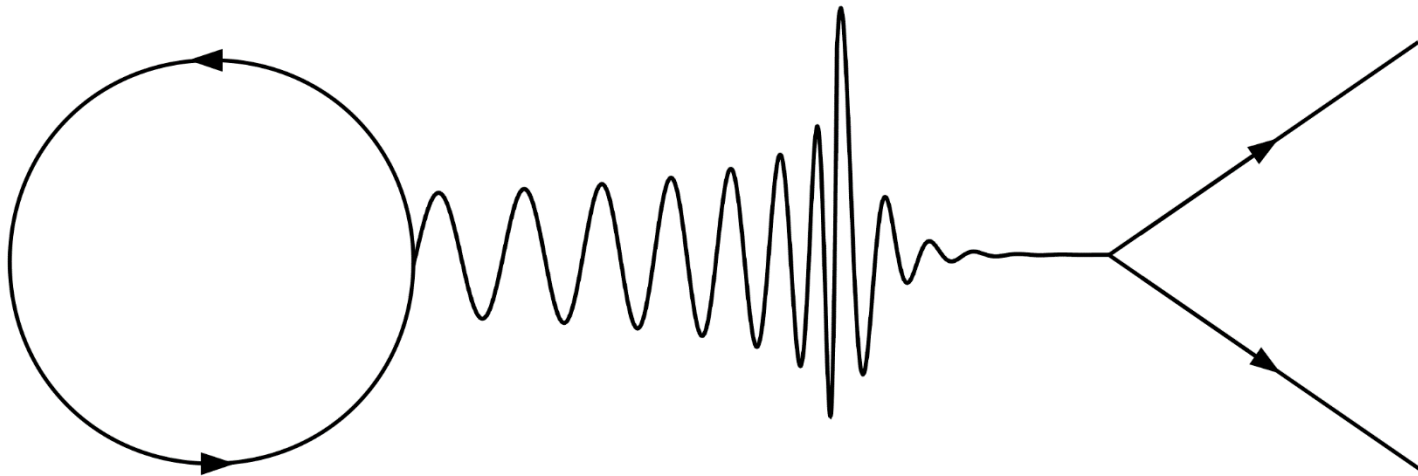


Testing GR with GWs



Vítor Cardoso
(Técnico, Lisboa)

*Image: Cardoso & Pani,
CERN Courier (2016)*

LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less
Agog Over Results of Eclipse
Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed
or Were Calculated to be,
but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

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DON'T WORRY OVER NEW LIGHT THEORY

Physicists Agree That It Can Be
Disregarded for Practical
Purposes.

NEWTON'S LAW IS SAFE

At Most It Suffers Only Slight
Correction, Says Prof. Bum-
stead of Yale University.

OTHER PROFESSORS' VIEWS

EINSTEIN EXPOUNDS HIS NEW THEORY

It Discards Absolute Time and
Space, Recognizing Them Only
as Related to Moving Systems.

IMPROVES ON NEWTON

Whose Approximations Hold for
Most Motions, but Not Those
of the Highest Velocity.

INSPIRED AS NEWTON WAS

Articles in The Times
from Nov. 10, 1919,
left; Nov. 16, 1919,
center; and Dec. 3,
1919.

Uniqueness: the Kerr solution

Theorem (Carter 1971; Robinson 1975; Chrusciel and Costa 2012):
A stationary, asymptotically flat, vacuum BH solution must be Kerr

$$ds^2 = \frac{\Delta - a^2 \sin^2 \theta}{\Sigma} dt^2 + \frac{2a(r^2 + a^2 - \Delta) \sin^2 \theta}{\Sigma} dt d\phi \\ - \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2 - \frac{\Sigma}{\Delta} dr^2 - \Sigma d\theta^2$$

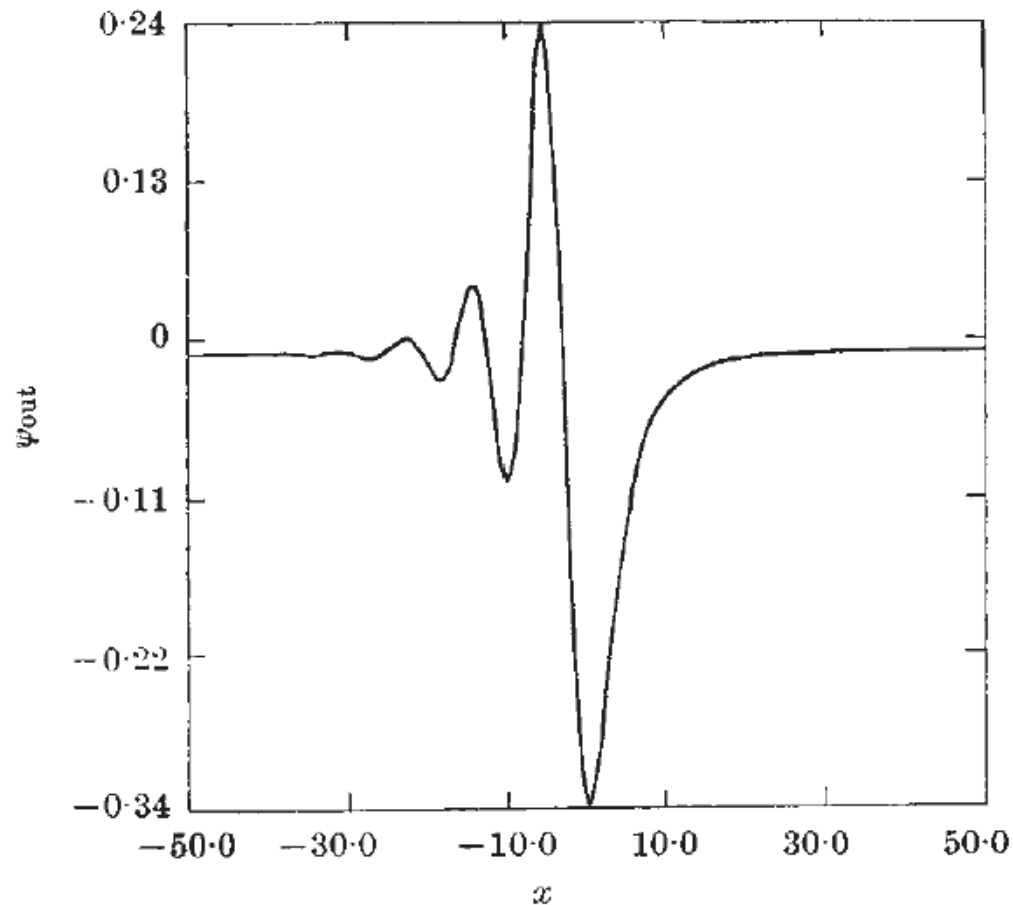
$$\Sigma = r^2 + a^2 \cos^2 \theta, \quad \Delta = r^2 + a^2 - 2Mr$$

Describes a rotating BH with mass M and angular momentum $J=aM$, iff $a < M$

“In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein’s equations of general relativity provides the *absolutely exact representation* of untold numbers of black holes that populate the universe.”

