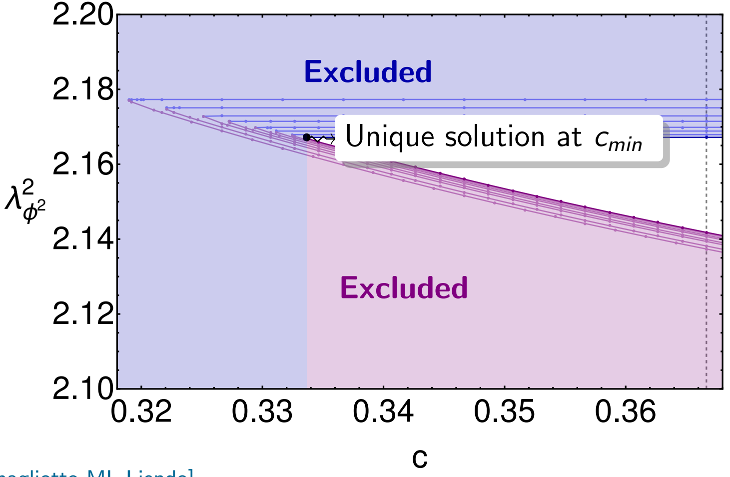

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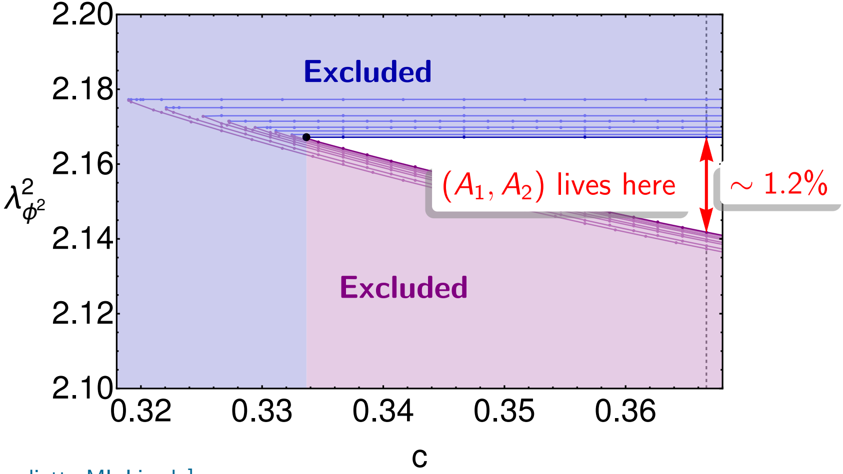

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Lorentzian inversion formula

$$\phi\phi \sim \lambda_{\phi^2}^2 \underbrace{\phi^2}_{\Delta=2\Delta_\phi} + \lambda_{\mathcal{C}_\ell}^2 \underbrace{\mathcal{C}_{\ell>0}}_{\Delta=2\Delta_\phi+\ell} + \cdots$$

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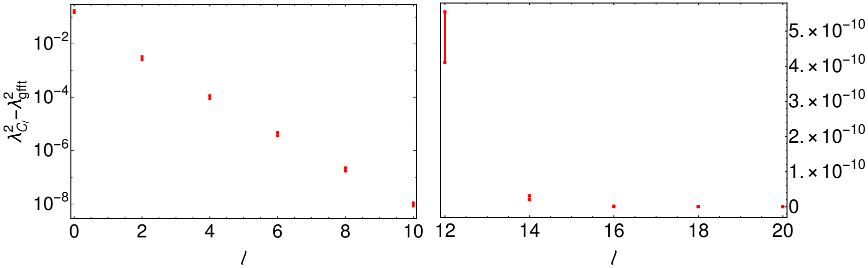

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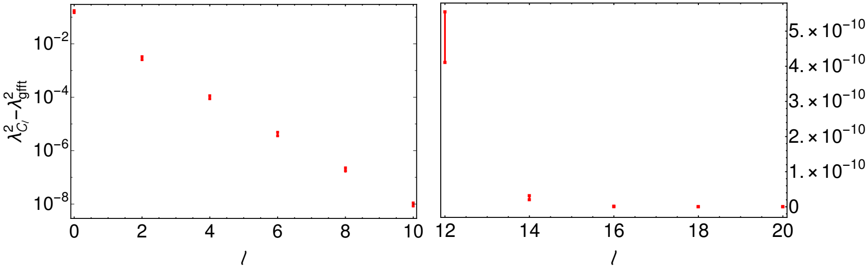

