Particle-hole symmetry for composite fermions

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in collaboration with J. K. Jain, Ref: Phys. Rev. B **96**, 245142 (2017)

June 21, 2018

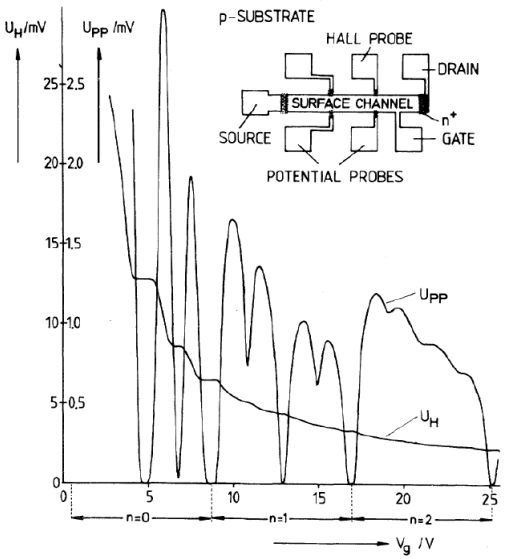
VILLUM FONDEN

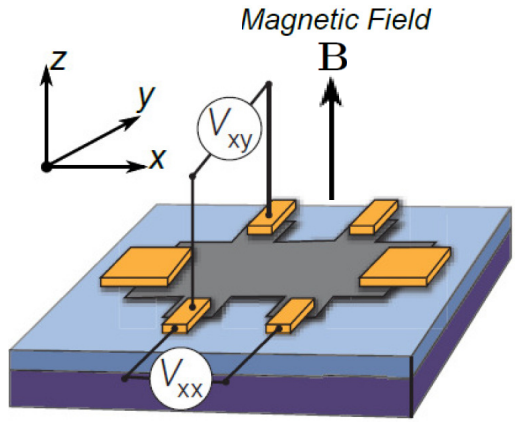
European Research Council
Established by the European Commission

Plan of the talk

- introduction to the fractional quantum Hall effect and composite fermions
- particle-hole symmetry of electrons - exact symmetry
- particle-hole symmetry of composite fermions - approximate emergent symmetry
- conclusion and outlook

Experimental discovery of the integer QHE

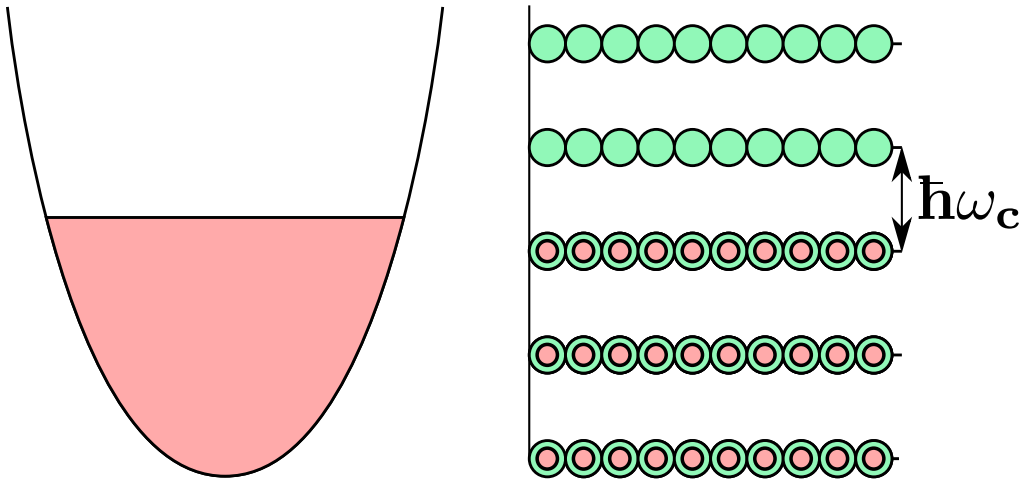




K. v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. **45**, 494 (1980)

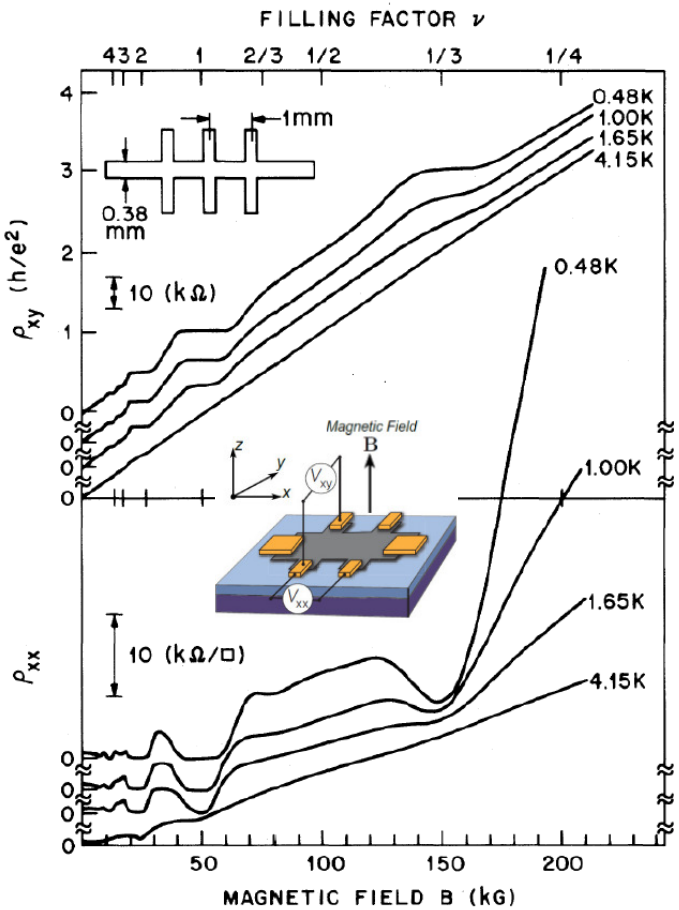
- Plateau in Hall resistance $R_{xy} = h/(ne^2)$ where n is an integer.
- Vanishing longitudinal resistance R_{xx} & R_{yy}: gapped excitations.
- Generic to two-dimensional electron and hole systems: independent of sample type and geometry and robust to disorder.
- forms the standard of resistance: Klitzing constant $R_{\rm K} = h/e^2 = 25812.807557(18)\ \Omega$.

IQHE arises from the formation of → Landau levels (LLs)



- Excitation gap is set by the cyclotron energy → $\hbar\omega_c = \hbar\frac{eB}{m_{\rm eff}}$.

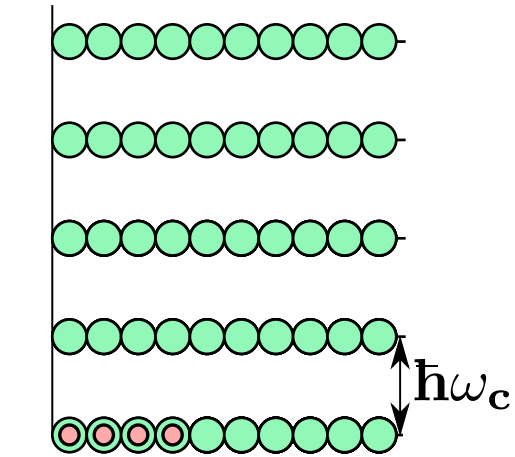
Plateau at $h/(\frac{1}{3}e^2)$: Quarks???



D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982)

- Vanishing longitudinal resistance R_{xx} & R_{yy}: gap to excitations.

FQHE arises from electron-electron interactions



- Electrons interacting via Coulomb forces:

$$\mathcal{H} = \sum_{i<j} \frac{1}{|r_i - r_j|}$$

- Quantum mechanics $\rightarrow$ *lowest Landau level* constraint.
- Interactions $\rightarrow$ a unique state from the degenerate manifold.

Laughlin made a brilliant ansatz for $\nu = 1/m$

- assumed a Jastrow (pairwise) correlated state.