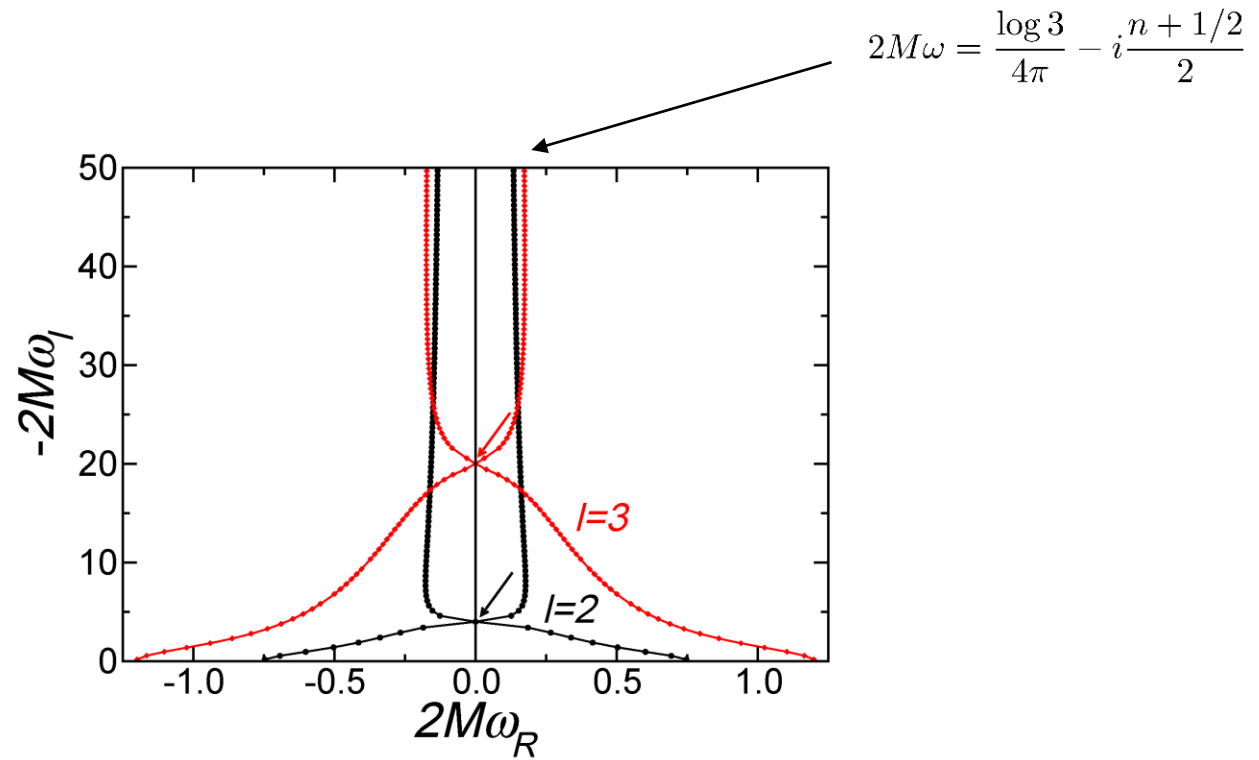
S. Chandrasekhar, The Nora and Edward Ryerson lecture, Chicago April 22 1975

Hair loss: the characteristic modes of black holes



C.V.Vishveshwara, Nature 227: 938 (1970)

Data and routines at blackholes.ist.utl.pt

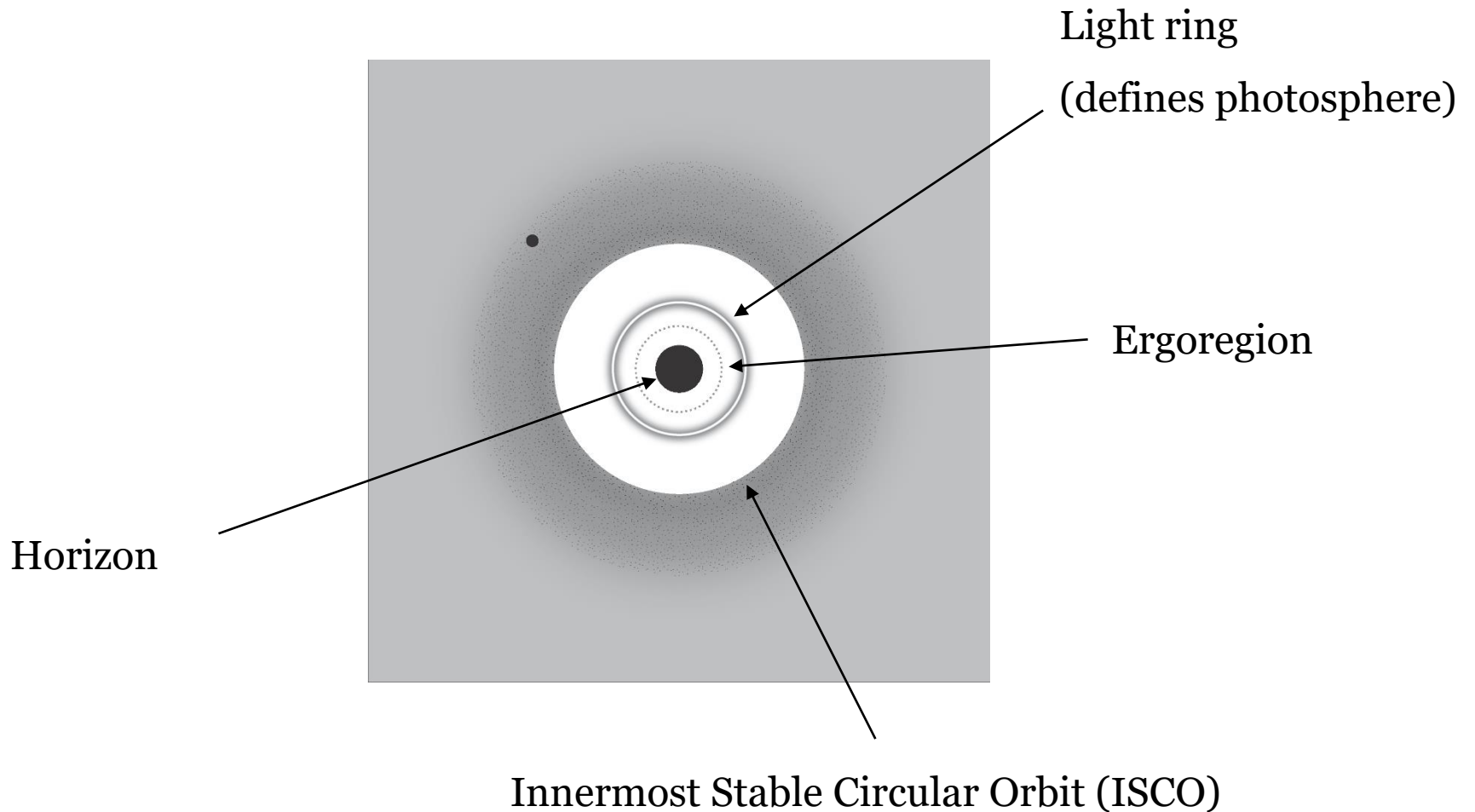


$$f = \omega_R/2\pi = 1.207 \left(\frac{10 M_\odot}{M} \right) \text{ kHz}$$

$$\tau = 1/|\omega_I| = 0.5537 \left(\frac{M}{10 M_\odot} \right) \text{ ms}.$$

Berti, Cardoso and Will PRD73: 064030 (2006)
Berti, Cardoso and Starinets, CQG 26: 163001 (2009)

