Emergence of gauge symmetries: a working example

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- New J. Phys. 16 (2014) no.12, 123028
- Phys. Rev. D89 (2014) no.12, 124019
- JHEP 1610 (2016) 084
- ...

- Condensed-matter systems display emergent relativistic behavior in a plethora of ways. Analogue experiments probing relativistic field equations in the laboratory? [Barceló, Liberati and Visser, Living Rev. Rel. (2011)]

- Partial analogies: External (non-relativistic) gravitational field. [Unruh, Phys. Rev. Lett. (1981)]

- Many questions: Is it possible to go beyond, and obtain fully relativistic field theories? How do gauge symmetries emerge? Can gravity be emergent?

Overview

- Introduction ✓
- Emergent electrodynamics in Helium-3
- A general definition of emergent gauge symmetries
- What about gravity?
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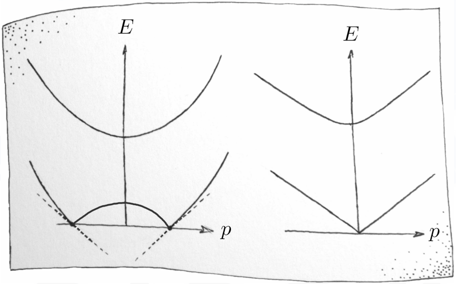
- Non-relativistic condensed-matter systems with Fermi points. Emergence of Lorentz invariance in a manageable mathematical formalism. [Volovik, Oxford University Press (2003)]
- Non-relativistic and relativistic dispersion relations:

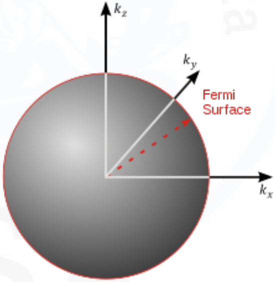


- Large number N of non-relativistic fermions with spin $1/2$. Small fluctuations δN: grand canonical energy operator of the free theory:

$$H - \mu N = \sum_{\boldsymbol{p}\,B} \left(\frac{p^2}{2m^*} - \mu \right) a^\dagger_{\boldsymbol{p}\,B} a_{\boldsymbol{p}\,B}$$

- The vacuum (lowest-energy) state is characterized by its Fermi surface. Fermi momentum:

$$p_{\mathrm{F}} = \sqrt{2m^* \mu}$$



- Including interactions (pairing channel): [Leggett, Oxford University Press (2006)]

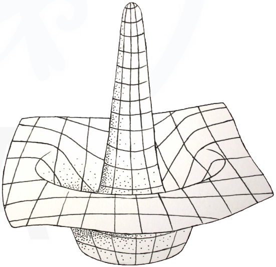
$$\frac{1}{2}\sum_{\boldsymbol{p}\,\boldsymbol{p}'\,B\,C}\tilde{V}\left(\boldsymbol{p}+\boldsymbol{p}'\right)a^{\dagger}_{-\boldsymbol{p}'\,C}a^{\dagger}_{\boldsymbol{p}'\,B}a_{-\boldsymbol{p}\,B}a_{\boldsymbol{p}\,C}$$

- Interaction potential depends on $|\boldsymbol{p}'+\boldsymbol{p}|$:

$$\tilde{V}(|\boldsymbol{p}'+\boldsymbol{p}|)=\sum_{l=0}^{\infty}(2l+1)\tilde{V}_l(p,p')P_l(\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{p}}')$$

- Spin-triplet pairing interaction ($l=1$) with $|\tilde{V}_1|\gg|\tilde{V}_{l\neq 1}|$ + close to the Fermi surface:

$$\tilde{V}\simeq -g\,\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{p}}'\simeq -\frac{g}{p_{\mathrm{F}}^2}\boldsymbol{p}\cdot\boldsymbol{p}'$$



- Grand canonical pairing Hamiltonian:

$$H_{\mathrm{P}} - \mu N = \sum_{\boldsymbol{p}\,B} \left(\frac{p^2}{2m^*} - \mu \right) a^{\dagger}_{\boldsymbol{p}\,B} a_{\boldsymbol{p}\,B} + \frac{g}{2p_{\mathrm{F}}^2} \sum_{\boldsymbol{p}\,\boldsymbol{p}'\,B\,C} (\boldsymbol{p}' \cdot \boldsymbol{p}) a^{\dagger}_{-\boldsymbol{p}'\,C} a^{\dagger}_{\boldsymbol{p}'\,B} a_{\boldsymbol{p}\,B} a_{-\boldsymbol{p}\,C}$$

