AdS$_3$/CFT$_2$ : Moduli and wrapping

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Based on work with D. Bombardelli, O. Ohlsson Sax and A. Torrielli,

IGST 2018, Copenhagen
Plan

1. Integrable AdS$_3$/CFT$_2$
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1. Integrable $\text{AdS}_3/\text{CFT}_2$
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3. Towards Wrapping in $\text{AdS}_3/\text{CFT}_2$
4. Outlook
Integrable $\text{AdS}_3/\text{CFT}_2$
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- Half max susy bkds $\text{AdS}_3 \times S^3 \times T^4$ and $\text{AdS}_3 \times S^3 \times S^3 \times S^1$
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- Integrability works for backgrounds with:
  - RR charges (n.h. D1/D5) \[ \text{Borsato+Ohlsson Sax+Sfondrini+BS+Torrielli} \]
  - NSNS+RR charges (n.h. D1+F1/D5+NS5)  
    \[ \text{Hoare+Tseytlin, Lloyd+Ohlsson Sax+Sfondrini+BS} \]
  - We expect it will also work with more general charges
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- Integrability ”works” means:
  - Wsheet S matrix known exactly in $\alpha'$ or $R_{AdS}$, satisfies YBE
  - S matrix fixed by famous central extension and crossing eqs
  - Bethe Equations for asymptotic states: protected spectrum
Moduli and Integrability
**AdS$_3 \times S^3 \times T^4$ Moduli**

IIB string theory on $T^4$ has 25 moduli:

\[ g_{ab} \times 10, \quad B_{ab} \times 6, \quad C_{ab} \times 6, \quad C_0, \quad C_{abcd}, \quad \phi \]
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Integrable results found when moduli zero

What happens away from the origin of moduli space?
Turning on moduli in $\text{AdS}_3 \times S^3 \times T^4$

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E.g. Pure RR charge bkd:
- 9 geometric moduli $g_{ab}$ of $T^4$
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E.g. Pure NSNS charge bkd has $C_0$ and $C_2^+$. 
Turning on $C_0$ in NSNS AdS$_3 \times S^3 \times T^4$

Set $C_0$ to a non-zero constant.
Turning on $C_0$ in NSNS AdS$_3 \times S^3 \times T^4$

Attractor mechanism: $C_4 = -C_0 \text{vol}(T^4)$
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Gauge-invariant RR field-strength:

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F_3 = dC_2 - C_0 H = -C_0 k \text{vol}(S^3) \neq 0
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Since $H \neq 0$, $F_3 \neq 0$, and all other $F = 0$

GS action same as mixed-charge background!
Integrability of NSNS AdS$_3 \times S^3 \times T^4$ with $C_0 \neq 0$

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Perhaps we can identify a new 't Hooft-like parameter $\lambda \sim C_0 k g s$.
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Parameters $q$ and $\tilde{q}$ related to $k$ and $C_0$

$$\tilde{q} \rightarrow -g_s C_0 k \frac{\alpha'}{R^2}, \quad q \rightarrow k \frac{\alpha'}{R^2}$$
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We have

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NSNS bkd integrable across moduli space away from $C_0 = 0$ point
Integrability of NSNS AdS$_3 \times S^3 \times T^4$ with $C_0 = 0$

At the $C_0 = 0$ WZW point:
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- Central extensions zero - is derivation of $S$ matrix valid?
Integrability of NSNS AdS$_3 \times S^3 \times T^4$ with $C_0 = 0$

At the $C_0 = 0$ WZW point:

S matrix remains finite and non-diagonal

Central extensions zero - is derivation of S matrix valid?

Pert. long strings appear - new sector in Hilbert space.
In summary:
An integrable $S$ matrix for $\text{AdS}_3 \times S^3 \times T^4$ backgrounds with any charges and non-zero moduli is known.
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$R^2$ and the coupling cst $h(R)$ depend on consequential moduli.
Towards Wrapping in $\text{AdS}_3/\text{CFT}_2$
Wrapping Corrections in AdS$_3$

BEs are asymptotic equations for the spectrum: $L \to \infty$
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In AdS$_{5,4}$ finite $L$ corrections are incorporated as a series in $e^{-mL}$

In AdS$_3$ have massless states $m = 0$

Hard to understand finite $L$ corrections

[Abbott+Aniceto]
Low energy states in RR AdS$_3$

Consider BMN limit

\[ p_i \rightarrow \frac{p_i}{h}, \quad h \rightarrow \infty \]
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match to pert. wsheet scattering. Leading order trivial.
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RR AdS$_3$ has gapless excitations, which at low-energies are relativistic (massless) left- or right- movers

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At small $p$, massive/massive and massive/massless scattering has pert. expansion. Ditto massless left/massless right scattering.
Massless/massless S matrix for wsheet l/l or r/r on the other hand

\[ S_{l/l} = S_{l/l}^{(0)} + h^{-2} S_{l/l}^{(1)} + h^{-4} S_{l/l}^{(2)} + \ldots \]

is non-trivial and non-diagonal even at leading order.
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In strict low-energy limit we have massless relativistic theory with $S_{l/r}$ trivial, and $S_{l/l}$, $S_{r/r}$ non-trivial.
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q-Poincaré properties and a geometric interpretation of the associated boost

[Torrielli+Fontanella+Stromwall+Borsato]
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Such integrable systems have been argued by Zamolodchikov to describe 2d CFTs. We call ours CFT$^{(0)}$. 

