Adding flavour to the S-matrix Bootstrap

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Perimeter Institute

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based on 1805.11143
w/ Pedro Vieira

Outline

- Introduction & simplest case
- Set-up $O(N)$ bootstrap
- Integrable examples
- Results away from integrability
- Large N
- Open questions

Introduction

S-matrix Theory

Non-perturbative S-matrix from general principles like:

- Analyticity
- Unitarity
- Crossing symmetry

Popular in the 60's

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$$\left\{ \begin{array}{l} \text{Analyticity} \\ \text{Unitarity} \\ \text{Crossing symmetry} \end{array} \right.$$

Popular in the 60's

[Paulos, Penedones, Toledo, van Rees, Vieira '16]

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Constrain the space of QFTs

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Establish bounds on couplings given some spectrum

Q: Given $\{m, m_B\}$, what is g_{max}?

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Constrain the space of QFTs

Establish bounds on couplings given some spectrum

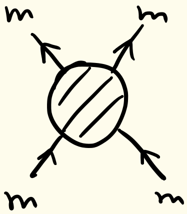
Q: Given $\{m, m_B\}$, what is g_{max}?

Physical intuition: increase coupling $\Rightarrow \begin{cases} \text{more bound states} \\ \text{smaller } m_B \end{cases}$

Review Simplest Case

[Paulos, Penedones, Toledo, van Rees, Vieira '16]

1+1 dimensions
$2 \to 2$ scattering



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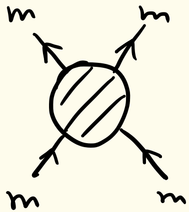
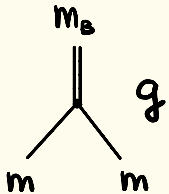


Spectrum
$\{m, m_B\}$



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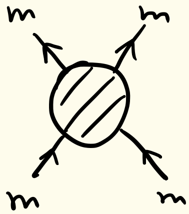

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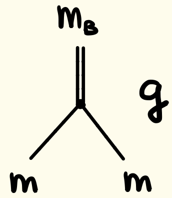

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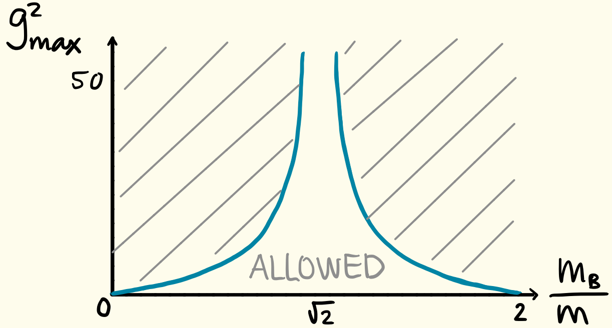

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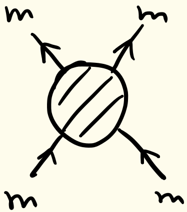

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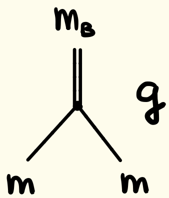

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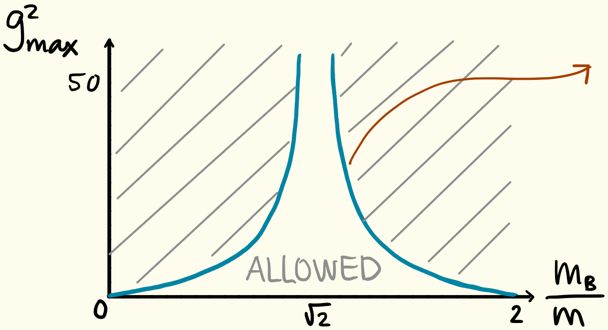

sine Gordon (breathers)

$$S_{SG}(\theta) = \frac{\operatorname{sh}\theta + i\sin\lambda}{\operatorname{sh}\theta - i\sin\lambda}$$

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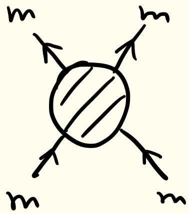

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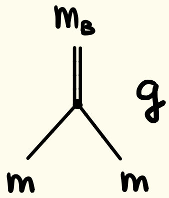

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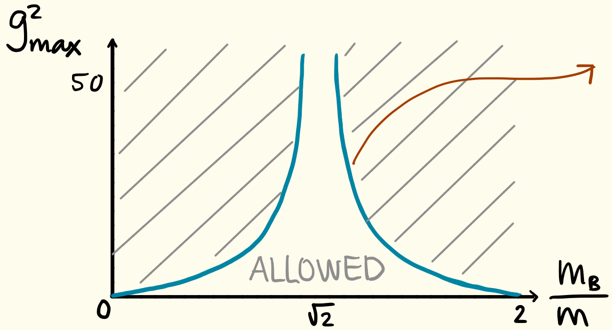


sine Gordon (breathers)

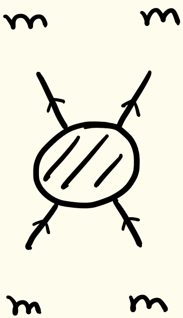
$$S_{SG}(\theta) = \frac{\operatorname{sh}\theta + i \sin\lambda}{\operatorname{sh}\theta - i\sin\lambda}$$

- Analytic derivation using max. modulus principle.

