Black Holes are Alpha-Bit Sup

arXiv:1706.09434, arXiv:1805.?????

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Preview

- ❑ Qubits are composite resources
- ❑ Another resource (that you have never heard of) is more fundamental than a qubit
- ❑ Quantum error correction in AdS/CFT is only approximate and bulk operators are state-dependent
- ❑ Sending qubits at the coherent information rate does not exhaust the ability of a channel to send quantum information
- ❑ There is no need to use classical bits to do entanglement-distillation, state-merging, remote state preparation, channel simulation or teleportation

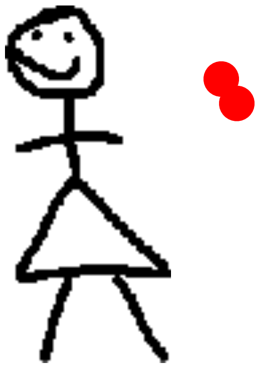
Quantum Communication Resource Inequalities

$$1 \ \text{qubit} > 1 \ \text{ebit}$$

zero-bits

$$1 \ \text{ebit} + 2 \ \cancel{\text{cbits}} > 1 \ \text{qubit}$$

weakened
version of qubits



Quantum Communication Resource Inequalities

$$1 \text{ cbit} < 1 \text{ qubit} > 1 \text{ ebit}$$

$$\underbrace{1 \text{ ebit}}_{\text{coherence}} + \underbrace{2 \text{ zero-bits}}_{\text{communication}} \overset{(a)}{=} 1 \text{ qubit}$$

weakened version of qubits

asymptotic

$$m \text{ qubits} \geq 2m \text{ zero-bits} \ ?$$

$$1 \text{ cbit} \overset{(a)}{>} 1 \text{ zero-bit} \ ?$$

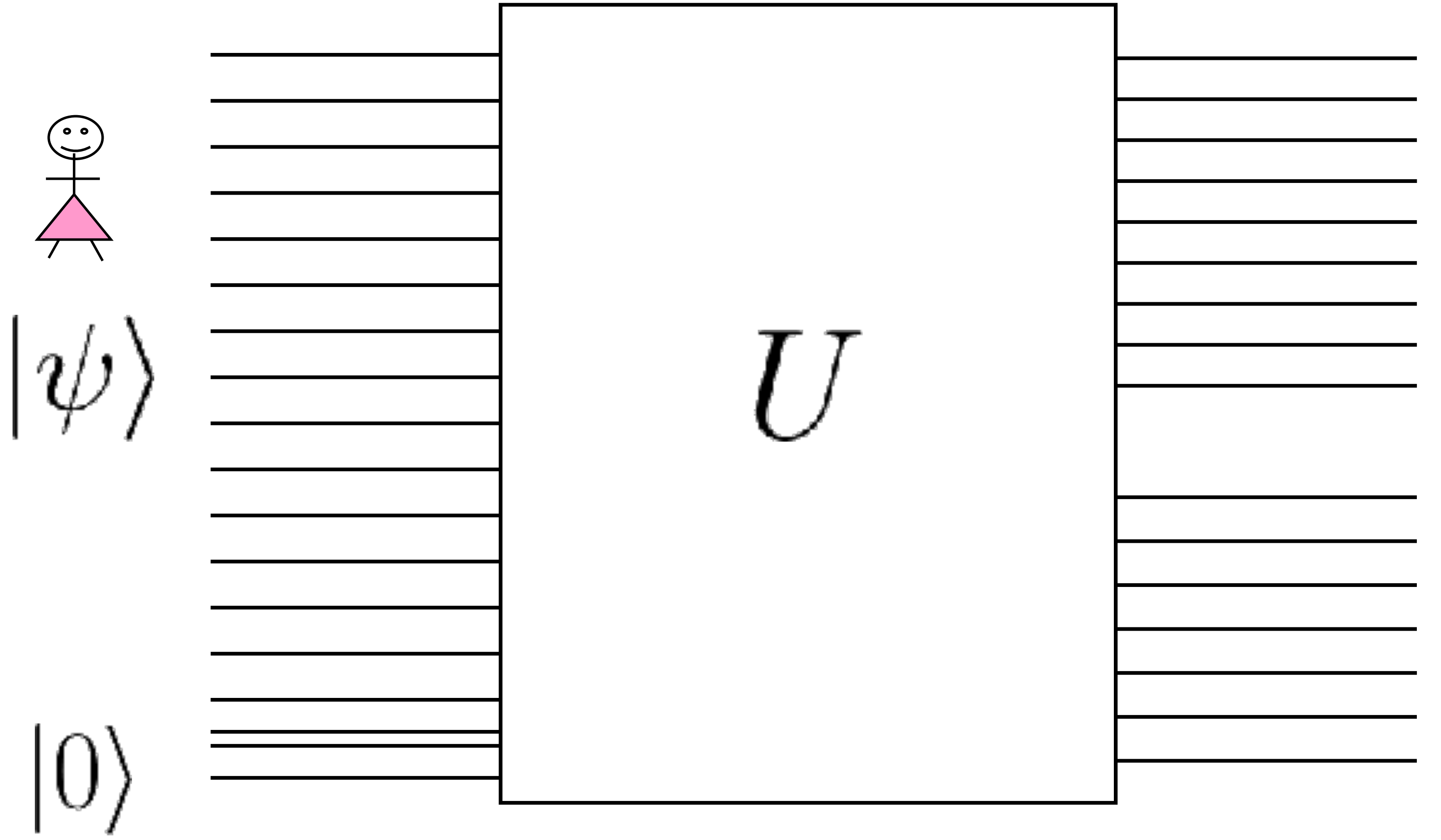
What are zero-bits?

$$1 \text{ zero-bit} \;=\; 1\ \alpha\text{-bit} \quad \text{with } \alpha = 0$$

What are zero-bits?

