Unifying correlators in $AdS_5 \times S_5$ supergravity

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on:
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Integrability in Gauge and String Theory 2018
August 20, Copenhagen

We'll study 4-point correlators in N=4 SYM at strong 't Hooft coupling

- Nonplanar effects ($1/N_c$) in a theory with *nice* planar limit. What remains of integrability?
- How does 10D supergravity on $AdS_5 x S_5$ emerge from CFT?!?
  Get concrete data about string theory at low energies
- Eventual comparison with integrability results

General CFT framework:

1. Spectrum:

-Protected single-traces:

$$\mathcal{O}^p \simeq \mathrm{Tr}[\phi^{i_1} \ldots \phi^{i_p}] \qquad \Delta = p \geq 2,$$

-~~Unprotected single~~-traces $\Delta \sim \lambda^{1/4} \gg 1$ heavy

-Multi traces $\mathcal{O}^p \partial^n \mathcal{O}^q$ $\Delta \approx p + q + n + \gamma/N_c^2$

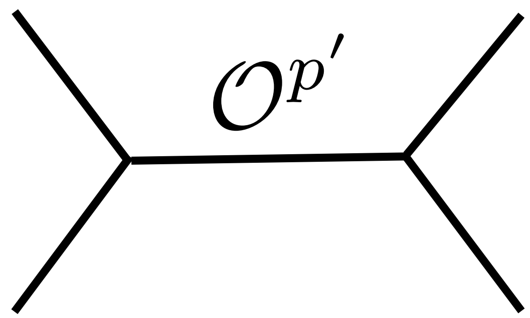
This talk's focus: composites of various single-traces

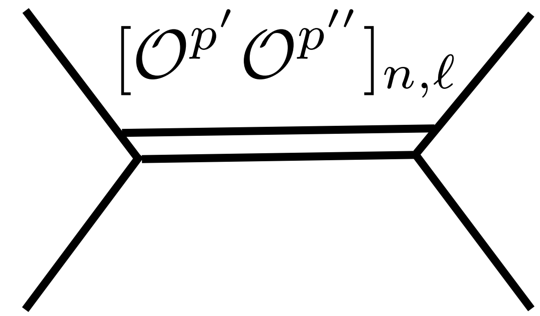
p=2: stress tensor

p>2: S_5 graviton spherical harmonics

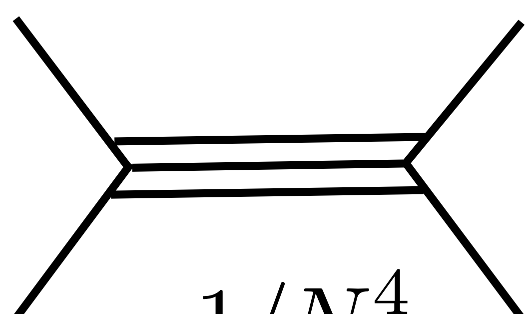
General CFT framework:

2. Operator Product Expansion (OPE)

$\langle \mathcal{O}^p \mathcal{O}^q \mathcal{O}^r \mathcal{O}^s \rangle = \sum$ 

+ 

$\sim 1/N_c^2$ **tree-level**

+ 
$\sim 1/N_c^4$ **one-loop**

+ ...

Multi-traces encode supergravity loop expansion

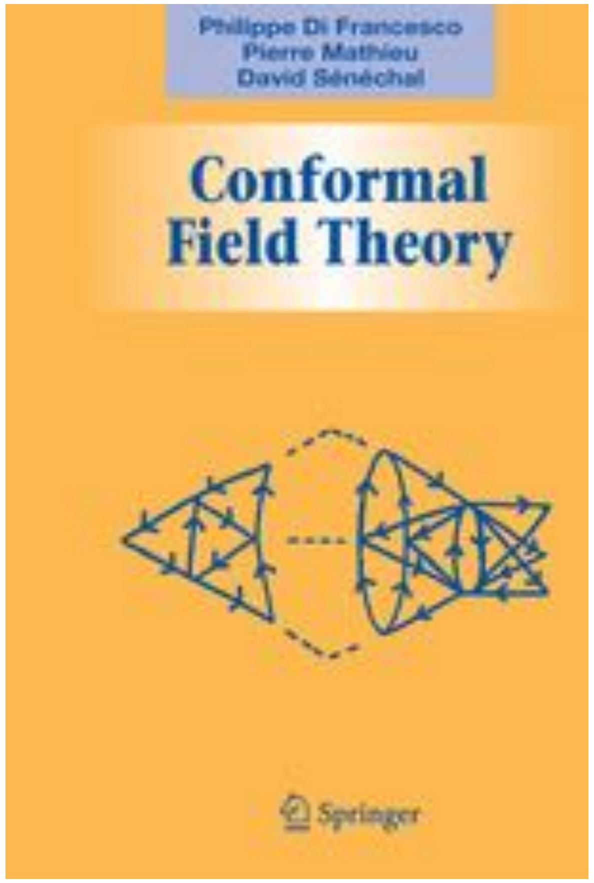
Main tool: recall Kramers-Kronig relation

$$f(E) = f(\infty) + \int \frac{dE' \, \mathrm{Disc}\, f(E')}{2\pi(E' - E - i0)}$$

Ex: Re(f) ~ phase velocity of light
Im(f) ~ absorption by medium

Determines propagation from absorption

consequence of causality (analyticity at complex energies)



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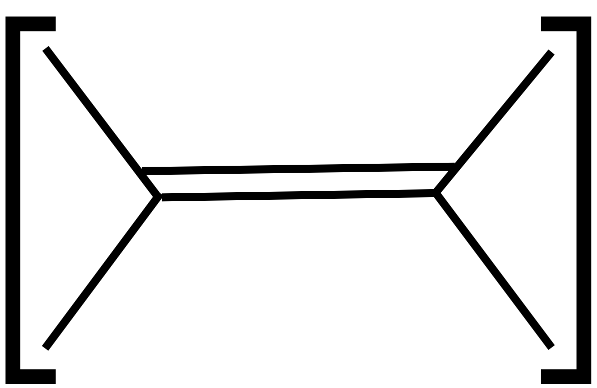
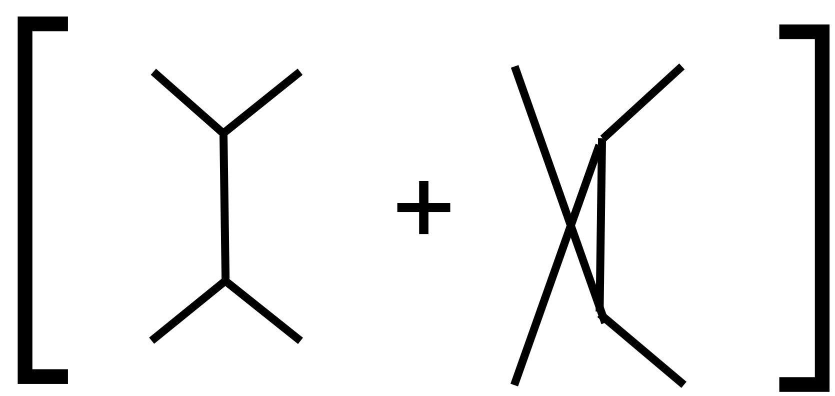
=CFT dispersion relation

[SCH '17]

Reconstructs double-traces from single traces:

coeff.  $= \sum\!\!\int$ 

finite sum

'absorptive part'

~ single traces

Part I

- Lorentzian inversion formula
- Superconformal Ward identities & OPE
- Bootstrapping double-trace dimensions

Part II

An unexpected SO(10,2) symmetry

Lorentzian inversion formula

$$c(J,\Delta) = \int_{\diamond} \left[G_{\Delta+1-d,j+d-1}\right] \times \left[\mathrm{dDisc}\, G\right]$$