O(N) Set-up

2d, 2→2 scattering

$s+t+\cancel{u}^{\,0}=4m^2$

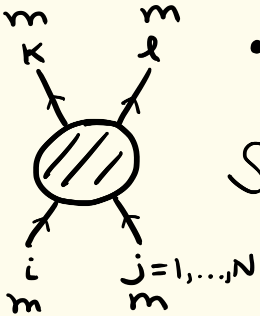


$S(s)$

O(N) Set-up

2d , 2→2 scattering

$s+t+\cancel{u}=4m^2$, with $u \to 0$



• O(N)

$S_{ij}^{kl}(s)$

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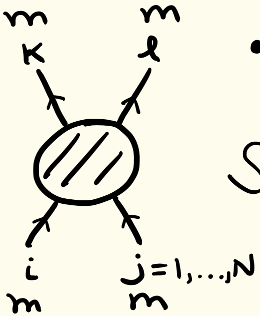

- O(N)

$\mathbb{K}$  $\mathbb{1}$  $\mathbb{P}$ 

$$S_{ij}^{kl}(s) = \sigma_1(s)\,\delta_{ij}\,\delta^{kl} + \sigma_2(s)\,\delta_i^{l}\,\delta_j^{k} + \sigma_3(s)\,\delta_i^{k}\,\delta_j^{l}$$

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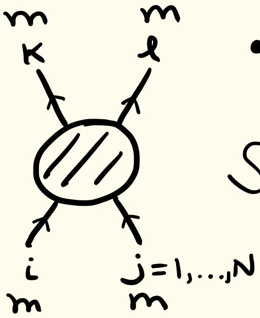

• O(N) $\qquad \mathbb{K}$  $\qquad \mathbb{1}$  $\qquad \mathbb{P}$ 

$$S_{ij}^{k\ell}(s) = \sigma_1(s)\,\delta_{ij}\,\delta^{k\ell} + \sigma_2(s)\,\delta_i^{\ell}\,\delta_j^{k} + \sigma_3(s)\,\delta_i^{k}\,\delta_j^{\ell}$$

$$= \sum_{\text{rep}} S_{\text{rep}}(s)\,\mathbb{P}_{\text{rep}}$$ ⇝ singlet, antisym, sym

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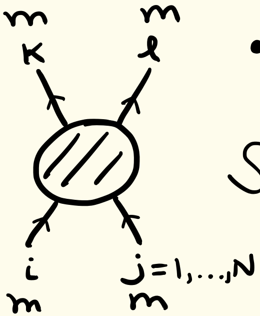

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$$|S_{\text{rep}}(s)|^2 \leq 1 \quad (s>4m^2)$$

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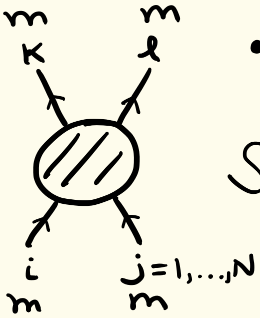


• O(N)

$$S_{ij}^{kl}(s)=\sigma_1(s)\,\delta_{ij}\,\delta^{kl}+\sigma_2(s)\,\delta_i^l\,\delta_j^k+\sigma_3(s)\,\delta_i^k\,\delta_j^l$$

$$=\sum_{rep}S_{rep}(s)\,\mathbb{P}_{rep}$$

⤳ singlet, antisym, sym

• Unitarity

$$|S_{rep}(s)|^2\leq 1\quad (s>4m^2)$$

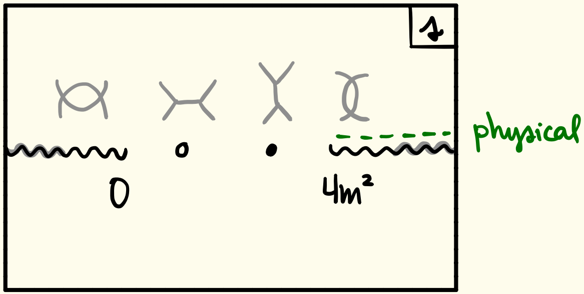
• Crossing

$$S_{rep}(t=4m^2-s)=\sum_{rep'}d_{rep}^{rep'}\,S_{rep'}(s)$$

$$d=\begin{pmatrix}\frac{1}{N} & -\frac{N}{2}+\frac{1}{2} & \frac{N}{2}+\frac{1}{2}-\frac{1}{N}\\ -\frac{1}{N} & \frac{1}{2} & \frac{1}{2}+\frac{1}{N}\\ \frac{1}{N} & \frac{1}{2} & \frac{1}{2}-\frac{1}{N}\end{pmatrix}$$

O(N) Set-up

- Analytic properties



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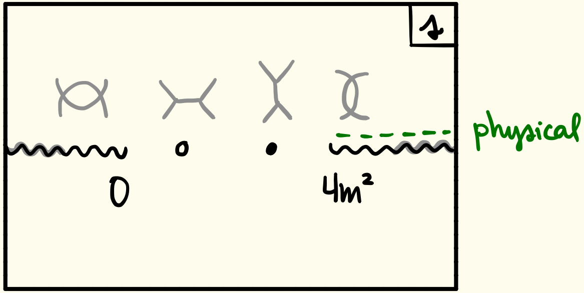

'Real analyticity'

$$S^*(s) = S(s^*)$$

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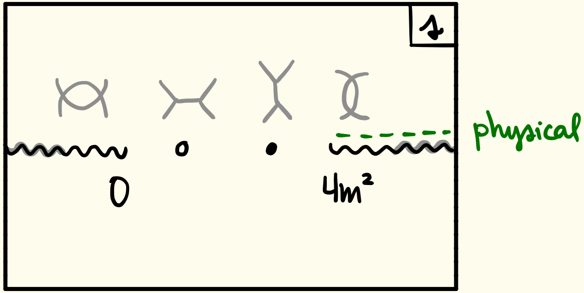