$d_B >> d_E$

B encodes the zero-bits of $|\psi\rangle$



$\forall |\psi_1\rangle, |\psi_2\rangle,$

$\|\psi_1^B - \psi_2^B\|_1 \approx \|\psi_1 - \psi_2\|_1$

$\Updownarrow$

$\forall |\psi\rangle,$

$\psi^E \approx \omega^E$

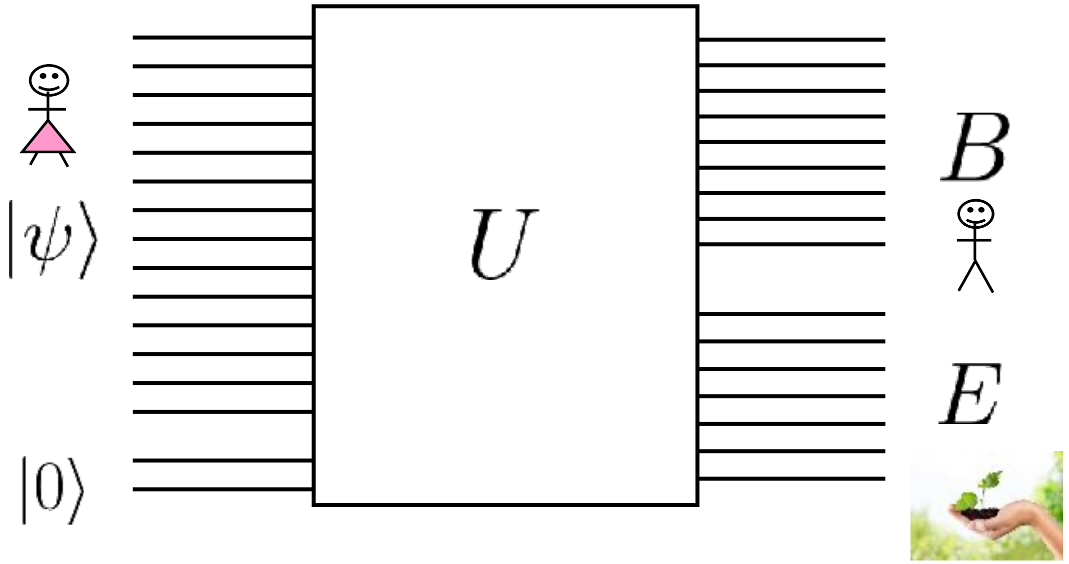
isometric

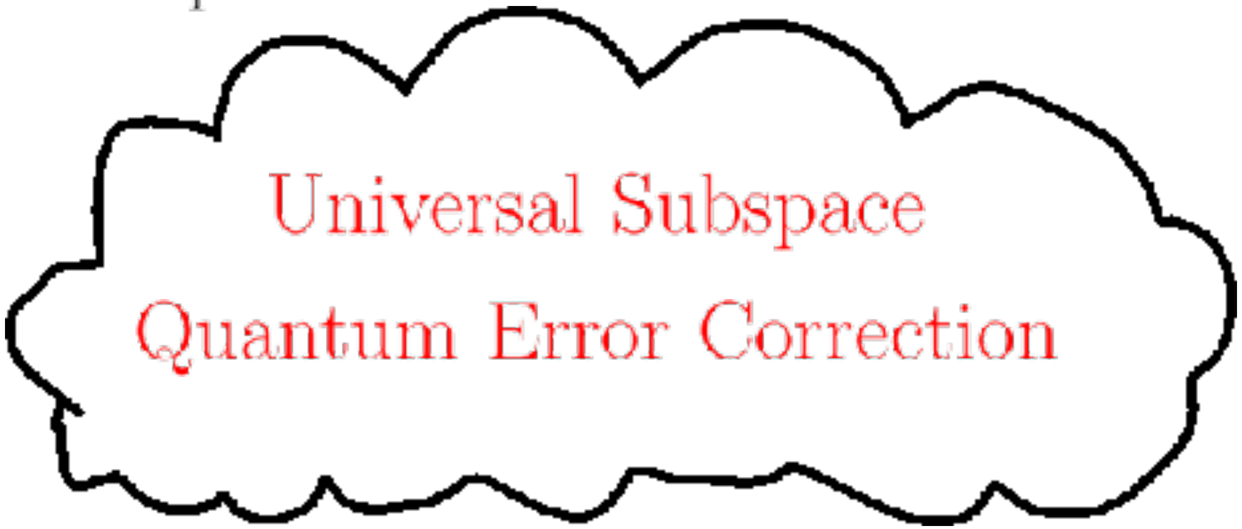
What can you do with zero-bits?

Not possible to recover the state $|\psi\rangle$ from B with no information about the state
- Error correction is not possible

However if we know $|\psi\rangle \in \text{span}\left(|\psi_1\rangle, |\psi_2\rangle\right)$ then we can determine $|\psi\rangle$

Able to error-correct any two-dimensional subspace S





Definition of zero-bits

Definition of qubits

"n qubits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

$$\exists \mathcal{D}$$

$$\mathcal{N} = \mathrm{Id} \quad ?$$

What do we need to be true about the channel?

Bob can always error correct so long as error correction is possible

Definition of zero-bits

"n zero-bits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

$$\forall S \subseteq A \quad d_S = 2$$

$$\exists \mathcal{D}_S$$
$$\mathcal{D}_S \circ \mathcal{N}|_S = \mathrm{Id}_S$$

OK now what about zero-bits?

Now Bob only has to be able to error correct any *two-dimensional subspace*

Definition of zero-bits

"n qubits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B)$$

$$d_A = 2^n$$

$$\forall S \subseteq A \qquad d_S = 2$$

$$\exists \mathcal{D}_S$$
$$\mathcal{D}_S \circ \mathcal{N}|_S = \mathrm{Id}_S$$

Definition of zero-bits

Why do I never get told anything interesting

"n zero-bits"

$$\mathcal{N} : \mathcal{S}(A) \to \mathcal{S}(B) \qquad\qquad \mathcal{N}^c : \mathcal{S}(A) \to \mathcal{S}(E)$$

[Hayden, Winter 2012]

$$d_A = 2^n$$

$$\forall S \subseteq A \quad d_S = 2$$

$$\exists \mathcal{D}_S$$

$$\|\mathcal{D}_S \circ \mathcal{N}|_S - \mathrm{Id}_S\|_\diamond \leq \delta$$

$$\Leftrightarrow$$

$$\forall |\psi\rangle \in A,$$

$$\|\mathcal{N}^c(\psi - \omega)\|_1 \leq \varepsilon$$

Need to make definition approximate if zero-bits are to be different from qubits

$$\boxed{\frac{1}{16}\delta^2 \leq \varepsilon \leq 8\sqrt{\delta}}$$

Definition of alpha-bits

"Subspace decoupling duality"

"n α-bits"

$\mathcal{N}:\mathcal{S}(A)\to\mathcal{S}(B)$

$\mathcal{N}^c:\mathcal{S}(A)\to\mathcal{S}(E)$

$d_A = 2^n$

$\Leftrightarrow$

$\forall\,|\psi\rangle\in AR,\quad d_R = 2^{\alpha n}$

$\left\|\mathcal{N}^c\otimes\mathrm{Id}_R\left(\psi-\omega\otimes\psi^R\right)\right\|_1\le\varepsilon$