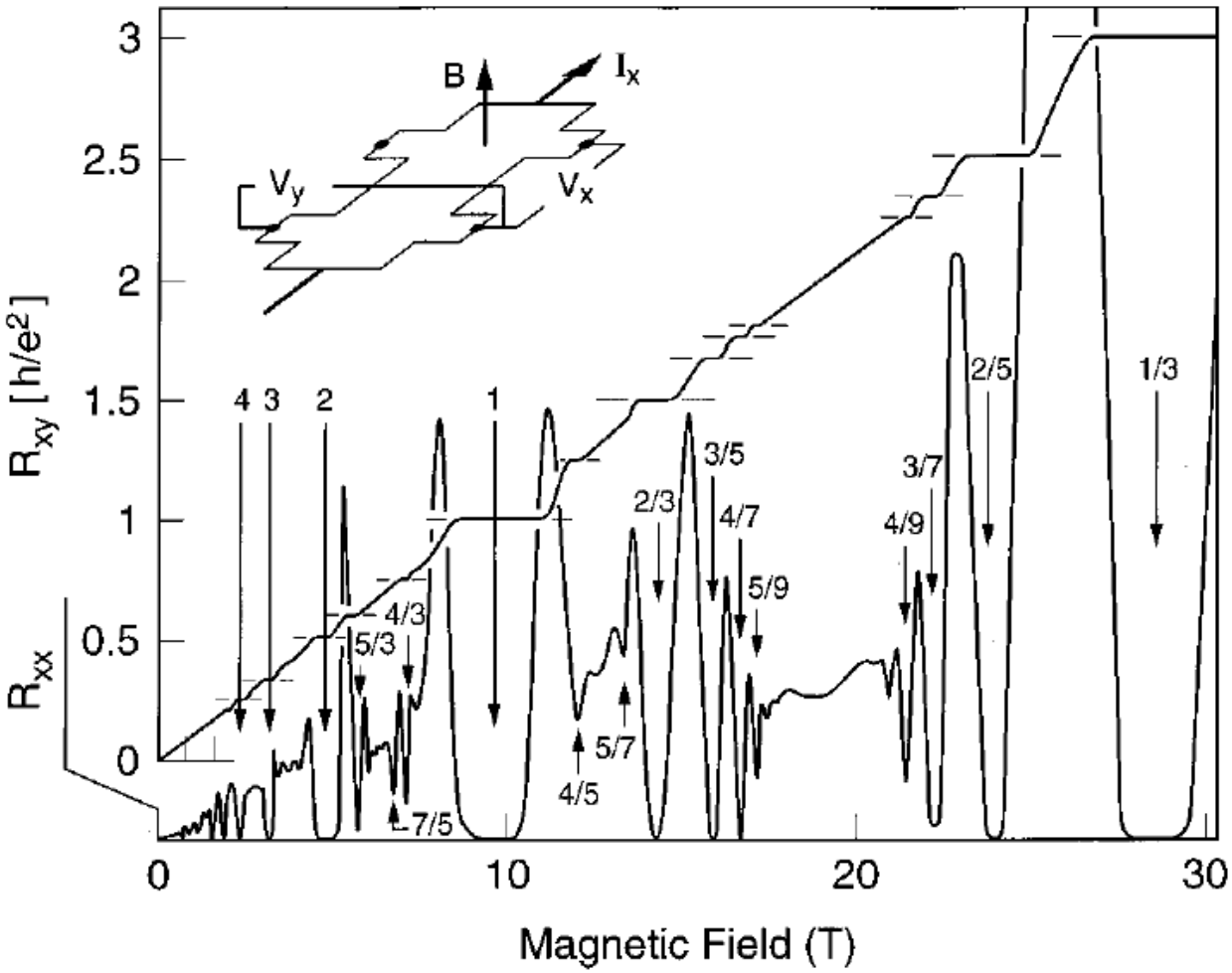
$$\Psi_{\frac{1}{m}} = \prod_{i<j}(z_i - z_j)^m \exp\left[-\frac{1}{4\ell^2}\sum_i |z_i|^2\right]$$

$$z = x - iy, \quad \text{magnetic length } \ell = \sqrt{\frac{\hbar}{eB}}$$

- fermionic (bosonic) wave functions must be antisymmetric (symmetric), hence m is odd (even) integer
- fluid with fractionally charged particles obeying fractional braid statistics

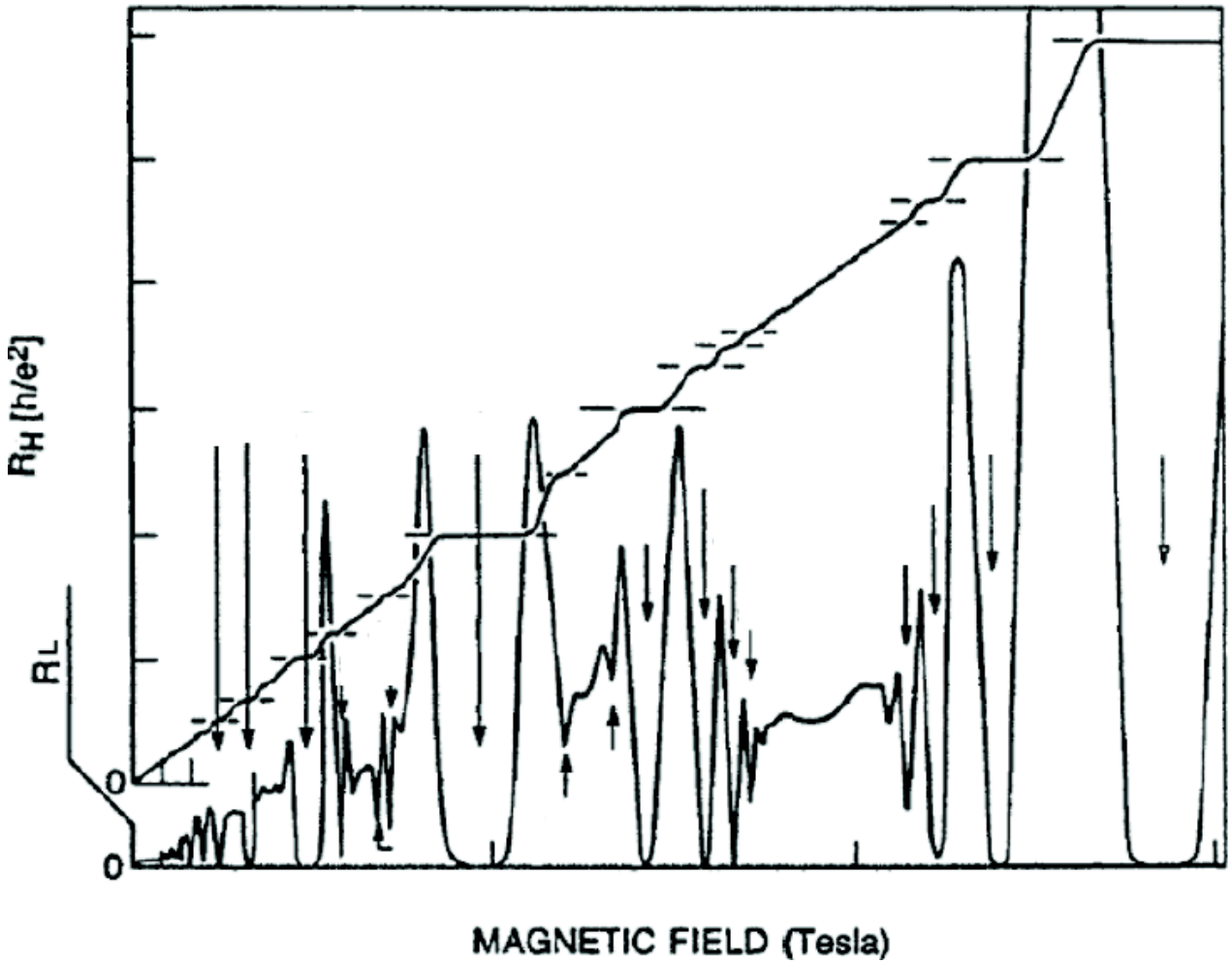
R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983)

Zoo of fractions following the sequence $n/(2n \pm 1)$



J. P. Eisenstein and H. L. Stormer, Science **248**, 4962, 1510-1516 (1990)

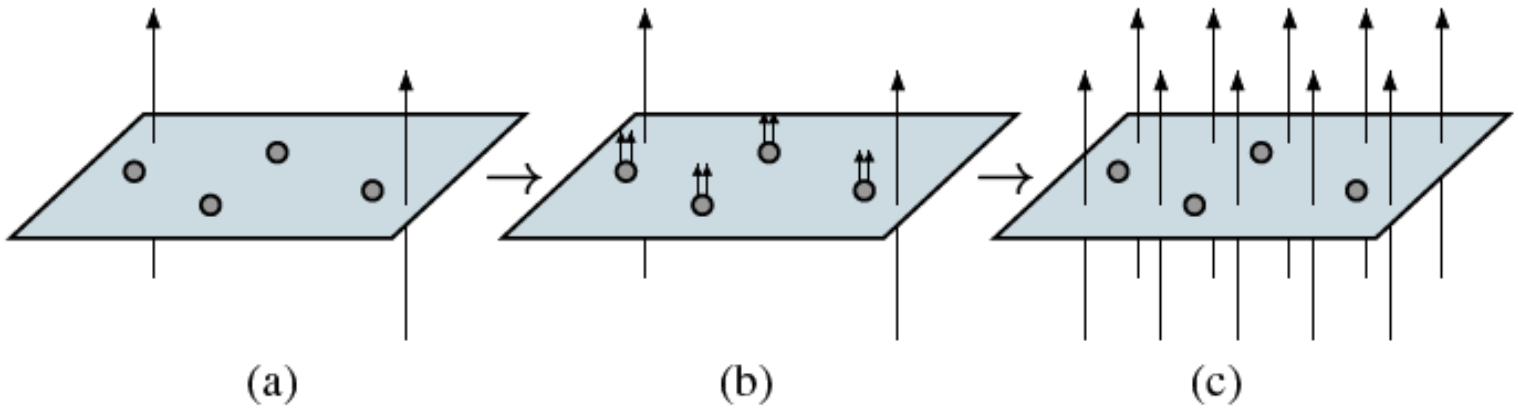
FQHE is qualitatively indistinguishable from IQHE



J. P. Eisenstein and H. L. Stormer, Science **248**, 4962, 1510-1516 (1990)

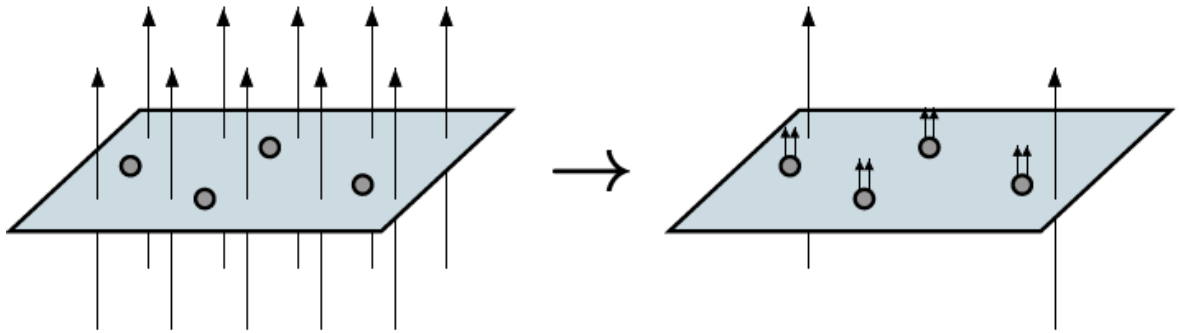
Composite fermion theory

FQHE is understood as the integer QHE of emergent fermions called composite fermions (CFs). A composite fermion is a bound state of an electron and even number of vortices/flux quanta.