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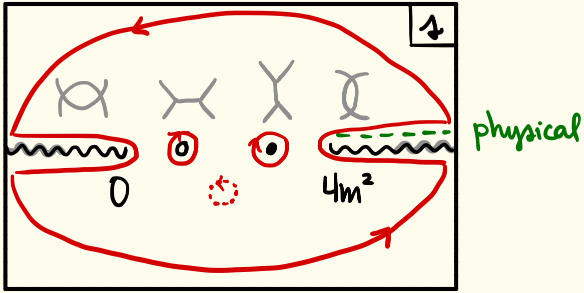

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Cauchy's thm

$$S(s) = \frac{1}{2\pi i} \oint_c \frac{S(x)}{x-s}\, dx$$

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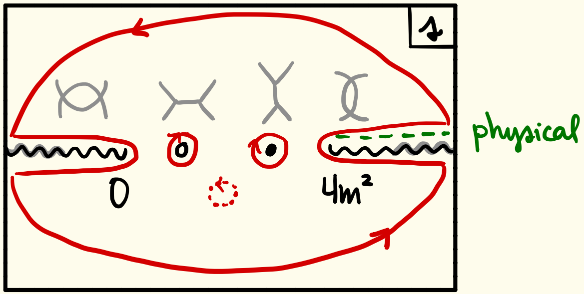

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- Dispersion relation (subtraction)

$$S_{rep}(s) = S_{rep}(2) + \frac{(s-2)}{(m_{rep}^2-2)}\left[\frac{g_{rep}^2}{s-m_{rep}^2} - \sum_{rep'} d_{rep}^{rep'} \frac{g_{rep'}^2}{4m^2-s-m_{rep'}^2}\right]$$

$$+ \int_{4m^2}^{\infty} \frac{(s-2)}{(x-2)}\left[\frac{\rho_{rep}(x)}{x-s} - \sum_{rep'} d_{rep}^{rep'} \frac{\rho_{rep'}(x)}{x-s}\right] dx$$

O(N) Set-up

Numerics

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- With dispersion relations $\begin{cases} \text{analyticity} & \checkmark \\ O(N) & \checkmark \\ \text{crossing sym} & \checkmark \end{cases}$

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- Ask Mathematica

$$\text{FindMaximum}\left[\underline{\qquad}\,,\ |S_{rep}(s_j)|^2 \le 1\right] \quad (s_j > 4m^2)$$

e.g.: g_{rep}, $S_{rep}(s^*)$

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- Can play with: N, # bound states, reps, masses

e.g. $N=7$; $m_{sing} = 1.3m$, $m_{anti} = 1.5m$; Max g_{anti}

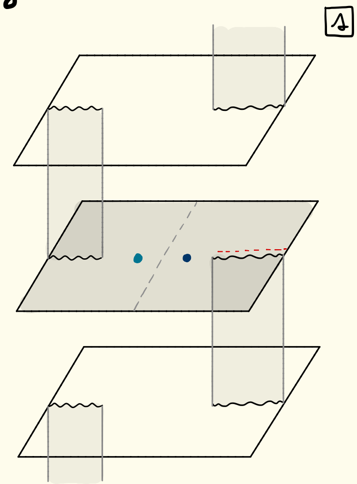
O(N) Integrable S-matrices

Rapidity θ

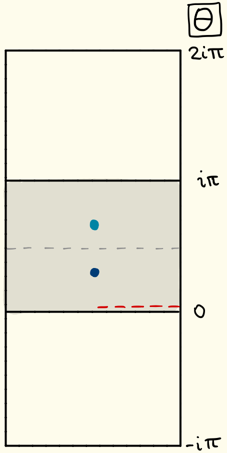
O(N) Integrable S-matrices

Rapidity θ

physical sheet



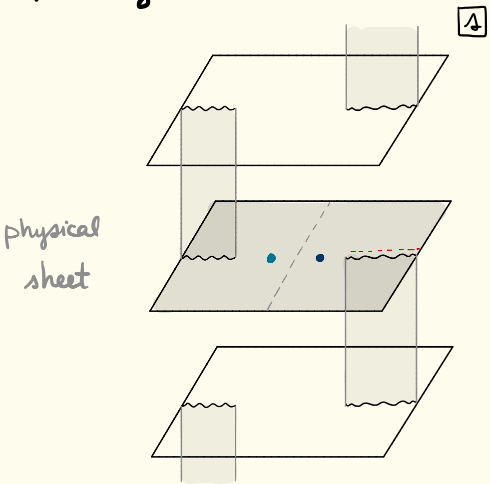
$$s = 4m^2\cosh^2(\theta/2)$$



physical strip

O(N) Integrable S-matrices

Rapidity θ



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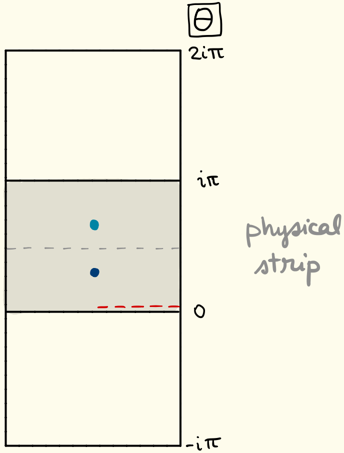

- Unitarity $\quad S_{rep}(\theta)\, S_{rep}(-\theta) = 1$

O(N) Integrable S-matrices

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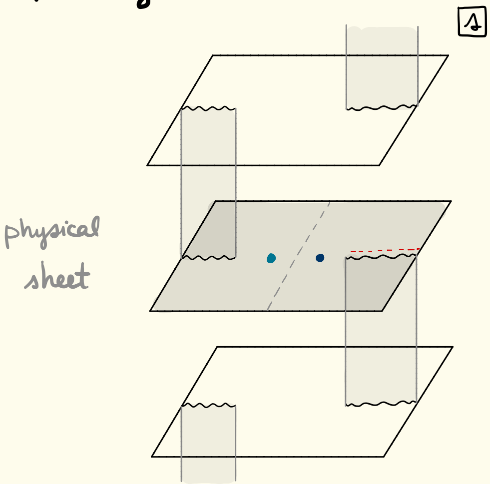

