Emergent Causality in Holography

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Based mostly on:

NE, Horowitz ‘16; NE ‘16; NE, Fischetti ‘17

Spacetime Emergence

Holography: the equivalence of a higher-dim system to a lower-dim one.

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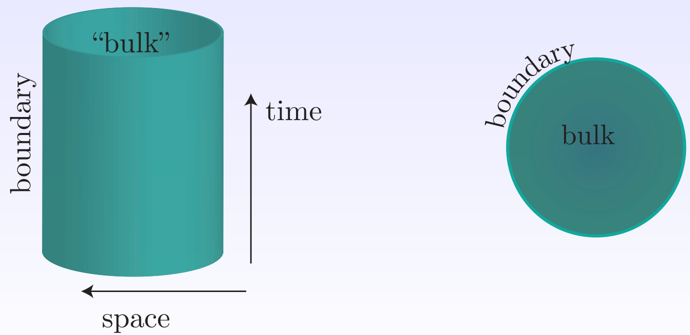
Spacetime itself is emergent

The holographic nature of quantum gravity suggests that spacetime geometry is itself emergent from fundamental quantum gravity degrees of freedom.

A Model of Spacetime Emergence

AdS/CFT

An equivalence between gravity "in a box" in $d+1$ dimensions (the bulk) and a quantum theory on the boundary.



Spacetime inside the cylinder emerges from the boundary quantum theory.

Geometry vs. Causality

- Regular geometry is just the spacetime metric $g_{ab}(p)$, which is the same as knowing the inner products $v(p) \cdot u(p)$ for any two vectors at p.

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- Causality is more primitive (more fundamental?).

- How and when is causal structure emergent in quantum gravity?

Assumptions about AdS/CFT

H1 *(Bulk)* When a semiclassical bulk M exists, it contains perturbative quantum fields

H2 *(Boundary)* The lower-dim theory is a QFT on a timelike geometry that can be embedded on ∂M;

H3 *(Bulk-to-boundary)* Correlators of local operators in the boundary are related to correlators of dual bulk fields:

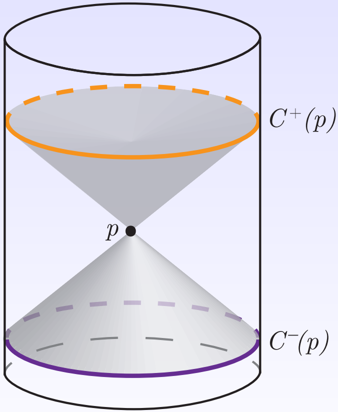
$$\lim_{x_i \to X_i} f_n(x_i) \langle \phi(x_1) \cdots \phi(x_n) \rangle_{\text{bulk}} = \langle \mathcal{O}(X_1) \cdots \mathcal{O}(X_n) \rangle_{\text{bdy}}$$

Table of Contents

1. Lightcone Cuts
2. Emergent Bulk Causal Structure
3. Which States are Dual to Semiclassical Causal Structure?
4. Emergence of the Extra Dimensions time permitting
5. Open Questions

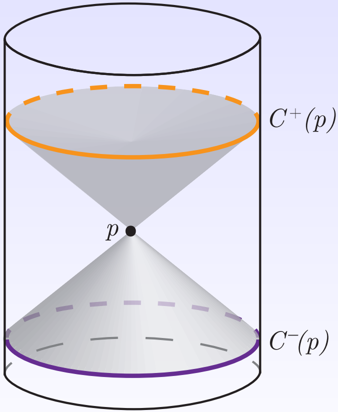
Lightcone Cuts

- Lightcone of a point = subset of the spacetime that can influence or be influenced by a point.
- The boundary of the intersection of the lightcone of p with ∂M defines the *lightcone cuts of p* Newman; NE, Horowitz .



Lightcone Cuts

- The part of the boundary between $C^+(p)$ and $C^-(p)$ is out of causal contact with p. Everything else is within causal contact of p.
- The location of lightcone cuts can be found from the singularity structure of Lorentzian correlators on the boundary. More on this in a moment.



