Decoherence for Fluctuations out of Equilibrium

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Outline

- Background and Motivation:
  - Tuning Example: Flatness Problem
  - Arrow of Time and Boltzmann Brain Problem
  - Decoherence
- Results:
  - Equilibrating/Fluctuation Toy qubit Model

FRW Universe

- 1st Friedmann Equation:

$$(\frac{\dot{a}}{a})^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + ...)$$

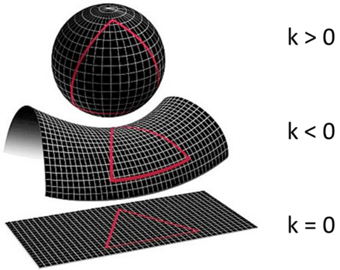
- Describes a homogeneous, isotropic universe expanding (or contracting) with scale factor, a
- Energy densities evolve based on:

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)$$

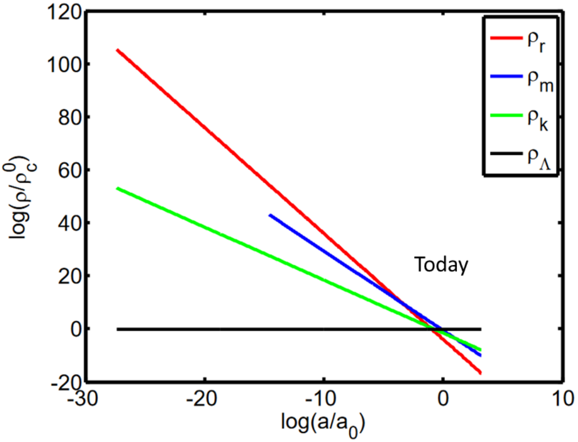
Curvature

- Critical density: ρ_c
  If $\rho_m + \rho_r + \ldots = \rho_c$ then flat
- Curvature, k:

$$\rho_k = \rho_c - (\rho_m + \rho_r + \ldots)$$
$$k = -a^2 \rho_k$$



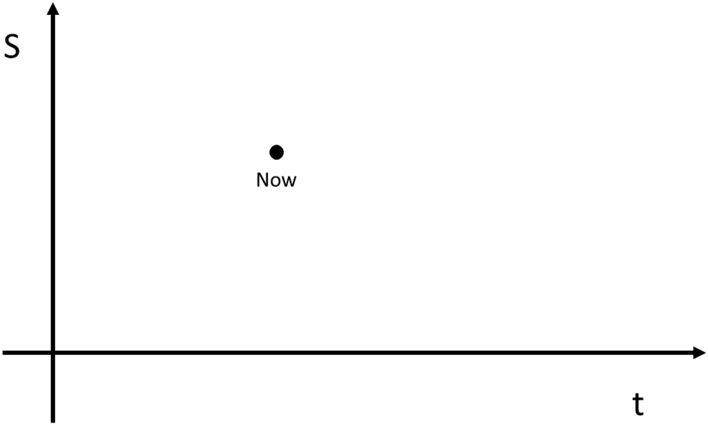
Flatness Problem



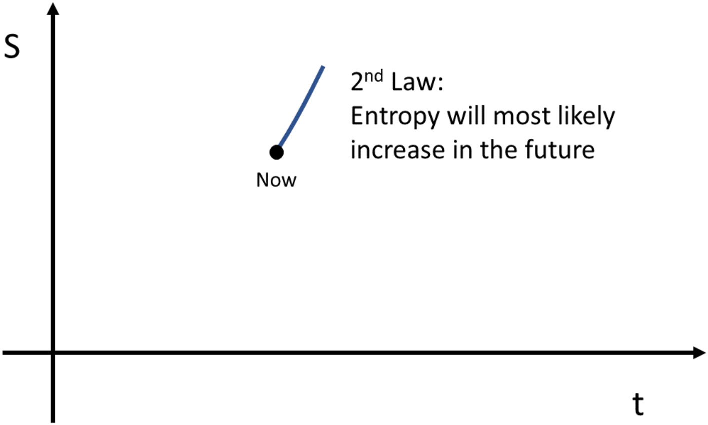
Today:
$< 1\%$

Early Universe:
$< 10^{50}$

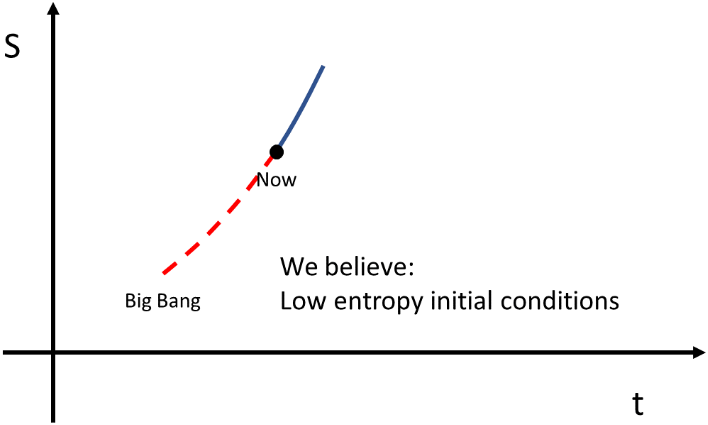
Explaining the Initial State



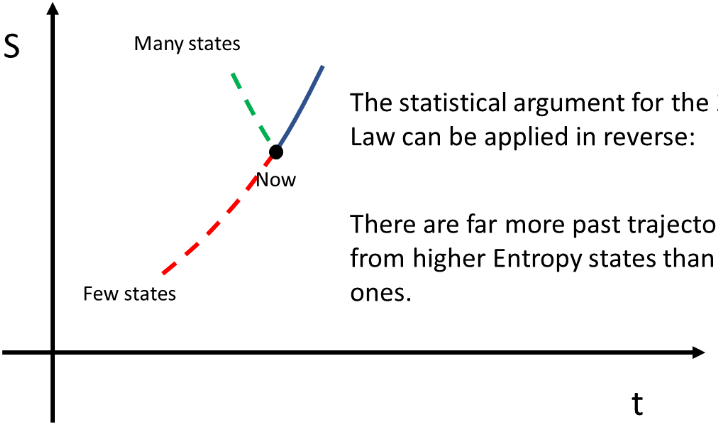
Explaining the Initial State



Explaining the Initial State



"Boltzmann Brain" Problem



The statistical argument for the 2nd Law can be applied in reverse:

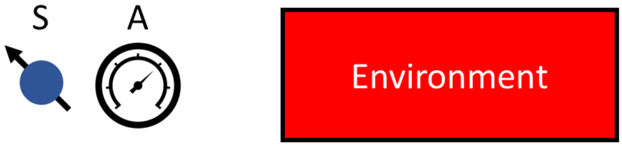
There are far more past trajectories from higher Entropy states than lower ones.

Quantum Measurement

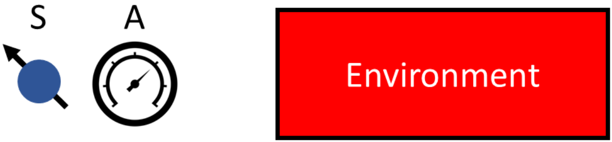
In the Decoherence picture, measurements have two main ingredients:

- Entangling interaction (between system and apparatus).

- Information loss to the environment.

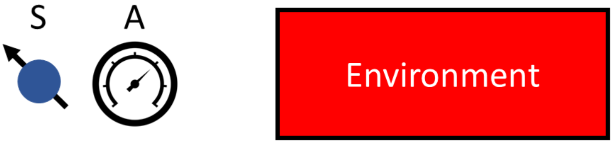


Quantum Measurement



$$(\alpha|0_S\rangle + \beta|1_S\rangle)|0_A\rangle|E(t_0)\rangle$$

Quantum Measurement



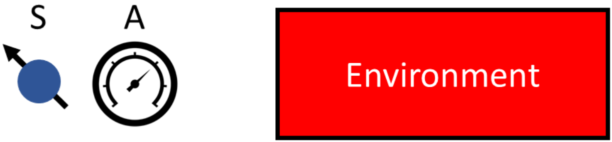
$$(\alpha|0_S\rangle + \beta|1_S\rangle)|0_A\rangle|E(t_0)\rangle$$

Apparatus measures system:

$$\downarrow$$

$$(\alpha|0_S\rangle|0_A\rangle + \beta|1_S\rangle|1_A\rangle)|E(t_1)\rangle$$

Quantum Measurement



$$(\alpha|0_S\rangle + \beta|1_S\rangle)|0_A\rangle|E(t_0)\rangle$$

Apparatus measures system:

$$(\alpha|0_S\rangle|0_A\rangle + \beta|1_S\rangle|1_A\rangle)|E(t_1)\rangle$$

Loses information to environment:

$$(\alpha|0_S\rangle|0_A\rangle|0_E(t)\rangle + \beta|1_S\rangle|1_A\rangle|1_E(t)\rangle)$$

Decoherence

$$\rho_{SA} = Tr_E |\Psi\rangle\langle\Psi|$$

Quantum superposition: $\rho_{SA}(t_1) = \begin{bmatrix} |\alpha|^2 & \cdots & \epsilon \\ \vdots & \ddots & \vdots \\ \epsilon^* & \cdots & |\beta|^2 \end{bmatrix}$

$\downarrow$

Classical probabilities:
$p_1 = |\alpha|^2, p_2 = |\beta|^2$

$$\rho_{SA}(t) = \begin{bmatrix} |\alpha|^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & |\beta|^2 \end{bmatrix}$$

Decoherence

$$\rho_{SA} = \begin{bmatrix} |\alpha|^2 & \dots & \epsilon \\ \dots & \dots & \dots \\ \epsilon^* & \dots & |\beta|^2 \end{bmatrix} \qquad S_{SA} = -tr(\rho_{SA} log\, \rho_{SA})$$

As Decoherence occurs:

- Off diagonals die off*.