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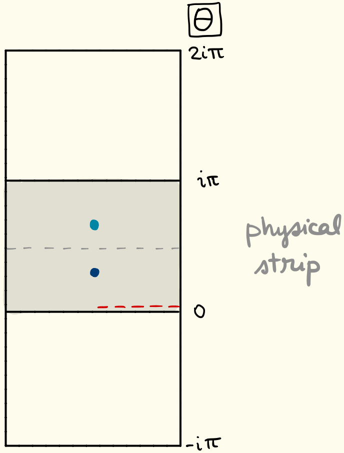

- Unitarity $\quad S_{rep}(\theta)\, S_{rep}(-\theta) = 1$
- Crossing $\quad S_{rep}(i\pi - \theta) = \sum_{rep'} d_{rep}^{rep'}\, S_{rep'}(\theta)$

O(N) Integrable S-matrices

[Zamolodchikov^2 '78] Unitarity + Crossing + Yang-Baxter

O(N) Integrable S-matrices

[Zamolodchikov² '78] Unitarity + Crossing + Yang-Baxter

- General form $(N>2)$

$$\boxed{\bar{S}^{\text{int}}(\theta) = \left(\prod_{j=1}^{M} \frac{\operatorname{sh}\theta + i\sin\alpha_j}{\operatorname{sh}\theta - i\sin\alpha_j}\right) \bar{S}^{\text{NLSM}}(\theta)}$$

$$\bar{S} = \begin{pmatrix} S_{\text{sing}} \\ S_{\text{anti}} \\ S_{\text{sym}} \end{pmatrix}$$

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$$\boxed{\bar{S}^{\text{NLSM}}(\theta) = \begin{array}{r} \text{sing} \\ \text{anti} \\ \text{sym} \end{array} \begin{pmatrix} -1 \\ -\frac{\theta - i\pi}{\theta + i\pi} \\ -\frac{\theta - i\pi}{\theta + i\pi}\,\frac{\theta - i\lambda_{GN}}{\theta + i\lambda_{GN}} \end{pmatrix} F_{\pi+\lambda_{GN}}(\theta)\, F_{2\pi}(-\theta)}$$

$$F_a(\theta) = \frac{\Gamma\left(\frac{a + i\theta}{2\pi}\right)\Gamma\left(\frac{a - i\theta + \pi}{2\pi}\right)}{(\theta \to -\theta)}$$

$$\lambda_{GN} = \frac{2\pi}{N-2}$$

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$\Rightarrow$ Very specific spectra

O(N) Integrable S-matrices

- Gross-Neveu

$$\boxed{\bar{S}^{GN}(\theta) = \frac{\operatorname{sh}\theta + i\sin\lambda_{GN}}{\operatorname{sh}\theta - i\sin\lambda_{GN}}\, \bar{S}^{NLSM}(\theta)}$$

bound states in sing & anti
(same mass)

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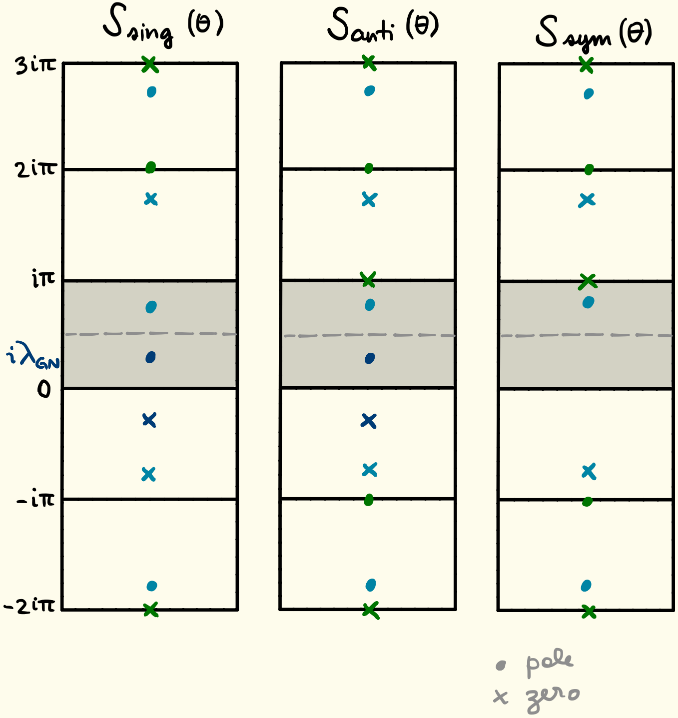

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bound states in sing & anti
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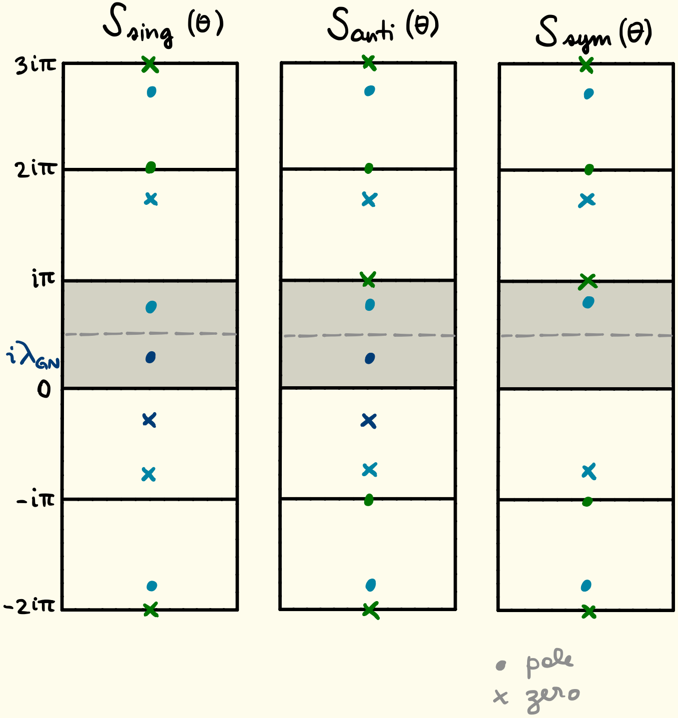
Γ's from crossing + unit.