Black holes are black

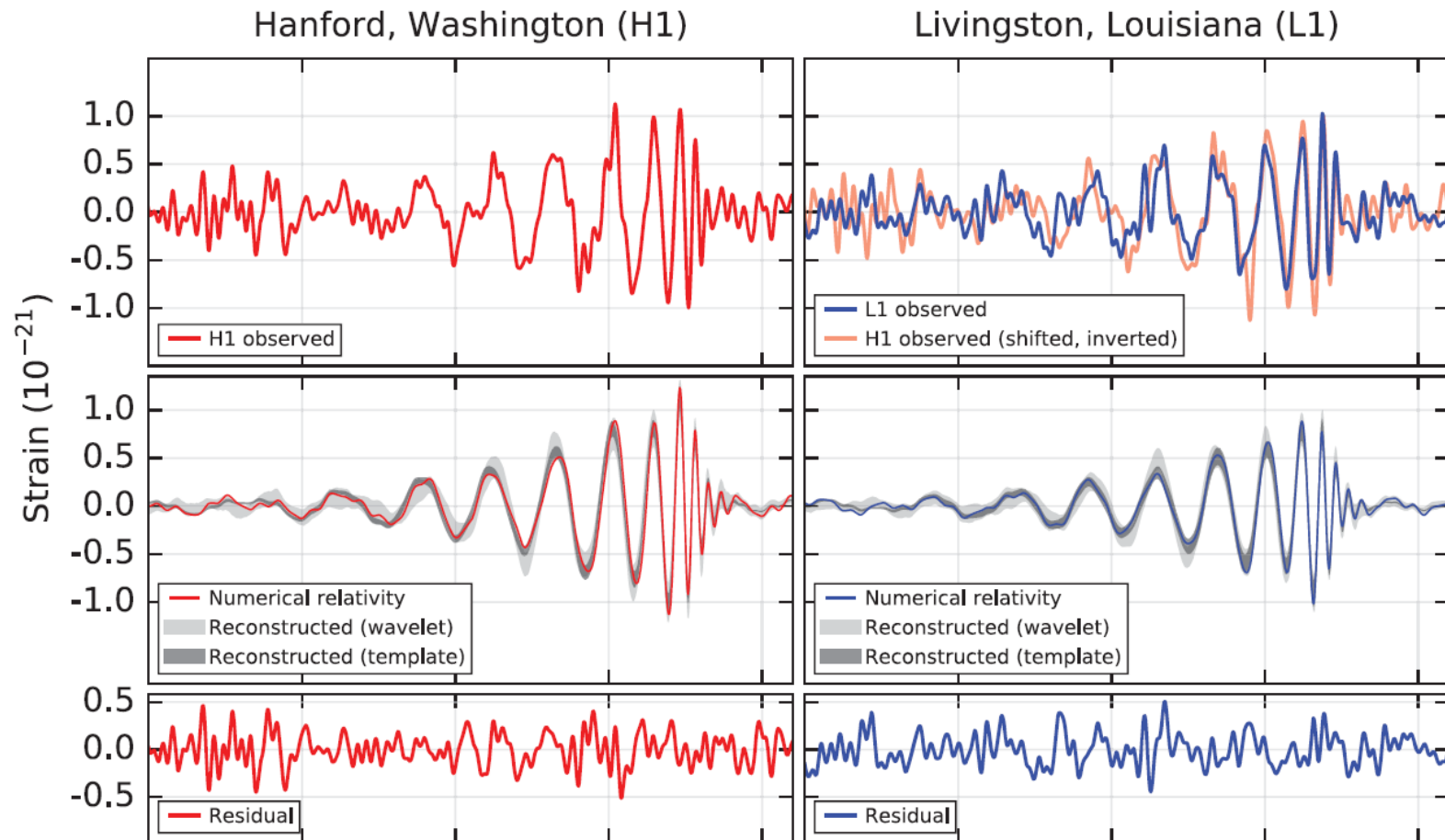


Cardoso & Pani, *Nature Astronomy* 1: 586 (2017)

Living Reviews in Relativity 22: 1 (2019)

Image: Ana Carvalho

They are out there, in isolation and in pairs



Fundamental questions

a. BH seeds, BH demography, galaxy co-evolution (how many, where, how?)

Barack+ CQG36:143001 (2019); arXiv:1806.05195

b. What is graviton mass or speed?

See review Barack+ CQG36:143001 (2019); arXiv:1806.05195

c. Is there near-horizon structure? Is the object really a BH?

Cardoso+ PRL116: 171101 (2016); Nature Astronomy 1: 586 (2017); LRR 22:1 (2019)

d. Is cosmic censorship preserved?

Sperhake+ PRL103:131102 (2009); Cardoso+ PRL120:031103 (2018)

e. Is it a Kerr BH? Can we do BH spectroscopy and constrain alternatives?

Berti+ PRL117:101102 (2016); Cardoso & Gualtieri CQG33:174001 (2016)

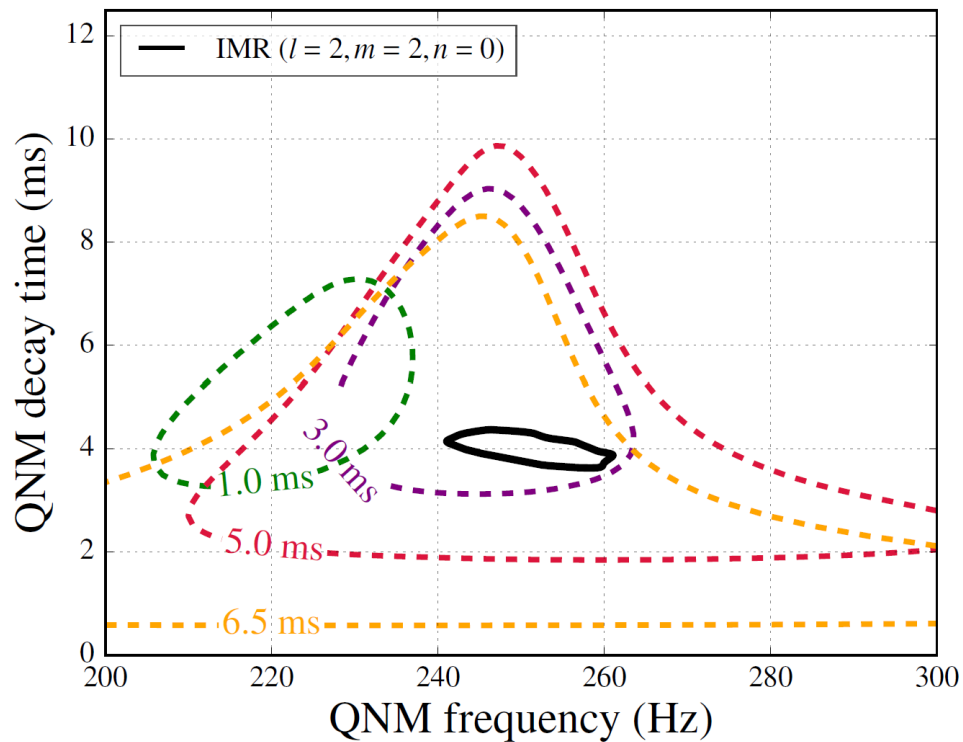
f. Are there extra radiation channels, corrections to gravity?

Barack+ CQG36: 143001 (2019); Barausse+PRL116:241104 (2016)

g. Can GWs from BHs inform us on fundamental fields/DM?

Macedo+ ApJ774: 48 (2013); Arvanitaki+ PRD95: 043001 (2016); Brito+ PRL119:131101 (2017); Brito+ Lect. Notes Phys. 906 (2015)