Useful Properties of Lightcone Cuts

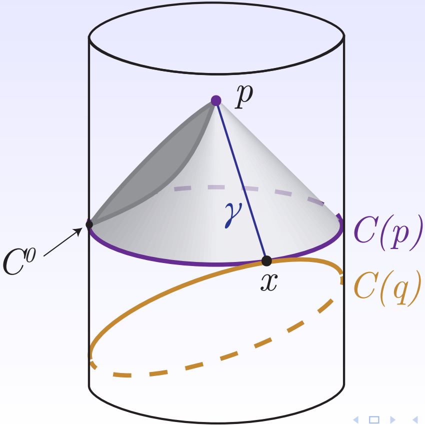
(1) Cut-Point Correspondence

The past and future lightcone cuts $C^{\pm}(p)$ of a bulk point p are a pair of unique, complete spatial slices of the boundary. $C^{\pm}(p)$ is nonempty whenever p can send future/past-directed null geodesics to ∂M.

Useful Properties of Lightcone Cuts

(2) Null Separation

If $C(p)$ and $C(q)$ are C^1 and tangent at a point x, then the corresponding points p and q are null-separated.

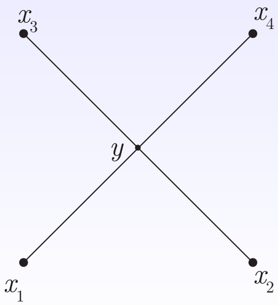


Lightcone Cuts from Lorentzian Correlators

For perturbatively interacting bulk fields, Lorentzian position space correlators are singular at null separations whenever energy momentum is conserved at the vertex:

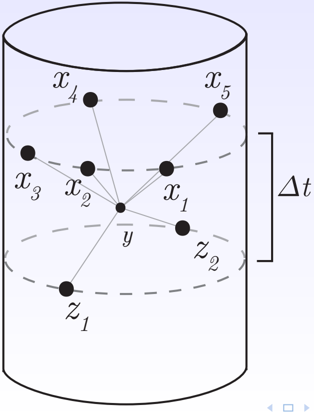
$$\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle \text{ is singular.}$$

In a Landau diagram:



Lightcone Cuts from Lorentzian Correlators

When we take the limit of the correlator as the bulk points go to the boundary, we get a bulk-sourced boundary singularity called a *bulk point singularity* [Maldacena, Simmons-Duffin, Zhiboedov; Gary, Giddings, Penedones; Okudas, Penedones; ...].



Lightcone Cuts from Lorentzian Correlators

Lightcone cuts are precisely the locations at which the Lorentzian correlators are singular: location of lightcone cuts (with some caveats) can be found from singularity structure of Lorentzian correlators.

- $d+1$ boundary points uniquely identify (at most) one bulk point p
- With $d+2$ points, can conserve energy momentum at p

  [Maldacena, Simmons-Duffin, Zhiboedov]
- With $d+3$ points, we can fix p, and move two points around while conserving energy-momentum at p: allows us to trace the lightcone cut

Using Lightcone Cuts

Lightcone cuts depend only the bulk *causal structure*. Any questions that can be answered with lightcone cuts can only depend on the causal structure.

1. Which object in the CFT encodes the bulk *causal structure*?
2. Which states are dual to semiclassical bulk causal structure?

Table of Contents

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Bulk Reconstruction NE, Horowitz '16

General Theorem (forget AdS/CFT for a moment)

The conformal metric at a point p can be found from a sufficiently large collection of null vectors $\{\eta_k\}$ at p, which includes a set of $(d+1)$ lin. indep. null vectors $\{\ell_i\}_{i=1}^{d+1}$.

Because the ℓ_i are null:

$$l_i^\mu l_i^\nu g_{\mu\nu} = 0$$

Expand the rest of the η_k's in ℓ_i basis:

$$\eta_k^\mu = \sum_i M_{ki} \ell_i^\mu$$

Because the η_k are null:

$$0 = \eta_k^\mu \eta_k^\nu g_{\mu\nu} = \sum_{i,j} M_{ki} M_{kj} \ell_i^\mu \ell_j^\nu g_{\mu\nu}$$

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- Get a set of equations for the conformal metric $\lambda^2 g_{\mu\nu}$.
- System is normally over-determined, but if we know that p is a point in a spacetime which has a well-defined metric, then we're guaranteed a solution.