- Condensation with order parameter:

$$\Psi^{i}_{BC} = \frac{g}{p_{\mathrm{F}}} \left\langle \sum_{\boldsymbol{p}} p^{i}\, a_{\boldsymbol{p}\,B} a_{-\boldsymbol{p}\,C} \right\rangle = i(\sigma_Q \sigma_2)_{BC}\, d^{Qi}$$

- Gor'kov factorization: [Gor'kov, Sov. Phys. JETP (1958)]

$$H_{\mathrm{P}} - \mu N = \sum_{\boldsymbol{p}\,B} \left(\frac{p^2}{2m^*} - \mu \right) a^{\dagger}_{\boldsymbol{p}\,B} a_{\boldsymbol{p}\,B} + \frac{1}{2p_{\mathrm{F}}} \sum_{\boldsymbol{p}\,B\,C} \boldsymbol{p} \cdot \boldsymbol{\Psi}_{BC} a^{\dagger}_{-\boldsymbol{p}\,C} a^{\dagger}_{\boldsymbol{p}\,B} + \frac{1}{2p_{\mathrm{F}}} \sum_{\boldsymbol{p}\,B\,C} \boldsymbol{p} \cdot \boldsymbol{\Psi}^{*}_{CB} a_{\boldsymbol{p}\,B} a_{-\boldsymbol{p}\,C}$$

- Planar state ($P = 1, 2, 3$):

$$d^{Pi}_{planar} = \Delta_0(\hat{s}^P \hat{m}^i + \hat{s}'^P \hat{n}^i)$$

- $\hat{\boldsymbol{m}}$, $\hat{\boldsymbol{n}}$ unit vectors in position space; $\hat{\boldsymbol{s}}$ and $\hat{\boldsymbol{s}}'$ unit vectors in spin space. Orthogonality conditions $\hat{\boldsymbol{m}} \cdot \hat{\boldsymbol{n}} = 0$, $\hat{\boldsymbol{s}} \cdot \hat{\boldsymbol{s}}' = 0$. Also $\Delta_0 \simeq k_{\mathrm{B}} T_{\mathrm{C}}$. Anisotropy axis $\hat{\boldsymbol{l}} = \hat{\boldsymbol{m}} \times \hat{\boldsymbol{n}}$.

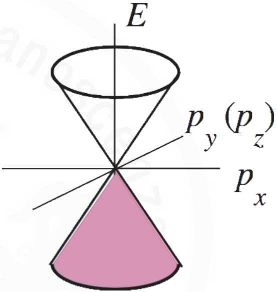
- Evolution equations of fermions ($c_\perp = \Delta_0 / p_{\mathrm{F}}$):

$$i\hbar \dot{a}_{\boldsymbol{p}\uparrow} = \left(\frac{p^2}{2m^*} - \mu\right) a_{\boldsymbol{p}\uparrow} - c_\perp \boldsymbol{p} \cdot (\hat{\boldsymbol{m}} - i\hat{\boldsymbol{n}}) a^\dagger_{-\boldsymbol{p}\uparrow}$$

$$i\hbar \dot{a}_{\boldsymbol{p}\downarrow} = \left(\frac{p^2}{2m^*} - \mu\right) a_{\boldsymbol{p}\downarrow} + c_\perp \boldsymbol{p} \cdot (\hat{\boldsymbol{m}} + i\hat{\boldsymbol{n}}) a^\dagger_{-\boldsymbol{p}\downarrow}$$

- Dispersion relation:

$$E^2(\boldsymbol{p}) = \left(\frac{p^2}{2m^*} - \mu\right)^2 + c_\perp^2(\boldsymbol{p} \times \hat{\boldsymbol{l}})^2$$



- Two Fermi points $\pm p_{\mathrm{F}}\hat{\boldsymbol{l}}$ around which ($c_\parallel = p_{\mathrm{F}}/m^*$):

$$E^2 \simeq c_\parallel^2 \mathfrak{p}_l^2 + c_\perp^2(\mathfrak{p}_m^2 + \mathfrak{p}_n^2)$$