J. K. Jain Phys. Rev. Lett. **63**, 199 (1989)

Composite fermions experience a reduced magnetic field

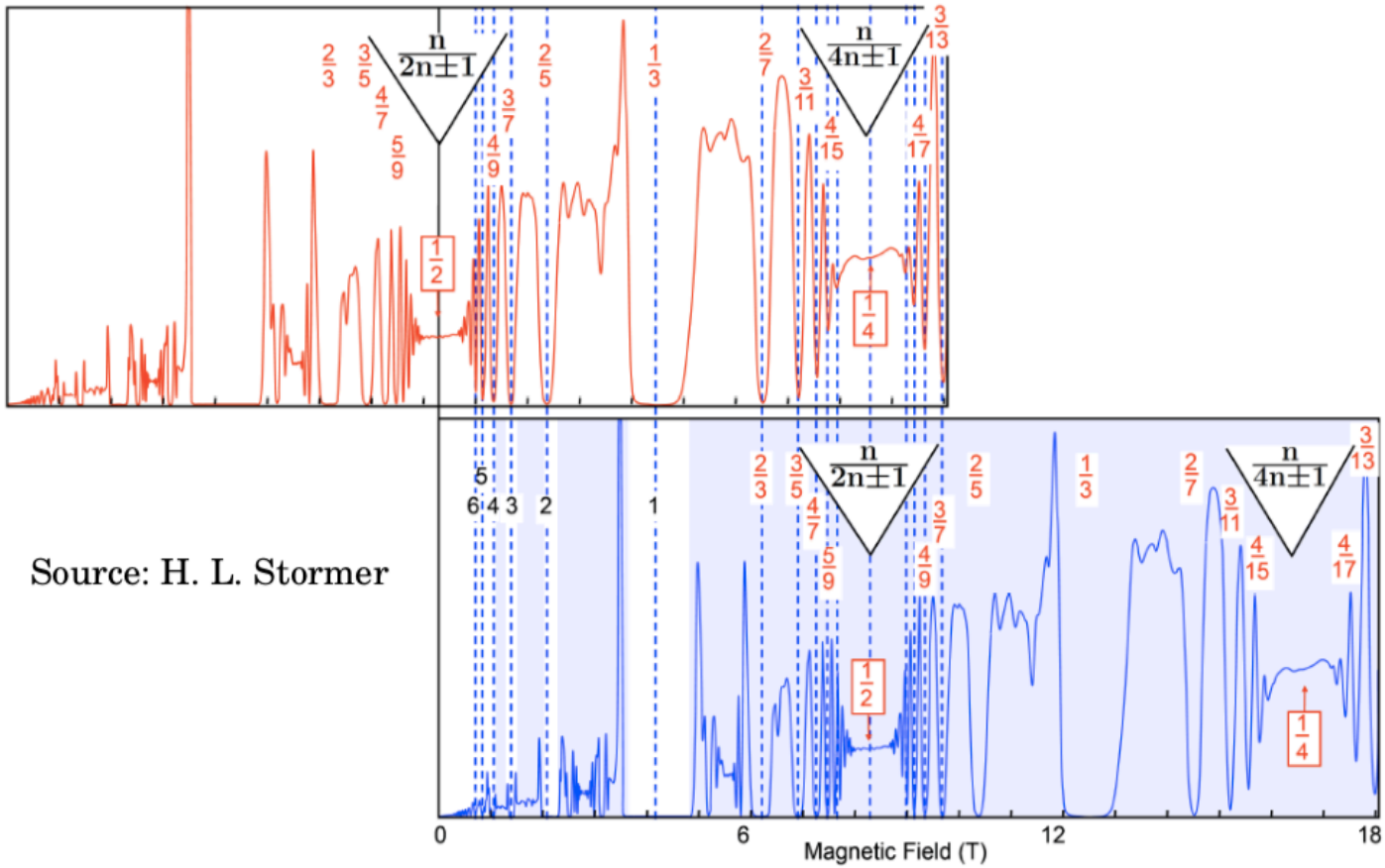


$$B^* = B - 2p\rho\phi_0, \quad \phi_0 = hc/e \text{ is a flux quantum}$$

$$\nu = \frac{\nu^*}{2p\nu^* + 1}$$

- interacting electrons at B = weakly interacting CFs at B^*
- weakly interacting CFs at B^* form their own Landau-like levels called ΛLs and when an integer number of these are filled, there is a finite gap to excitations.

Mapping from IQHE to FQHE



Source: H. L. Stormer

$$B^* = B - 2\rho\phi_0$$

FQHE wave functions are analogous to IQHE ones

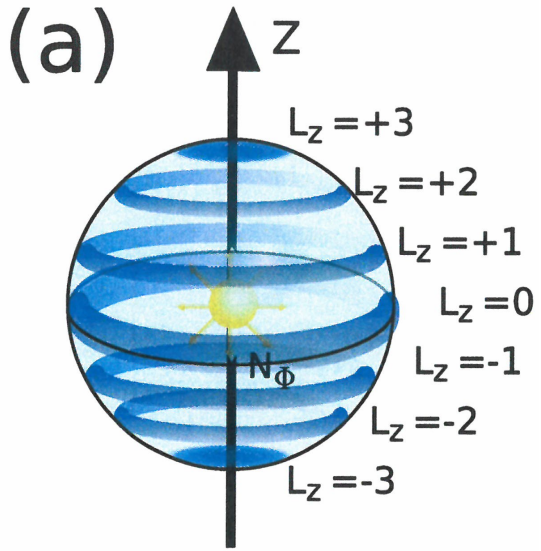
- Jain wave functions at $\nu = n/(2pn \pm 1)$:

$$\Psi_{\nu=\frac{n}{2pn\pm1}} = \mathcal{P}_{\mathrm{LLL}}\left(\Phi_{\pm n}\prod_{i<j}(z_i - z_j)^{2p} e^{-\frac{1}{4\ell^2}\sum_i |z_i|^2}\right).$$

  Φ_n wave function of n filled LLs.
  $\mathcal{P}_{\mathrm{LLL}}$ implements lowest Landau level projection.

- no adjustable parameters in these wave functions
- not just a qualitative theory but a quantitative one

Spherical geometry

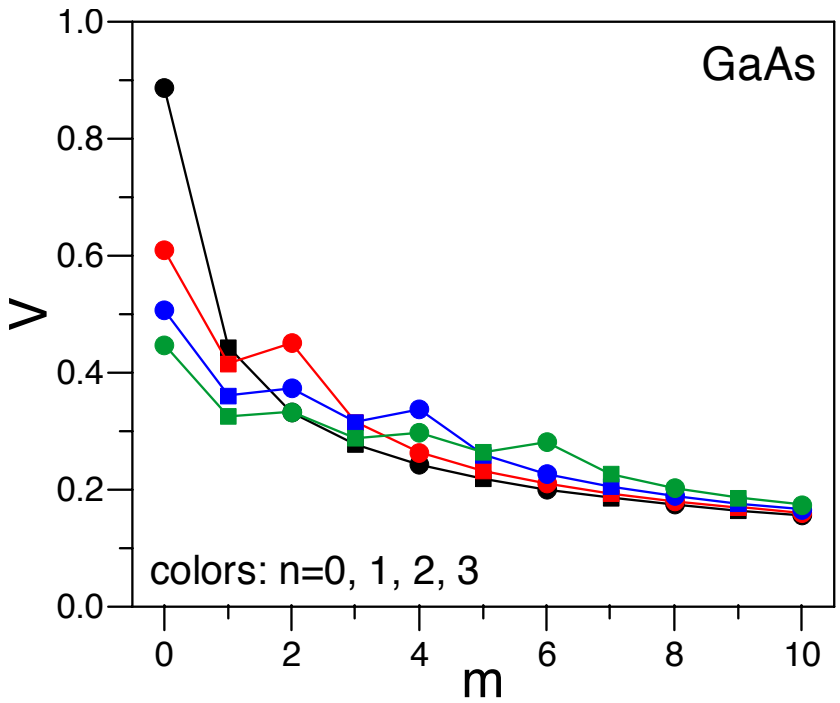


$$I = |Q|, |Q|+1, |Q|+2, \cdots \quad I_n = |Q| + n \quad m = -I, -I+1, \cdots, I-1, I$$