$\forall S\subseteq A\quad d_S\le 2^{\alpha n}+1$

$\exists\mathcal{D}_S$

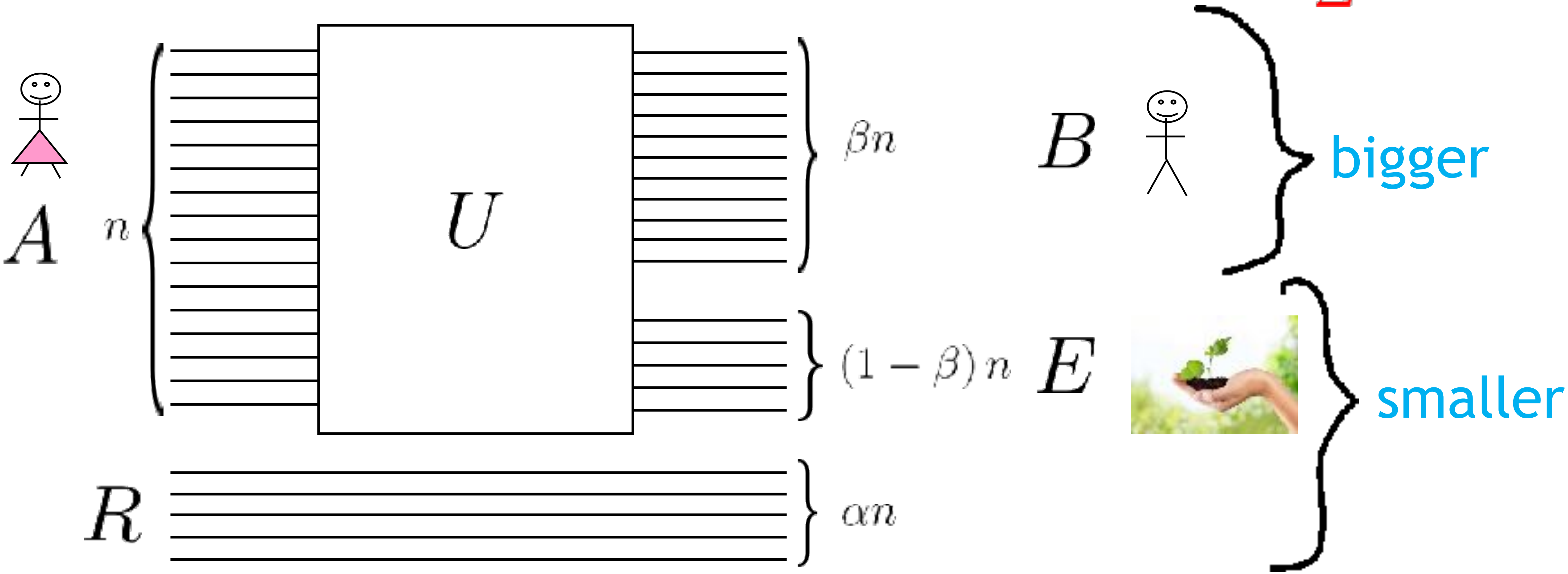
$\alpha=1\Rightarrow$ qubits

$\|\mathcal{D}_S\circ\mathcal{N}|_S-\mathrm{Id}_S\|_\diamond\le\delta$

$$\boxed{\frac{1}{16}\delta^2\le\varepsilon\le 8\sqrt{\delta}}$$

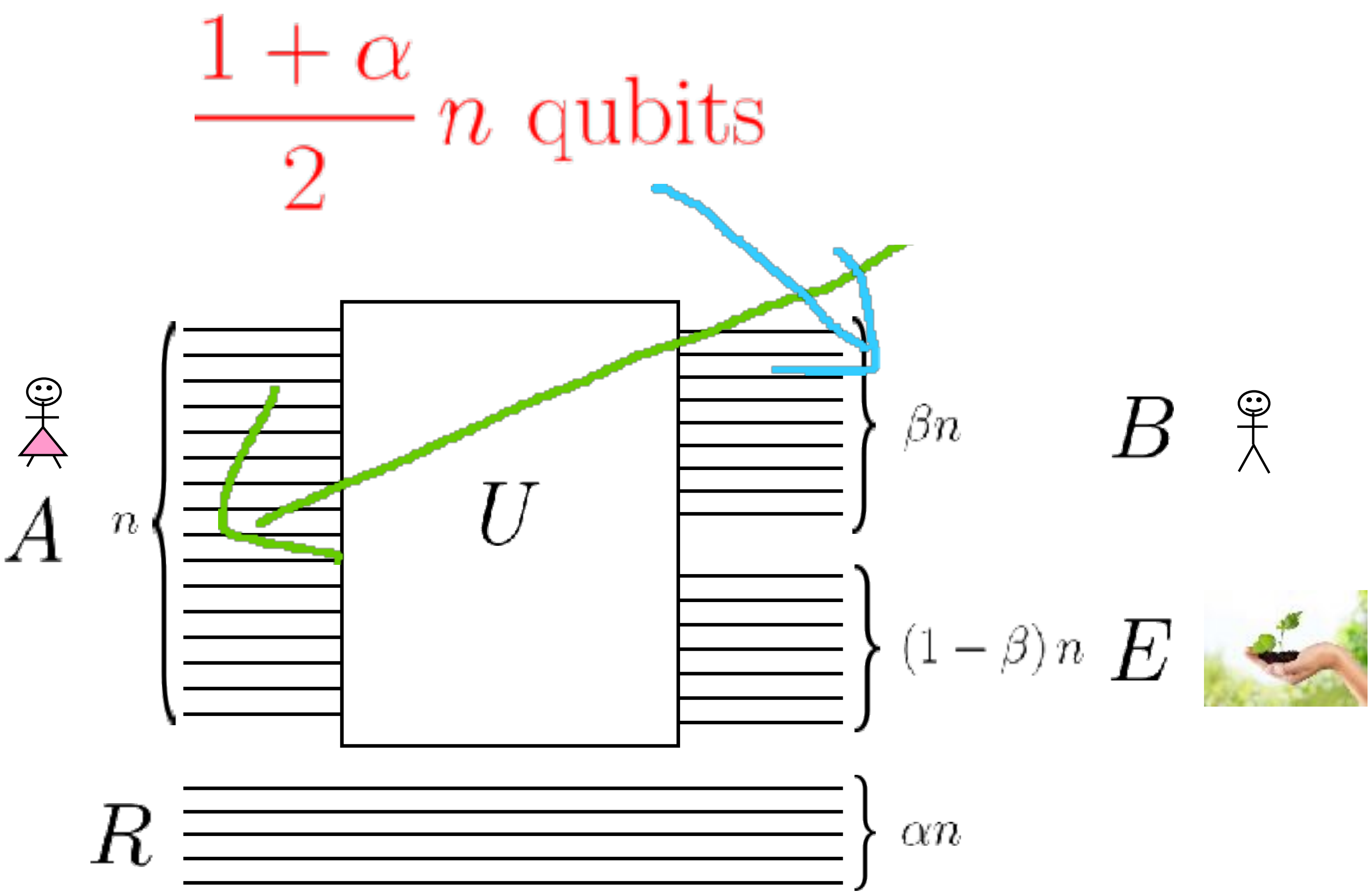
Transmitting alpha-bits

$$\beta > \frac{1+\alpha}{2}$$



Necessary condition to send alpha-bits. Also sufficient (with some subtleties about needing to use shared randomness and block