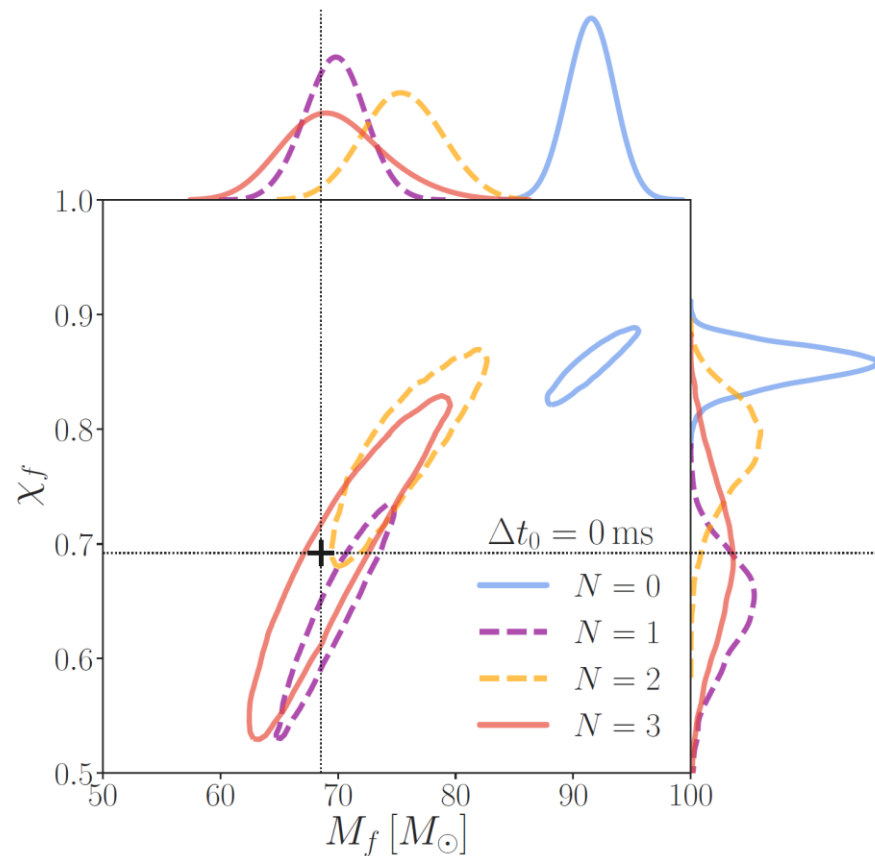
BH spectroscopy I



90% posterior distributions.

Black solid is 90% posterior of QNM as derived from the posterior mass and spin of remnant

The importance of overtones



Giesler+ arXiv:1903:08284

Baibhav + PRD97:044048 (2018); Brito+ PRD98: 084038 (2019)

BH spectroscopy II: testing Kerr nature with ringdown

Need to measure two or more modes: disentangle frequencies, damping times and amplitudes

$$\rho_{\text{GLRT}}^{l=2,3} \approx 17.6 + \frac{15.5}{q-1} - \frac{1.7}{q}$$
$$\rho_{\text{GLRT}}^{l=2,4} \approx 37.9 + \frac{83.6}{q} + \frac{44.1}{q^2} + \frac{50.1}{q^3}$$

Berti + PRD76: 104044 (2007)
Berti + PRL117: 101102 (2016)

Extra couplings with spectroscopy

Example: BH charge

(mini-charged DM predict heavy, fractional “electrons” and RN geometry:

Rujula + 1990; Perl+ 1997; Holdom 1986; Sigurdson + 2004)

$$\mathcal{L} = \sqrt{-g} \left(\frac{R}{16\pi} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + 4\pi e j_{\text{em}}^\mu A_\mu + 4\pi e_h j_h^\mu B_\mu + 4\pi \epsilon e j_h^\mu A_\mu \right)$$

Ringdown bound $\frac{Q}{M} \lesssim 0.1 \sqrt{\frac{100}{\rho}}$

Or 1 electron per 10^{19} neutrons. Bound can be generalized to other theories, provided spectra is known

Cardoso + JCAP 1605: 054 (2016)

Blázquez-Salcedo + PRD94:104024 (2016)

Environment: ringdown properties

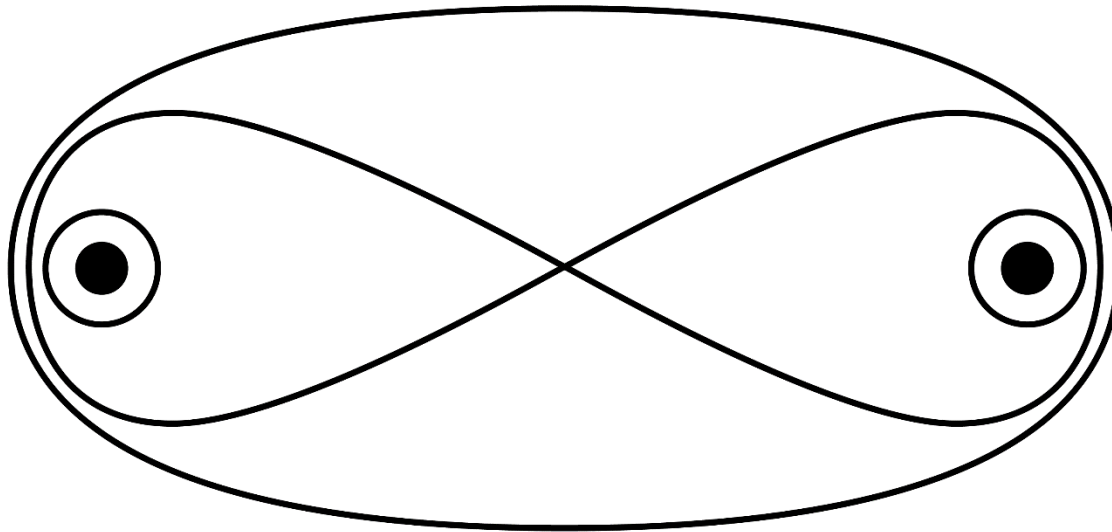
Correction	$ \delta_R [\%]$	$ \delta_I [\%]$
spherical near-horizon distribution	0.05	0.03
ring at ISCO	0.01	0.01
electric charge	10^{-5}	10^{-6}
magnetic field	10^{-8}	10^{-7}
gas accretion	10^{-11}	10^{-11}
DM halos	$10^{-21} \rho_3^{\text{DM}}$	$10^{-21} \rho_3^{\text{DM}}$
cosmological effects	10^{-32}	10^{-32}

Barausse + PRD89: 104059 (2014)

Binaries: molecular spectroscopy?

$$ds^2 = -\frac{dt^2}{U^2} + U^2 (d\rho^2 + \rho^2 d\phi^2 + dz^2)$$

$$U(\rho, z) = 1 + \frac{M}{\sqrt{\rho^2 + (z - a)^2}} + \frac{M}{\sqrt{\rho^2 + (z + a)^2}}$$



Gravitational molecules: a toy model

Change to prolate confocal elliptic coordinates

$$\rho^2 + (a - z)^2 = a^2(\chi + \eta)^2$$

$$\rho^2 + (a + z)^2 = a^2(\chi - \eta)^2$$

$$\partial_\eta \left((1 - \eta^2) \partial_\eta S \right) + \left(-a^2 \omega^2 \eta^2 - \frac{m^2}{1 - \eta^2} + \Lambda \right) S = 0$$

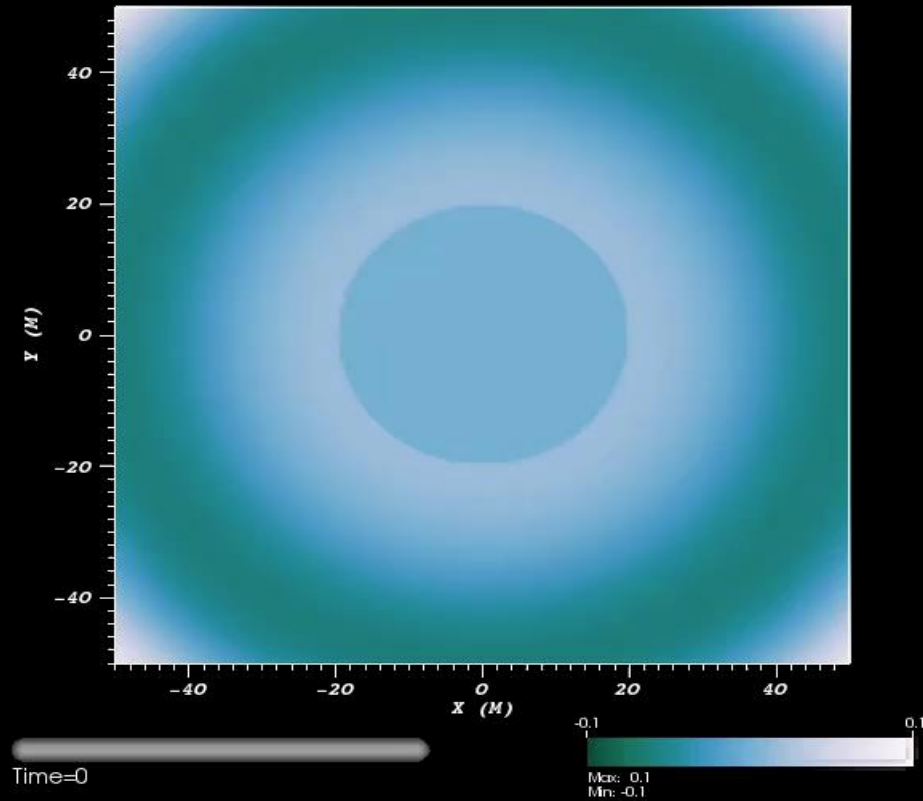
$$\partial_\chi \left((\chi^2 - 1) \partial_\chi R \right) + \left(a^2 \omega^2 \chi^2 + 8Ma\chi\omega^2 - \frac{m^2}{\chi^2 - 1} - \Lambda \right) R = 0$$