L and its z-component L_z are good quantum numbers

$$N_\phi = 2Q = \nu^{-1} N - \mathcal{S}$$

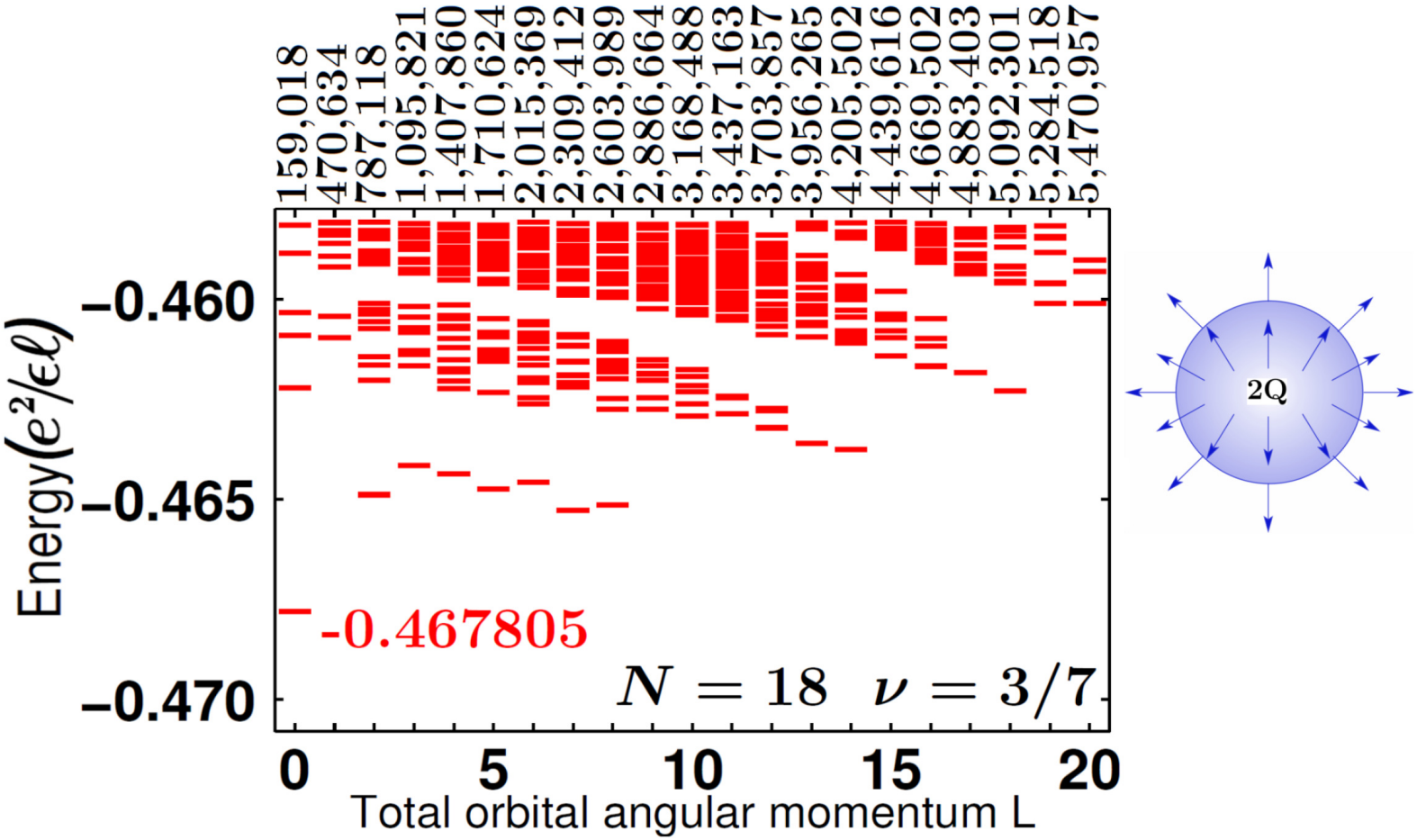
Haldane pseudopotentials



V_m energy of two electrons in a state of relative angular momentum m

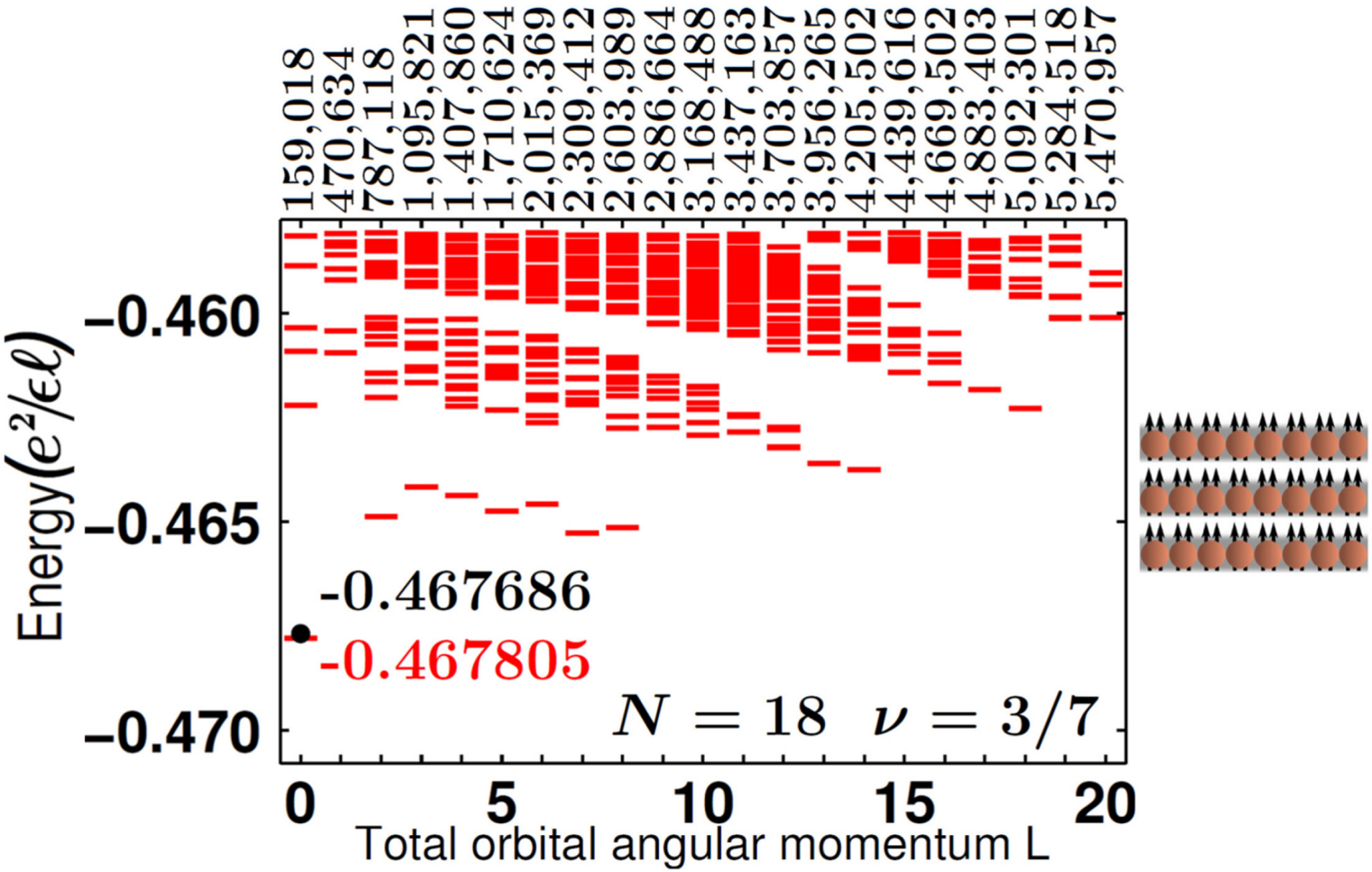
F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983)