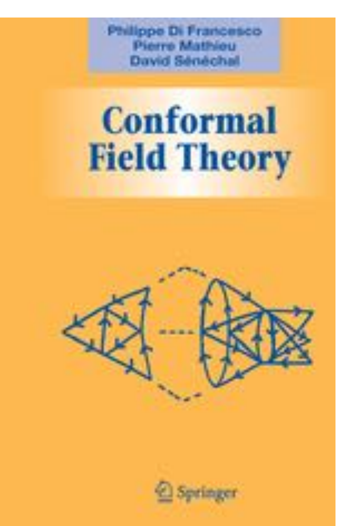
s-channel OPE coefficients

causal diamond

block with J and Δ exchanged

absorptive part

[SCH '17]



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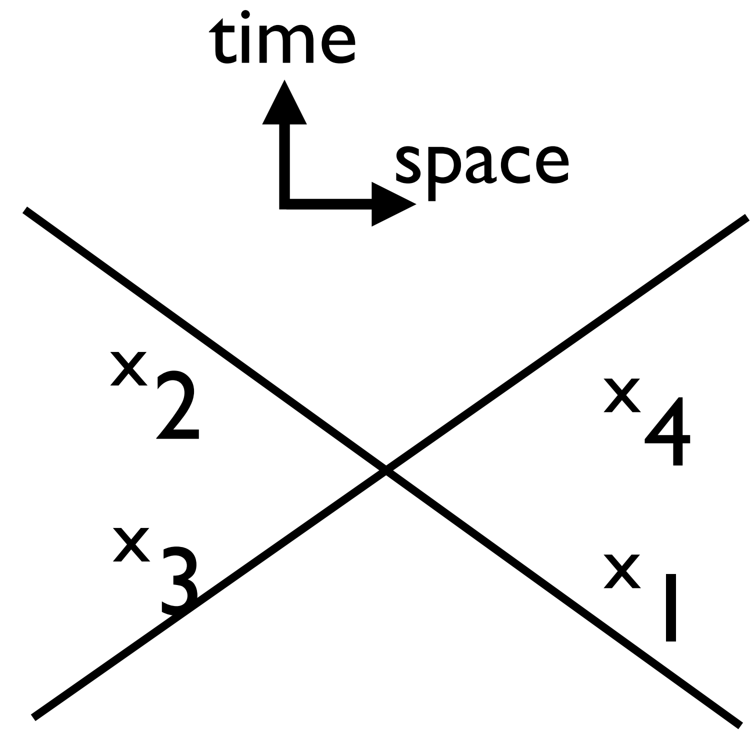
converges for J>1
(boundedness in Regge limit)

[see also: Simmons-Duffin, Stanford& Witten; Kravchuk& Simmons-Duffin '18]

Input: double commutator



$$\mathrm{dDisc}\ G \equiv -\frac{1}{2}\langle 0|[\phi_4,\phi_1][\phi_2,\phi_3]|0\rangle$$

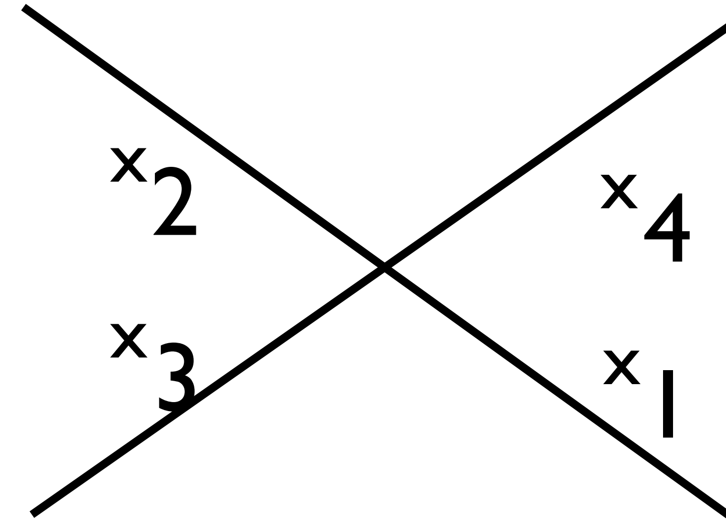
Positive-definite and bounded object. Simplifications:

1. Large J ($z \to 1$): low twists in cross-channel

$\Rightarrow$ 1/J expansion [cf Fernando Alday's talk!]

2. Large N_c: dominated by single-traces

Commutators kill double-traces:

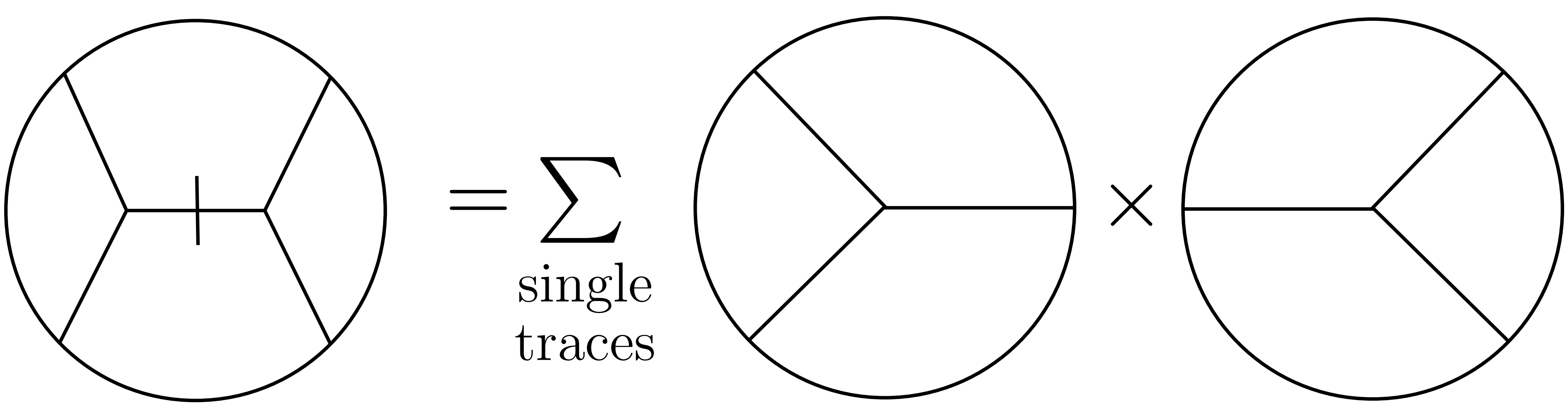


$$\phi_1\phi_4 \sim \sum_{j,\Delta} c_{j,\Delta}((x_1-x_4)^2)^{\frac{\Delta-\Delta_1-\Delta_4}{2}}$$