$$S_{rep}(i\pi-\theta) = \sum_{rep'} d\!\!I_{rep}^{rep'}\, S_{rep'}(\theta)$$

$$S_{rep}(\theta)\, S_{rep}(-\theta) = 1$$



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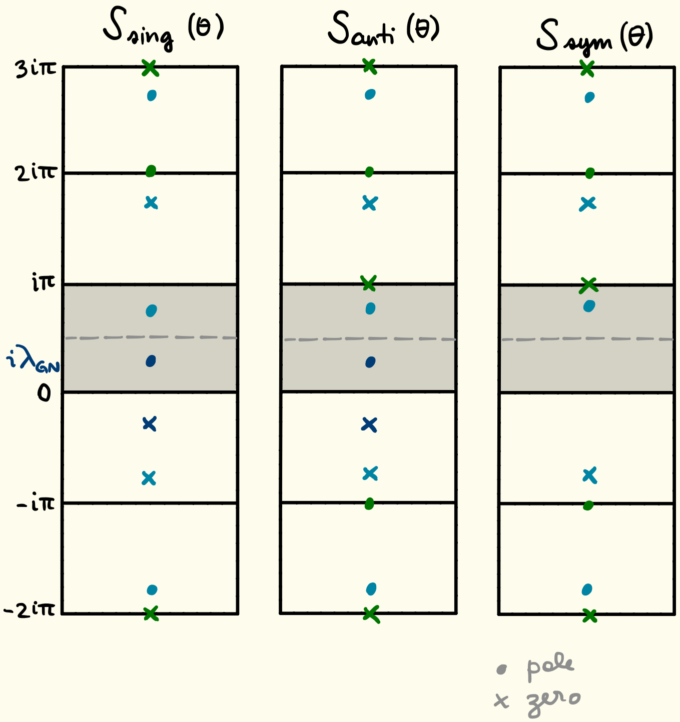
$$S_{rep}(\theta)\, S_{rep}(-\theta) = \mathbb{1}$$



- (N=2) sine Gordon kinks

O(N) Integrable S-matrices

[Hortacsu, Schroer, Thun '79]

- Special case: $S_{anti} = -S_{sym}$ $(\sigma_2 = 0)$

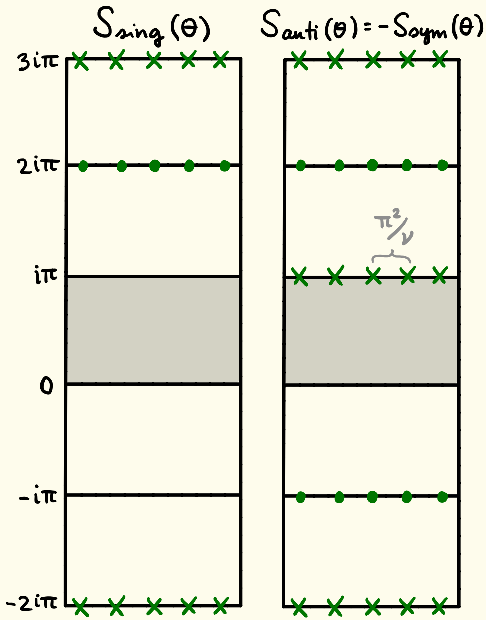
O(N) Integrable S-matrices

[Hortacsu, Schroer, Thun '79]

- Special case: $S_{anti} = -S_{sym}$ $(\sigma_2 = 0)$

$$\boxed{\bar{S}(\theta) = \begin{pmatrix} \dfrac{\mathrm{sh}\left[\nu\left(1-\frac{i\theta}{\pi}\right)\right]}{\mathrm{sh}\left[\nu\left(1+\frac{i\theta}{\pi}\right)\right]} \\ -1 \\ +1 \end{pmatrix} \prod_{n=-\infty}^{\infty} F_{\pi + \frac{in\pi^2}{\nu}}(-\theta)}$$

$N = 2\cosh\nu$



O(N) Integrable S-matrices

[Hortacsu, Schroer, Thun '79]

- Special case: $S_{anti} = -S_{sym}$ $(\sigma_2 = 0)$

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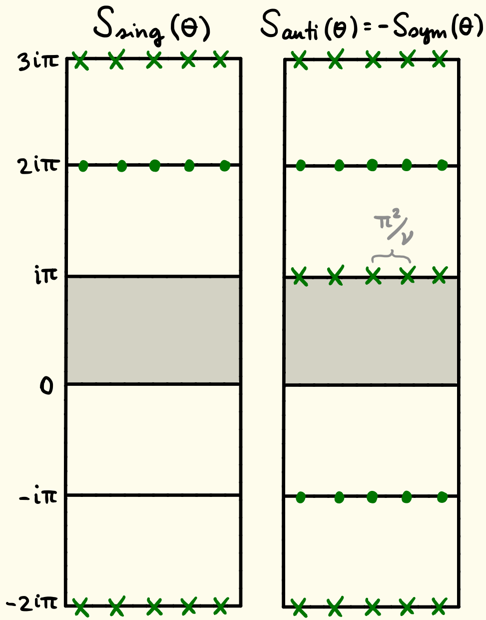
[Doroud, Elias Miró '18]