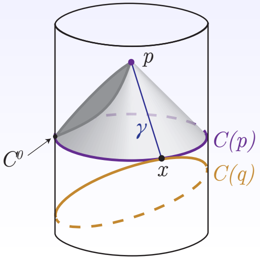
Bulk Reconstruction NE, Horowitz '16

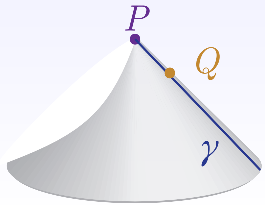
- If we had a way of writing down a set of null vectors at any bulk point p in terms of boundary quantities, we'd have an *explicit* pointwise construction of the bulk causal structure.
- Cue in the lightcone cuts: they allow us to "translate" null vectors from the bulk interior to the boundary.

Bulk Reconstruction NE, Horowitz '16

Bulk Conformal Metric

Endow the space of lightcone cuts with the same Lorentzian structure as the bulk: if $C(p)$ and $C(q)$ are tangent, they are null-separated in the space of cuts. A set of $(d+1)$ cuts tangent to $C(p)$ gives the conformal metric on cut space. And this is the same as the bulk conformal metric.





Bulk Geometry from Lightcone Cuts NE, Horowitz '16

Corollary

The (conformal) metric at any point p in causal contact with the boundary (past or future) can be found from a set of $(d+1)$ tangent lightcone cuts.

So, once you determine the set of lightcone cuts of the bulk from the correlators, you can immediately find the bulk metric everywhere outside of an event horizon*.

Table of Contents

1. Lightcone Cuts
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Emergence of Spacetime

A semiclassical conformal metric emerges from the boundary if and only if the set of singularities of $(d+3)$-pt correlators admits a nontrivial solution to the equation for the conformal metric.

Knowing the state is unnecessary: we only need to know its singularity structure.

Discarding Extra Information

$$
\begin{aligned}
W_\psi &: \partial M^{d+3} \to \{0,1\} \\
(X_1, \ldots, X_{d+3}) &\mapsto \begin{cases} 1 & \text{if } \langle \mathcal{O}(X_1) \cdots \mathcal{O}(X_{d+3}) \rangle_\psi \ \text{ is singular} \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

- Define $|\psi\rangle \sim |\phi\rangle$ if $W_\psi(X_i) = W_\phi(X_i)$ for all X_i.
- This coarse-grains over all information except that which is both necessary and sufficient to determine whether a semiclassical causal structure exists and if so, what it is.

Causal States

- The equivalence class of $|\psi\rangle$, denoted $\widetilde{|\psi\rangle}$ is *the causal state.*
- We can quotient out the Hilbert space $\mathcal{H}$ by $\sim$. Intuitive arguments indicate that $\mathcal{H}/\sim$ is itself a Hilbert space.
- The causal states are members of a new state space, the causal Hilbert space. This is defined independently of any bulk dual, and most causal states do not have bulk duals. For those that do, the support of W will solve the equations for the conformal metric nontrivially.

Causal States

- Have we done anything nontrivial?

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- Yes, states are specified by the full set of expectation values, not just by the singularities.

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- Have we done anything nontrivial?
- Yes, states are specified by the full set of expectation values, not just by the singularities.
- Can also see this from intuition in AdS/CFT: if $|\psi\rangle$ is a boundary state dual to an asymptotically AdS semiclassical bulk, we can perturb $|\psi\rangle$ by $O(N^0)$. This will change the state, but not the singularity structure of the correlators.

Properties of the Causal Hilbert Space

1. Any operator A on the full Hilbert space $\mathcal{H}$ where

$$A|\psi\rangle \sim A|\phi\rangle$$

whenever $|\psi\rangle \sim |\phi\rangle$ gives rise to an operator on the causal Hilbert space.

2. Can define the *causal density matrix* using the inner product on the causal Hilbert space:

$$\tilde{\rho} = \widetilde{|\psi\rangle}\widetilde{\langle\psi|}$$

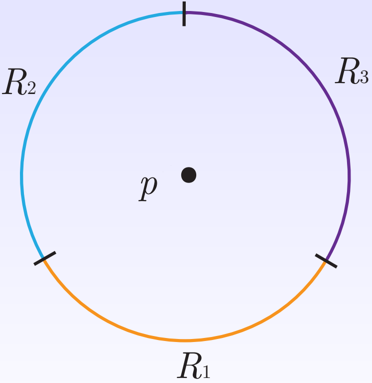
Reduced Causal States

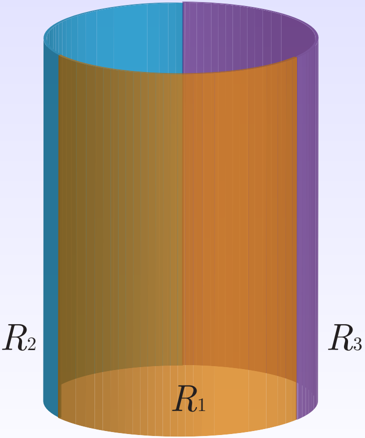
Restrict the range of W_ψ to a particular boundary subregion R:

$$\begin{aligned} W_\psi : R^{d+3} &\to \{0,1\} \\ (X_1, \ldots, X_{d+3}) &\mapsto \begin{cases} 1 & \text{if } \langle \mathcal{O}(X_1) \cdots \mathcal{O}(X_{d+3}) \rangle_\psi \text{ is singular} \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

Then $|\psi\rangle \sim_R |\phi\rangle$ if $W_\psi|_R = W_\phi|_R$. The equivalence class $\widetilde{|\psi\rangle}_R$ is the reduced causal state of R.

Quantum Error Correction and Secret Sharing





Properties of the Reduced Causal State

- $\widetilde{|\psi\rangle}_{R_i}$ for any one R_i is insuffiient for recovering the conformal metric at p.
- $\widetilde{|\psi\rangle}_{R_i \cup R_j}$ is sufficient for any two R_i.
- The third R_i is redundant.

Quantum Error Correction and Secret Sharing

- The conformal metric at p is protected against erasure of any one of the R_i. (Quantum error correction)
- Any one of the R_i is ignorant of the conformal metric at p. (Quantum secret sharing)

Table of Contents

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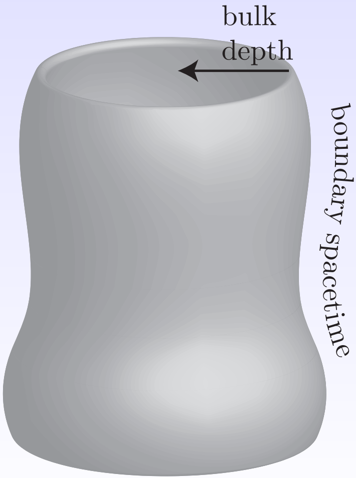
Extra Holographic Dimension

1. **Where does the extra “holographic” dimension come from?**



Emergence of the Extra Dimension

The CW in AdS/CFT

Intuition: the extra dimension emerges from RG flow in the QFT. Near-boundary region is dual to UV physics; going deeper into the bulk corresponds to RG flow towards the IR.

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Emergence of the Extra Dimension

The CW in AdS/CFT

Intuition: the extra dimension emerges from RG flow in the QFT. Near-boundary region is dual to UV physics; going deeper into the bulk corresponds to RG flow towards the IR.

... deeper in the bulk??

When is one point deeper in the bulk than another?

Emergence of the Extra Dimension

“p is deeper than q if the ‘r’ coordinate at p is smaller than the ‘r’ coordinate at q.”

Emergence of the Extra Dimension

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$$ds^2 = -(r^2+1)dt^2 + (1+r^2)^{-1}dr^2 + r^2 d\Omega$$

Emergence of the Extra Dimension

"p is deeper than q if the 'r' coordinate at p is smaller than the 'r' coordinate at q."

$$ds^2 = -(r^2+1)dt^2 + (1+r^2)^{-1}dr^2 + r^2 d\Omega$$

But:

- Most spacetimes don't admit a well-defined 'r' coordinate anywhere but in the asymptotic region
- Even for those that do, such a coordinate isn't unique
- This isn't covariant.