- Lowest energy/momentum scale in which violations of the emergent Lorentz symmetry are of order one:

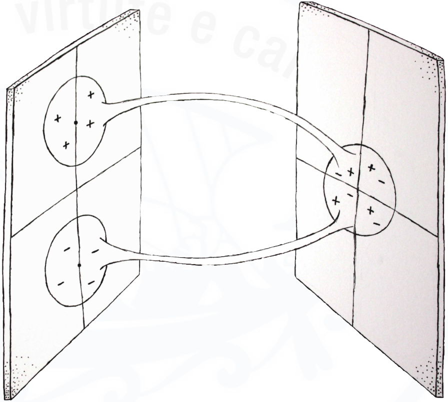
$$E_{\mathrm{L}} = m^* c_\perp^2 \qquad\qquad p_{\mathrm{L}} = p_{\mathrm{F}}\frac{c_\perp}{c_\parallel}$$

- Note that $c_\perp/c_\parallel \ll 1$!

- Homogeneous planar state: emergent Dirac equation.

- Dirac spinor:

$$\psi_{\mathfrak{p}} = \begin{pmatrix} a_{\mathfrak{p}\uparrow+} \\ a^{\dagger}_{-\mathfrak{p}\uparrow-} \\ a_{\mathfrak{p}\downarrow+} \\ a^{\dagger}_{-\mathfrak{p}\downarrow-} \end{pmatrix}$$

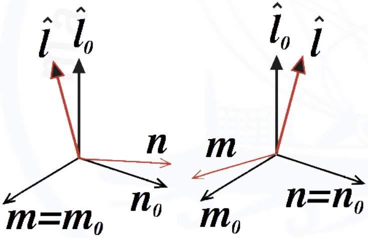


- Combination of degrees of freedom of the two Fermi points. Emergent notion of charge:

$$Q = Q_{\uparrow} + Q_{\downarrow} = N_{+} - N_{-}$$

- The effective fermions are accompanied by effective bosons, composed of the so-called textures: local variations of the order parameter.

- Orbital waves:



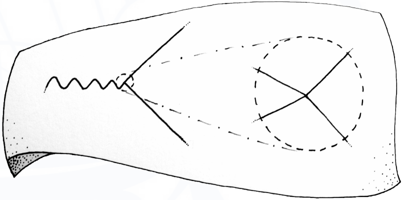
- Coherence length (Cooper pair size):

$$\xi_0 = \frac{\hbar}{p_\mathrm{F}} \frac{c_\parallel}{c_\perp}$$

- Add superfluid velocity and density fluctuations: four degrees of freedom.

- The effects of textures are represented through a potential A_a with four components.
- Low-energy theory of effective fermions and bosons with momentum cutoff

$$\Lambda_{\rm C} = \frac{\hbar}{\xi_0} = p_{\rm F}\frac{c_\perp}{c_\parallel} \ll p_{\rm F}$$



- Coherence length associated with condensation $\longrightarrow$ radiative stability of Lorentz invariance ($\Lambda_{\rm C} = p_{\rm L}$). The low-energy effective field theory is Lorentz invariant.

- The effective theory is Lorentz invariant:

$$\mathscr{L}(\psi, A_a) = -\frac{1}{4\mu_0} F^{ab} F_{ab} + \frac{\beta}{2\mu_0} (\partial_a A^a)^2 + \mathscr{L}_{\mathrm{D}}(\psi, A_a)$$

- Either the theory is gauge invariant ($\beta = 0$) or

$$\Box(\partial_a A^a) = 0$$

- Decoupling of some of the degrees of freedom if natural boundary conditions are imposed.

We recover (massless) electrodynamics as the effective low-energy description of the system.

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- Physical symmetries display non-trivial (non-zero) Noether charges that permit to parametrize the space of solutions of a given theory.

- On the other hand, gauge symmetries lead to trivial (zero) Noether charges.

- Definition: emergent gauge symmetries arise from the trivialization of the Noether currents associated with physical symmetries, due to the decoupling and lack of excitation of some degrees of freedom.

- System with set of fields Φ and space of solutions $\mathcal{S}$. No gauge symmetries: $\mathcal{S}$ parametrized by Noether charges $\mathcal{Q}$.