Klein-Gordon equation is identical to Schrodinger for Di-Hydrogen ionized molecule

Bernard+ PRD100: 044002 (2019)

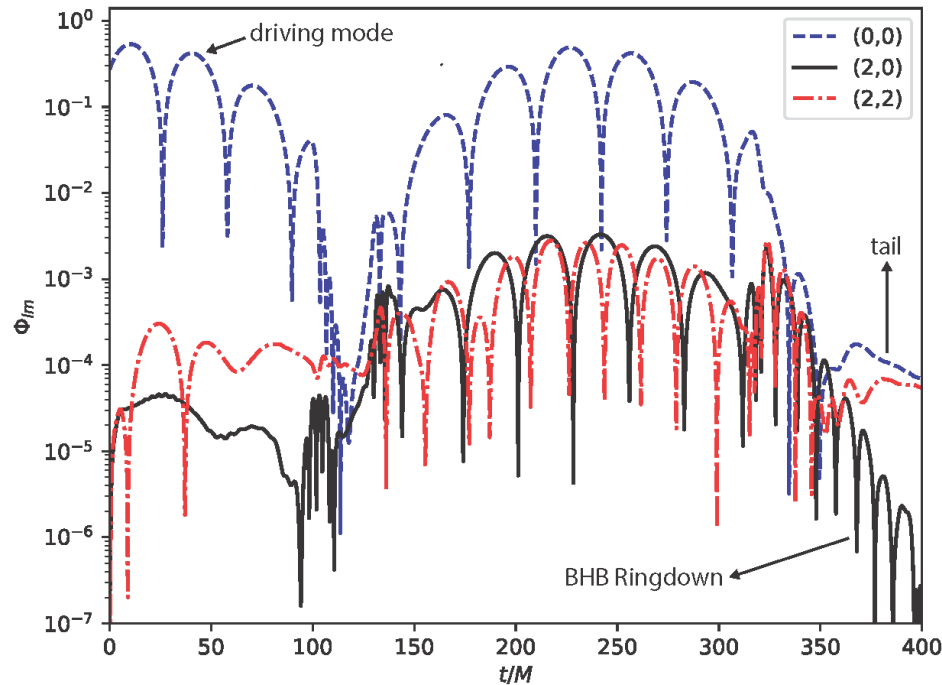
for Hydrogen molecule see Burrau M7: 1 (1928); Wilson PRSLA118:635 (1929); Hylleraas ZfP71: 739 (1931)

Gravitational molecules: a real BH binary



Bernard + PRD100: 044002 (2019); arXiv:1905.05204

Gravitational molecules: a real BH binary



$$T = (1.03 \pm 0.04) L + (8 \pm 1) M$$

Global BHB modes may be resonantly excited?

Bernard + PRD100: 044002 (2019); arXiv:1905.05204

DM I

Inspiral occurs in DM rich environment and may modify inspiral rate, given dense-enough media: accretion and drag may play role.

Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013); Barausse+PRD 2014

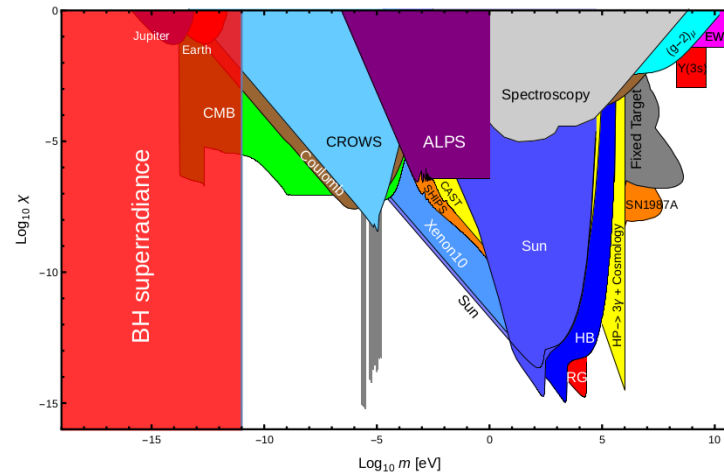
Self-gravity:

$$\rho_0 = 10^3 M_\odot \text{pc}^{-3} \sim 10^4 \text{GeV cm}^{-3}$$
$$\frac{M_{\text{inside r}}^{\text{DM}}}{M_{\text{BH}}} = 10^{-19} \left(\frac{M_{\text{BH}}}{10^6 M_\odot} \right)^2 \left(\frac{r}{100 M} \right)^3 \frac{\rho_{\text{DM}}}{\rho_0}$$

Accretion:

$$\dot{M}_{\text{BH}} = \frac{16\pi G^2 M_{\text{BH}}^2 \rho_{\text{DM}}}{v_{\text{DM}} c^2} \left(\dot{M} = \sigma \rho v \right)$$
$$\frac{\Delta M_{\text{BH}}}{M_{\text{BH}}} = 10^{-16} \left(\frac{M_{\text{BH}}}{10^6 M_\odot} \right) \frac{\rho_{\text{DM}}}{\rho_0} \frac{T}{1 \text{ year}} \left(\frac{\sigma_v}{220 \text{ Km/s}} \right)^{-1}$$

DM II. Light fields



Cardoso+ 2018, adapted from Sigl (2017) and Jaeckel arXiv:1303.1821

Interesting as effective description; proxy for more complex interactions;
arise as interesting extensions of GR* (*BD or generic ST theories, $f(R)$, etc*)

Bosons do exist (Higgs) and lighter versions may as well

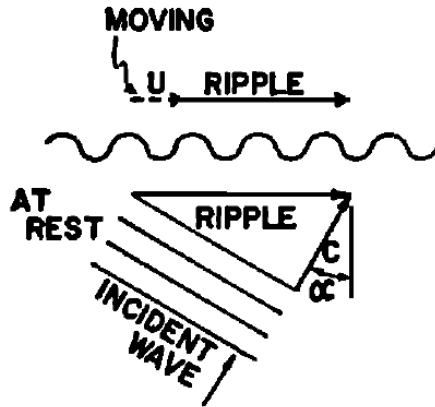
*Peccei-Quinn (interesting because not invented to solve DM problem),
axiverse (moduli and coupling constants in string theory)*

$$\mathcal{L} = \frac{R}{k} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi - \frac{\mu_S^2}{2} \Psi \Psi - \frac{k_{\text{axion}}}{2} \Psi * F^{\mu\nu} F_{\mu\nu}$$