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Inverting the $\phi\phi$ OPE

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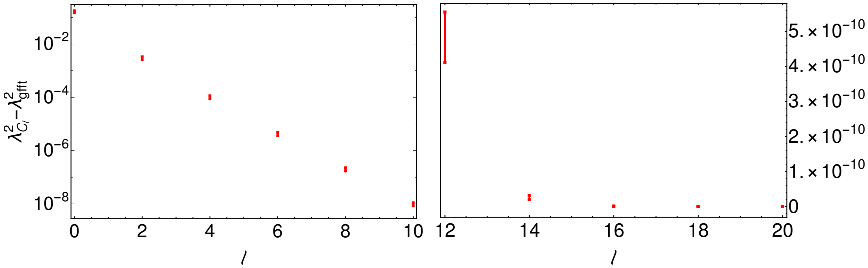

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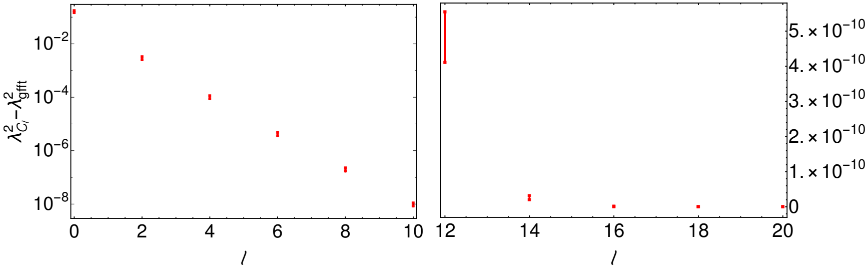

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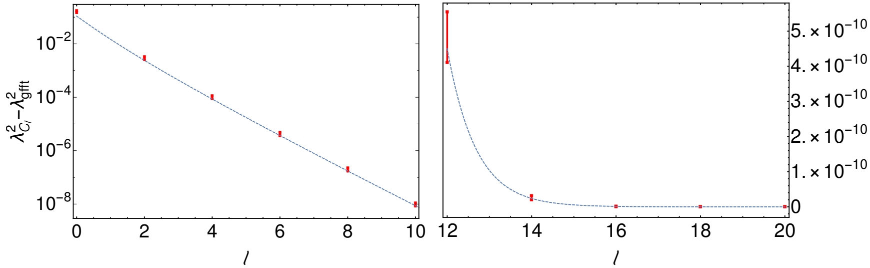

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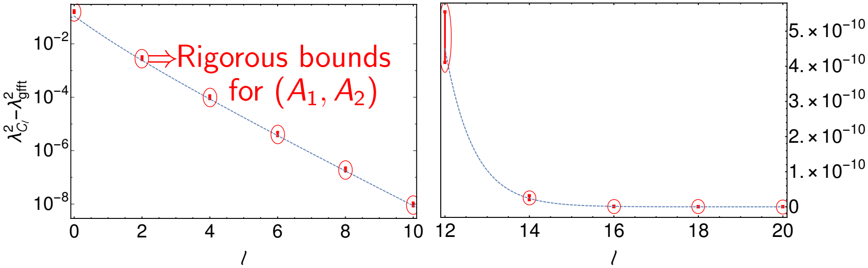

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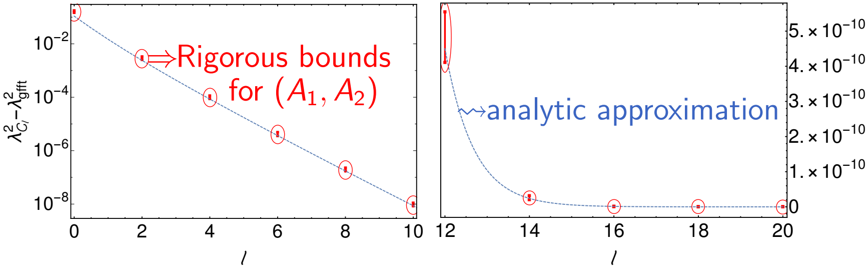