Exact spectrum in the spherical geometry: $\nu = 3/7$



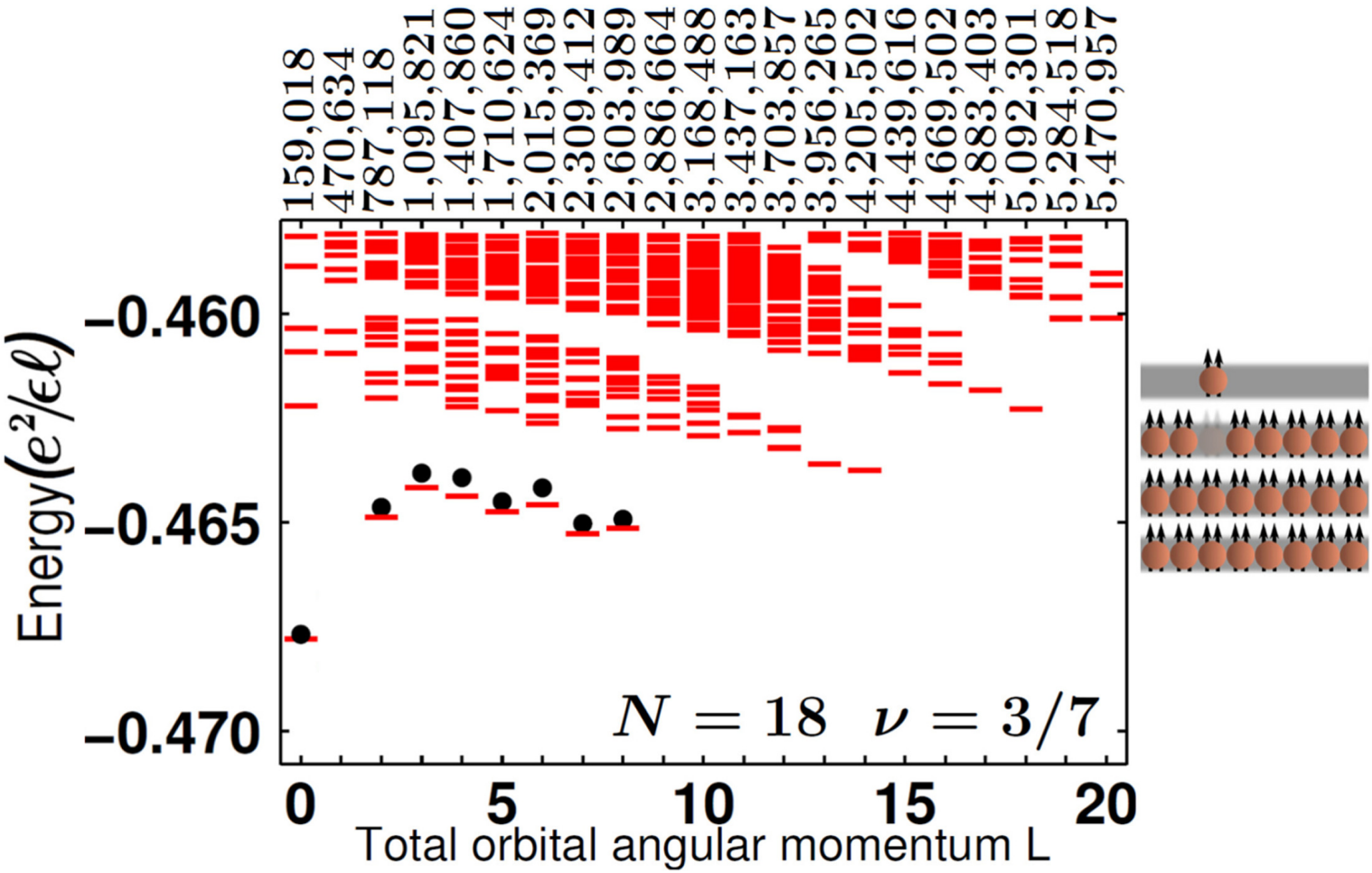
Ajit C. Balram, A. Wójs and J. K. Jain, Phys. Rev. B **88**, 205312 (2013)

Exquisite agreement for ground state: CF vs exact



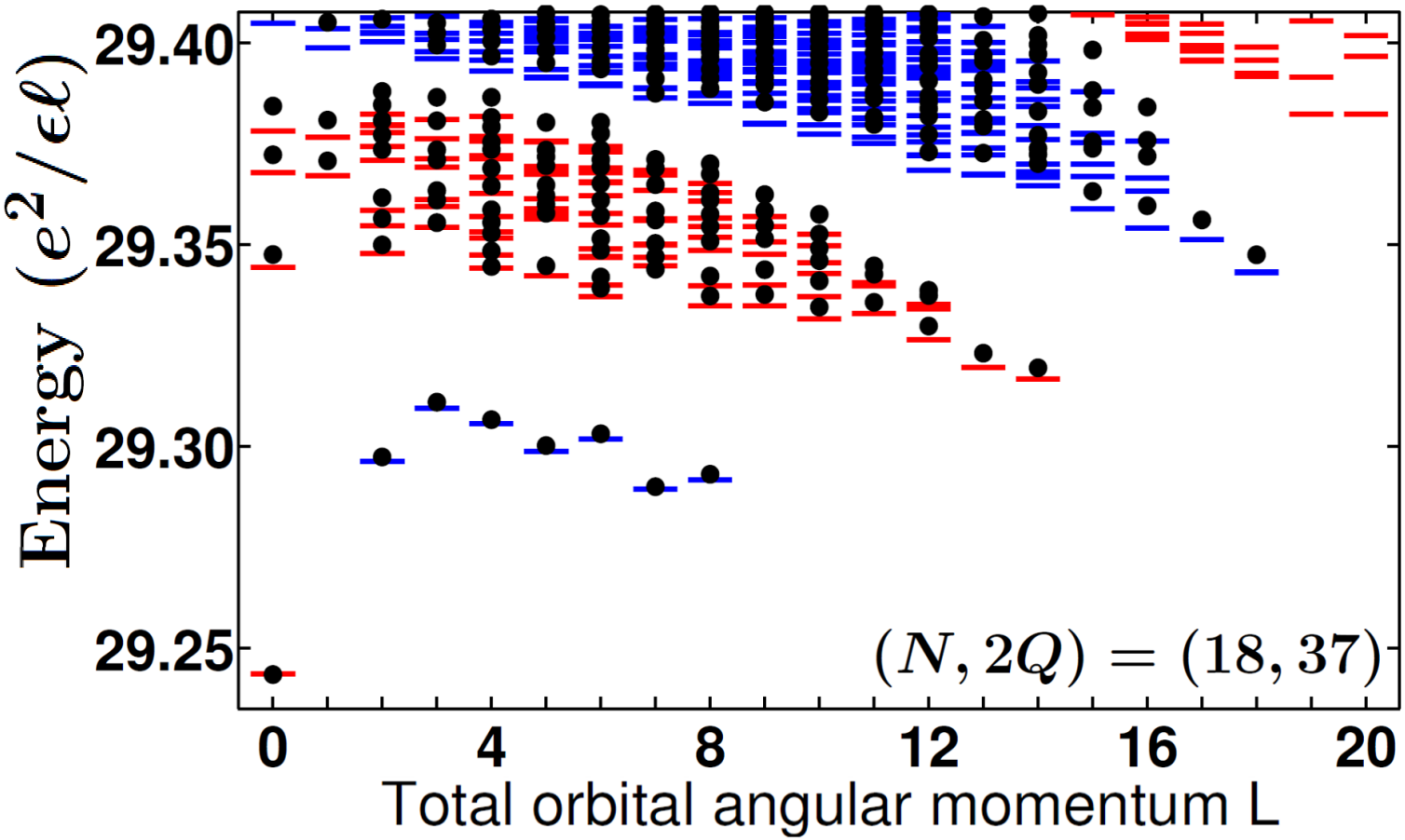
Ajit C. Balram, A. Wójs and J. K. Jain, Phys. Rev. B **88**, 205312 (2013)

Exquisite agreement for excited state: CF vs exact



Ajit C. Balram, A. Wójs and J. K. Jain, Phys. Rev. B **88**, 205312 (2013)

Build the full spectrum by constructing CF excitions



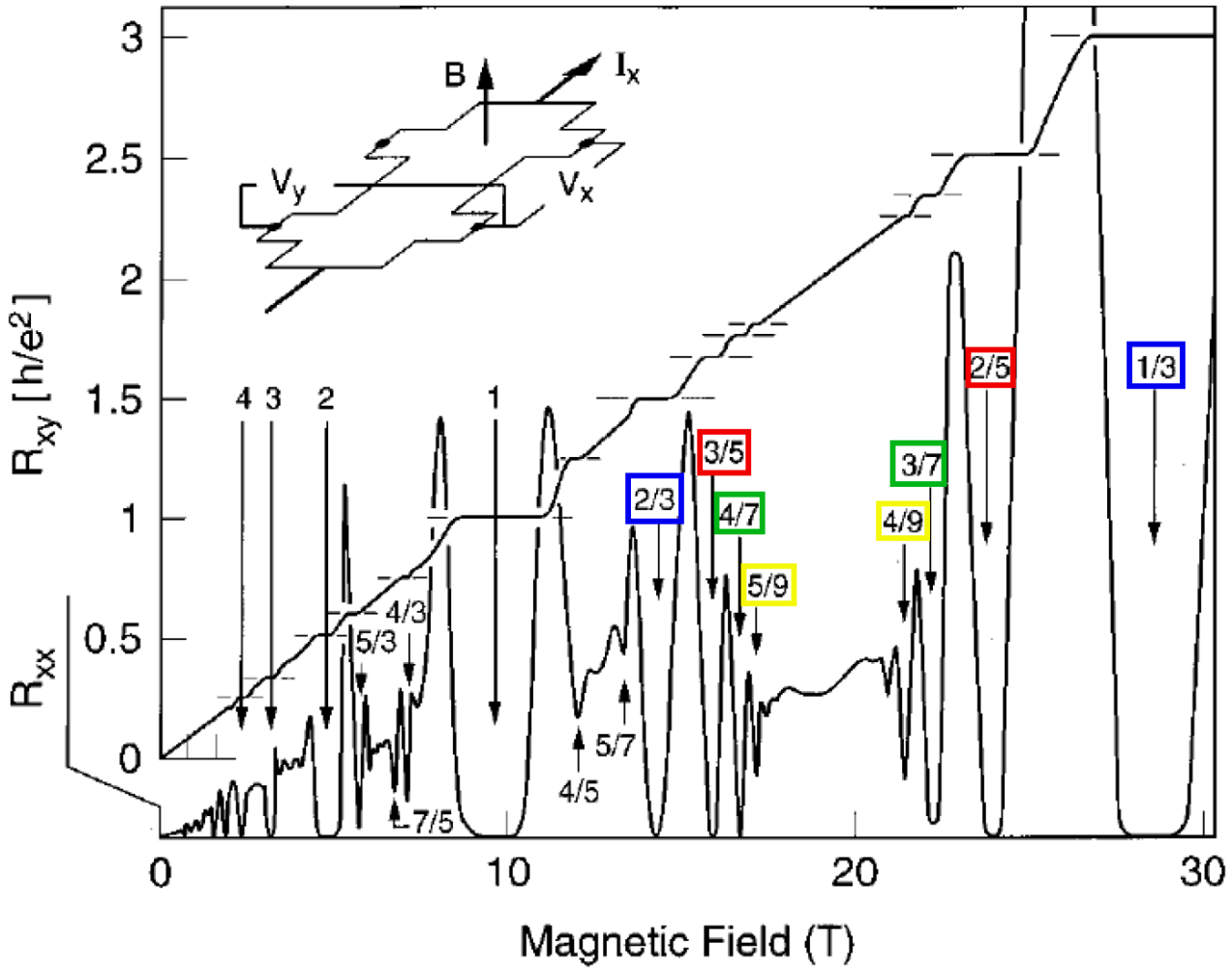
Ajit C. Balram, A. Wójs and J. K. Jain, Phys. Rev. B **88**, 205312 (2013)