...and one or more could be a component of DM. D. Marsh, Phys. Repts. 2016

Superradiance

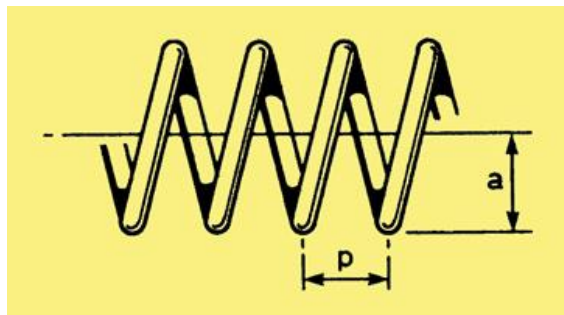
Zel'dovich JETP Lett. 14:180 (1971); Brito+ Lect. Notes Phys.906 (2015)



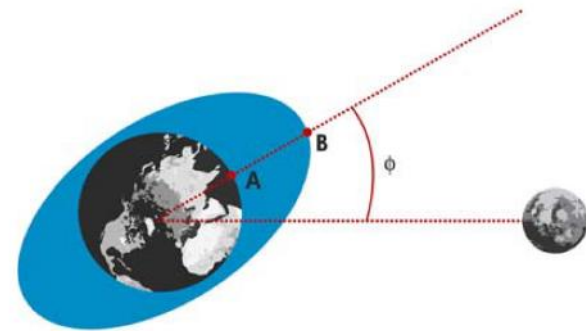
Ribner, J. Acous. Soc. Amer.29 (1957)



Tamm & Frank, Doklady AN SSSR 14 (1937)

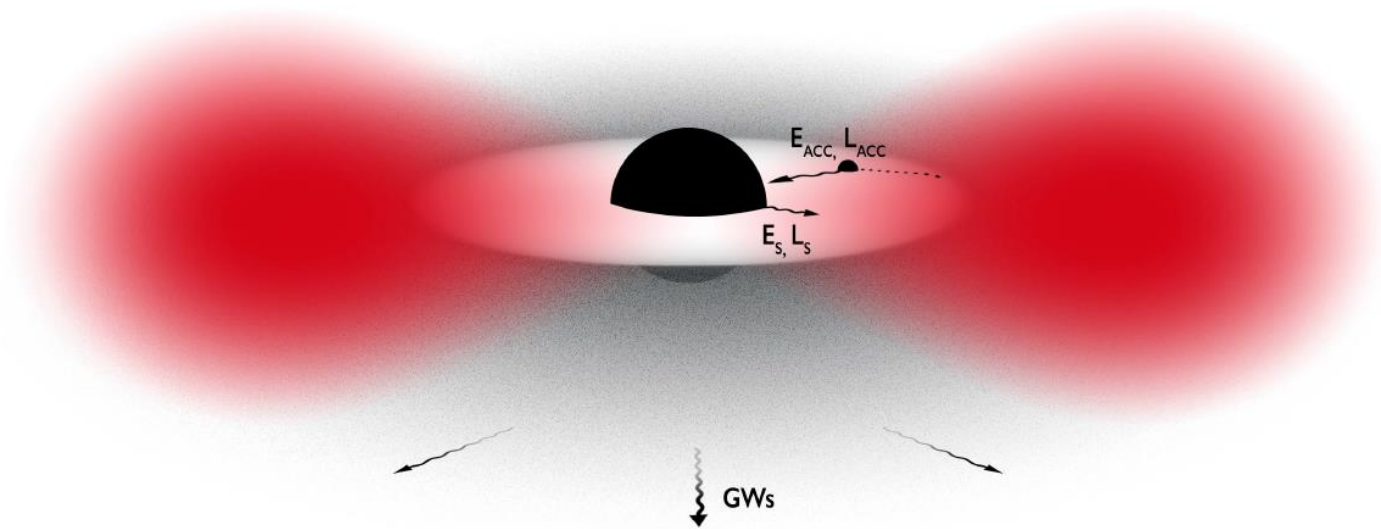


Pierce (& Kompfner), Bell Lab Series (1947)
Ginzburg, anomalous Doppler year



G. H. Darwin, Philos. Trans. R. Soc. London 171 (1880)

Fundamental fields: bounding the boson mass



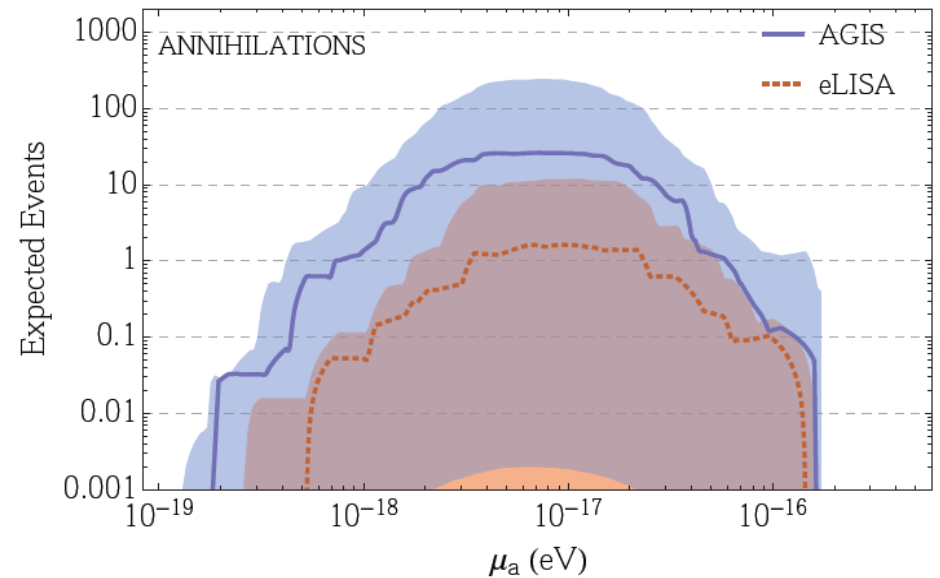
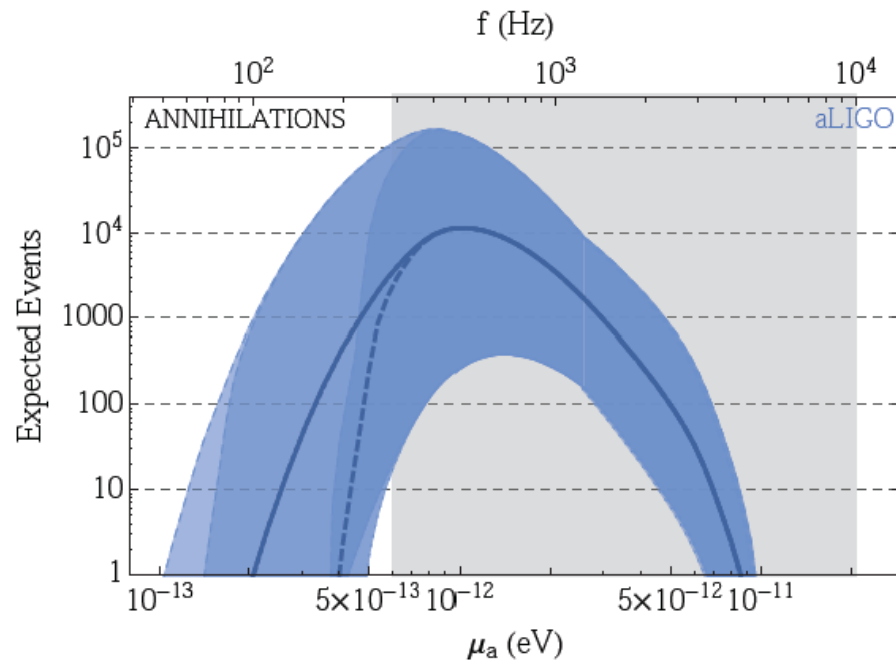
© a.s./grit

$$\tau \sim 100 \left(\frac{10^6 M_{\odot}}{M} \right)^8 \left(\frac{10^{-16} \text{eV}}{\mu} \right)^9 \text{ seconds}$$

Wonderful sources of GWs

Brito, Cardoso, Pani, Lecture Notes Physics 906: 1-237 (2015)