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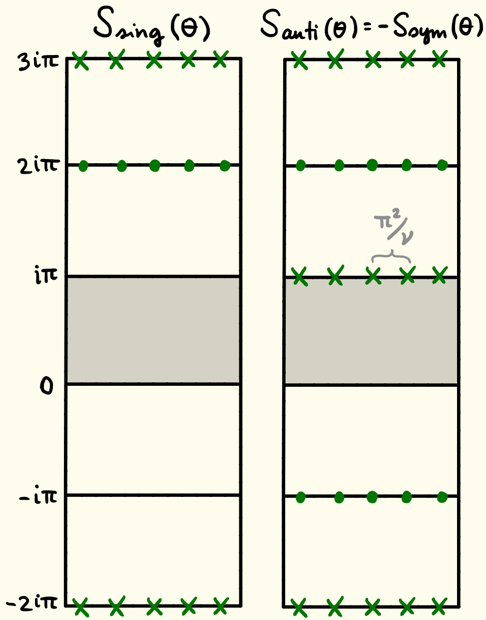

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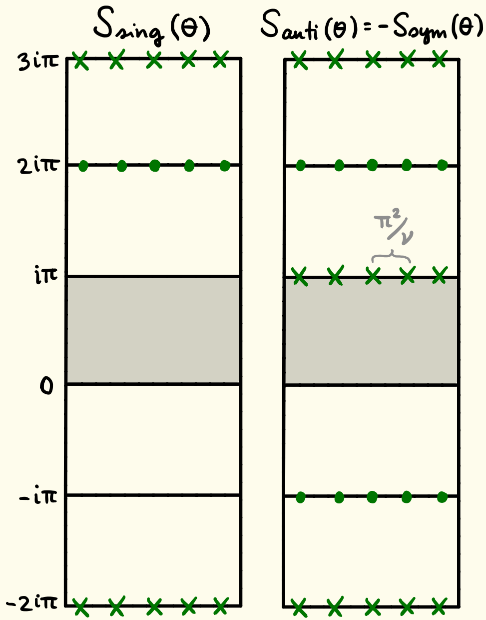
✓ Can reproduce integrable solns.



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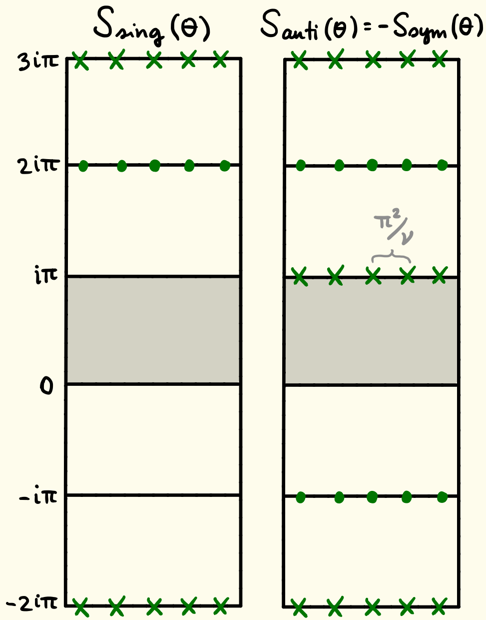
✓ Can reproduce integrable solns.

- See also [He, Irrgang, Kruczenski; Paulos, Zheng '18]



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Away from integrability, play w/ spectra

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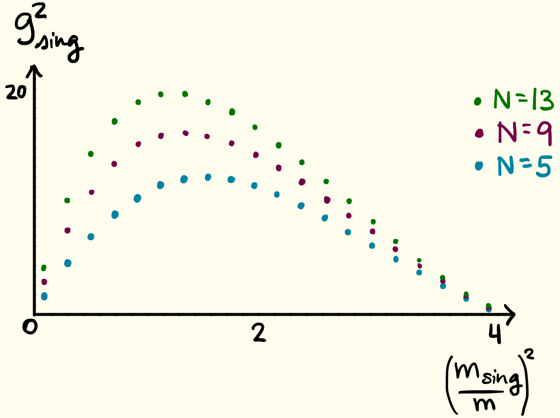
e.g.

$\boxed{1 \text{ b.s. } m_{sing}}$



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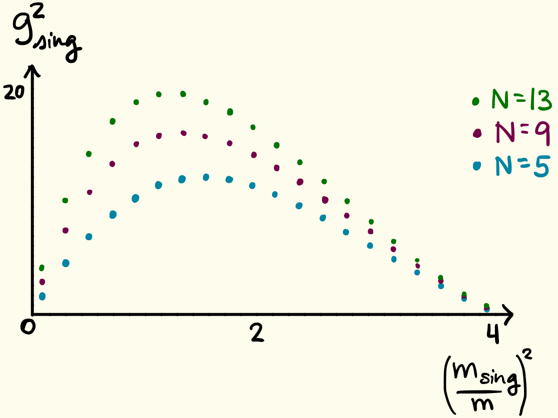