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Similar S matrices invastigated in the 90s by Zamolodchikov, Fendley and Intriligator, Dunning...
Dressing factor in low-energy limit

Dressing factors have an expansion

\[ \sigma = h^2 \sigma_{AFS} + h^0 \sigma_{HL} + \ldots \]
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\[ \sigma_{\text{HL}}^2 \equiv e^{i \theta_{\text{HL}}} \] reduces to

\[ \theta_{\text{rel}}(\vartheta) = \frac{2i}{\pi} \int_{-\infty}^{\infty} d\phi \frac{e^{\vartheta + \phi}}{e^{2\vartheta} + e^{2\phi}} \log \left( \frac{1 - ie^\phi}{1 + ie^\phi} \right) - \frac{\pi}{2} \]

\( \vartheta = \theta_1 - \theta_2 \) with \( p_i = e^{\theta_i} \)

as expected for a relativistic theory.
HL dressing factor in low-energy limit

\( \sigma_{HL}^{\text{rel}} \) satisfies the famous sine-Gordon crossing equation

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\sigma_{HL}(\vartheta)\sigma_{HL}(\vartheta + i\pi) = i \tanh \frac{\vartheta}{2}
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In physical strip \( \sigma_{HL}^{rel} \) is equal to Zamolodchikov scalar factor in SG soliton/anti-soliton scattering (with \( \gamma = 16\pi, \beta^2 = 16\pi/3 \))

\[
\sigma_{HL}^{rel}(\vartheta) = \prod_{l=1}^{\infty} \frac{\Gamma^2(l-\tau)\Gamma(l+\tau+1/2)\Gamma(l+\tau-1/2)}{\Gamma^2(l+\tau)\Gamma(l-\tau+1/2)\Gamma(l-\tau-1/2)}
\]

where \( \vartheta = 2\pi \tau i \)
Relativistic Bethe Equations

The BEs reduce to (two copies of)

\[ 1 = \prod_{j=1}^{K_0} \tanh \frac{\beta_{1,k} - \theta_j}{2}, \]

\[ e^{iLp_k} = (-1)^{K_0-1} \prod_{j=1, j \neq k}^{K_0} \sigma^2(\varphi_{kj}) \prod_{j=1}^{K_1} \coth \frac{\beta_{1,j} - \theta_k}{2} \prod_{j=1}^{K_3} \coth \frac{\beta_{3,j} - \theta_k}{2}, \]

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Relativistic TBA for CFT\(^{(0)}\)

Introduce rapidity density \( \rho_0 = \frac{\Delta n}{\Delta \theta} \), and analogously \( \rho_{\pm 1,3} \) for level-1 magnons, (solutions come in pairs: \[\text{[Fendley+Intriligator]}\]
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The thermodynamic limit of BEs imply a set of TBA equations

$$\varepsilon_0 = \nu_0(\theta) - \sum_{n=1,3} \phi^* (L_n + L_{-n}); \quad \varepsilon_{\pm n} = -\phi^* L_0,$$

where $\phi = \frac{\text{sech} \theta}{2\pi}$, $n = 1, 3$ and $A = 0, \pm n$

$$\nu_0(\theta) \equiv MR e^\theta, \quad \varepsilon_A \equiv \log \frac{\rho_A^h}{\rho_A^r}, \quad L_A \equiv \log(1 + e^{-\varepsilon_A}),$$
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\]

The exact ground-state energy for right-movers is given by

\[
E_{0,\text{right}}(R) = -\frac{M}{2\pi} \int d\theta e^\theta \log(1 + e^{-\varepsilon_0(\theta)}),
\]
Properties of $\text{CFT}^{(0)}$ from TBA

Ground-state energy with anti-periodic boundary conditions for fermions (a.k.a. Witten’s index) is zero. So BMN vacuum receives no corrections.
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Ground state energy with periodic boundary conditions for fermions is related to the central charge of $\text{CFT}^{(0)}$

$$c \equiv \lim_{L \to \infty} \frac{6R}{\pi L} \log \text{Tr} e^{-R H_L} = -\frac{6R}{\pi} E_0(R) = 6,$$
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$$E_n(R) = \frac{2\pi}{R} (1 - n)^2, \quad n = 2, 3, \ldots$$
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These properties suggest CFT($^{(0)}$) is just a free CFT on $T^4$ or $R^4$. 
Conclusions
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- AdS$_3 \times S^3 \times T^4$ spectrum with $p_{T^4} = w_{T^4} = 0$ integrable for any background charges across whole moduli space*

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- At origin, NSNS S matrix finite and non-diagonal. Need to understand long string sector.

[Reference: Giribet+Hull+Kleban+Porrati+Rabinovici, Gaberdiel+Gopakumar]

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- Match short strings to Maldacena Ooguri spectrum? Connect to recent conjecture by Baggio and Sfondrini? Relation to low $k$ results?

  [Giribet+Hull+Kleban+Porrati+Rabinovici,Gaberdiel+Gopakumar]

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Conclusions: Wrapping

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- Generalise to $\text{AdS}_3 \times S^3 \times T^4$ backgrounds with other charges.
Thank you