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Outline

1. The Superconformal Bootstrap Program
2. (A_1, A_2) Argyres-Douglas Theory
3. **Landscape of $4d$ $\mathcal{N} = 2$ SCFTs**
4. Summary & Outlook

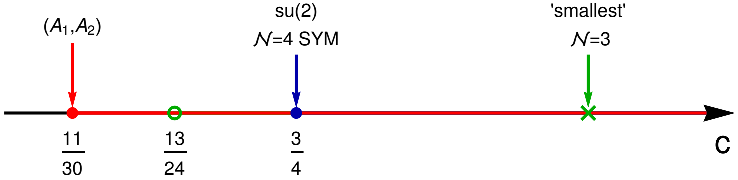
Landscape of $4d\ \mathcal{N} \geqslant 2$ SCFTs

Projection of space of SCFTs to an axis

→ $4d\ \mathcal{N} = 4$ SCFTs $c = a \geqslant \frac{3}{4}$ [Beem Rastelli van Rees]

→ $4d\ \mathcal{N} \geqslant 3$ SCFTs $c = a > \frac{13}{24}$ [Cornagliotto ML Schomerus]

→ $4d\ \mathcal{N} \geqslant 2$ SCFTs $c \geqslant \frac{11}{30}$ [Liendo Ramirez Seo]



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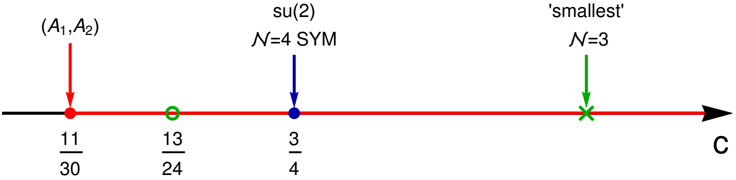

Finer view of the space of theories:

⇒ Organize theories by flavor symmetry

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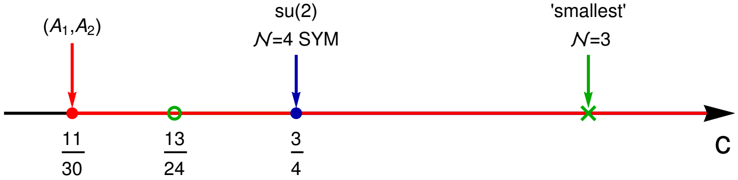


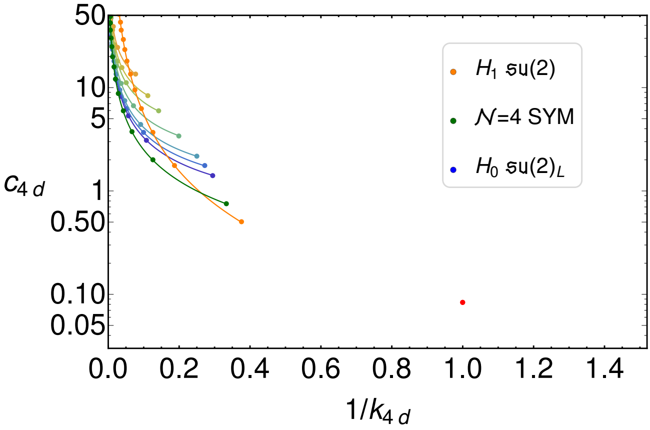
Finer view of the space of theories:

$\Rightarrow$ Organize theories by flavor symmetry

$\langle TT \rangle \propto c, \qquad \langle JJ \rangle \propto k$

$4d$ $\mathcal{N}=2$ SCFT with $su(2)$ flavor symmetry

- $4d$ Flavor current supermultiplet



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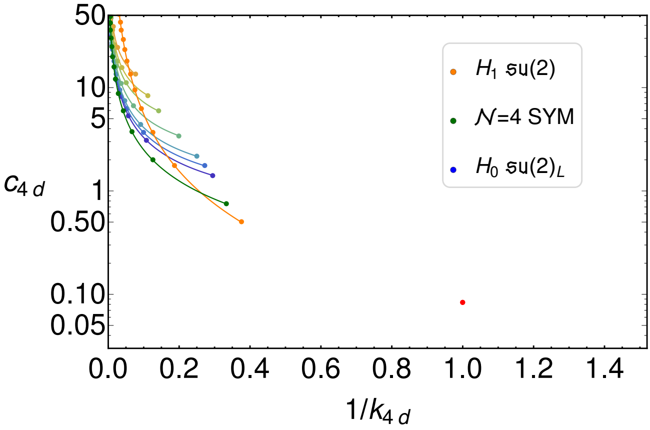