particle-hole symmetry in a Landau level

- Particle-hole symmetry of electrons confined to a single Landau level is an exact symmetry of any *two-body* interaction such as the Coulomb one.
- under particle-hole symmetry:
  - creation operator $c^\dagger \rightarrow$ annihilation operator c and vice-versa
  - fully filled Landau level $\rightarrow$ empty one and vice-versa
  - filling factor $\nu \rightarrow$ filling factor $1-\nu$
  - any two-body Hamiltonian remains unchanged (up to a constant)
- an exact mapping of eigenstates and eigenenergies for the Coulomb Hamiltonian from filling factor ν to $1-\nu$

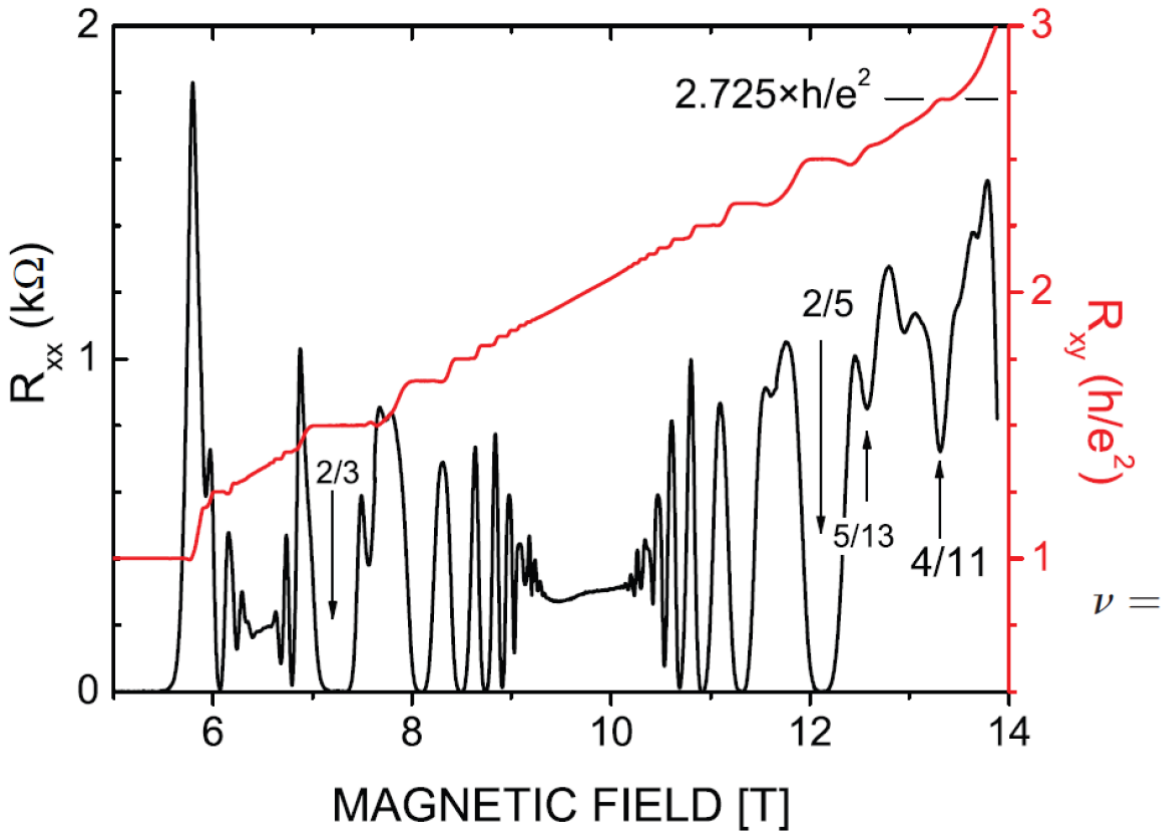
$$[\mathrm{oo}-\!-\!-\!-] \rightarrow [-\!-\mathrm{oooo}]$$

particle-hole symmetry in the lowest Landau level



J. P. Eisenstein and H. L. Stormer, Science **248**, 4962, 1510-1516 (1990)

FQHE states which are not IQHE states of CFs



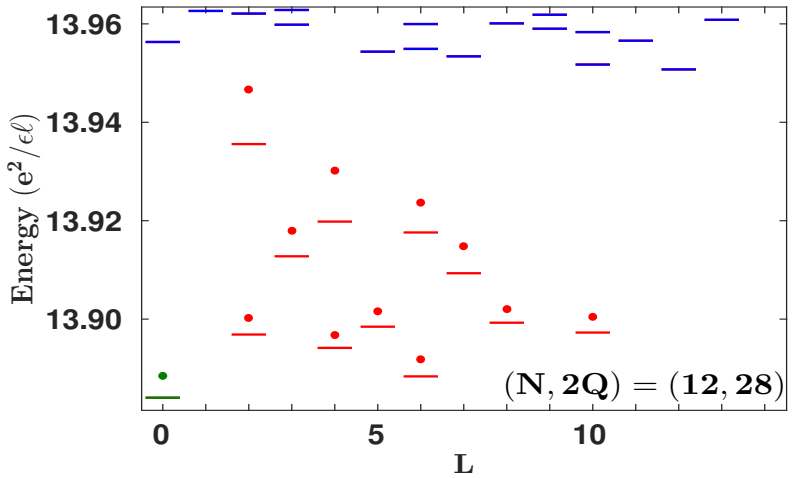
$$\nu = \frac{n}{2pn \pm 1}$$

Pan *et al.*, Phys. Rev. B **91**, 041301(R) (2015)

$4/11$ maps into $\nu^* = 1 + 1/3 = 4/3$

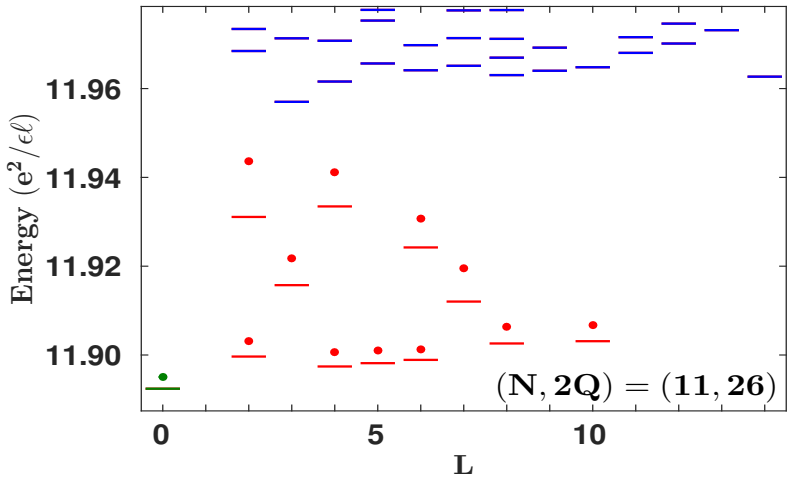
$5/13$ maps into $\nu^* = 1 + (1 - 1/3) = 5/3$