Wonderful sources for different GW-detectors



Arvanitaki+ PRD91:084011 (2015)

Brito+CQG32:134001 (2015); Lect.Notes Physics 906 (2015)

Wonderful sources for different GW-detectors

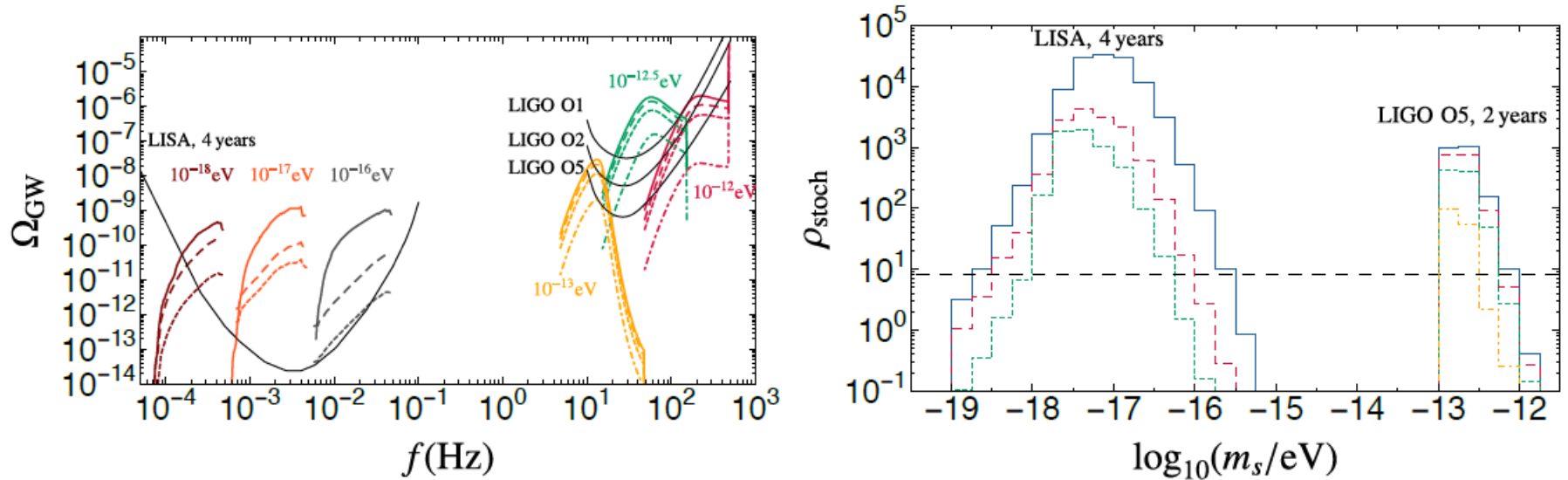
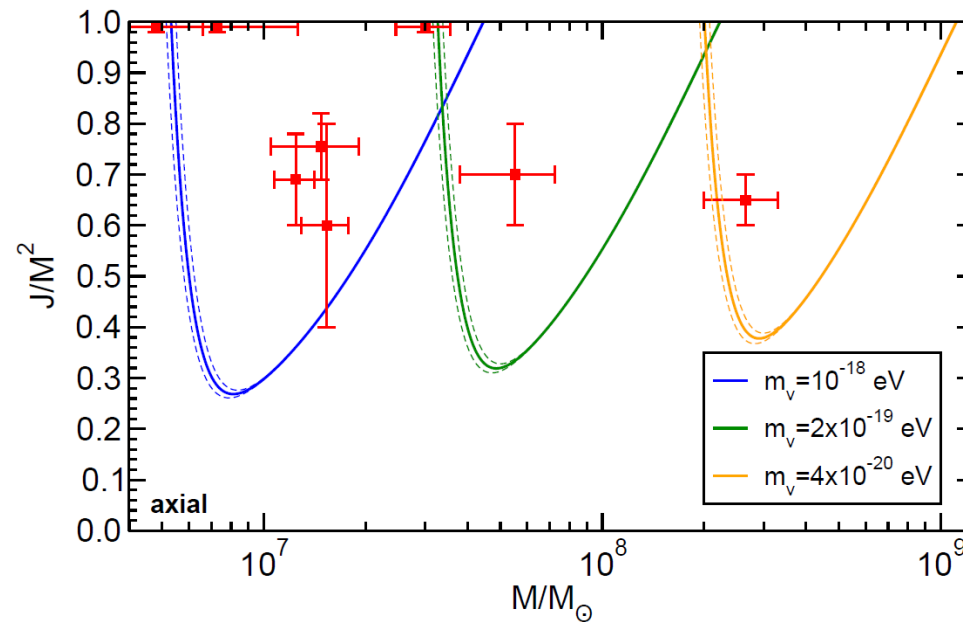


FIG. 2. Left panel: stochastic background in the LIGO and LISA bands. For LISA, the three different signals correspond to the “optimistic” (top), “less optimistic” (middle) and “pessimistic” (bottom) astrophysical models. For LIGO, the different spectra for each scalar field mass correspond to a uniform spin distribution with (from top to bottom) $\chi_i \in [0.8, 1]$, $[0.5, 1]$, $[0, 1]$ and $[0, 0.5]$. The black lines are the power-law integrated curves of Ref. [61], computed using noise PSDs for LISA [9], LIGO’s first two observing runs (O1 and O2), and LIGO at design sensitivity (O5) [62]. By definition, $\rho_{\text{stoch}} \geq 1$ when a power-law spectrum intersects one of the power-law integrated curves. Right panel: ρ_{stoch} for the backgrounds shown in the left panel. We assumed $T_{\text{obs}} = 2$ yr for LIGO and $T_{\text{obs}} = 4$ yr for LISA.

Bounding the boson mass with EM observations

Pani + PRL109, 131102 (2012)



Bound on photon mass is model-dependent: details of accretion disks or intergalactic matter are important... but gravitons interact very weakly!

$$m_g < 5 \times 10^{-23} \text{ eV}$$

Brito + PRD88:023514 (2013); Review of Particle Physics 2014

Couplings and EM bursts

Consider couplings to SM

$$\mathcal{L} = \frac{R}{k} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi - \frac{\mu_S^2}{2} \Psi \Psi - \frac{k_{\text{axion}}}{2} \Psi * F^{\mu\nu} F_{\mu\nu}$$

