Nonrelativistic Naturalness and the question of Emergent Lorentz Symmetry

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Emergent Symmetries in Particle Physics, Cosmology and Condensed Matter Systems
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Collaborators

Nonrelativistic Short-Distance Completions of a Naturally Light Higgs, [arXiv:1608.06937] with

Petr Hořava and Christopher J. Mogni
Berkeley Center for Theoretical Physics

Ziqi Yan
Perimeter Institute for Theoretical Physics

Nonrelativistic Yang-Mills Theory for a Naturally Light Higgs Boson, Phys. Rev. D 96, 095030 (2017), [arXiv:1705.04701] with Z.Y. and

Laure Berthier
Niels Bohr Institute

Technical Naturalness

Technical naturalness à la 't Hooft (1979):

"at any energy scale μ, a physical parameter or set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ would increase the symmetry of the system."

Naturalness Puzzles

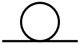
- Cosmological constant problem: $R_{\text{universe}} \gg \ell_{\text{Planck}}$.
- Eta problem: $\eta \ll 1$ or $m_{\text{inflaton}} \ll H$.
- Strange metals: $\rho \sim T$.
- Higgs hierarchy problem: $M_{\text{Higgs}} \ll M_{\text{Planck}}$.

Naturalness in ϕ^4 theory

Relativistic scalar in $3+1$ dimensions:

$$S = \frac{1}{2}\int d^4x \left(\partial_\mu \phi \, \partial^\mu \phi - m^2\phi^2 - \frac{\lambda}{12}\phi^4 \right)$$

EFT below "naturalness scale" M

 $\Longrightarrow \delta m^2 \sim \lambda M^2$

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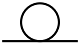 $\Longrightarrow \delta m^2 \sim \lambda M^2$

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Higgs Hierarchy Problem: $\lambda \sim 1 \Longrightarrow M \sim m \sim 1$ TeV.

Nonrelativistic Naturalness in $z = 2$ ϕ^4 theory

UV: $$S = \frac{1}{2}\int dt\, d^3\mathbf{y}\left\{\dot{\phi}^2 - \zeta^2(\partial_i\partial_j\phi)^2 - c^2(\partial_i\phi)^2 - m^2\phi^2 - \frac{\lambda}{12}\phi^4\right\}$$

$z = 2$ scaling symmetry: $\mathbf{y} \to b\mathbf{y}$ and $t \to b^2 t$

$$\phi \to \phi + b_{ij}y^i y^j \implies \zeta^2 \sim 1, \qquad c^2 \sim \varepsilon_1$$

$$\phi \to \phi + b \implies \lambda \sim \varepsilon_0, \qquad m^2 \sim \varepsilon_0, \qquad \varepsilon_0 \ll \varepsilon_1 \ll 1$$

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IR: $$S = \frac{1}{2}\int d^4x\left\{(\nabla_\mu\Phi)^2 - m^2\Phi^2 - \frac{\lambda_h}{12}\Phi^4 - \tilde{\zeta}^2(\nabla_i^2\Phi)^2\right\}$$

Good: $\lambda_h = \frac{\lambda}{c^3} \sim \frac{\varepsilon_0}{\varepsilon_1^{3/2}} \sim 1$ while $m^2 \sim \varepsilon_0$

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Good: $\lambda_h = \dfrac{\lambda}{c^3} \sim \dfrac{\varepsilon_0}{\varepsilon_1^{3/2}} \sim 1$ while $m^2 \sim \varepsilon_0$

Bad: $\tilde{\zeta}^2 = \dfrac{\zeta^2}{c^4} \sim \dfrac{1}{\varepsilon_1^2 M^2} = \dfrac{\varepsilon_0}{\varepsilon_1^2 m^2} \sim \dfrac{1}{\varepsilon_1^{1/2} m^2}$

Nonrelativistic Solution to the Hierarchy Problem

$$S = \frac{1}{2}\int dt\, d^3\mathbf{y}\left\{\dot{\phi}^2 - \zeta_3^2(\partial^3\phi)^2 - \zeta_2^2(\partial^2\phi)^2 - c^2(\partial\phi)^2 - m^2\phi^2 - \frac{\lambda}{12}\phi^4\right\}$$

$$\zeta_3^2 \sim 1, \qquad \zeta_2^2 \sim \varepsilon_2, \qquad c^2 \sim \varepsilon_1,$$

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$$\begin{array}{lll} \zeta_3^2 \sim 1, & \zeta_2^2 \sim \varepsilon_2, & c^2 \sim \varepsilon_1, \\ \lambda \sim \varepsilon_0, & m^2 \sim \varepsilon_0, & \varepsilon_0 \ll \varepsilon_1 \ll \varepsilon_2 \ll 1. \end{array}$$

$$S = \frac{1}{2}\int d^4x\Big\{(\nabla_\mu\Phi)^2 - m^2\Phi^2 - \frac{\lambda_h}{12}\Phi^4 - \tilde\zeta_3^2(\nabla_i^3\Phi)^2 - \tilde\zeta_2^2(\nabla_i^2\Phi)^2\Big\}$$

$$\lambda_h \sim \frac{\varepsilon_0}{\varepsilon_1^{3/2}} \sim 1, \qquad \tilde\zeta_3^2 \sim \frac{1}{m^4}, \qquad \tilde\zeta_2^2 \sim \frac{\varepsilon_2}{\varepsilon_1^{1/2}}\frac{1}{m^2} \Longrightarrow \frac{\varepsilon_2}{\varepsilon_1^{1/2}} \le 1$$

Lorentz Invariance in the IR

Need same c for ALL species in the IR!

Force c for all species to be c_0 in UV.

$\beta_i = \left(-\frac{4}{3} + \# \underbrace{\frac{e^4}{c_0^2}}_{\sim\varepsilon_1 \ll 1}\right)\rho_i$ where $c_i^2 = \rho_i M^{4/3}$

Matching of c's is preserved in the IR up to $\varepsilon_1 \log(M/m) \ll 1$

EFT cannot explain the "initial" condition of c_0

Emergent Lorentz Symmetry

LIV tends to decay in the IR. [Chadha and Nielsen, 1983]

Scalar (c_b) and spinor (c_f) + Yuk. g. [Anber and Donoghue 1102.0789]

Generic g: Lorentz symmetry emerges (slowly) at low energies.

Strongly coupled fixed point g_*: $c_b - c_f \to 0$ as a power law.

Favorable evidence in holography. [Bednik, Pujolàs and Sibiryakov 1305.0011]

Recap and Outlook

Nonrelativistic UV $\rightarrow$ relativistic IR.

Shift symmetries produce natural hierarchies among couplings.

Mass hierarchy even with $O(1)$ IR couplings.

Accommodate SM Yukawas and Yang-Mills couplings.

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Apply to other problems (e.g., strange metals).

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Thank you!