Coulomb spectra at 4/11 and 5/13



$$\nu = \frac{4}{11} = \frac{\frac{4}{3}}{2\frac{4}{3}+1}$$

1+1/3 of composite fermions



$$\nu = \frac{5}{13} = \frac{\frac{5}{3}}{2\frac{5}{3}+1}$$

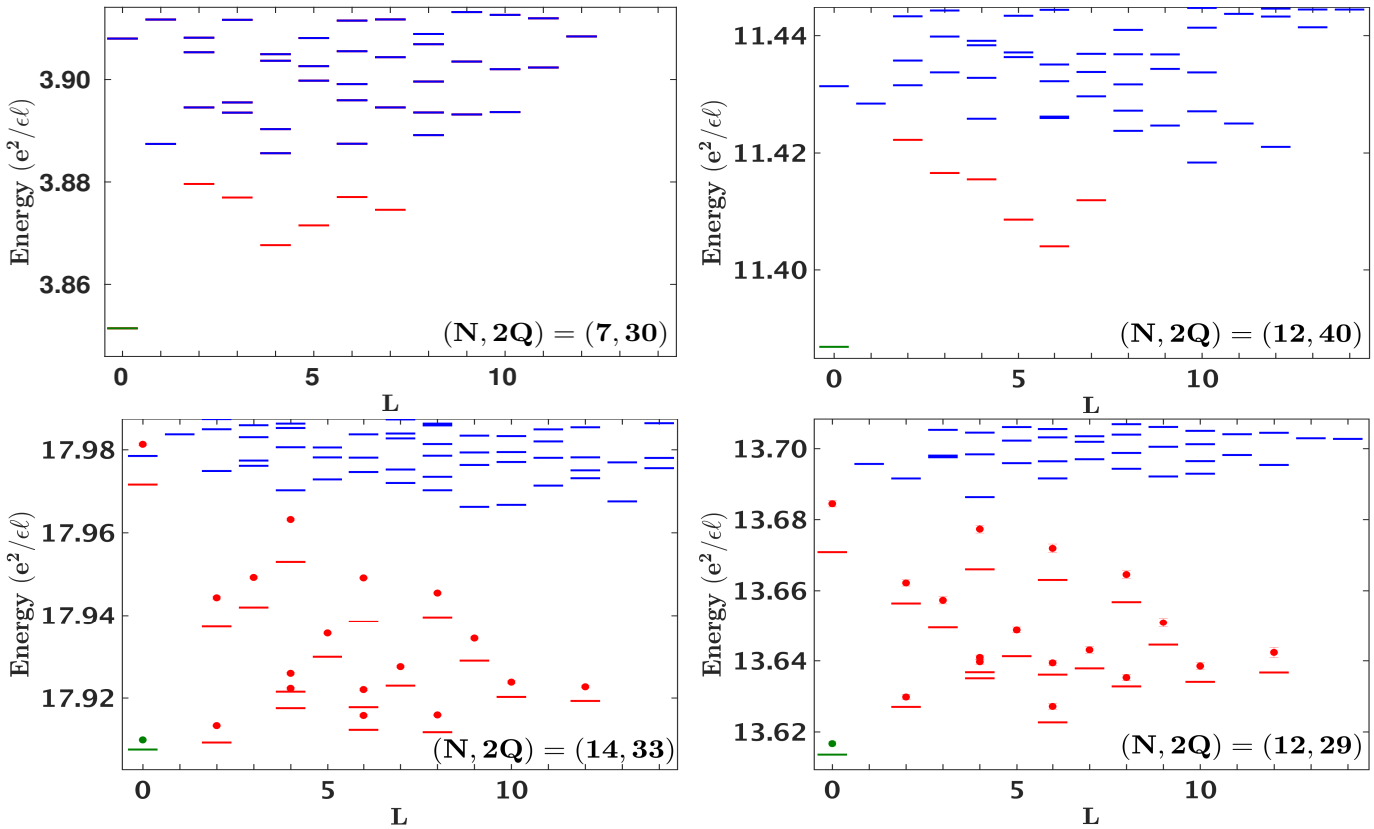
2-1/3 of composite fermions

particle-hole symmetry in a Λ level

- Is there an analog of particle-hole symmetry for composite fermions confined to a single Λ level?
- The two-body electron interaction induces higher-body interaction between composite fermions
- Is the two-body interaction between CF particles and CF holes identical?
- Are the three and higher-body interactions between CFs negligible?
- Relate states at

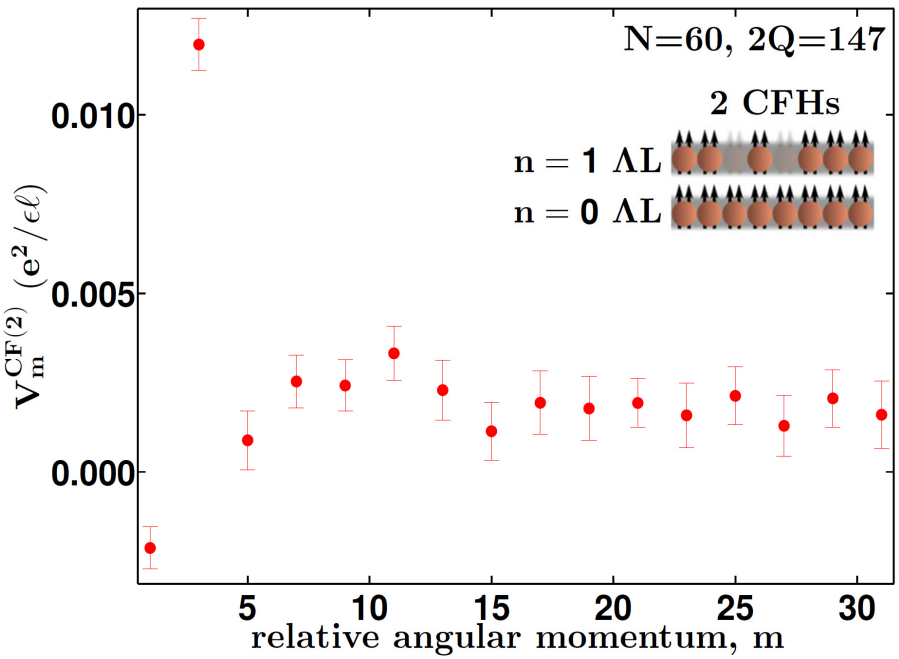
$$\frac{n+\bar{\nu}}{2(n+\bar{\nu})\pm 1} \rightarrow \frac{n+1-\bar{\nu}}{2(n+1-\bar{\nu})\pm 1}$$

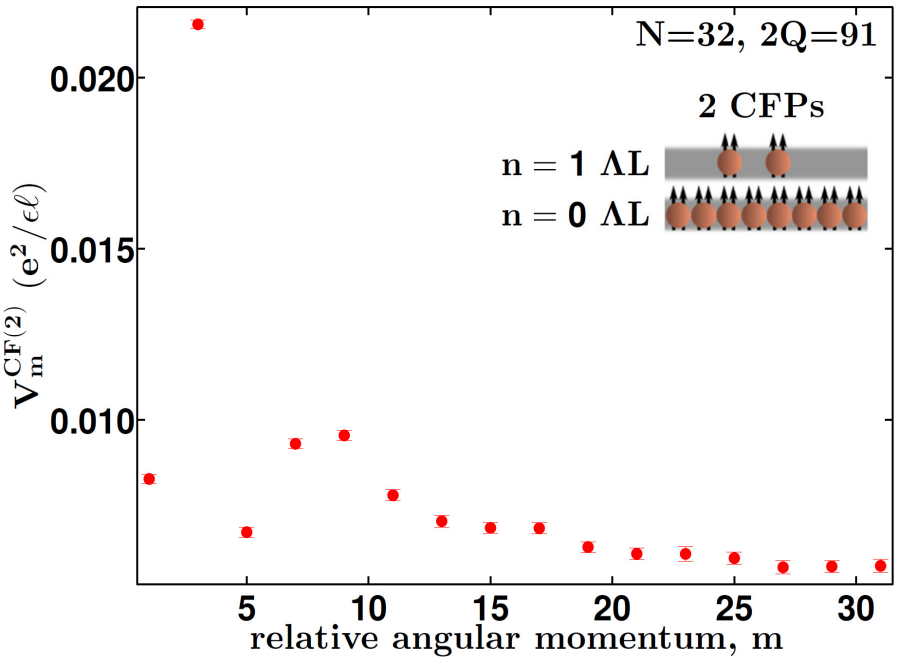
particle-hole symmetry in a Λ level: some examples



$$\frac{n+\bar{\nu}}{2(n+\bar{\nu})\pm 1} \qquad \frac{n+1-\bar{\nu}}{2(n+1-\bar{\nu})\pm 1}$$