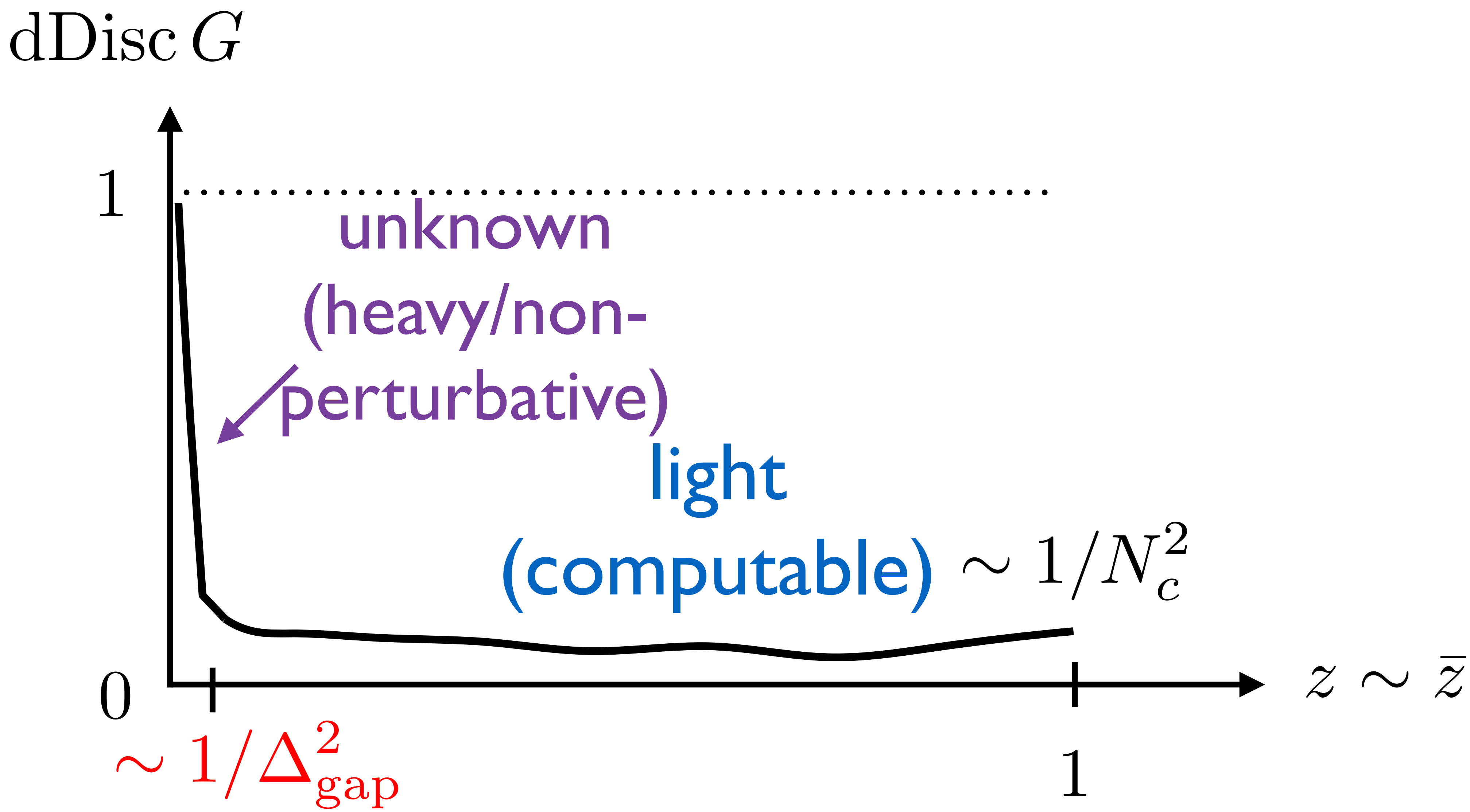
$$[\phi_1,\phi_4] \sim \sum_{j,\Delta} c_{j,\Delta}|(x_1-x_4)^2|^{\frac{\Delta-\Delta_1-\Delta_4}{2}} \sin(\pi\frac{\Delta-\Delta_1-\Delta_4}{2})$$

$\propto \gamma/N_c^2$

dDisc ~ imaginary part of Witten diagrams!



Nonperturbative picture:



Knowing just light single-traces, dispersion relation yields:
$\Rightarrow$ full OPE data w/ controlled perturbative+nonperturbative corrections!

[see also: Alday, Bissi&Perlmutter; Li, Meltzer&Poland]

Superconformal OPE

We study correlator of four half-BPS primaries in $[0,p,0]$

$$\mathcal{O}^p(x,y) \equiv y^{i_1}\cdots y^{i_p}\ \mathrm{Tr}[\phi^{i_1}\cdots\phi^{i_p}] - (\text{multi traces})$$

Null six-vectors *y* conveniently tracks R-symmetry indices

Correlator depends on cross-ratios:

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}, \qquad v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z}),$$

$$\sigma = \frac{y_{12}^2 y_{34}^2}{y_{13}^2 y_{24}^2} = \alpha\bar{\alpha}, \qquad \tau = \frac{y_{23}^2 y_{14}^2}{y_{13}^2 y_{24}^2} = (1-\alpha)(1-\bar{\alpha}),$$

In principle we should use superconformal blocks

For four half-BPS ops, really only need Ward identity:

the z dependence disappears when α=z

[Nirschl& Osborn '04]

General solution:

$$\mathcal{G}_{\{p_i\}}(z,\bar{z},\alpha,\bar{\alpha}) = k\chi(z,\alpha)\chi(\bar{z},\bar{\alpha}) + \frac{(z-\alpha)(z-\bar{\alpha})(\bar{z}-\alpha)(\bar{z}-\bar{\alpha})}{(\alpha-\bar{\alpha})(z-\bar{z})} \times\left(-\frac{\chi(\bar{z},\bar{\alpha})f(z,\alpha)}{\alpha z(\bar{z}-\bar{\alpha})} + \frac{\chi(\bar{z},\alpha)f(z,\bar{\alpha})}{\bar{\alpha}z(\bar{z}-\alpha)} + \frac{\chi(z,\bar{\alpha})f(\bar{z},\alpha)}{\alpha\bar{z}(z-\bar{\alpha})} - \frac{\chi(z,\alpha)f(\bar{z},\bar{\alpha})}{\bar{\alpha}\bar{z}(z-\alpha)}\right) + \frac{(z-\alpha)(z-\bar{\alpha})(\bar{z}-\alpha)(\bar{z}-\bar{\alpha})}{(z\bar{z})^2(\alpha\bar{\alpha})^2} H_{\{p_i\}}(z,\bar{z},\alpha,\bar{\alpha}),$$

k, $f(z,\alpha)$ =protected

$H(z,\bar{z},\alpha,\bar{\alpha})$ =dynamical

Superconformal Casimir commutes with decomposition

$$f_{\{p_i\}}(z,\alpha) = \sum_{j,m} b_{\{p_i\}}(j,m) k_{1+m+j}(z) k_{-m}(\alpha)$$

$$H_{\{p_i\}}(z,\bar{z},\alpha,\bar{\alpha}) = \sum_{j,\Delta,m,n} a_{\{p_i\}}(j,\Delta,m,n) G_{j,\Delta}(z,\bar{z}) Z_{m,n}(\alpha,\bar{\alpha})$$

These are standard bosonic blocks:

$$k \sim {}_2F_1, \quad G \sim \frac{z\bar{z}}{z-\bar{z}}(kk - kk), \quad Z \sim \text{similar}$$

[cf Bissi& Lukowksi '15]

In practice, we won't need 'superconformal blocks':

a(j,Δ;m,n) and b(j;m) contain all information!

Disconnected correlator

At order N[0], correlator is very simple:

$$\mathcal{G}^{(0)}_{pqqp} = \delta_{p,q} + \left(\frac{u}{\sigma}\right)^{\frac{p+q}{2}} \left[\left(\frac{\tau}{v}\right)^{q} + \delta_{p,q}\right]$$

SUSY decomposition gives mess, however

Trick: magic differential operator which kills protected stuff

Supermultiplets which can appear in [0,p,0] correlators:

Multiplet	Dynkin labels	Dimension Δ and spin ℓ
Half-BPS $\mathcal{B}_{0,n}$	$[0,n,0]$	$\Delta = q,\ \ell = 0$
Quarter-BPS $\mathcal{B}_{m,n}$	$[m,n{-}m,m]$	$\Delta = m+n,\ \ell = 0,\ m \geq 1$
Semi-short $\mathcal{C}_{\ell,m,n}$	$[m,n{-}m,m]$	$\Delta = m+n+2+\ell$
Long $\mathcal{A}_{\ell,\Delta,m,n}$	$[m,n{-}m,m]$	$\Delta > m+n+2+\ell$

All but longs are killed by a 8th order operator:

$$\Delta^{(8)} H_{\{p_i\}} \equiv \frac{z\bar{z}\alpha\bar{\alpha}}{(z-\bar{z})(\alpha-\bar{\alpha})} \left(\mathcal{D}_z - \mathcal{D}_\alpha\right)\left(\mathcal{D}_z - \mathcal{D}_{\bar{\alpha}}\right)\left(\mathcal{D}_{\bar{z}} - \mathcal{D}_\alpha\right)\left(\mathcal{D}_{\bar{z}} - \mathcal{D}_{\bar{\alpha}}\right) \frac{(z-\bar{z})(\alpha-\bar{\alpha})}{z\bar{z}\alpha\bar{\alpha}} H_{\{p_i\}}$$

$$\mathcal{D}_x \equiv x^2\partial_x(1-x)\partial_x - \tfrac{1}{2}(r+s)x^2\partial_x - \tfrac{1}{4}rsx$$