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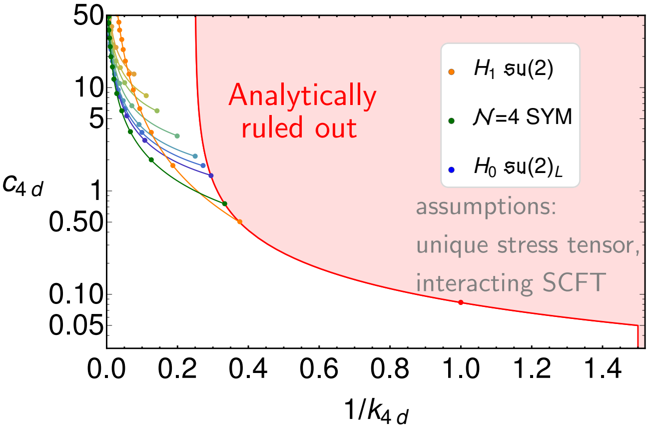

[Beem ML Liendo Peelaers Rastelli van Rees]

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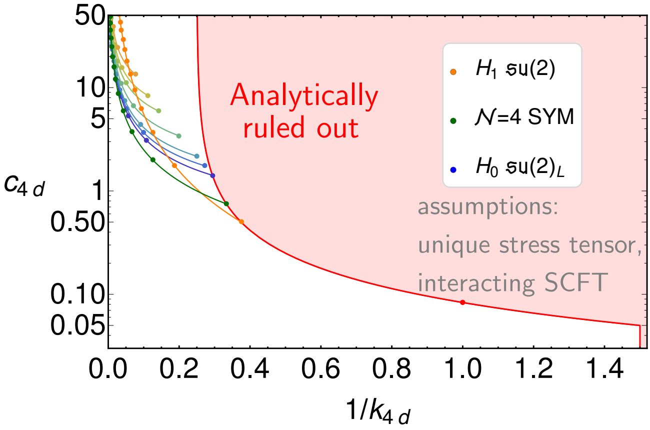

[Beem ML Liendo Peelaers Rastelli van Rees]

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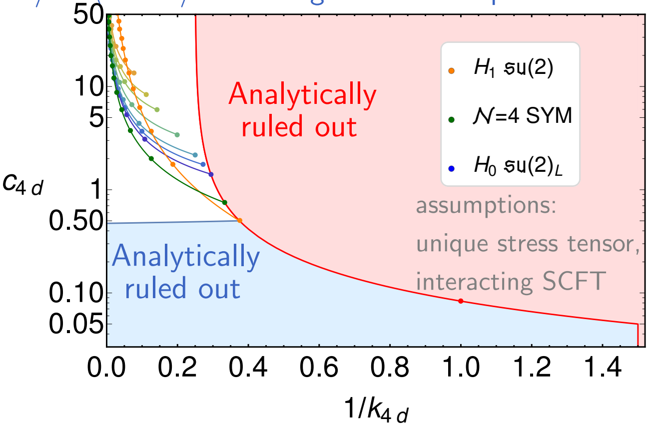

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]

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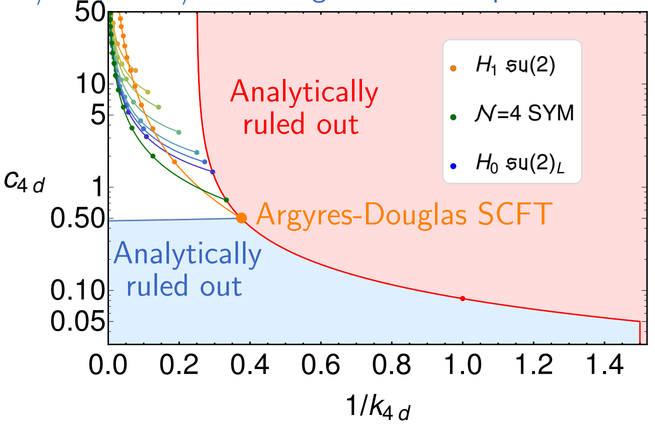