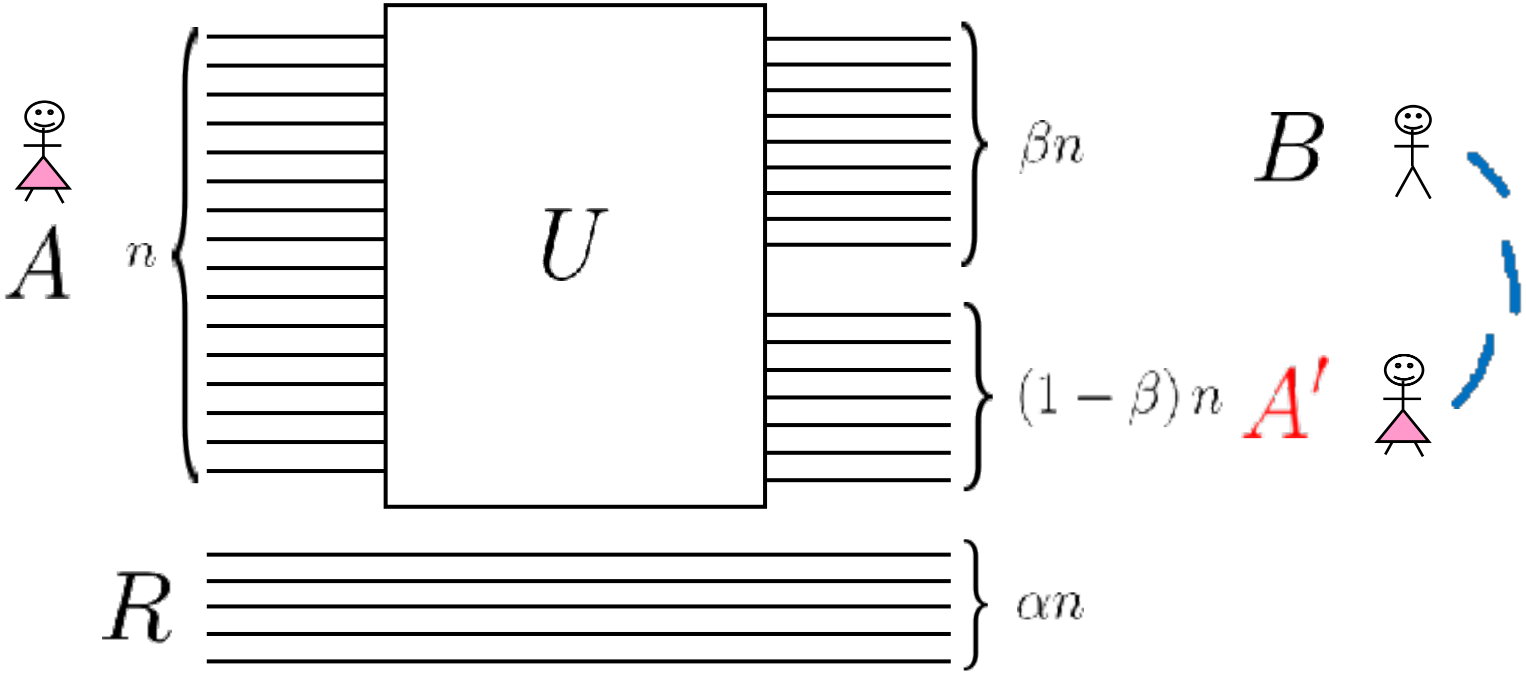
Transmitting alpha-bits



$$\beta > \frac{1+\alpha}{2}$$

Transmitting alpha-bits

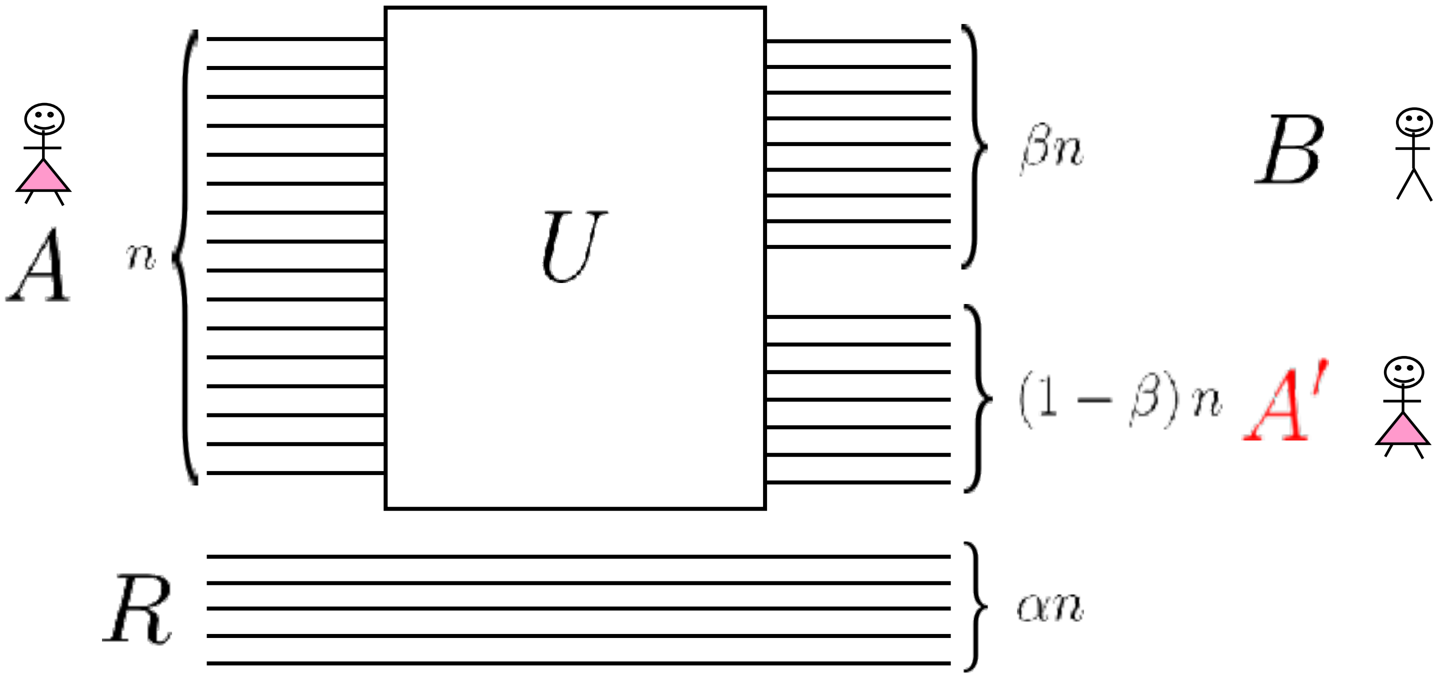
$$\frac{1+\alpha}{2} n \text{ qubits} \geq n \ \alpha\text{-bits} + \frac{1-\alpha}{2} n \text{ ebits}$$



$$\beta > \frac{1+\alpha}{2}$$

Transmitting alpha-bits

$$(1+\alpha)\text{ qubits} \overset{(a)}{\geq} 2\ \alpha\text{-bits} \ + \ (1-\alpha)\text{ ebits}$$

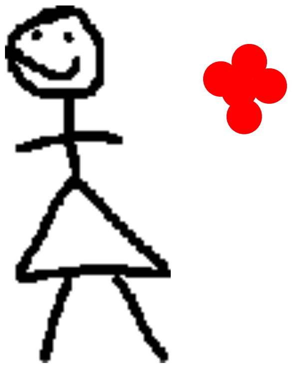


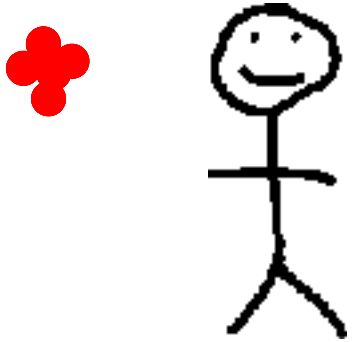
$$\beta > \frac{1+\alpha}{2}$$

(Coherent) super-dense coding

1. Alice and Bob share n ebits

$$\sum_{k=0}^{2^n-1} |k\rangle_A |k\rangle_B$$

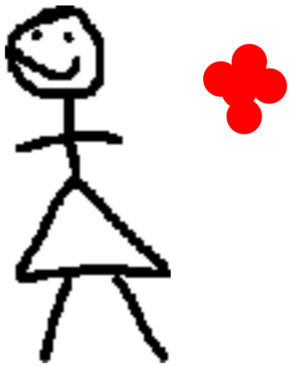




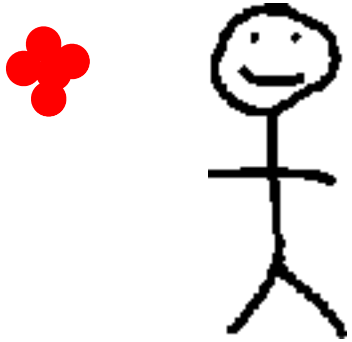
(Coherent) super-dense coding

1. Alice and Bob share n ebits
2. Alice applies an operation to her qubits
3. Alice sends her qubits to Bob

$$\sum_{k=0}^{2^n-1} e^{\frac{2\pi k r i}{2^n}} |k \oplus s\rangle_A |k\rangle_B$$