Physically, gives correlator of chiral Lagrangians (10D axidilaton)

[Drummond, Gallot & Sokatchev '06]

$$\Delta^{(8)} H^{(0)}_{pqqp} = \frac{u^{\frac{p+q}{2}+2}}{\sigma^{\frac{p+q}{2}-2}} \left(\frac{\tau^{q-2}}{v^{q+2}} + \delta_{p,q}\right) \times C(p)C(q) \qquad C(p) = p^2(p^2-1)$$

Since $\Delta^{(8)}H$ =generalized free field, OPE takes simple form:

$$a_{\{p_i\}}(j,\Delta,m,n) = \frac{1}{\Delta^{(8)}_{m,n}(h,\bar{h})} \times (\Gamma\text{-functions})$$

Here we have divided by the eigenvalue:

$$\Delta^{(8)}_{m,n}(h,\bar{h}) = \left(h - \frac{n-m+2}{2}\right)\left(h + \frac{n-m}{2}\right)\left(h - \frac{m+n+4}{2}\right)\left(h + \frac{m+n+2}{2}\right)$$
$$\times \left(\bar{h} - \frac{n-m+2}{2}\right)\left(\bar{h} + \frac{n-m}{2}\right)\left(\bar{h} - \frac{m+n+4}{2}\right)\left(\bar{h} + \frac{m+n+2}{2}\right)$$

$$h = 1 + \frac{\Delta - j}{2}, \qquad \bar{h} = 2 + \frac{\Delta + j}{2}$$

[cf Drummond et al '18]

From this N^0 OPE data, can *derive* formulas for various superconformal blocks

Ex: half-BPS block with [0,p+q,0]
= sum of all contributions with Casimir eigenvalue 0

$$\mathcal{B}^{r,s}_{0,\Delta} = \begin{cases} k=1, \qquad f(z,\alpha) = \displaystyle\sum_{i=\max(|r|,|s|)}^{\Delta-2} k^{r,s}_{1+\frac{i}{2}}(z)\; k^{-r,-s}_{-\frac{i}{2}}(\alpha), \\ H(z,\bar{z},\alpha,\bar{\alpha}) = \displaystyle\sum_{i=\max(|r|,|s|)}^{\Delta-4} \sum_{j=0}^{(\Delta-i)/2} G^{r,s}_{j,i+j+4}(z,\bar{z})\; Z^{r,s}_{j,i+j}(\alpha,\bar{\alpha}). \end{cases}$$

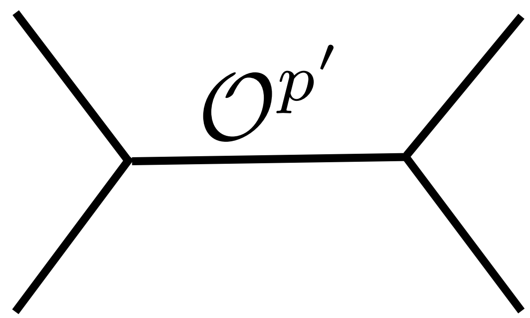
Agrees with earlier formulas for superconformal blocks

[Dolan& Osborn, ...]

Order 1/N^2

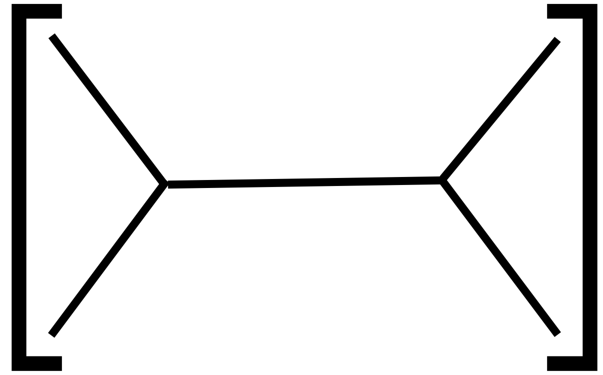
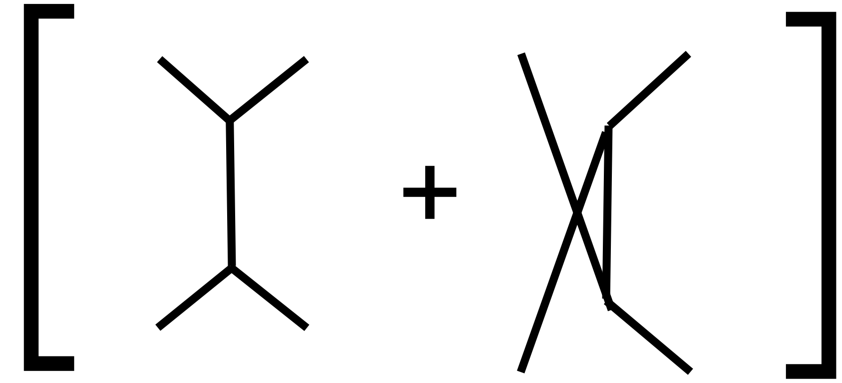
At this order, dDisc saturated by single-traces (thus half-BPS)

$$\mathrm{dDisc}\; G = \sum$$

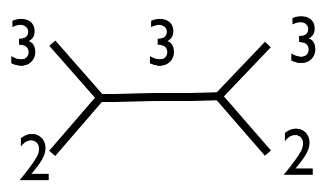


$$\underset{\text{finite sum}}{=} \sum f_{pqp'} f_{rsp'} \mathcal{B}_{p'}$$

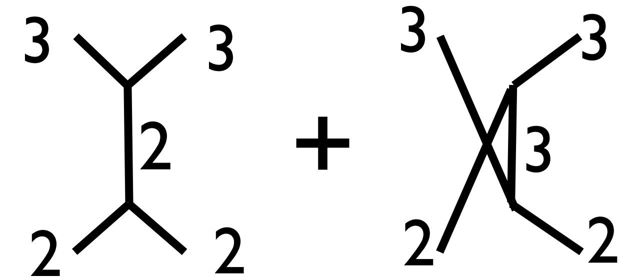
Inversion formula (*thank you SUSY!*) converges for J>-2

In particular, we get a crossing equation for the half-BPS f's coeff.  $= \sum \int$ 

Ex:



$$\Big|_{1/2-\mathrm{BPS}} = f_{233}^2 \; \mathcal{B}_{0,3} \quad = \int$$



$$= \left(\frac{1}{3} f_{222} f_{233} + \frac{7}{9} f_{233}^2 \right) \mathcal{B}_{0,3} \quad + \ldots$$

From this we deduce: $f_{233} = \frac{3}{2} f_{222}$ ✓

In this way we bootstrap all 3-point couplings!