Checklist for a Definition

Want a definition of bulk depth which is:

1. Covariant
2. Generic: makes sense with minimal assumptions about the bulk
3. Matches expectations in spacetimes that do have an 'r' coordinate
4. Has a field theory dual on the boundary with a connection to RG flow

Checklist for a Definition

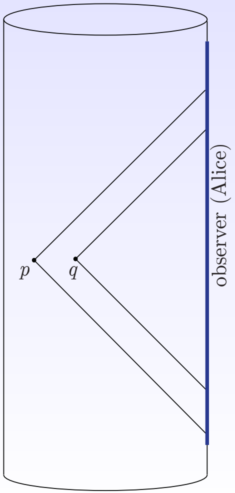
Will give a definition which gets 4/4: it is covariant, matches expectations in spacetimes with r coordinate, has a connection to energy scale, and can be formulated under the following minimal assumptions:

1. The bulk is C^2.
2. The bulk has good causal structure ("AdS hyperbolic")
3. The bulk obeys the Achronal Averaged Null Curvature Condition:

$$\int_\gamma R_{ab}k^a k^b \geq 0.$$

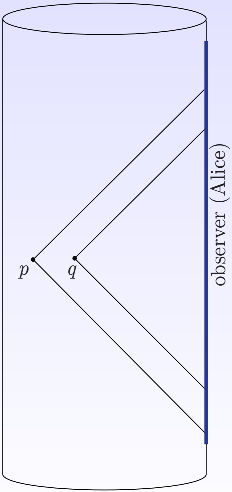
Intuition: Bulk Depth from Boundary Observers

When does a timelike boundary observer perceive p to be deeper than q?



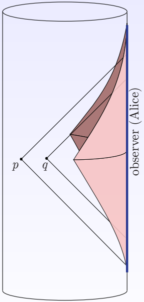
Bulk Depth from Boundary Observers

Alice sees p as farther away from her than q whenever she spends more time out of contact with p than with q: **when the q-signals are "sandwiched" by the p-signals on Alice's worldline**.



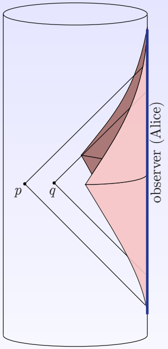
Bulk Depth Relative to Subregions

- In AdS/CFT, we like to think in terms of boundary subregions or in terms of the entire boundary.
- Can we apply the intuition of a timelike observer to a boundary subregion?



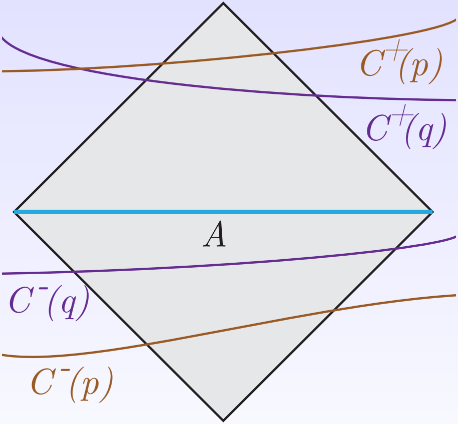
Bulk Depth Relative to Subregions

- If every observer in the causal diamond of a boundary region A sees the p signals sandwiching the q signals, then A perceives p as deeper than q.
- Lightcone cuts give a natural way of making this precise.



Bulk Depth from Sandwiches

If $C^{\pm}(p)$ "sandwich" $C^{\pm}(q)$, then the subset of the causal diamond of A which is spacelike to q is properly contained in the subset which is spacelike to p.



Observers on the causal diamond of A perceive p as deeper than q.

Bulk Depth: A Definition

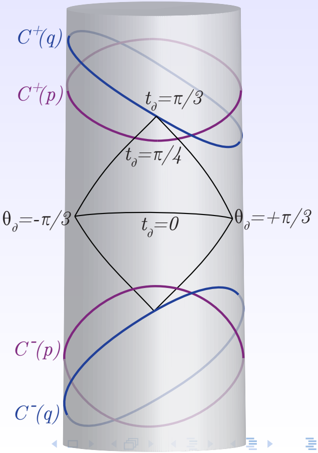
Definition of Bulk Depth

A bulk point p is deeper than a bulk point q relative to a boundary region A if $C^{\pm}(p)$ sandwich $C^{\pm}(q)$ on the causal diamond of A.

Example: Bulk Point Depth in Pure AdS

$$ds^2 = -(1+r^2)dt^2 + (1+r^2)^{-1}dr^2 + r^2 d\Omega^2$$

- p, q at $r = 1$, $t = 0$, $\theta = 0,\ \pi/4$.
- A at $t = 0$, $\theta \in [-\pi/3, \pi/3]$. f
- A perceives q as deeper.