$$0 \leq r < 2^n$$
$$0 \leq s < 2^n$$

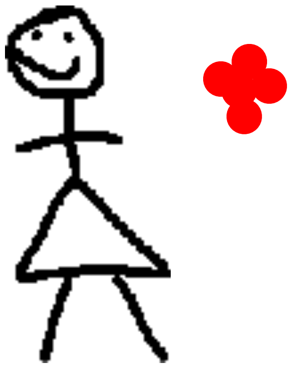


(Coherent) alpha-bit super-dense coding

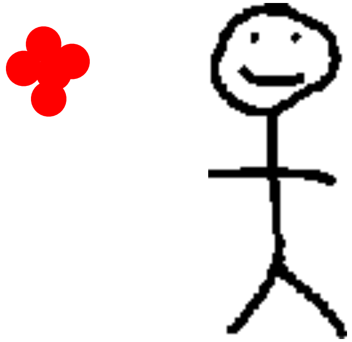
1. Alice and Bob share n ebits
2. Alice applies an operation to her qubits
3. Alice sends her qubits to Bob as α-bits

Done ✓

$$\sum_{k=0}^{2^n-1} e^{\frac{2\pi k r i}{2^n}} |k \oplus s\rangle_A |k\rangle_B$$



$$0 \leq r < 2^n$$
$$0 \leq s < 2^{\alpha n}$$



Zero-bits and ebits as fundamental resources

All noiseless quantum resources (qubits, α-bits, cobits...) can be rewritten in terms of zero-bits and ebits

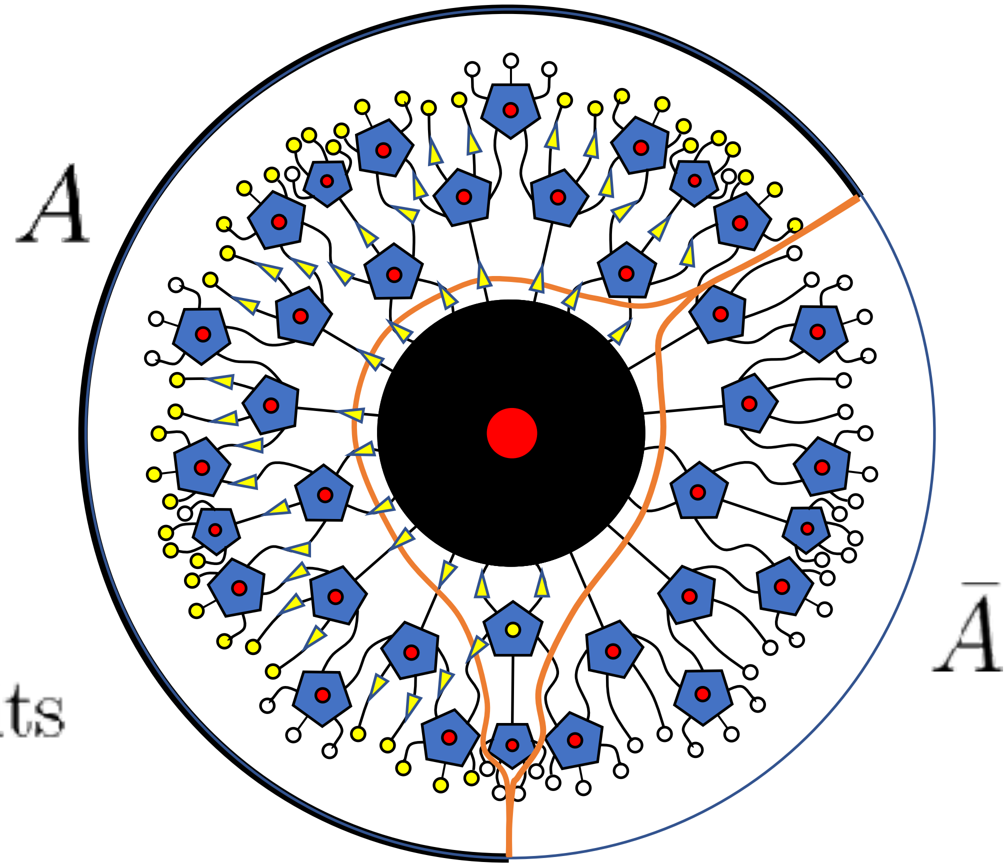
$$\text{e.g.} \quad 1 \ \alpha\text{-bit} \stackrel{(a)}{=} (1+\alpha) \text{ zero-bits} + \alpha \text{ ebits}$$

When rewritten in this basis, the quantum resource ordering becomes the product ordering:

$$(a, b) \geq (a', b') \iff (a \geq a') \wedge (b \geq b')$$

Alpha-bits and Black Holes

Alpha-bits arise naturally when studying black holes in AdS/CFT

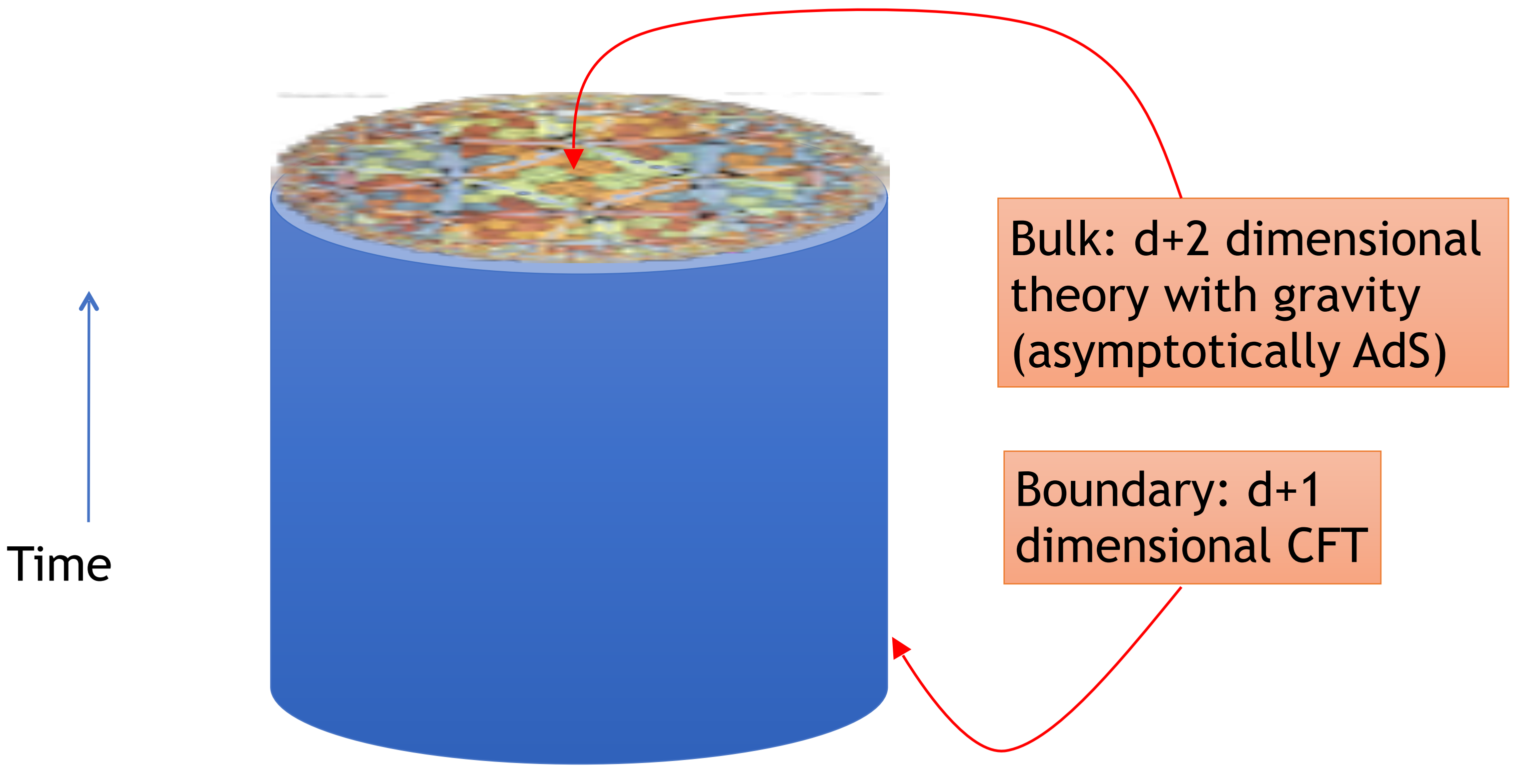


Boundary subregion may encode α-bits of a bulk region

Implications: Error-correction is only approximate, reconstructed operators are state-dependent

Anti-de Sitter/Conformal Field Theory Correspondence

Example:
Type IIB string theory on $AdS_5 \times S^5$
$\cong$
Supersymmetric N=4 Yang-Mills

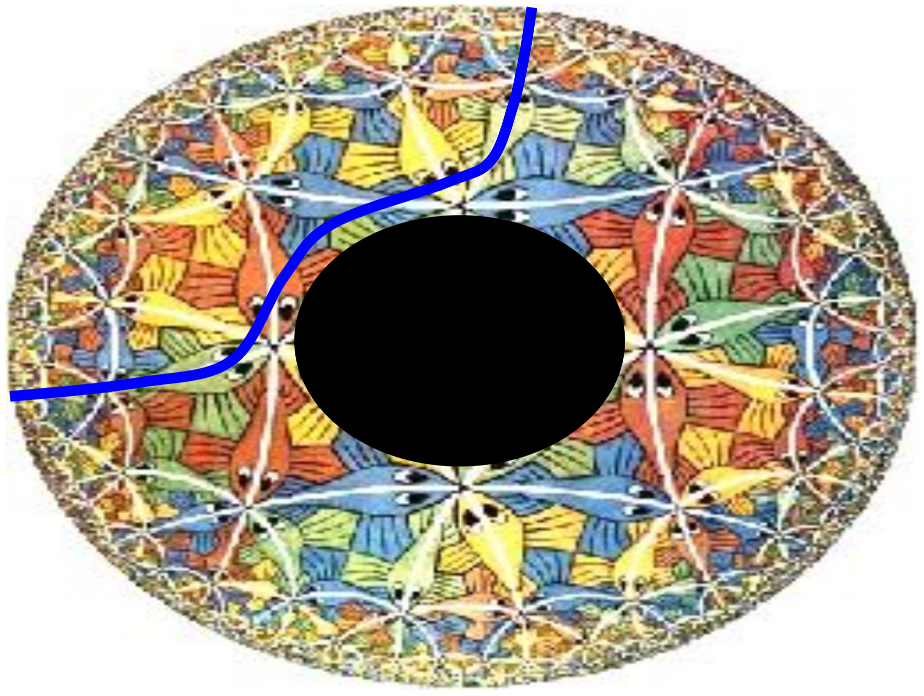


Conjecture: Equivalence of string (gravity) theory in bulk with CFT on boundary [Maldacena’97]

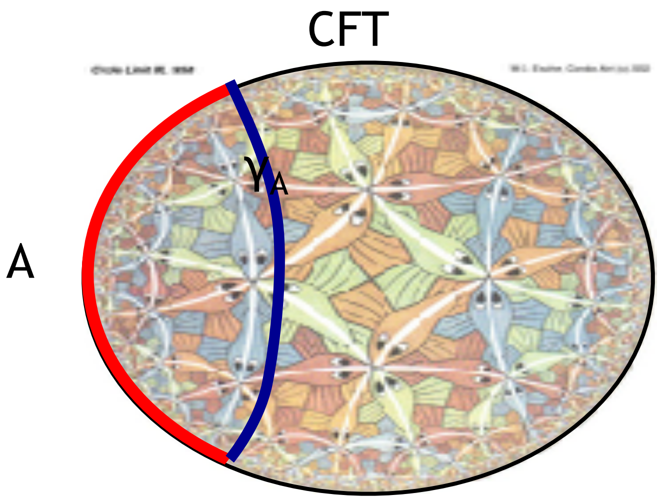
Question: How are bulk degrees of freedom encoded in the boundary?

Spatial slice of anti-de Sitter space

- Hyperbolic space
- Fish-counting metric
- Geodesics (straight lines) follow fish
- Negatively curved: sum of angles in triangle < 180^{o}
- Can place matter deep inside AdS



Entropy in AdS/CFT



Ryu-Takayanagi proposal for bulk formula:

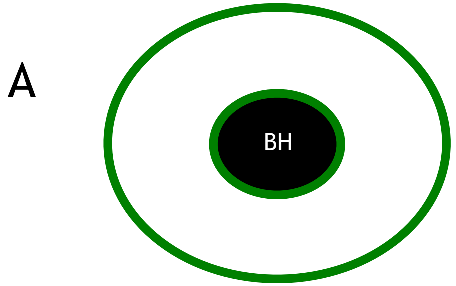
QM

$$S(A) = \frac{1}{4G_N} \min_{\gamma_A}(\mathrm{area}(\gamma_A))$$

GR

Minimize over spatial bulk surfaces γ_A homologous to A.

Generalizes black hole entropy to wide class of spatial regions!



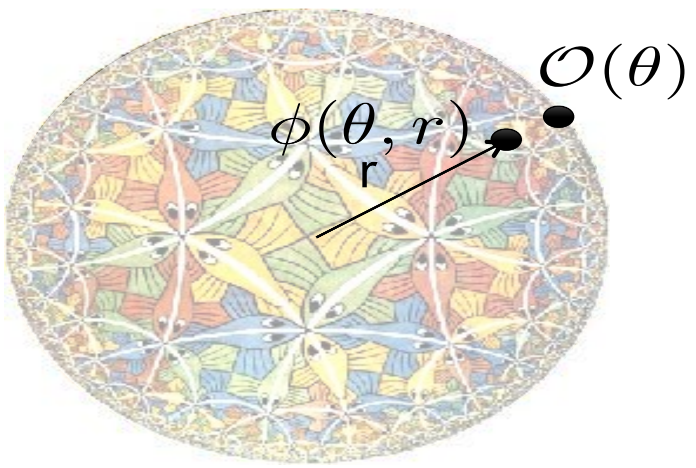
Deep message:
Entanglement structure of boundary reflects (encodes?) geometry of bulk.