Matches known @weak&strong coupling

$$f_{pqr} = \sqrt{\frac{pqr}{4c}}$$

Free theory

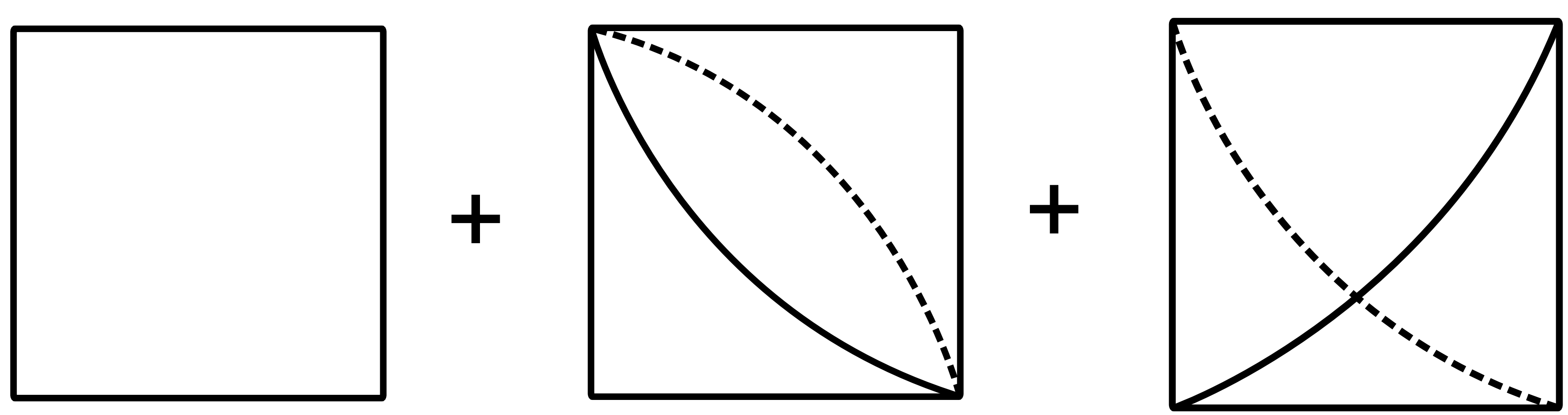
Having bootstrapped all 3-pt couplings,
get full protected part $f(z,\alpha)$ at order 1/N²

[1D inversion: Simmons-Duffin, Stanford& Witten]

Expect to match free theory = poly($u/\sigma, \tau/v$)

In fact enough to fully reconstruct free theory! ✓

$$G^{\text{free}} = \sum$$



where each line represents ≥0 Wick contractions

Double-trace mixing

from the dDisc of the correlator, get all OPE data:

Ex: $$c_{2442}(\ell,\Delta,0,2)=\int_0^1\frac{dz}{z^2}(1-z)^2k^{2,2}_{1-h}(z)z^{-1}\int_0^1\frac{d\bar z}{\bar z^2}(1-\bar z)^2\kappa(\bar h)k^{2,2}_{\bar h}(\bar z)\bar z^{-1}$$
$$\times\left[2\left(\frac{z}{1-z}\right)^3-4\left(\frac{z}{1-z}\right)^4-4\left(\frac{z}{1-z}\right)^5\log(z)\right]\mathrm{dDisc}\left(\frac{\bar z}{1-\bar z}\right)^3$$

=dDisc G_{2442}

The log(z) term gives anomalous dimensions

All integrals give just bunch of Γ's

$$\int_0^1\frac{d\bar z}{\bar z^2}(1-\bar z)^{\frac12(p_{21}+p_{34})}\tilde\kappa\left(\bar h/2\right)k^{p_{21},p_{34}}_{\bar h/2}(\bar z)\mathrm{dDisc}\left[\left(\frac{1-\bar z}{\bar z}\right)^\lambda\bar z^{-p_{34}/2}\right]$$
$$=\frac{r^{p_{21},p_{34}}_{\bar h}}{\Gamma(-\lambda)\Gamma(-\lambda-\frac{p_{21}+p_{34}}{2})}\frac{\Gamma(\bar h-\lambda-p_{34}/2-1)}{\Gamma(\bar h+\lambda+p_{34}/2+1)}$$

Anomalous dimension are really mixing matrices

For example, at twist 6 and R-symmetry rep [0,2,0]:

$$\gamma^{(1)}_{6,0,2} = \begin{pmatrix} \gamma^{(1)}_{24,24} & \gamma^{(1)}_{24,33} \\ \gamma^{(1)}_{33,24} & \gamma^{(1)}_{33,33} \end{pmatrix} \quad \text{with:} \quad \gamma^{(1)}_{pq,rs} \equiv \frac{\langle a^{(0)}\gamma^{(1)}\rangle_{pqrs}}{\sqrt{\langle a^{(0)}\rangle_{pqqp}\langle a^{(0)}\rangle_{rssr}}}$$

For even spins this evaluates to:

$$\left(\gamma^{(1)}\right)^{+}_{6,0,2} = \frac{-60}{(\bar{h}-4)(\bar{h}-1)(\bar{h})(\bar{h}+3)} \begin{pmatrix} 12+\bar{h}(\bar{h}-1) & 6\sqrt{\bar{h}(\bar{h}-1)} \\ 6\sqrt{\bar{h}(\bar{h}-1)} & 6+\bar{h}(\bar{h}-1) \end{pmatrix}$$

Amazingly, the *eigenvalues* are simple:

$$\left(\gamma^{(1)}\right)^{+}_{6,0,2} = \left\{ \frac{-\Delta^{(8)}_{0,2}(4,\bar{h})}{(\bar{h}-4)_6}, \; \frac{-\Delta^{(8)}_{0,2}(4,\bar{h})}{(\bar{h}-2)_6} \right\} \qquad (\bar{h} = j+5)$$

Straightforward to look at other cases, ie twist=8 [0,2,0]:

$$\begin{pmatrix} (2442) & (2433) & (2435) & (2444) \\ (3342) & (3333) & (3335) & (3344) \\ (3542) & (3533) & (3553) & (3544) \\ (4442) & (4433) & (4453) & (4444) \end{pmatrix}$$

In all cases we reproduce a recent conjecture: ✓

[Aprile, Drummond, Heslop&Paul '18]

All eigenvalues take the form:

$$\gamma^{(1)} = -\frac{1}{c}\frac{\Delta^{(8)}}{(j+1+\text{integer})_6} \quad !$$

Part II

An unexpected SO(10,2) symmetry

First, let us emphasize conjectured formula:

[Aprile, Drummond, Heslop&Paul '18]

$$\gamma = -\frac{\Delta^{(8)}}{c} \times \frac{1}{(j+1+m)_6} + O(1/c^2)$$

Crazy that complicated matrix has rational eigenvalues!

Crazier: Take flat space 10D dilation scattering:

Expand over10D Legendre polynomials

First, let us emphasize conjectured formula:

[Aprile, Drummond, Heslop&Paul '18]

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Crazier: Take flat space 10D dilation scattering:

$$A^{(10)}(s,t) = 8\pi G_N s^4 \times \frac{1}{stu}$$

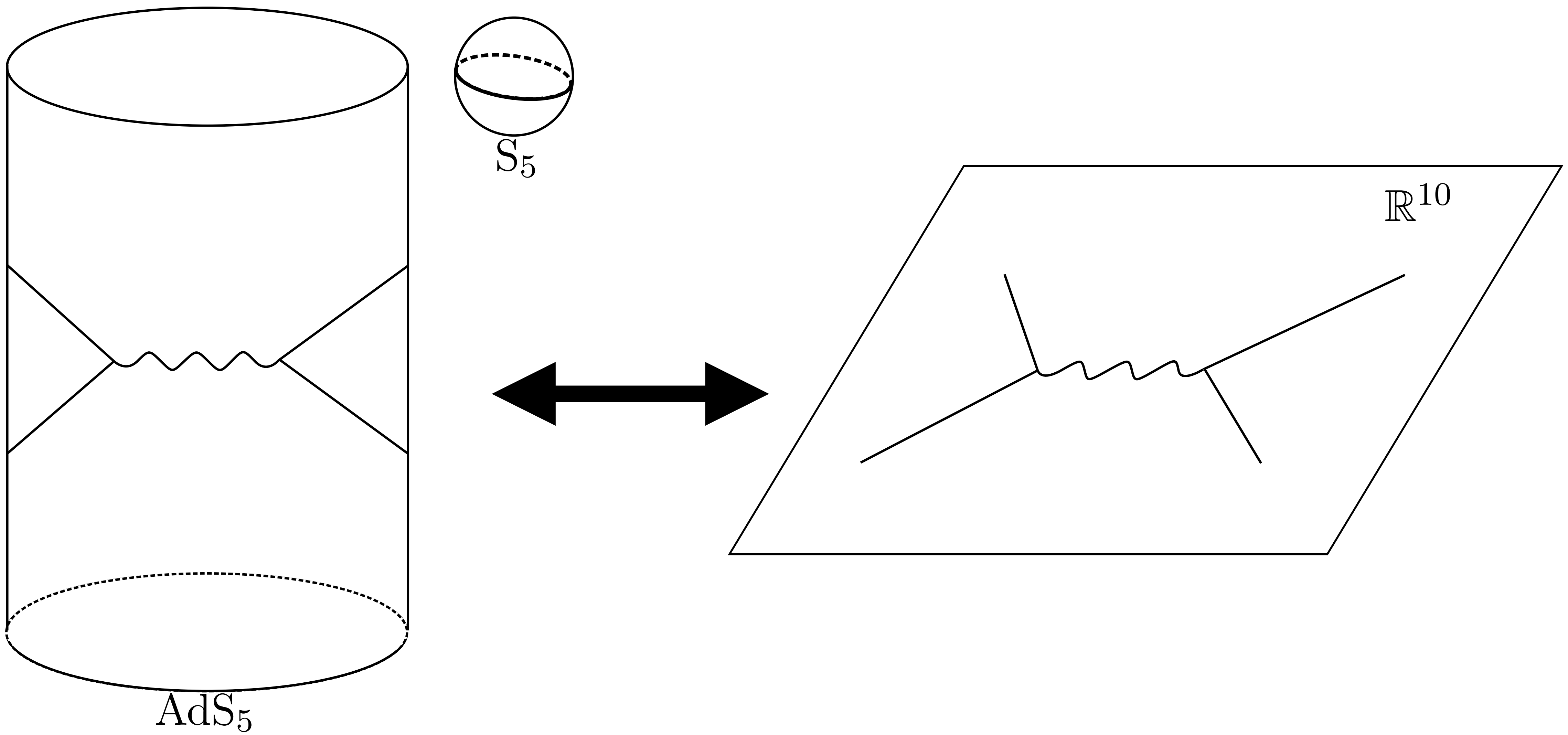
Expand over10D Legendre polynomials $\left(G_N = \frac{\pi^4 L^8}{8c}\right)$