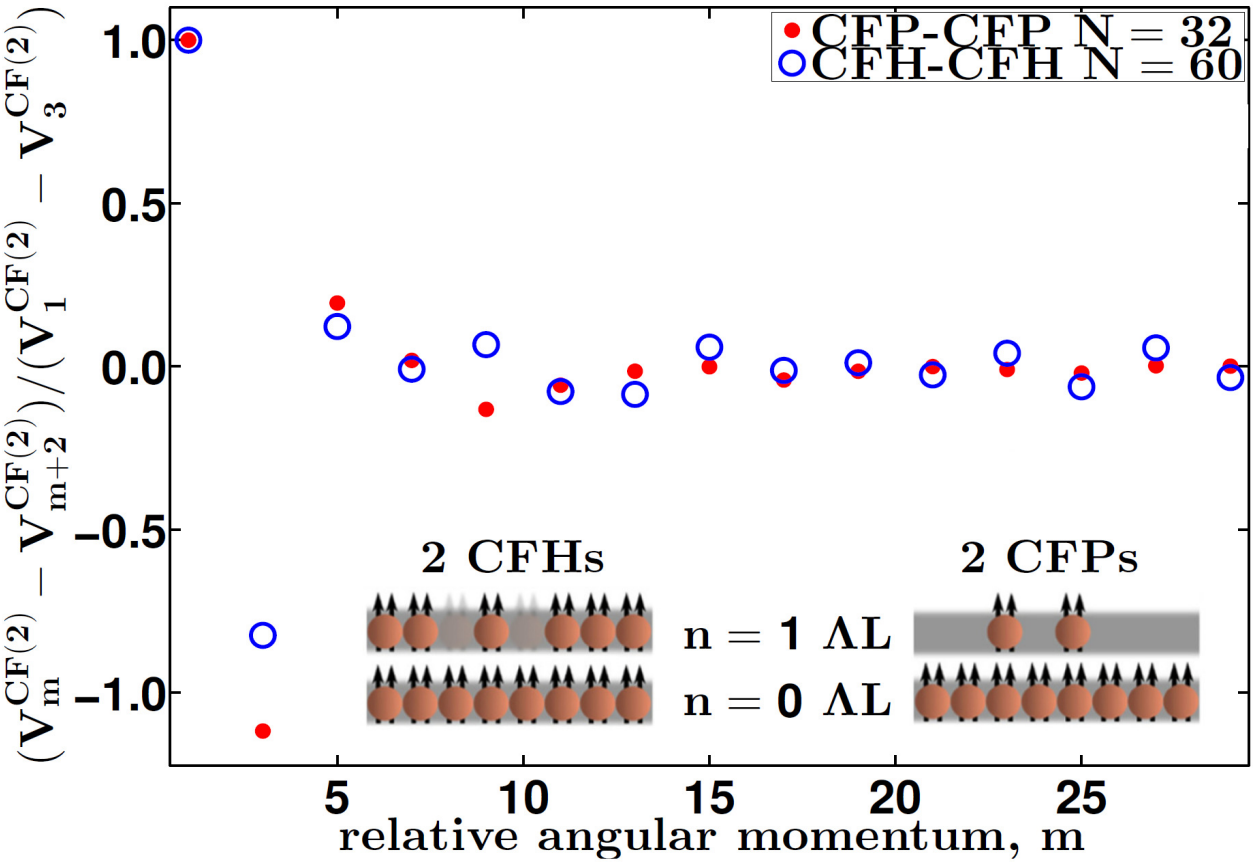
Composite fermion pseudopotentials





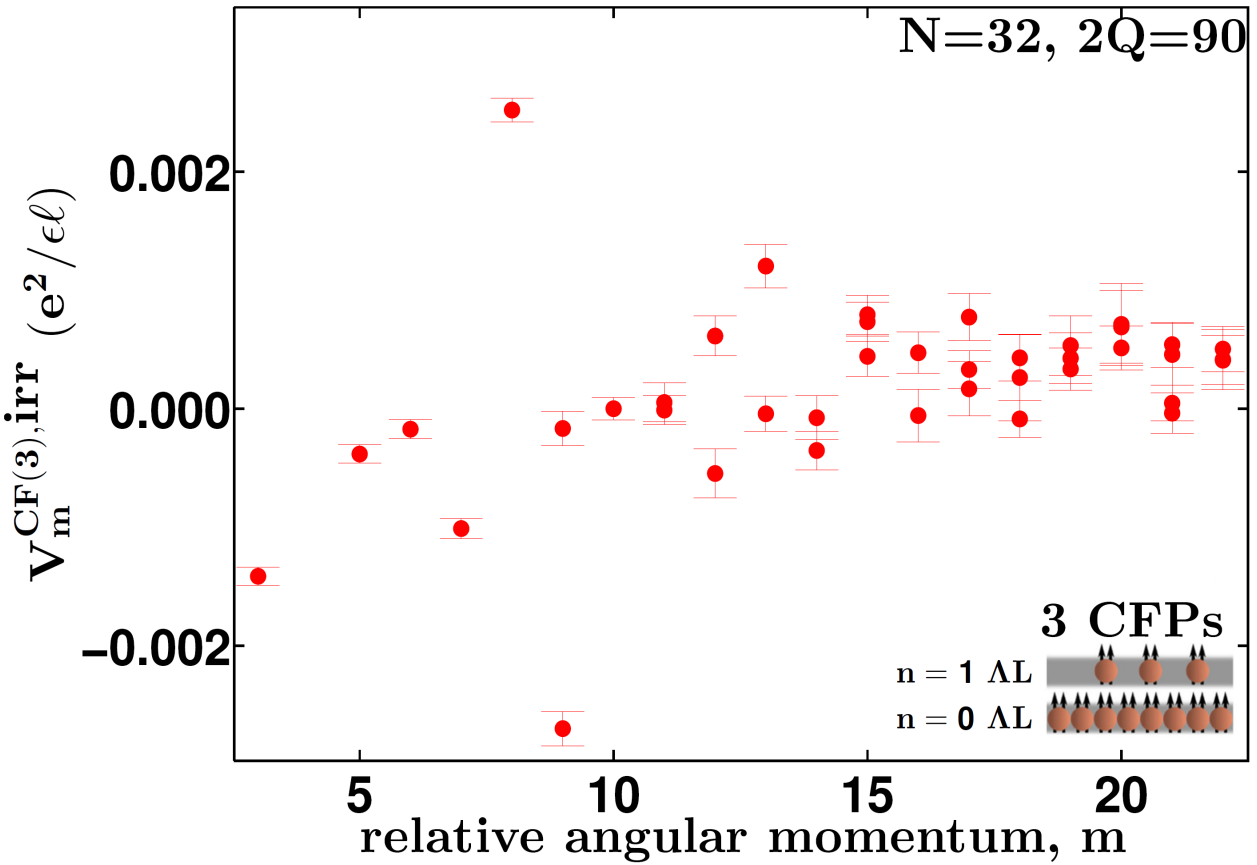
energy of two composite fermions in relative angular momentum m

Two-body interactions between CF particles and holes



interaction between CF particles and CF holes is nearly the same

Three-body interactions between composite fermions



an order of magnitude smaller than the two-body interaction

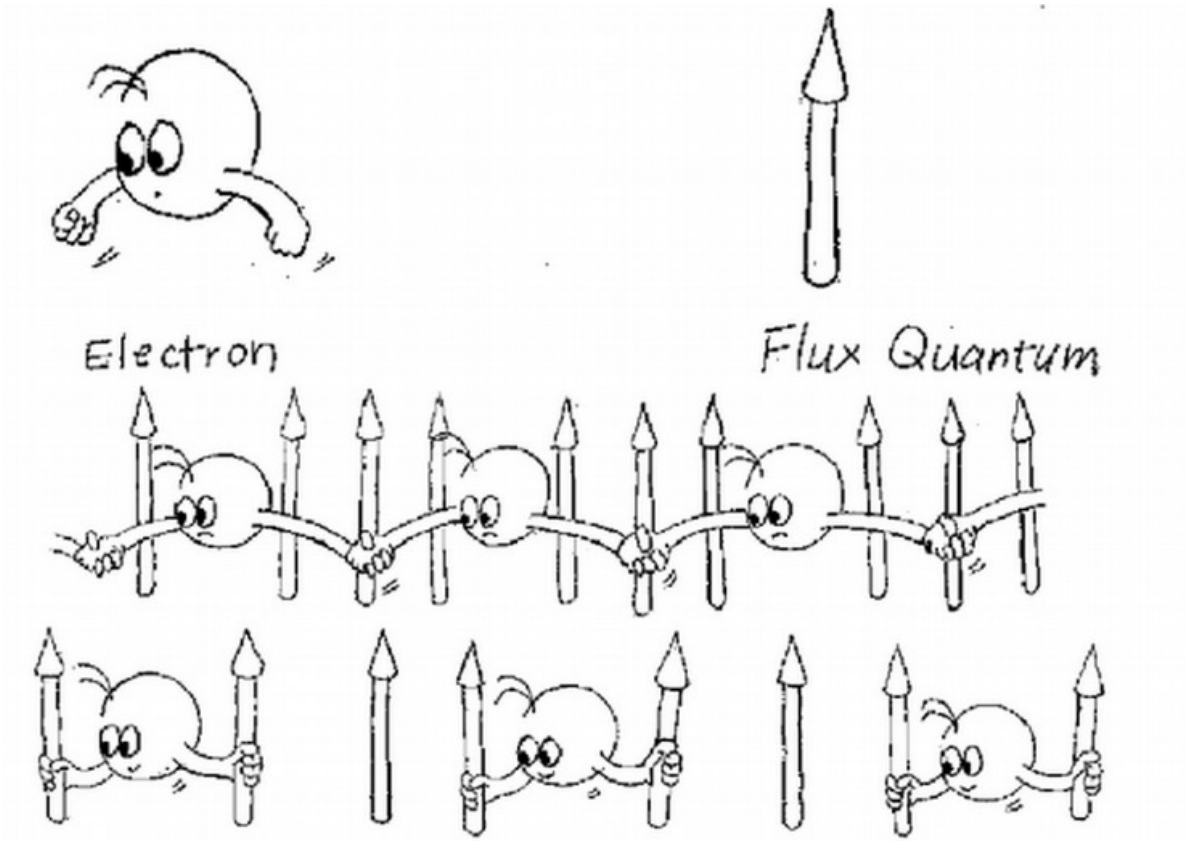
Outlook

- Approximate symmetry relating states at:

$$\frac{n+\bar{\nu}}{2(n+\bar{\nu})\pm 1} \rightarrow \frac{n+1-\bar{\nu}}{2(n+1-\bar{\nu})\pm 1}$$

  for small values of n.
- This symmetry is not present in the original electronic Hamiltonian and arises entirely due to the formation of composite fermions: particlle-hole symmetry of composite fermions.
- Two-body interactions between composite fermion holes is approximately the same as that between composite fermion particles.
- Three- and higher-body interactions between composite fermions is negligible.

Thank you for your attention!



Source: Kwon Park

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