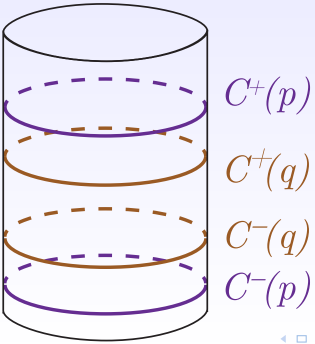


Global Bulk Depth

By taking A to be an entire slice of ∂M, get a definition qualifying when a bulk point is *globally* deeper than another.

Global Bulk Depth

A bulk point p is globally deeper than a bulk point q if $C^{\pm}(p)$ sandwich $C^{\pm}(q)$.



Global Sandwich

When are there points that are absolutely deeper in the bulk than others?

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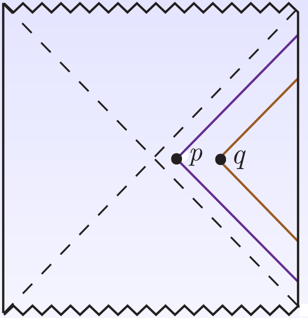
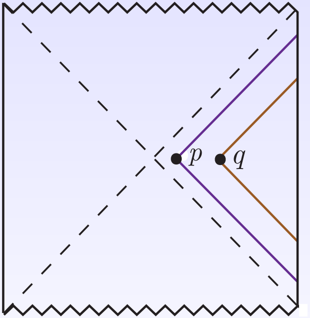


Global Sandwich

When are there points that are absolutely deeper in the bulk than others?



Global Sandwiches Theorem

Any spacetime with an event horizon admits points that are absolutely deeper than others.

Temporal vs Spatial Separation

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Temporal vs Spatial Separation

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Temporal vs Spatial Separation

- Definition says that points whose lightcone cuts are at larger time separations are deeper in the bulk
- Usual intuition is that larger *spatial* distances on the boundary correspond to probing deeper into the bulk, rather than longer time separation.
- Can the definition via the lightcone cuts match this expectation?

(Hint: answer is yes!)

The Causal Wedge

- The causal wedges of nested boundary regions are nested
  [Hubeny, Rangamani, Tonni]
- Increasing the region on the boundary increases the size of corresponding bulk region: intuitively think of this as probing deeper in the bulk.
- Does this intuitive notion agree with definition?

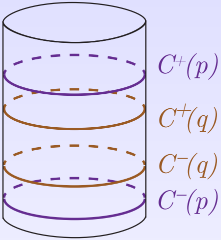
The Causal Wedge

Causal Wedge Inclusion Theorem

p is deeper in the bulk than q if and only if every causal wedge containing p also contains q.

Can prove for both relative and global bulk depth definitions.

Connection to Energy Scales and RG Flow



- p is deeper than q if it corresponds to longer time separations on the boundary.
- Longer time separation means lower energy scales: moving deeper into the bulk corresponds to lower energy scales
- By the causal wedge inclusion theorem, bulk depth corresponds to coarse-graining over larger distances
- A connection with energy/momentum scale through both distance and time scales.

Bulk Depth from Lorentzian Correlators

Lightcone cuts are found from the singularity structure of

$$\langle \mathcal{O}(X_1) \cdots \mathcal{O}(X_{d+3}) \rangle. \tag{1}$$

Going deeper into the bulk corresponds to looking at bulk-point singularities at longer time separation.

Table of Contents

1. Lightcone Cuts
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Weak Holography

H1 *(Bulk)* When a semiclassical bulk M exists, it contains perturbative quantum fields

H2′ *(Boundary)* The lower-dim theory is a dynamical theory on a timelike or null geometry that can be embedded on ∂M (need not be an asymptotic boundary);

H3′ *(Bulk-to-boundary)* Correlators of bulk fields are related to some object on the boundary via an appropriate limit:

$$\lim_{x_i \to X_i} f_n(x_i) \langle \phi(x_1) \cdots \phi(x_n) \rangle_{\text{bulk}} = O(X_1 \cdots X_n)$$

Everything we have done works under these assumptions.

Open Questions

1. Dual and existence of efficient curves and measurement of bulk depth?
2. Causal entanglement entropy? Would require some sort of causal Hilbert space factorization.
3. Multiple Dual geometries?
4. Relationship to entanglement and the reduced density matrix?
5. Pathological geometries?
6. Determining the conformal factor in broad generality?
7. Understanding causal states for spacelike boundaries - can this lead us to understanding time emergence?
8. ...