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]

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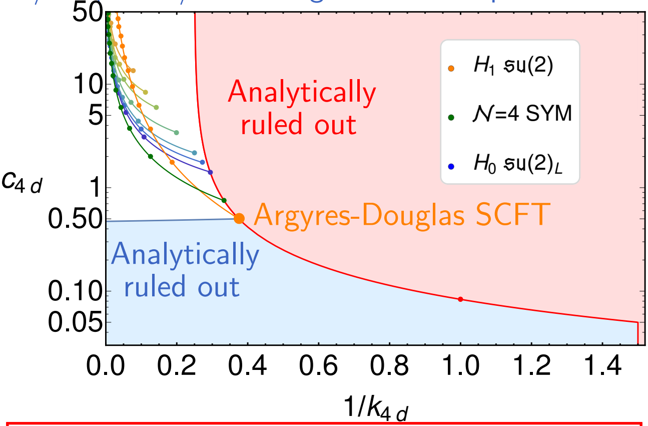

Only for $su(2)$, $su(3)$, $so(8)$, g_2, f_4, e_6, e_7, e_8

[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]

Outline

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Constrained the "simplest" Argyres-Douglas theory

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What is the "smallest" $\mathcal{N} = 3$ SCFT?

Thank you!

Backup slides

Outline

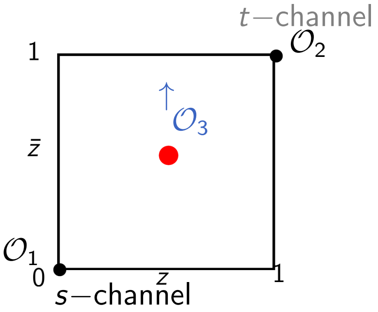
Inversion formula

5 **Lorentzian inversion formula for (A_1, A_2)**

6 **A solvable subsector**

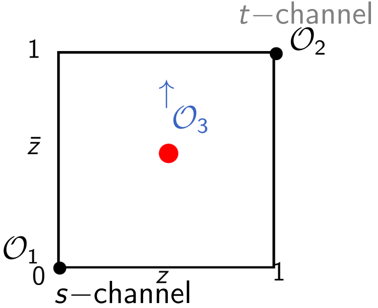
7 **Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs**

The lightcone bootstrap



The lightcone bootstrap

$\bar{z} \to 1$ **at fixed** z



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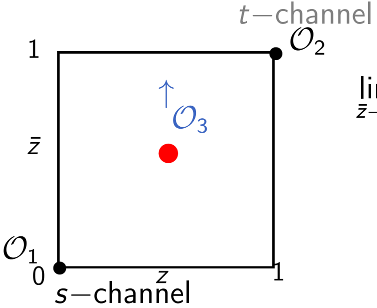

$\bar{z} \to 1$ at fixed z

$$\lim_{\bar{z}\to 1}(1-\bar{z})^{2\Delta_\phi}\sum_{\Delta,\ell}\lambda^2_{\phi\phi\mathcal{O}_{\Delta,\ell}}\quad\underbrace{g_{\Delta,\ell}(z,\bar{z})}$$

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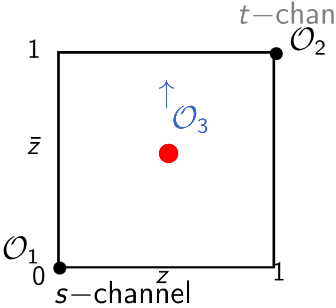

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$$= \left(\underbrace{f(z)} + \underbrace{(1-\bar{z})^{\tau_m} f_{\tau_m,\ell}(z)} + \dots \right)$$

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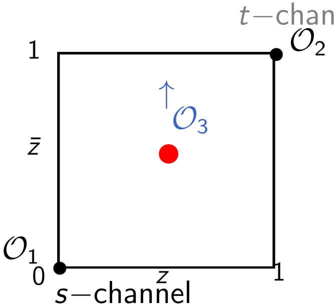