- Subset $\Psi \subset \Phi$ decoupled (maybe only approximately). Hence, non-trivial solutions with $\Psi = 0$ exist. This defines a projection

$$\Pi : \mathcal{S} \longrightarrow \mathcal{U} \subset \mathcal{S}$$

- The subset $\mathcal{U}$ of the space of solutions can be defined as $\Psi = 0$ or as $\mathcal{Q}_\Psi = 0$, where $\mathcal{Q}_\Psi \subset \mathcal{Q}$.

A. The symmetries with charges $\mathcal{Q}_\Psi$ do not preserve the subset $\mathcal{Q}_\Psi = 0$.

$\rightarrow$ All the symmetries in $\mathcal{U}$ are physical.

B. Some of the symmetries with charges $\mathcal{Q}_\Psi$ preserve the subset $\mathcal{Q}_\Psi = 0$.

$\rightarrow$ $\mathcal{U}$ has gauge symmetries!

> Abstract definition of emergent gauge symmetries that is applicable to diverse settings. Applications?

- Lagrangian density for vector (with mass m) and fermion fields:

$$\mathscr{L}(\psi, A_a) = -\frac{1}{4\mu_0} F_{ab}F^{ab} + \frac{\zeta}{2}(\partial_a A^a)^2 - \frac{m^2}{2} A^a A_a + \mathscr{L}_{\mathrm{D}}(\psi, A_a)$$

- Symmetries T_χ...

$$A_a \longrightarrow A_a + \partial_a \chi \qquad \psi \longrightarrow e^{-i\chi}\psi \qquad (\zeta\Box + m^2)\chi = 0$$

- ...with Noether currents ($\varphi = \partial_a A^a$)

$$J^a_\chi\big|_{\mathcal{S}} = -\partial_b(F^{ab}\chi) + \zeta\varphi\partial^a\chi - \zeta\chi\partial^a\varphi$$

- The field equations imply:

$$(\zeta\Box + m^2)\varphi = 0$$

- We can define:

$$\Phi = \{\psi, A_a\} \qquad \Psi = \{\varphi = \partial_a A^a\}$$

- Natural boundary conditions lead to $\varphi = 0$, which defines the projection Π. Under the symmetries T_χ,

$$\varphi \longrightarrow \varphi - \frac{m^2}{\zeta}\chi$$

A. $m \neq 0$: there are no gauge symmetries in $\mathcal{U}$ (3 degrees of freedom).

B. $m = 0$: there are gauge symmetries in $\mathcal{U}$ (2 degrees of freedom).

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- Fundamental difference between electrodynamics and general relativity: the latter is a nonlinear theory.

- But there is a well-known recursive to obtain these nonlinearities, starting from the free representation of gravitational waves.

[Deser, Gen. Rel. Grav. (1970)]

- Starting point: most general theory of h_{ab} $(h = \eta^{ab} h_{ab})$,

$$\frac{1}{2}\Box h^{ab} + c_2 \partial^c \partial^{(a} h^{b)}_{\ c} - \frac{c_3}{2}\eta^{ab}\partial_c\partial_d h^{cd} - \frac{c_3}{2}\partial^a\partial^b h + \frac{1}{2}c_4 \eta^{ab}\Box h = 0$$

- Now we add a conserved and traceless source T_{ab}:

$$\partial_b T^{ba} = 0 \qquad \eta^{ab} T_{ab} = 0$$

- These five conditions lead generally (under natural boundary conditions) to the five decoupling equations

$$\partial_b h^{ba} = 0 \qquad \eta^{ab} h_{ab} = 0$$

- This defines the projection Π and leads to the emergence of linear gauge symmetries.

- Is it possible to use bootstrap arguments in order to find general relativity? What is the nonlinear generalization of the decoupling equations above? Interplay with Weinberg-Witten theorem?

[Weinberg and Witten, Phys. Lett. B (1980)]

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1. Constructing emergent models provides new insights about the low-energy properties: Lorentz and gauge symmetries, the nature of the electric charge and force carriers,...

[New J. Phys. 16 (2014) no.12, 123028]

2. Gauge invariance can be seen as the result of Lorentz invariance and natural boundary conditions.

[JHEP 1610 (2016) 084]

3. This picture can be applied to linearized gravity, but no explicit construction of its nonlinear completion yet.

[Phys. Rev. D89 (2014) no.12, 124019]

Mange tak!