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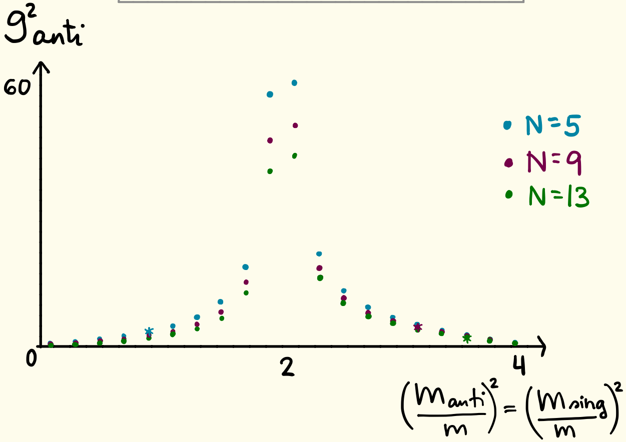

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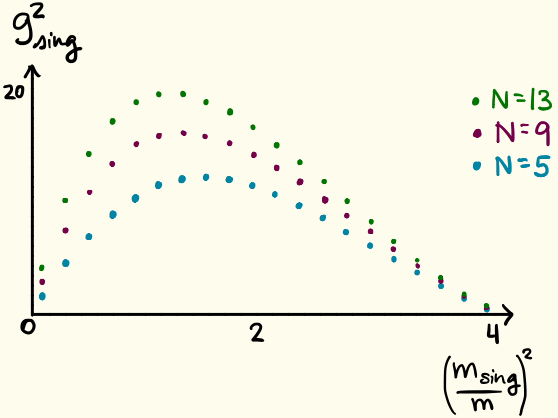

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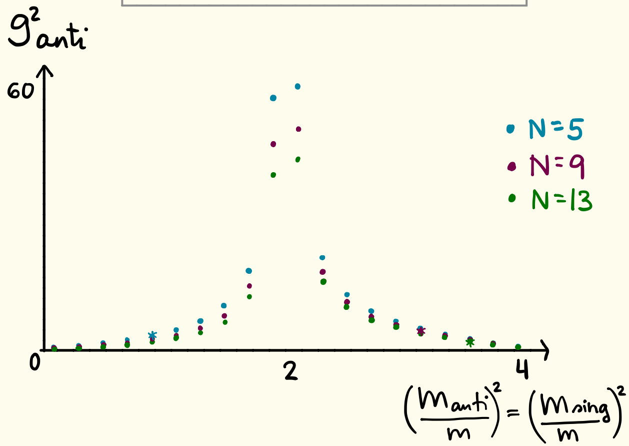

- Not only g_{max}, but full $S(\theta)$

Non-Integrable Results

- One bound state
  SINGLET

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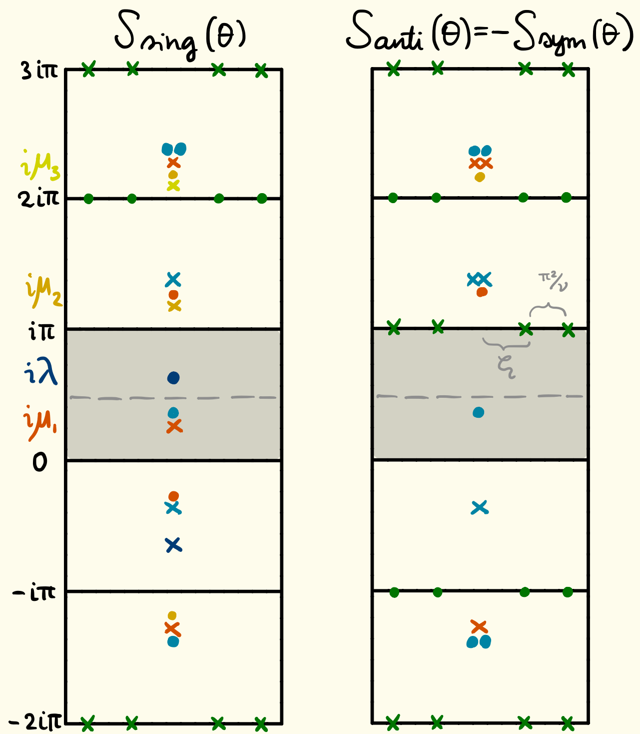
Input: N, λ (m_{sing})



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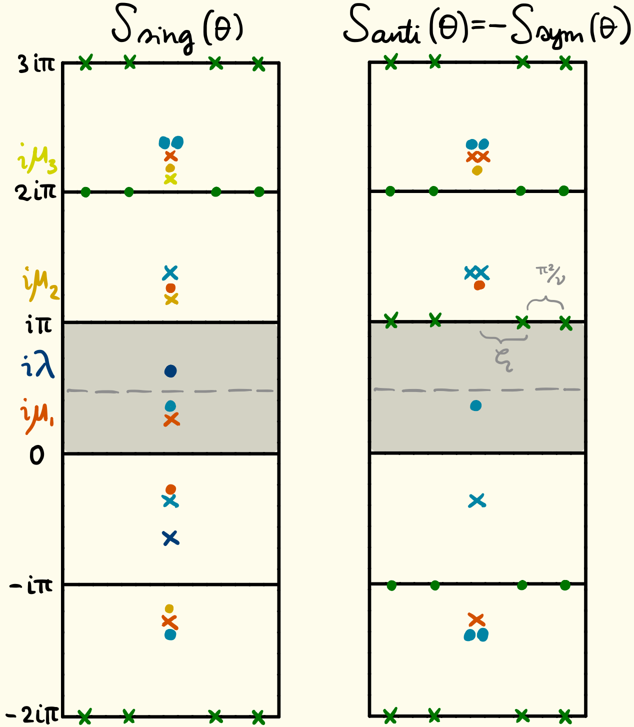