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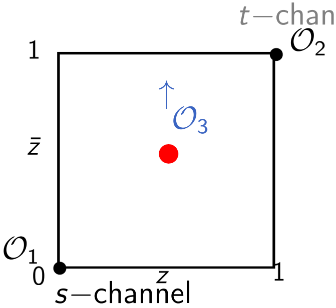

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$\rightarrow$ $t-$channel dominated by lowest twist τ_m operators

The lightcone bootstrap

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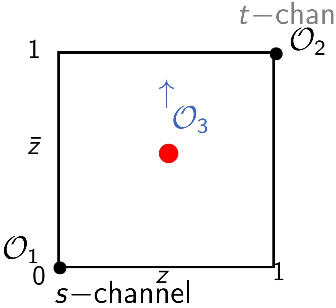

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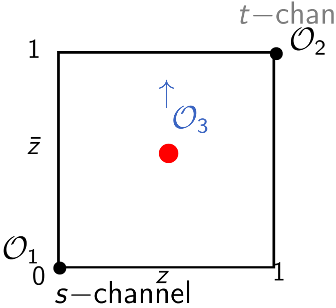

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$\rightarrow$ $t-$channel dominated by lowest twist τ_m operators

$\rightarrow$ behavior reproduced by infinite sum over $s-$channel spins

The lightcone bootstrap

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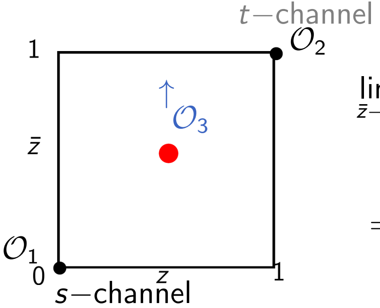

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- $\rightarrow$ $t-$channel dominated by lowest twist τ_m operators
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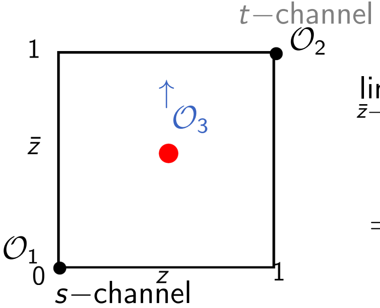
- $\rightsquigarrow$ Large spin spectrum of CFT

The lightcone bootstrap

$\bar{z} \to 1$ at fixed z



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$\mathbb{1} \Rightarrow \Delta \to 2\Delta_\phi + 2n + \ell \qquad (\phi\Box^n\partial_{\mu_1}\dots\partial_{\mu_\ell}\phi)$

A Lorentzian inversion formula

Large spin perturbation theory

→ Very successful for $3d$ Ising model

[Alday Zhiboedov, Simmons-Duffin]

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- $\rightarrow$ Operators organize in trajectories
- $\rightarrow$ large ℓ dominated by low $t-$channel twists

Outline

Inversion formula

5. **Lorentzian inversion formula for (A_1, A_2)**

6. **A solvable subsector**

7. **Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs**

Lorentzian inversion formula: Superconformal case

Invert $\phi\phi$ OPE

- $\rightarrow$ Same as bosonic inversion, valid for $\ell > 1$
- $\rightarrow$ Feed in $\bar{\phi}\phi \sim 1 +$ Stress tensor multiplet $+ \dots$

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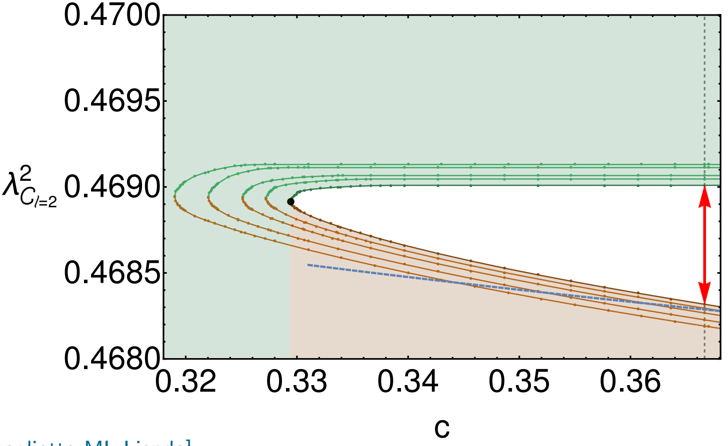
- $\rightarrow$ and in *u*-channel ($\phi\phi$)
  - $\hookrightarrow$ $\phi\phi \sim \phi^2 + \dots$

