Decoherence for Fluctuations out of Equilibrium

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Outline

- Background and Motivation:
  - Tuning Example: Flatness Problem
  - Arrow of Time and Boltzmann Brain Problem
  - Decoherence

- Results:
  - Equilibrating/Fluctuation Toy qubit Model
FRW Universe

▶ 1st Friedmann Equation:

\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_r + \ldots)
\]

▶ Describes a homogeneous, isotropic universe expanding (or contracting) with scale factor, \(a\)

▶ Energy densities evolve based on:

\[
\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + p)
\]
Curvature

- Critical density: $\rho_c$
  If $\rho_m + \rho_r + ... = \rho_c$ then flat

- Curvature, $k$:

$$\rho_k = \rho_c - (\rho_m + \rho_r + ...)$$
$$k = -a^2 \rho_k$$
Flatness Problem

Today: $< 1\%$

Early Universe: $< 10^{50}$
Explaining the Initial State
Explaining the Initial State

2\textsuperscript{nd} Law:
Entropy will most likely increase in the future
Explaining the Initial State

We believe: Low entropy initial conditions
"Boltzmann Brain" Problem

The statistical argument for the 2\textsuperscript{nd} Law can be applied in reverse:

There are far more past trajectories from higher Entropy states than lower ones.
Quantum Measurement

In the Decoherence picture, measurements have two main ingredients:

- Entangling interaction (between system and apparatus).
- Information loss to the environment.
Quantum Measurement

\((\alpha |0_S\rangle + \beta |1_S\rangle)|0_A\rangle|E(t_0)\rangle\)
Quantum Measurement

Apparatus measures system:

\[ \langle \alpha | 0_S \rangle \langle 0_A | + \beta | 1_S \rangle \langle 1_A | | E(t_1) \rangle \]

\[ \langle \alpha | 0_S \rangle | 0_A \rangle + \beta | 1_S \rangle | 1_A \rangle | E(t_1) \rangle \]
Quantum Measurement

Apparatus measures system:

\[ (\alpha |0_S\rangle + \beta |1_S\rangle)|0_A\rangle|E(t_0)\rangle \]

\[ (\alpha |0_S\rangle|0_A\rangle + \beta |1_S\rangle|1_A\rangle)|E(t_1)\rangle \]

Loses information to environment:

\[ (\alpha |0_S\rangle|0_A\rangle|0_E(t)\rangle + \beta |1_S\rangle|1_A\rangle|1_E(t)\rangle) \]
Decoherence

\[ \rho_{SA} = Tr_E |\Psi\rangle\langle \Psi| \]

Quantum superposition: \( \rho_{SA}(t_1) = \begin{bmatrix} |\alpha|^2 & \cdots & \epsilon \\ \vdots & \ddots & \vdots \\ \epsilon^* & \cdots & |\beta|^2 \end{bmatrix} \)

Classical probabilities: \( p_1 = |\alpha|^2, p_2 = |\beta|^2 \)

\( \rho_{SA}(t) = \begin{bmatrix} |\alpha|^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & |\beta|^2 \end{bmatrix} \)
Decoherence

\[ \rho_{SA} = \begin{bmatrix} |\alpha|^2 & \ldots & \epsilon \\ \ldots & \ldots & \ldots \\ \epsilon^* & \ldots & |\beta|^2 \end{bmatrix} \]

\[ S_{SA} = -\text{tr}(\rho_{SA} \log \rho_{SA}) \]

As Decoherence occurs:

- Off diagonals die off*.

\[ |\epsilon| \rightarrow 0 \]

- Accompanied by Entropy increase.

\[ \Delta S_{SA} > 0 \]
Toy Model

Environment:

8 qubits
Hilbert space dim: $2^8 = 256$
Time independent Hamiltonian with finite dimensional Hilbert space:

\[ E_j = \frac{2\pi j}{256}, \quad j = 0, 1, 2, \ldots 255 \]

\[ |\psi(t)\rangle = \sum_j c_j(t = 0) e^{-iE_j t} |E_j\rangle \]

Exact recurrence guaranteed every 256 time steps:

\[ |\psi(0)\rangle = |\psi(256)\rangle \]
Energy Eigenstates

\[ |E_j\rangle = b_j0 |00000000\rangle + b_j1 |00000001\rangle + b_j2 |00000010\rangle + ... \]

Energy Eigenstates chosen to have lots of entanglement:

\[ S(\rho_{q_i}) = 1 \quad \forall i \]
\[ S(\rho_{q_i,q_j}) = 2 \quad \forall i,j \]

Every qubit and every combination of two qubits are maximally mixed.
Initial State

Environment: 

“Thermal”

System + Apparatus: Pure
Initial State
Toy Model

Allowed to interact:
Toy Model

Allowed to interact:

Equilibrates:
Notes about Toy Model

- Evolution is completely determined by initial state and dynamics. (Fully Unitary)

- Apparent equilibration and fluctuations only arise due to tracing out the environment.
Evolution of 1 qubit
Other initial states
Initial Decoherence in Toy Model

Off diagonals die off as entropy increases.

\[
\rho_A(t) = \begin{pmatrix}
|\alpha|^2 & \epsilon \\
\epsilon^* & |\beta|^2
\end{pmatrix}
\]
Off diagonals during Fluctuations

![Graph showing Off diagonals during Fluctuations](image-url)
Off diagonals during Fluctuations
Remaining Questions and Work in Progress

- Evolution of the off diagonals during fluctuations may indicate different decoherent story.

- Need to properly diagnose whether classical description makes sense for the fluctuations.

- Still exploring more initial states (for system and environment). Further splitting to System + Apparatus + Environment.