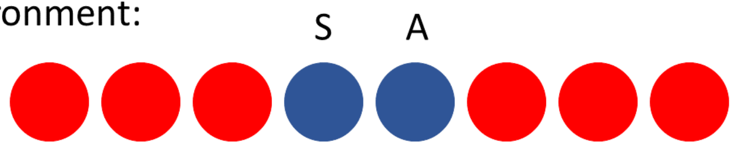
$$|\epsilon| \rightarrow 0$$

- Accompanied by Entropy increase.

$$\Delta S_{SA} > 0$$

Toy Model

Environment:



8 qubits

Hilbert space dim: $2^8 = 256$

Recurrence

Time independent Hamiltonian with finite dimensional Hilbert space:

$$E_j = 2\pi j/256, \qquad j = 0, 1, 2, ...255$$
$$|\Psi(t)\rangle = \sum_j c_j(t=0) e^{-iE_j t} |E_j\rangle$$

Exact recurrence guaranteed every 256 time steps:

$$|\Psi(0)\rangle = |\Psi(256)\rangle$$

Energy Eigenstates

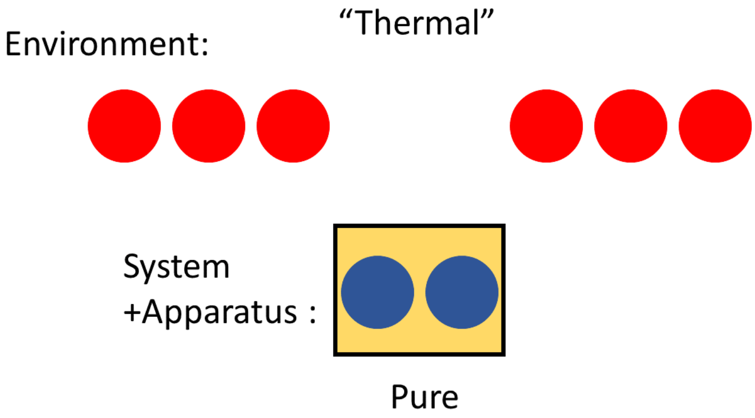
$$|E_j\rangle = b_j0\,|00000000\rangle + b_j1\,|00000001\rangle + b_j2\,|00000010\rangle + \ldots$$

Energy Eigenstates chosen to have lots of entanglement:

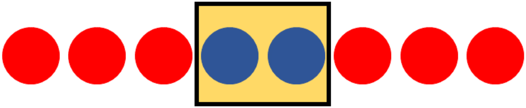
$$S(\rho_{q_i}) = 1 \quad \forall i$$
$$S(\rho_{q_i,q_j}) = 2 \quad \forall i,j$$

Every qubit and every combination of two qubits are maximally mixed.

Initial State

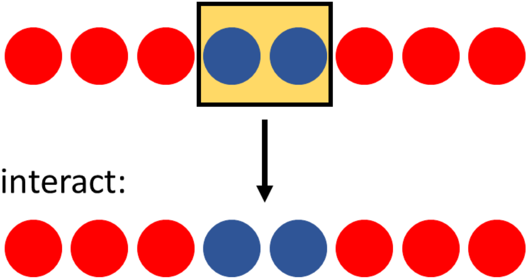


Initial State

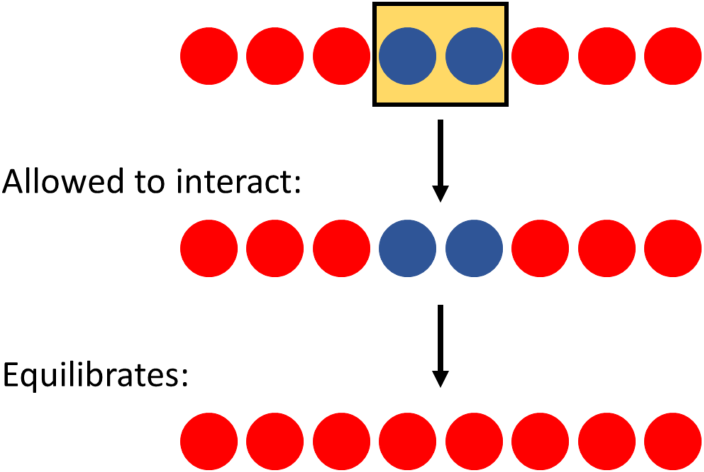


Toy Model

Allowed to interact:



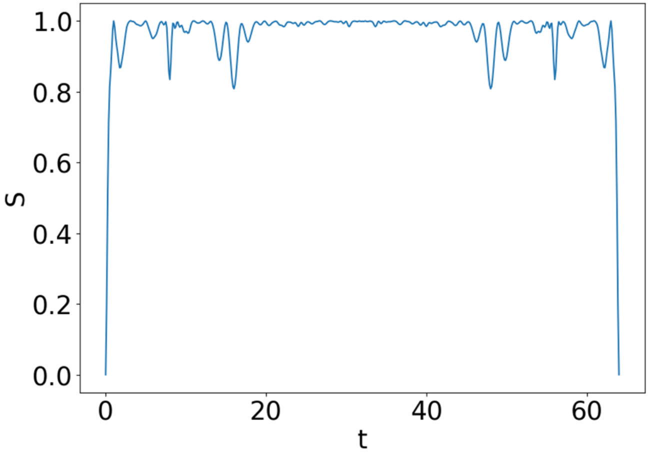
Toy Model



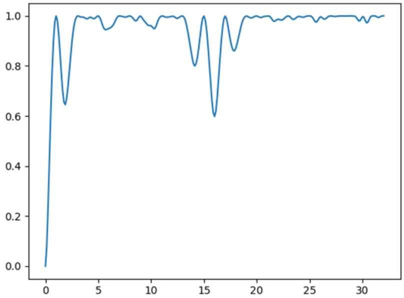
Notes about Toy Model

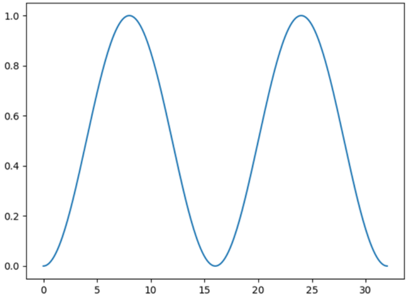
- Evolution is completely determined by initial state and dyanmics. (Fully Unitary)
- Apparent equilibration and fluctuations only arise due to tracing out the environment.

Evolution of 1 qubit

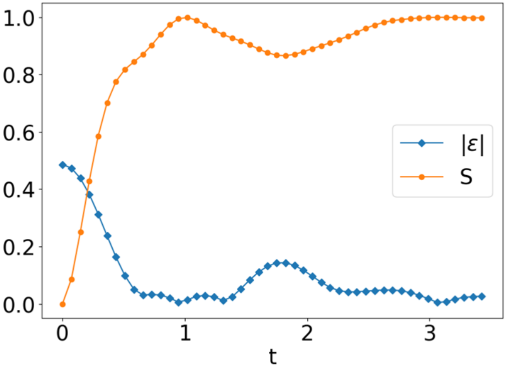


Other initial states





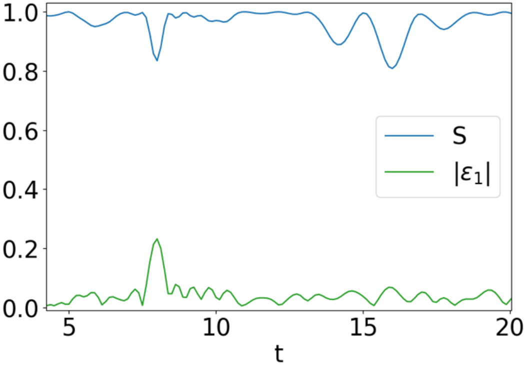
Initial Decoherence in Toy Model



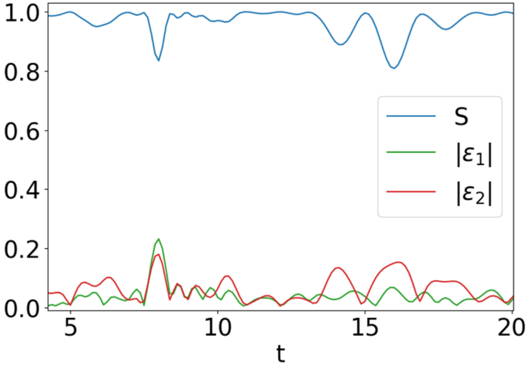
$$\rho_A(t) = \begin{bmatrix} |\alpha|^2 & \epsilon \\ \epsilon^* & |\beta|^2 \end{bmatrix}$$

Off diagonals die off as entropy increases.

Off diagonals during Fluctuations



Off diagonals during Fluctuations



Remaining Questions and Work in Progress

- Evolution of the off diagonals during fluctuations may indicate different decoherent story.
- Need to properly diagnosis whether classical description makes sense for the fluctuations.
- Still exploring more initial states (for system and environment). Further splitting to System+Apparatus+Environment.