Bounding OPE coefficients

$$\phi\phi \sim \lambda^2_{\phi^2} \underbrace{\phi^2}_{\Delta=2\Delta_\phi} + \lambda^2_{\mathcal{C}_\ell} \underbrace{\mathcal{C}_{\ell>0}}_{\Delta=2\Delta_\phi+\ell} + \cdots$$



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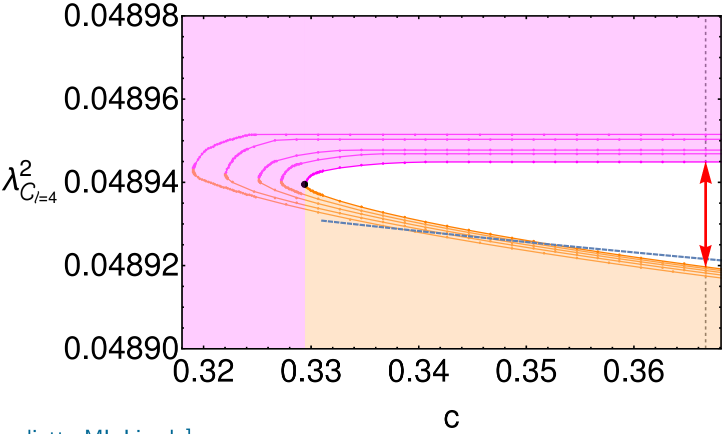

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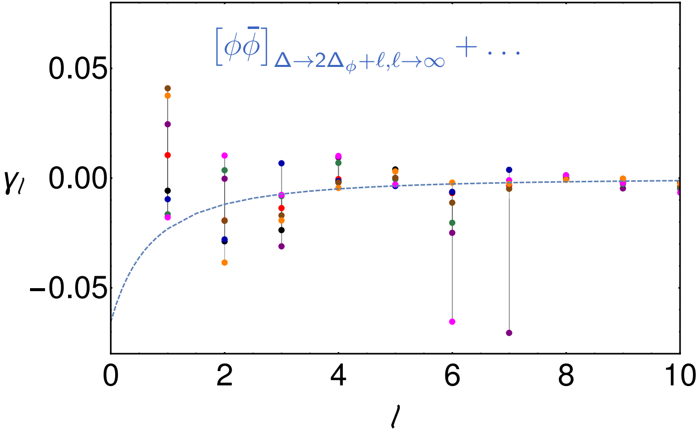

Outline

Chiral algebra

Organize operators in representations of superconformal algebra

$\{\mathcal{O}_{\Delta,(j_1,j_2),} \qquad \}$

Chiral algebra

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$$\{\mathcal{O}_{\Delta,(j_1,j_2),\underbrace{R}_{SU(2)_R},\underbrace{r}_{U(1)_r},f}\}$$

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→ Meromorphic!

Chiral algebra

Why?

- Subsector = Cohomology of nilpotent $\mathbb{Q}$

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Example: free hypermultiplet

Complex scalars in hypermultiplet are in the cohomology

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$4d\ \mathcal{N} \geqslant 2$ SCFT $\longrightarrow$ chiral algebra

Which operators are in the cohomology?

