Tutorial on Algebraic Geometry and Representation Theory

Varieties and their equations. Linear algebra from a tensor perspective. Segre and Veronese embeddings. Secant varieties.

Basics on representations of the Symmetric Group and General Linear Group; their dimensions. Kronecker and Littlewood-Richardson coefficients.

G. Ottaviani

Lecture I: Basics on Tensor Spaces

The $2 \times 2 \times 2$ tensors, the symmetric $2 \times 2 \times 2 \times 2$ tensors.

Dual variety, dimension of dual of k-secant variety, identifiability, generic k-identifiability.

Space of symmetric tensors. Symmetric rank. Alexander-Hirschowitz Theorem. Generically k-identifiable cases. Galuppi-Mella Theorem on generic symmetric tensors which are identifiable.

Comon Conjecture on symmetric rank, Strassen Conjecture on additivity of rank, some partial results and their spectacular failure due to recent Shitov Counterexamples.

Lecture II: Selected Results on Tensors

Sylvester Pentahedral Theorem, "Classical" and "Modern" approach.

Vector bundles as a tool to understand tensor spaces. Apolarity and Nonabelian Apolarity.

Space of Tensors of format $k_1 \times \cdots \times k_d$. The binary case. Conjectures about dimension of k-secant varieties and generic k-identifiability. Their numerical confirmation. Sketch on numerical homotopy method.

Matrix multiplication algorithm, its complexity and the rank of the matrix multiplication tensor. The exponent Omega. Symmetrization of matrix multiplication tensor.

Lecture III: Metric Properties in Tensor Spaces

Revisiting Singular Value Decomposition from the geometric point of view.

Metric properties in the space of real tensors. Spectral theory of tensors according to Lim and Qi. Best rank k approximation and Banach Theorem in the symmetric case.

Cartwright-Sturmfels formula for symmetric tensors. t-ples of singular tensors.

Friedland-Ottaviani formula, connections with the Hyperdeterminant and with the Euclidean Distance degree (EDdegree). The critical space. The Nuclear Norm and the Entanglement.

J. M. Landsberg

Tensors with continuous symmetry

Computer scientists have conjectured that it is nearly as easy to multiply large matrices as it is to add them. They define the exponent of matrix multiplication ω to be the infimum of the numbers τ such that $n \times n$ matrices may be multiplied using $O(n^\tau)$ arithmetic operations. The conjecture is that $\omega = 2$. The problem was posed in 1969 and there was steady progress on proving upper bounds for ω that ended in 1989. As a first attempt to unblock research on the exponent, I will discuss variants of the conjecture from a geometric perspective. Independent of matrix multiplication, it leads to new, previously uninvestigated properties of tensors of interest in their own right. This is joint work with A. Conner, F. Gesmundo, E. Ventura and Y. Wang.

P. Vrana

Lecture I: Tensor Restriction Problem and Entanglement

The restriction preorder, connection to rank, asymptotic restriction and transformation rates, connection to the exponent omega.

Quantum information basics: states, channels, local operations assisted by classical communication, entangled states and transformations.

Pure two-party entangled states, classification and transformation rates.

Lecture II: The Asymptotic Spectrum

The preordered semiring of tensor classes, asymptotic preorder. Stone property and the Stone-Kadison theorem. The asymptotic spectrum of tensors. Rate formula.

Examples of spectral points, spectral points for subrings, oblique tensors and the support functionals.

Lecture III: Universal Spectral Points

Marginals of a quantum state, entanglement polytope and its characterisation using representation theory. Equivalent definitions of quantum functionals.

Basic properties of the von Neumann entropy. Semigroup property of the Kronecker and Littlewood-Richardson coefficients. Entropic vanishing conditions. Consequences for the quantum functionals.

P. Bürgisser

Geometric invariant theory and complexity (overview)

Setting of geometric invariant theory, orbit closure intersection problem, basic examples, invariant ring and null cone, overview on recent results.

M. Walter

Lecture I: Geometric Invariant Theory and Noncommutative Duality

Null cone and Hilbert-Mumford criterion. Moment map and Kempf-Ness theorem. Interpretation as a noncommutative duality. Examples: matrix, operator, tensor scaling. Tensor scaling algorithm for uniform marginals.

Lecture II: Tensor Scaling Algorithms

Recap and analysis of tensor scaling algorithm. Extension to entanglement polytopes (general marginals). Geodesic convexity and optimization. Continuous scaling algorithms.