$$A_\ell^{(10)}(s) = -\frac{(L\sqrt{s}/2)^8}{c} \times \frac{1}{(j+1)_6} + O(1/c^2)$$

$$\gamma = -\frac{\Delta^{(8)}}{c}\frac{1}{(j+1+m)_6} \Leftrightarrow A_\ell^{(10)}(s) = -\frac{(L\sqrt{s}/2)^8}{c}\frac{1}{(j+1)_6}$$

CFT correlator is just flat 10D amplitude!?!?!?!

$$(L\sqrt{s}/2)^8 \leftrightarrow \Delta^{(8)}$$



Our proposed explanation:

1. 10D supergravity [@4-pt] ≃ CFT (coupling = $G_N s^4$)

$$A^{(10)}(s,t) = 8\pi G_N s^4 \frac{1}{stu}$$

scale-invariant & 10D conformal

2. $AdS_5 x S_5$ is conformal to flat space

Conjecture:

All 4-pt SYM correlators stem from a common 10D-conformal object

$$SO(4,2) \times SO(6)_R \subset SO(10,2)$$

Conjecture:

All 4-pt SYM correlators stem from a common 10D-conformal object

The object is 'dilaton correlator'

$$\langle \phi(w_1)\phi(w_2)\bar{\phi}(w_3)\bar{\phi}(w_4)\rangle_{10} \equiv \frac{G_{10}(u_{10}, v_{10})}{\left((x_{12}^2 - y_{12}^2)(x_{34}^2 - y_{34}^2)\right)^4}$$

depends only on 10D distances $x_{ij}^2 - y_{ij}^2$:

$$u_{10} \equiv \frac{(x_{12}^2 - y_{12}^2)(x_{34}^2 - y_{34}^2)}{(x_{13}^2 - y_{13}^2)(x_{24}^2 - y_{24}^2)}, \qquad v_{10} \equiv \frac{(x_{23}^2 - y_{23}^2)(x_{14}^2 - y_{14}^2)}{(x_{13}^2 - y_{13}^2)(x_{24}^2 - y_{24}^2)}$$

To extract SYM correlator H_{pqrs} ,

series-expand G_{10} in y's and take term with correct weight

$$\tilde{H}_{p_1p_2p_3p_4}(u,v,\sigma,\tau) = \oint \prod_{i=1}^{4}\left[\frac{da_i\, a_i^{1-p_i}}{2\pi i}\right] \frac{(u/\sigma)^{\frac{p_1+p_2}{2}-2}}{(1-\frac{\sigma}{u}a_1a_2)^4(1-a_3a_4)^4} \times G_{10}\left(u\frac{(1-\frac{\sigma}{u}a_1a_2)(1-a_3a_4)}{(1-a_1a_3)(1-a_2a_4)}, v\frac{(1-\frac{\tau}{v}a_2a_3)(1-a_1a_4)}{(1-a_1a_3)(1-a_2a_4)}\right)$$

SO(10,2) symmetry thus predicts differential relations:

$$\tilde{H}_{pqrs} = \mathcal{D}_{pqrs}\tilde{H}_{2222}$$

$$\mathcal{D}_{2222} = 1,$$

$$\mathcal{D}_{2332} = -\frac{\sqrt{u}}{\sqrt{\sigma}}\tau\partial_v,$$

$$\mathcal{D}_{2233} = 4 - u\partial_u,$$

$$\mathcal{D}_{3333} = 16 - 8u\partial_u + \frac{u+\sigma}{\sigma}(u\partial_u)^2 + 2\frac{u}{\sigma}u\partial_u v\partial_v + \frac{u(v+\tau)}{\sigma v}(v\partial_v)^2\,.$$

...

correct object is $\tilde{H}^{(1)} \equiv \text{``}\frac{\Delta^{(8)} H}{\Delta^{(8)}}\text{''} \equiv H^{(1)} - H^{(1),\text{free}}$

1. Check against classic results:

$$\tilde{H}^{(1)}_{2222} = -u^4 \bar{D}_{2,4,2,2}(u,v)$$

$$\tilde{H}^{(1)}_{2332} = -\frac{u^{9/2}}{\sqrt{\sigma}} \tau \bar{D}_{2,5,3,2},$$

$$\tilde{H}^{(1)}_{2233} = -u^4 (\bar{D}_{2,4,2,2} - \bar{D}_{2,4,3,3}),$$

$$\tilde{H}^{(1)}_{3333} = \dots$$

[D'Hoker, Freedman, Mathur, Matusis& Rastelli '99;
Arutyunov, Dolan, Osborn&Sokatchev '02-;
Berdichevsky& Naaijkens '03;
Dolan,Nirschl&Osborn '06;
Uruchurtu '08-]

2. More generally: suffices to check the dDisc=pole terms

$$\tilde{H}^{(1)}_{pqrs}\Big|_{v-\text{poles}} \stackrel{?}{=} \mathcal{D}_{pqrs}\left[-\frac{2u^4 \log u}{(1-u)^3 v} - \frac{u^3(1+u)}{(1-u)^2 v}\right]$$

easy to check to high order p,q,r,s~10!!

Derive Rastelli-Zhou's formula: write Mellin rep for H2222

$$G_{10}(w_i) = \int dsdt \frac{\Gamma(2-\frac{s}{2})^2\Gamma(2-\frac{s}{2})^2\Gamma(\frac{s+t}{2})^2}{(s-4)(t-4)(s+t-2)} \times (x_{12}^2 - y_{12}^2)^{\frac{s}{2}-2}\cdots$$

Expand in y: each « Gamma times power » just gets shifted!