$\rightarrow$ Stress tensor $T_{\mu\nu}$

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$4d$ $\mathcal{N} \geqslant 2$ **SCFT** $\longrightarrow$ **Chiral algebra**

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Outline

Inversion formula

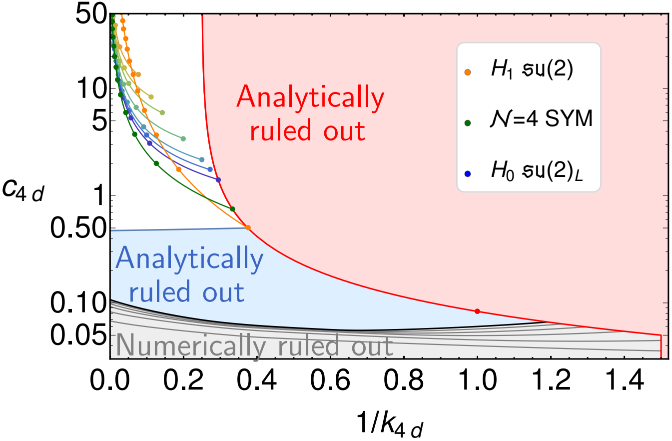
5 **Lorentzian inversion formula for** (A_1, A_2)

6 **A solvable subsector**

7 **Constraining the space of** $4d$ $\mathcal{N}=2$ **SCFTs**

Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs

$su(2)$ flavor symmetry



[Beem, ML, Liendo, Peelaers, Rastelli, van Rees; ML, Liendo]

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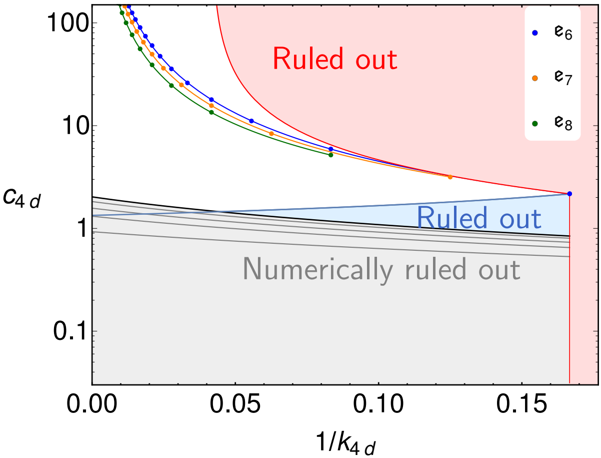
Constraining the space of $4d$ $\mathcal{N}=2$ SCFTs

e_6 flavor symmetry



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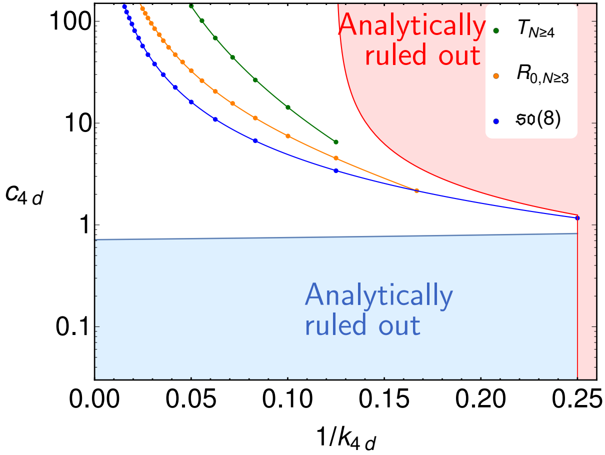
Constraining the space of $4d\ \mathcal{N}=2$ SCFTs

$su(4)$ flavor symmetry



[Beem, ML, Liendo, Peelaers, Rastelli, van Rees; ML, Liendo]

Constraining the space of $4d$ $\mathcal{N} = 2$ SCFTs

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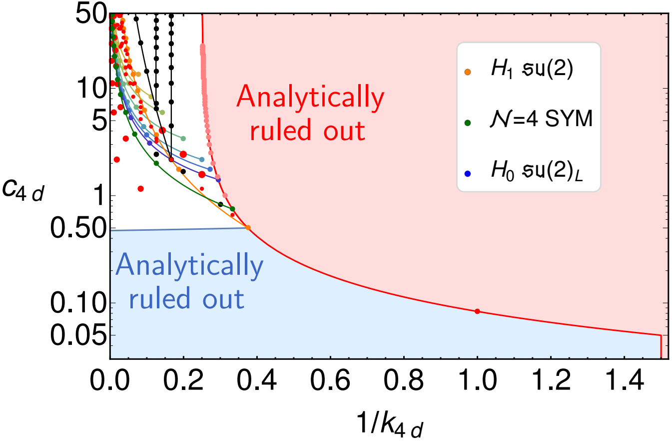

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