- Analytical agreement in AdS_3/CFT_2 [RT'06]
- Satisfies strong subaddivitiy [Headrick-T'07]
- Proof for spherical A [Casini-Huerta-Myers'11]
- General explanation [Lewkowycz-Maldacena'13]

Relating bulk and boundary observables

Boundary in terms of bulk: *extrapolate*

$$\mathcal{O}(\theta) = \lim_{r\to\infty} r^{\Delta}\phi(\theta, r)$$



Bulk in terms of boundary: *smearing*

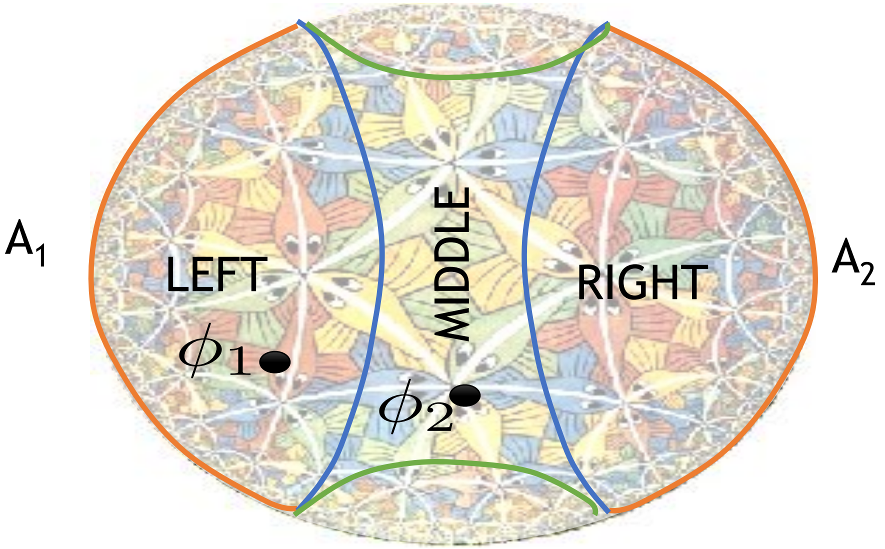
$$\phi(\theta, r) = \int K(\theta, r; \tilde{\theta}, \tilde{t})\, \mathcal{O}(\tilde{\theta}, \tilde{t})\, d\tilde{\theta} d\tilde{t}$$

K arises from solving some PDE's (Green's fn for classical bulk field equations)

Don't always need the whole boundary to reconstruct a given $\phi(\theta,r)$.

[Hamilton, Kabat, Lifschytz, Lowe 2006]

The causal and entanglement wedges



$$S(A) = \frac{1}{4G_N} \min_{\gamma_A}(\text{area}(\gamma_A))$$

Causal wedge: LEFT + RIGHT

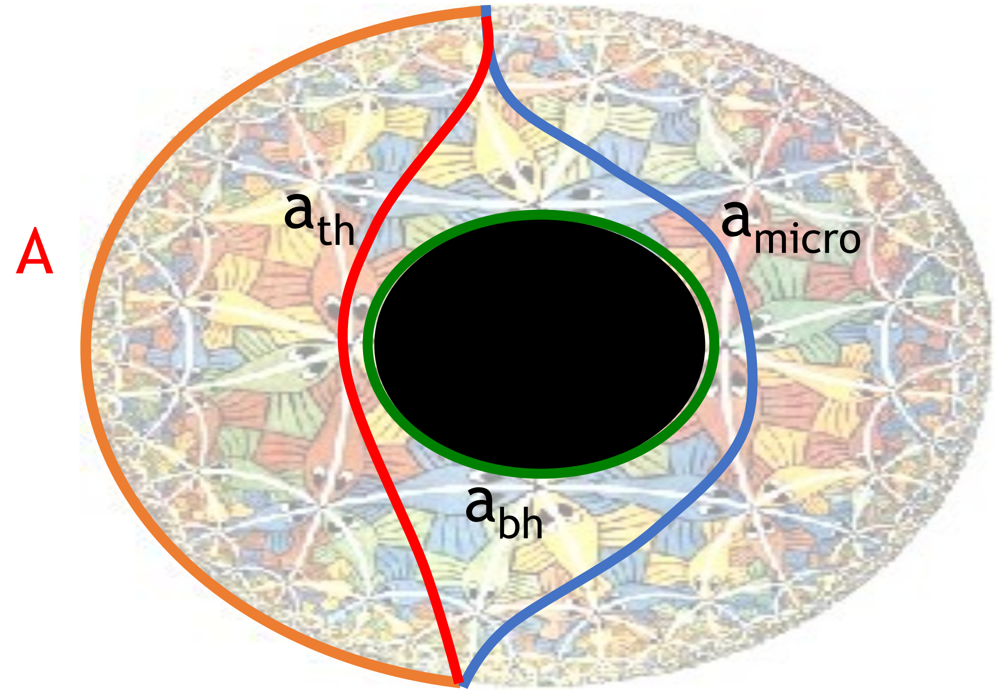
Entanglement wedge: LEFT + MIDDLE + RIGHT

Hamilton, Kabat, Lifschytz, Lowe:
Can reconstruct all bulk operators in the causal wedge

Quantum information arguments:
Can reconstruct all bulk operators in the full entanglement wedge

[Jafferis, Lewkowycz, Maldacena, Suh'16][Dong,Harlow,Wall'16][Cotler,Hayden,Salton,Swingle,Walter '17]

Decoding black hole microstates



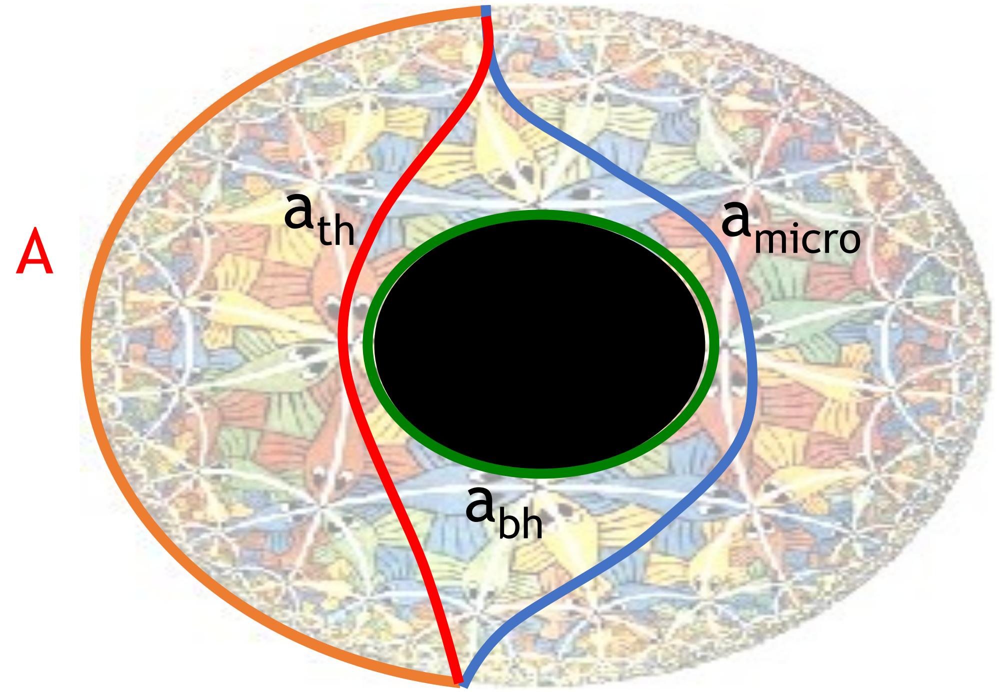
Pure state: a_{micro} is minimal
BH observables are mapped to A