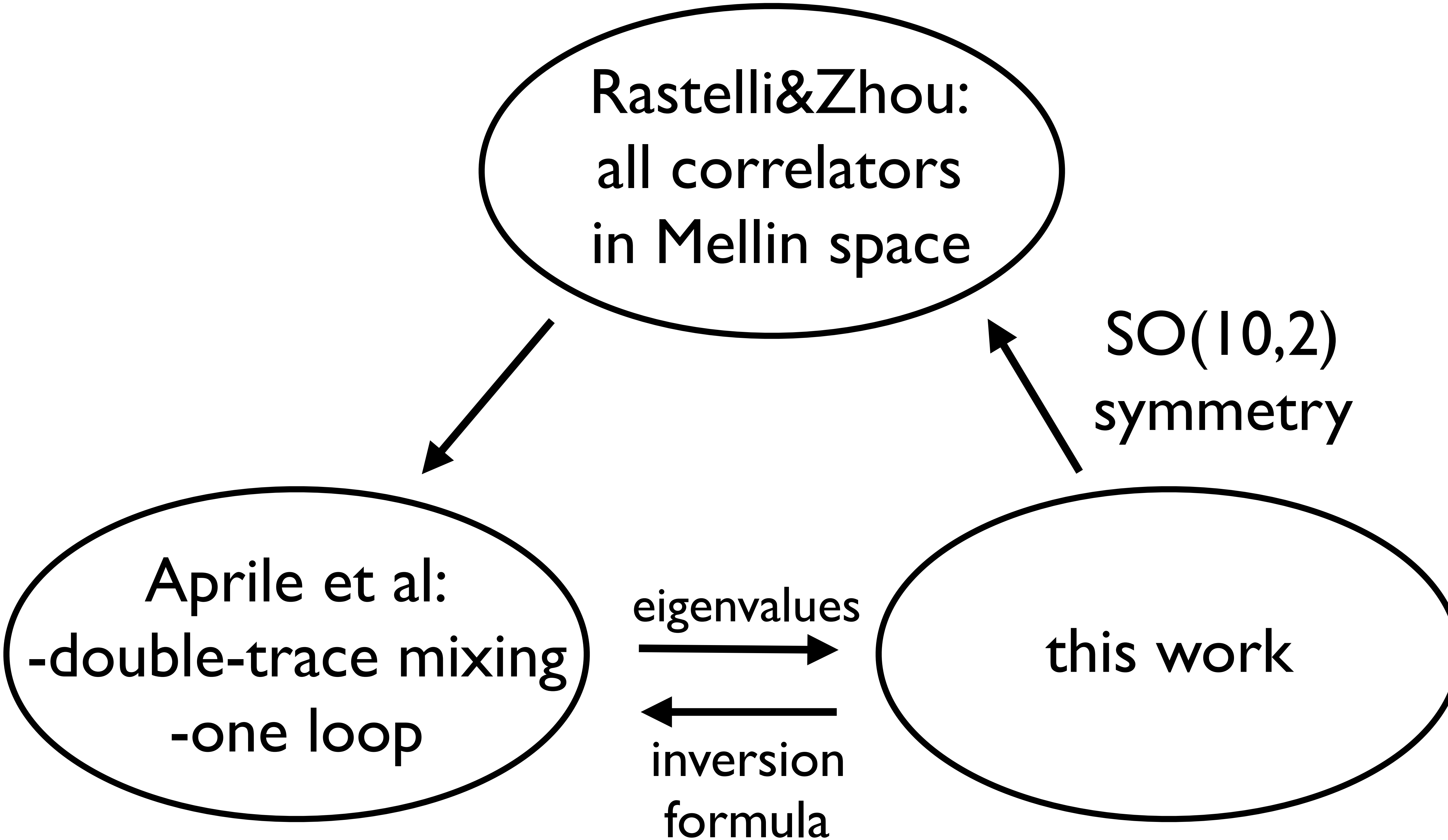
$$\Gamma\left(-\frac{s}{2}\right)(x_{12}^2 - y_{12}^2)^{\frac{s}{2}} = \sum_{p=0}^{\infty} \frac{(y_{12}^2)^p}{p!} \Gamma\left(-\frac{s}{2} - p\right)(x_{12}^2 - y_{12}^2)^{\frac{s}{2}-p}$$

General Mellin space correlator = sum of shifted 1/(stu)'s:

$$M_{pqrs}(s,t) = \sum \frac{\#}{(s-\#)(t-\#)(u-\#)}$$

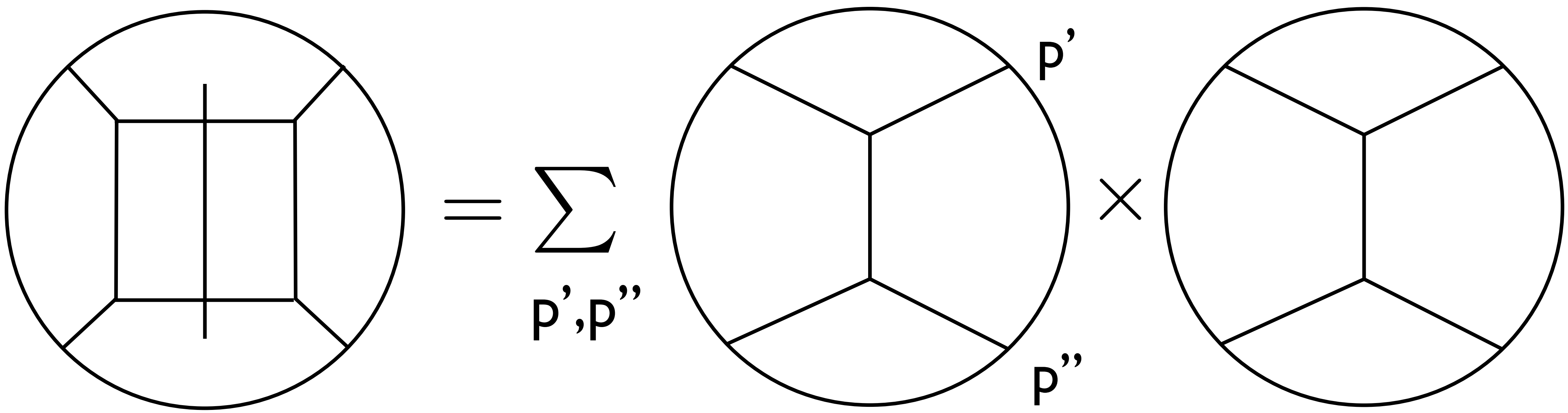
[Rastelli&Zhou '16]

each coefficient is product of six (1/p!)



perfect agreement between different methods!

From trees to loops



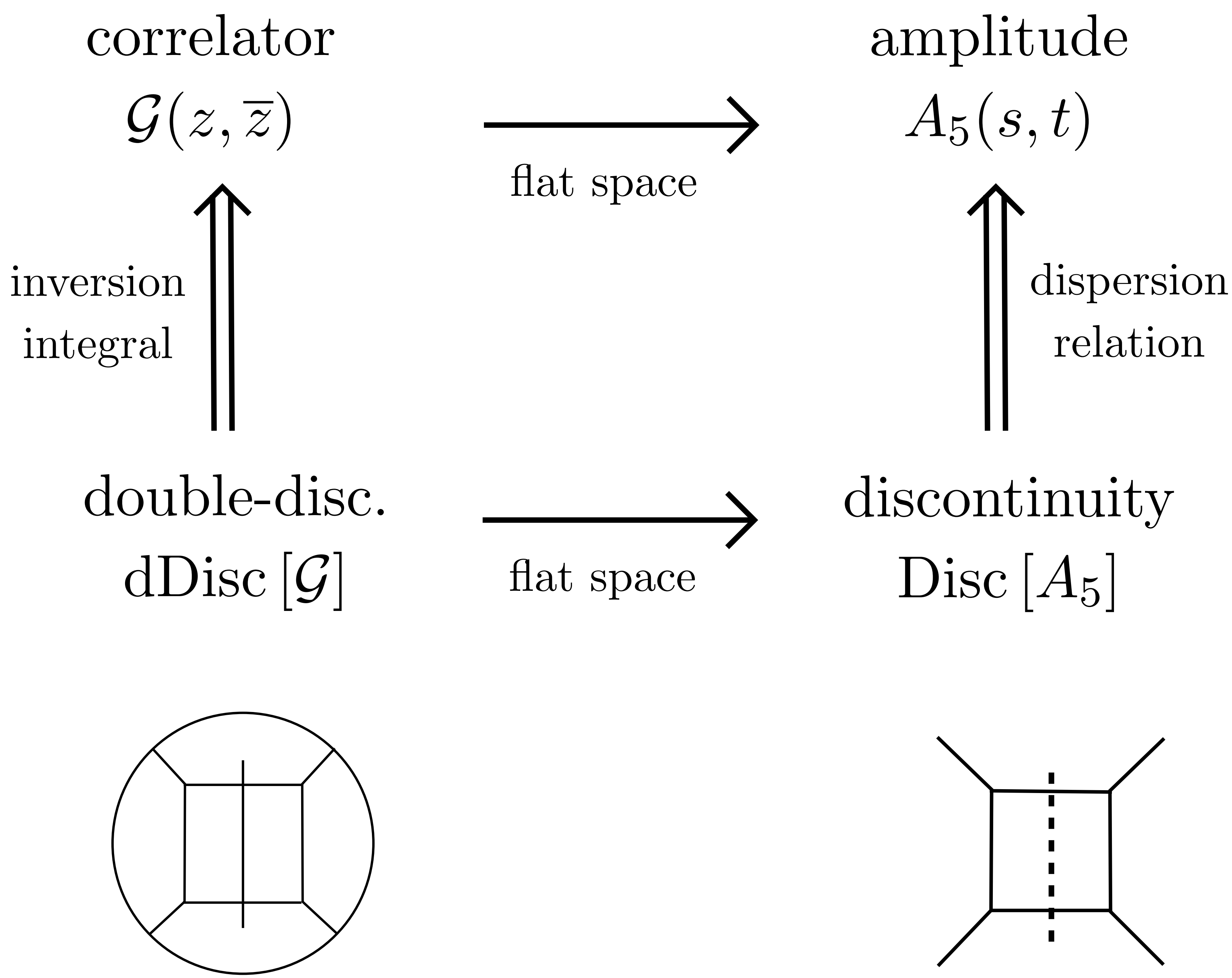
Trees predict the dDisc (~$\log^2 v$ terms)
=all one needs for the Kramers-Kronig relation

