Exercises in Tracking Algorithms (Nordic Detector school Oct 2018)

1)Consider our standard spectrometer with 6 planes:
http://www.nbi.dk/~phansen/nordforsk/Spectrometer.h
http://www.nbi.dk/~phansen/nordforsk/Spectrometer.C
http://www.nbi.dk/~phansen/nordforsk/Hit.h
http://www.nbi.dk/~phansen/nordforsk/RecoTrack.h
http://www.nbi.dk/~phansen/nordforsk/TruthTrack.h
http://www.nbi.dk/~phansen/nordforsk/SMgr.C
http://www.nbi.dk/~phansen/nordforsk/MakeFile

This is a C++ program simulating a compact spectrometer with very thin CMOS pixel layers made of 50mu thick MIMOSA chips (2 by 1 cm). It has a small magnet in the center as in slide 36. The goal is to measure positron production in the range 50MeV-1GeV from an electron beam hitting a diamond target just upstream of the first plane. This is of interest for future e+e- colliders. The instrument is actually in use by an Aarhus group.

The program simulates single mono-energetic positrons emitted along the horizontal x-axis and being bent in the vertical y direction. The simulated positrons are traced and digitized through the spectrometer, including multiple scattering, noise, and realistic measurement resolutions and inefficiencies.

One single track is attempted to be reconstructed, first using a Kalman Filter in the x-z plane for pattern recognition and then a global linear chi-squared fit for finding the best among the candidate tracks. These methods are implemented exactly as written in the slides using the ROOT TMatrixD class.

The reconstruction requires hits-on-track in all planes, except for the last two planes, where only one good hit is required. This requirement could be changed, but of course all planes are needed in the case of a spectrometer with only four planes. Six planes are probably necessary, however, to provide sufficient redundancy in the case of multi-particle impacts.

The usage is:
(edit the Spectrometer constructor to have the desired parameters.
Units are cm, GeV and Tesla)
>make
>./SMgr.exe | tee spectrometer.out
(look at output file)
> root
root> TBrowser b
(look at histograms)

1a) Derive from the spread of the measured inverse momenta, 1/p, at p= 0.05 GeV and p=1 GeV **the relative contribution** to the momentum resolution, $\Delta p/p$, **of multiple scattering** at the two beam momenta. Use default configuration.

1b) Check if the spectrometer configuration is optimal. The length (try e.g. 60cm)**?** The quality cuts? The B field (assuming it is possible to vary it by 50%)? The noise occupancy (which is determined by the charge threshold for a pixel hit)? Use the 1/p accuracy, the reconstruction efficiency and the computing time as deciding factors. Try both 0.05 and 1 GeV in each case.

1c) The single plane efficiency assumed in the simulation is the "hitEfficiency" in the Spectrometer constructor. **How would you measure the single plane hit efficiency** using this program on real data?

1d) The bremsstrahlung on slide 59 (Bethe-Heitler formula) is not taken into account. **When implemented in the simulation in the three planes before the magnet, how much loss of performance do you see**?

2 Let us consider a tracking detector fully immersed in a uniform magnetic field. **Show** that for circular tracks passing through the origin, the conformal transformation:

$u=x/(x^2+y^2)$ $v=y/(x^2+y^2)$

maps a circle to a straight line with the equation

$v = 1/2b - (a/b)u$

where (a,b) is the center of the circular track with radius $\sqrt{a^2+b^2}$.

A straight line can be characterized by its distance at closest approach, *d*, to the origin and the direction, θ, to its point at closest approach.
Show that any straight line passing through the conformal coordinates of some hit (u_i,v_i) fulfills the equation:

$$d = u_i \cos\theta + v_i \sin\theta$$

The idea is now to plot all the possible d and θ (the Hough transform) for all the hits in a 2D histogram and thus to find the (d,θ) of local peaks corresponding to circular tracks passing through (0,0). These could be used for seeds and/or triggering.

Assuming that this works (I tried unsuccessfully. You are very welcome to try also, but that is not required), **how do you translate** the found (d, θ) back to an estimate of the circle parameters a and b?

3 Let us again consider our standard spectrometer, but now being traversed by two oppositely charged pions from a K0 decay which has happened somewhere upstream. The measured state vectors are the two fitted tracks (z0, z', y0, y', q/p) with their covariance matrix. Take x0 as the position of the first plane. In this notation z'=dz/dx.

3a) Write down an initial estimate of the decay vertex v0 and the derivative matrices D and E from a Billoir fit to the decay vertex.
You now have the material for building a χ^2, which you, for example, can give to MINUIT or, alternatively, for inserting into the analytic solution to the linear problem on slides 79 and 80.

3b) Let us assume that the measured particles are really coming from K0 decay. **How could you take advantage** of this assumption in the fit to the K0 momentum and its decay point? Please be as explicit as you can.

4) A script for our standard spectrometer with added alignment code is found in
http://www.nbi.dk/~phansen/nordforsk/AlignSpectrometer.C
http://www.nbi.dk/~phansen/nordforsk/AMgr.C

Here we use a 10 GeV pion beam with no magnet and no target.

We start with each plane shifted randomly in y and z with a sigma of 100 microns.

Both the local and global method (see slides) are implemented. The first and second derivatives of the global χ^2 , as well as the average residuals, are accumulated in the function GlobalChi2(RecoTrack& t), again exactly as written in slide 98, and then used in the function Align() via the formula in slide 100.

Usage of this software:
Edit Makefile and the AlignSpectrometer constructor
>make
>./AMgr.exe | tee align.out
(look at output file and histogram file)

What accuracy seems to be obtained**?** Are there signs of "weak modes" giving trouble**?** Does it help to make iterations**?** Try numiter=3. Is global alignment better than local**?** Comment on the observed residuals and the pattern of "Alignment errors".

Since our spectrometer is not coupled to any other measurement, we actually do not care what the true coordinate system is. So try just to translate and rotate it to get rid of the "Alignment errors" (this is easiest done by adding a suitable linear function of the x-coordinate to corr_y[i] and corr_z[i] in the function Align() . But only do it after the first iteration, not the next ones also..).

I will need answers by December 22. They do not need to be long. Send to phansen@nbi.dk with the words "Detector course" appearing in the title.
Peter Hansen