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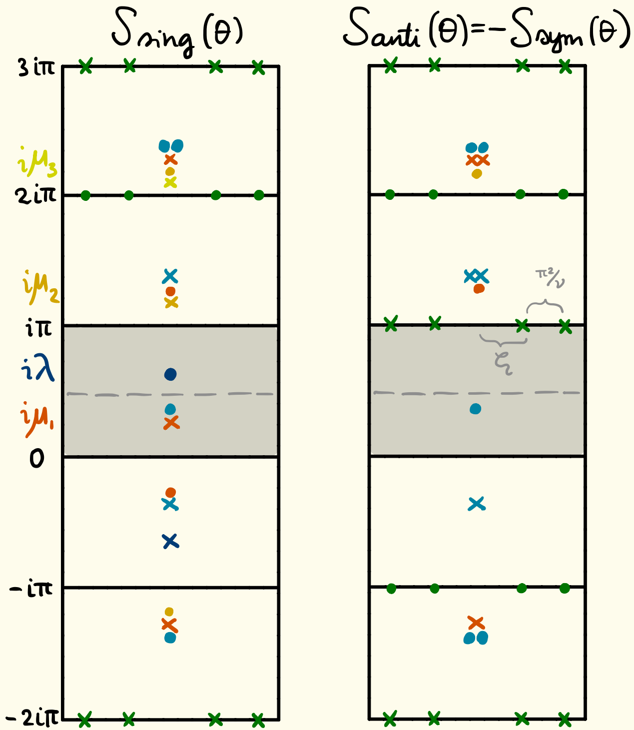

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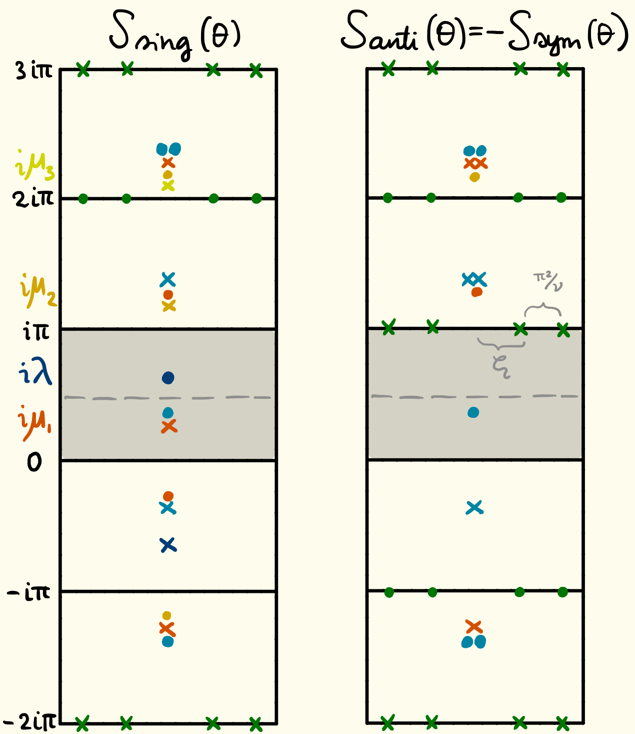
- More complicated structures
  in other cases



Large N

$$d\mathbb{I} = \begin{pmatrix} \frac{1}{N} & -\frac{N}{2}+\frac{1}{2} & \frac{N}{2}+\frac{1}{2}-\frac{1}{N} \\ -\frac{1}{N} & \frac{1}{2} & \frac{1}{2}+\frac{1}{N} \\ \frac{1}{N} & \frac{1}{2} & \frac{1}{2}-\frac{1}{N} \end{pmatrix}$$

Simplification in crossing eqns.

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$$S_{sing}(i\pi-\theta) = \frac{1}{N} S_{sing}(\theta) - \left(\frac{N}{2}-\frac{1}{2}\right) S_{anti}(\theta) + \left(\frac{N}{2}+\frac{1}{2}-\frac{1}{N}\right) S_{sym}(\theta)$$

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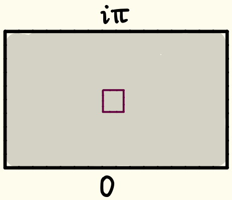

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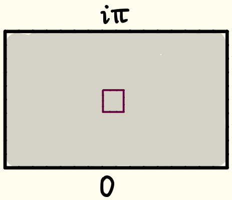

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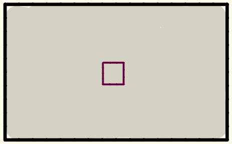
$i\pi$



0

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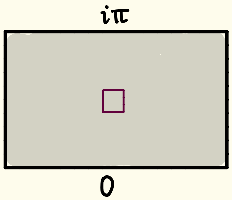

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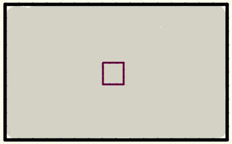

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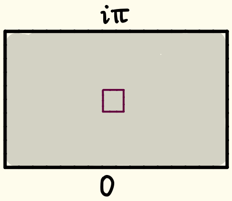

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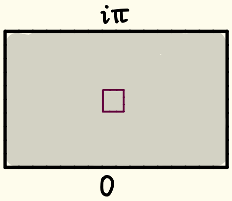

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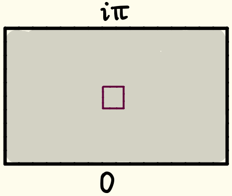

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✓ Large N extrapolation num. e.g. 1 b.s. Max $g^2_{sing} = 4 N^{\lambda/\pi} \sin^2 \lambda$

Summary / Open Questions

- Bounds on couplings of 2d QFTs w/ $O(N)$ sym.
- Made contact w/ known integrable solns.
- Explored analytic structure of S-matrices away from integrability (full solution in one case).
- Got some insights from large N analysis.

- How does YB arise at cusps? [He, Irrgang, Kruczenski '18]
- Particle production, physical models?
- Many people working: higher d, multiple processes, SUSY, resonances, ...

Thank you!