[SCH& Alday, '17]

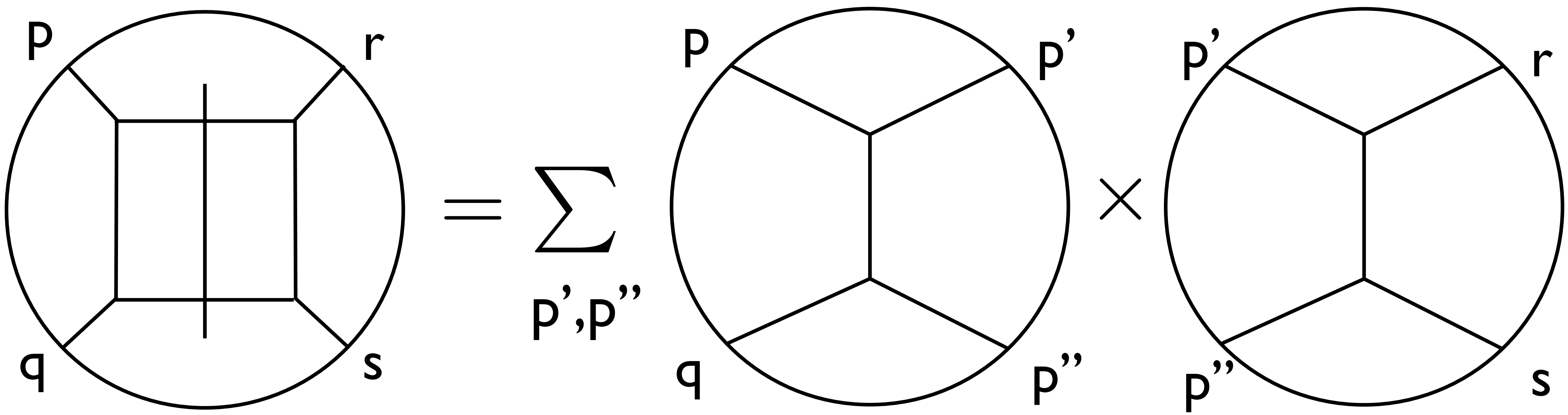
At one-loop, studied 2222 correlator

[Alday& SCH, '17]

Flat space limit perfectly matches 1-loop supergravity



For general correlators, really a matrix product:



The 10D symmetry trivially diagonalizes this sum

Start with OPE decomposition of 10D free field:

$$G_{10}^{(0)}(u,v) = 144\left(u^4 + \frac{u^4}{v^4}\right) = \sum_{j=0,\text{even}}^{\infty} \frac{8\Gamma(j+4)^2}{\Gamma(2j+7)}(j+1)_6\ G_{\ell,8+\ell}^{(10D)}(u,v)$$

A single block for each per spin: 10D dilatons have Δ=4

When reduced to 4D, 10D blocks are orthogonal!

[SCH,&Trinh, to appear]

⇒Just need to add powers of $1/(j+1)_6$ in the above!

Note: 10D extremal blocks extremely simple:

$$G_{\ell,8+\ell}^{(d=10)}(z,\bar{z}) = \mathcal{D}_{(3)} \cdot \frac{120}{(j+1)(j+2)(j+3)}\ z^{j+1}{}_2F_1(j+1,j+4,2j+8,z)$$

[Dolan&Osborn '11]

$$\mathcal{D}_{(3)}f(z) \equiv \left[\left(\frac{z\bar{z}}{\bar{z}-z}\right)^7 f(z) + \left(\frac{z\bar{z}}{\bar{z}-z}\right)^6 \frac{z^2}{2}\partial_z f(z) + \left(\frac{z\bar{z}}{\bar{z}-z}\right)^5 \frac{z^3}{10}\partial_z^2(zf(z)) + \left(\frac{z\bar{z}}{\bar{z}-z}\right)^4 \frac{z^4}{120}\partial_z^3(z^2 f(z))\right]$$

Explicit formula for leading-log at each loop order:

$$\mathcal{H}^{(k)}_{pqrs}(z,\bar{z},\alpha,\bar{\alpha})\Big|_{\log^k u} = \left[\Delta^{(8)}\right]^{k-1} \cdot \mathcal{D}_{pqrs} \cdot \mathcal{D}_{(3)} \cdot h^{(k)}(z).$$

Ex: $$h^{(2)}(z) = \frac{\mathrm{Li}_2(z) - (1-z)^5 \mathrm{Li}_2(z/(1-z))}{4z^5} - \frac{(1-z)(2z^2-7z+7)\log(1-z)}{8z^4} + \frac{(z-2)(1-z)}{z^3} + \frac{235}{576}\frac{z-2}{z}.$$

gives one-loop log² terms for *all* correlators

matches 2222 from: [Alday & Bissi '17, Aprile,Drummond,Heslop&Paul '17]

general formula:

$$h^{(k)}(z) \equiv \frac{1}{k!}\left(\frac{-1}{2}\right)^k \sum_{\ell=0,\ \mathrm{even}}^{\infty} \frac{960\Gamma(j+1)\Gamma(j+4)}{\Gamma(2j+7)} \frac{1}{[(\ell+1)_6]^{k-1}} z^{j+1}{}_2F_1(j+1,j+4,2j+8,z).$$

Summary

-Studied double-trace mixing in strongly coupled N=4 SYM using Lorenzian inversion formula

-SO(10,2) symmetry: formula for all spherical harmonics!

-Leading logs to all orders in $1/N_c$

Further questions

-What more is true at higher loops/higher points?

cf: [Loebbert, Mojaza& Plefka '18: hidden conformal symmetry]
[cf Maldacena' 11: Einstein vs conformal gravity]

-Other theories: 6D (2,0), ABJM?

‘Heavy’ part depends on nonperturbative UV completion.

It’s weighed by $\sim (\rho\bar{\rho})^{J/2}$. Use positivity + boundedness:

$$\left|c(j, \tfrac{d}{2} + i\nu)_{\text{heavy}}\right| \leq \frac{1}{c_T} \frac{\#}{(\Delta^2_{\text{gap}})^{j-2}}$$

This establishes, from CFT, an EFT power-counting in AdS.