Thermal state:
Minimizing with homology constraint
$a_{micro} + a_{bh} > a_{th}$

BH is outside A's thermal entanglement wedge:
BH observables not mapped to A

Location of entanglement wedge depends
on how uncertain decoder is about the microstate

Decoding black hole microstates



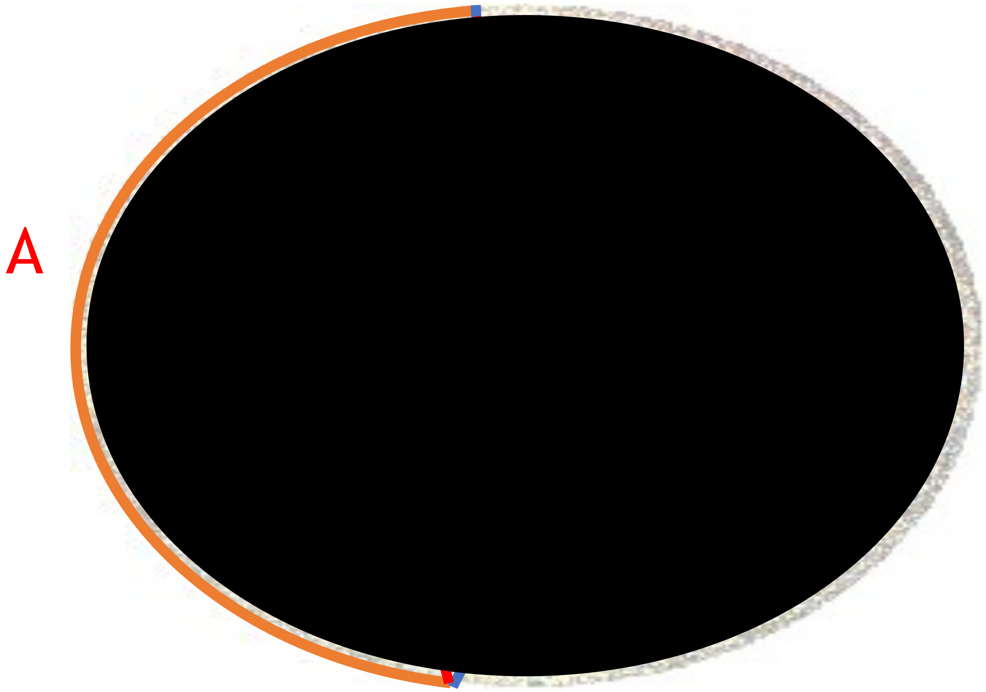
Consider mixture of BH microstates
Entropy: $r = \alpha\, a_{bh}$

Minimizing with homology constraint
Choose α such that
$a_{micro} + \alpha\, a_{bh} < a_{th}$

a_{micro} is minimal:
BH observables mapped to A

Any mixture of microstates will do: universal subspace quantum error correction!
A contains the black hole's α-bits!

Black holes are alpha-bit sup



Consider mixture of BH microstates
Entropy: $r = \alpha\, a_{bh}$

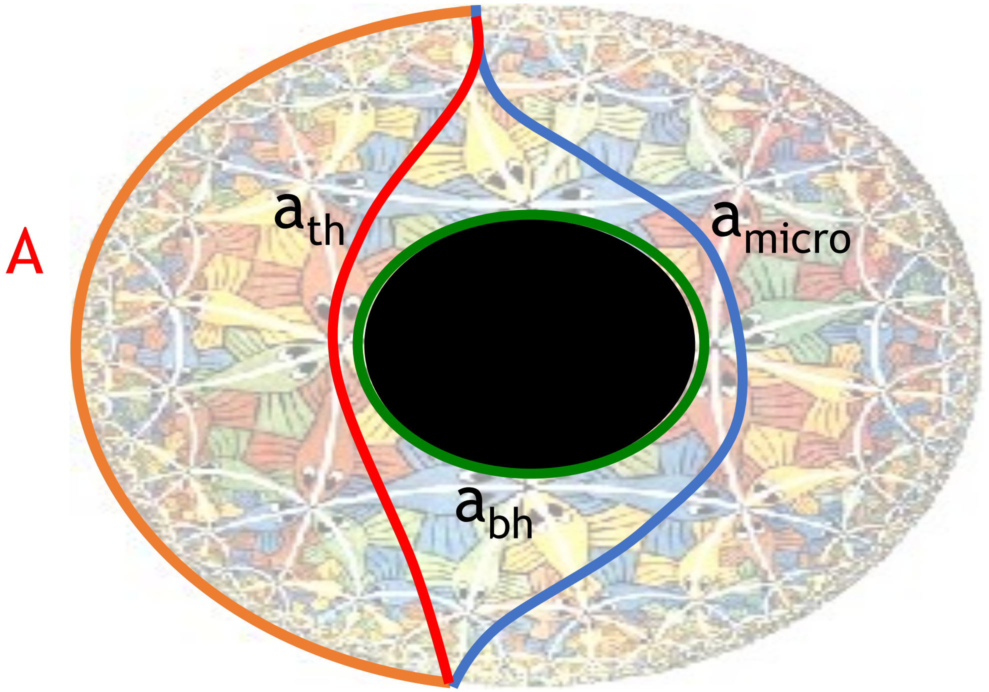
Minimizing with homology constraint
Choose α such that
$\alpha < (a_{th} - a_{micro})/a_{bh}$

$a_{th} = A = x\, a_{bh}$ $a_{micro} = A^c = (1-x)\, a_{bh}$

$x > (1+\alpha)/2$: saturates α-bit capacity!

Large black holes in AdS/CFT give explicit (but #$%! complicated) optimal α-bit codes for all α

Consequences



- Bulk to boundary mapping must be approximate
  - Otherwise, no difference between alpha-bits and qubits
  - Corrections $\exp(-O(S_{bh}))$
  - Nonperturbative?
- Bulk to boundary mapping is state-dependent, even in a fixed geometry

Summary

- Alpha-bits quantify asymptotically distinct forms of approximate quantum error correction
- Qubits are composite resources
  - Ebits and 0-bits are the fundamental independent resources of correlation and communication
- Alpha-bits arise naturally in AdS/CFT
  - Inequivalence of alpha-bits and qubits implies AdS -> CFT error correction is approximate
- 0-bits can substitute for cbits in entanglement distillation, teleportation, state merging, channel simulation, …
- Amortized capacity is singular at $\alpha=1$ (quantum capacity point)

Thank you