$$\Psi \sim \Psi_0 e^{-i\mu_S t}, \quad A_\mu \sim y(t) e^{ip_\mu x^\mu}$$

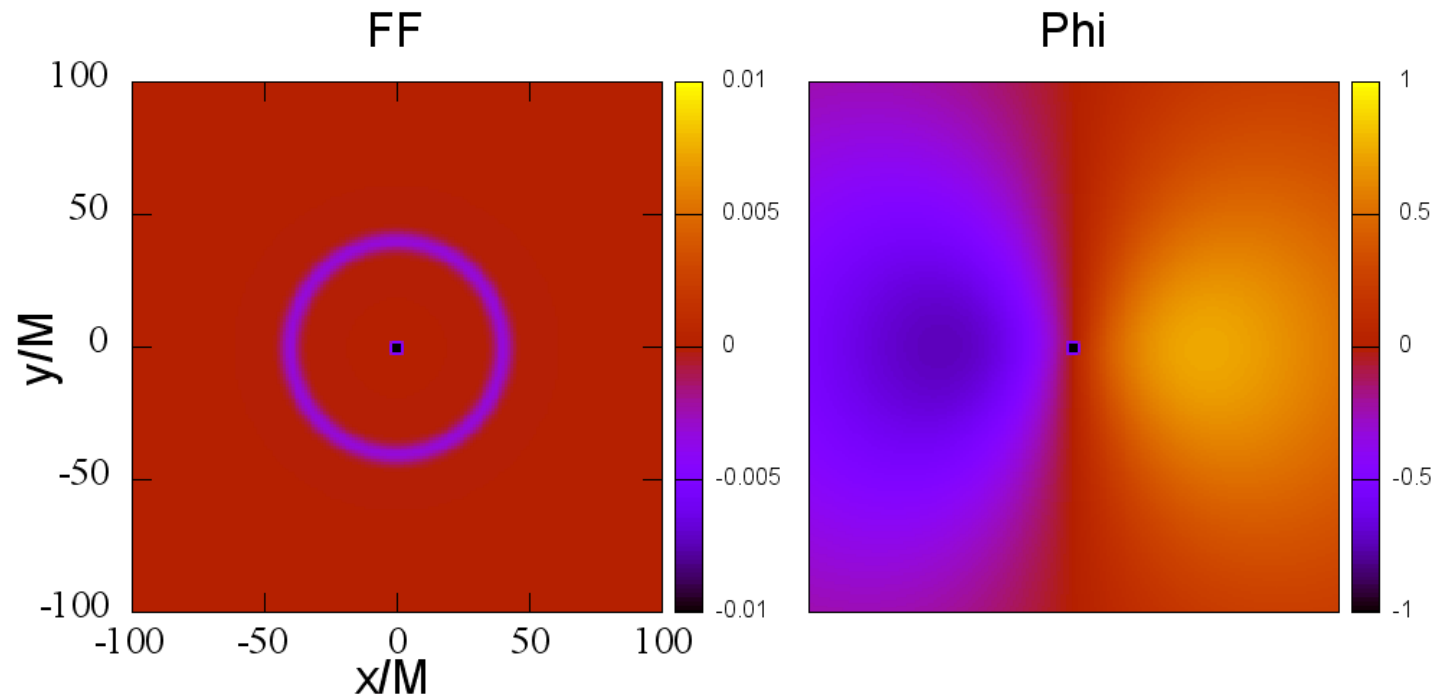
$$\frac{\partial^2 y}{\partial t^2} + (\omega^2 - 2\mu_S \Psi_0 k_{\text{axion}} |p| \cos \mu_S t) y = 0$$

Blasts of EM, laser-like radiation for

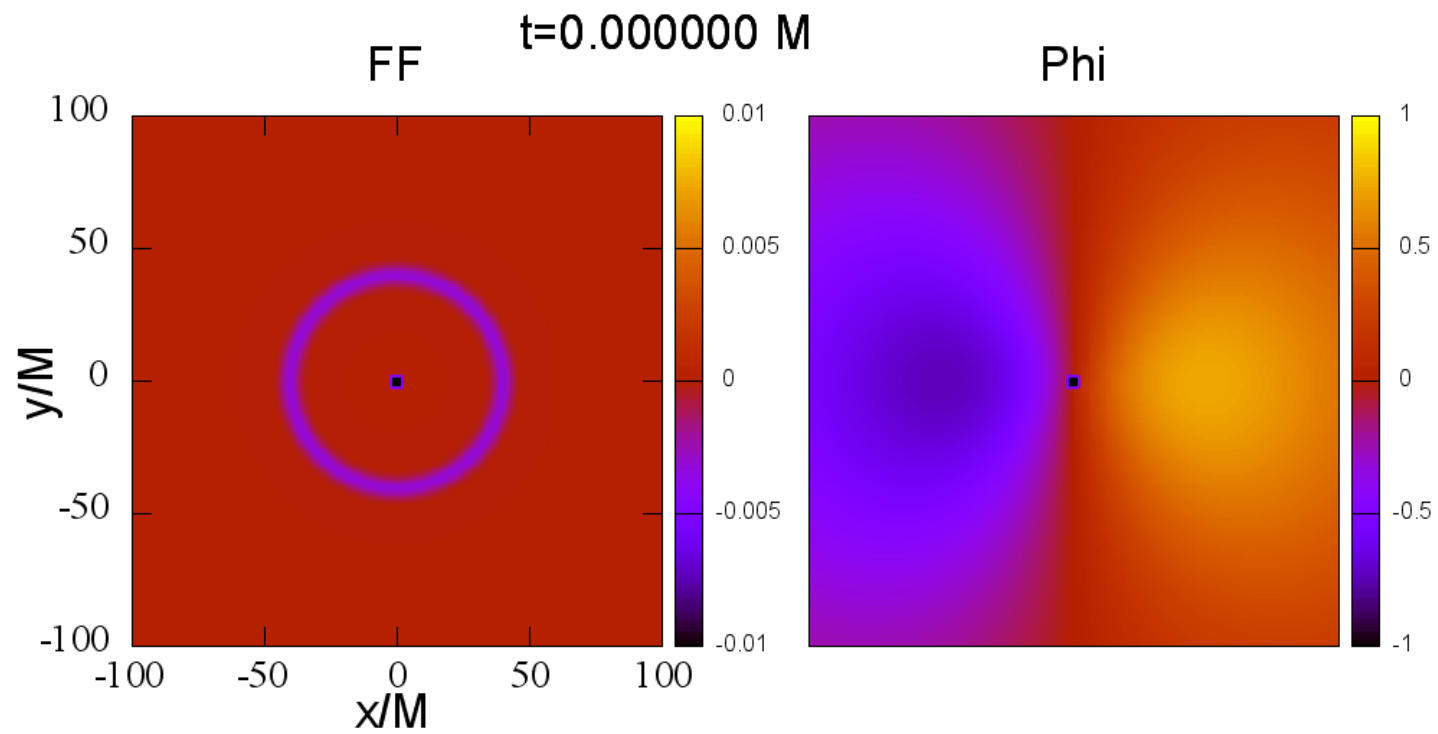
$$\mu > 10^{-8} \text{eV}, M < 0.01 M_{\text{sun}}$$

Boskovic+ PRD99:035006 (2019); Ikeda+ PRL122:081101 (2019)

$t=0.000000\ M$

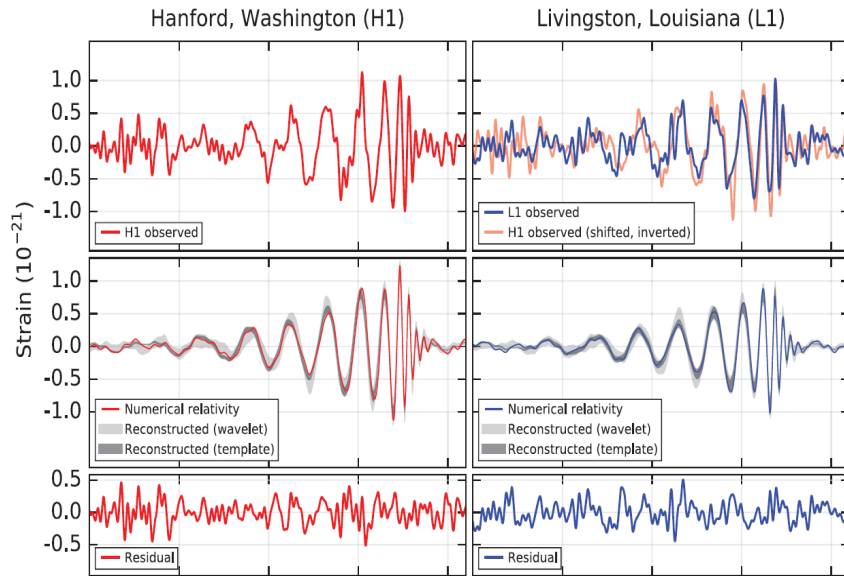


Ikeda, Brito and Cardoso PRL122: 081101 (2019)



Ikeda, Brito and Cardoso PRL122: 081101 (2019)

The nature of dark compact objects



$$f_{GW}^{-8/3}(t) = \frac{(8\pi)^{8/3}}{5} \left(\frac{G\mathcal{M}}{c^3} \right)^{5/3} (t_0 - t)$$
$$\mathcal{M} = (\mu^3 M^2)^{1/5}$$

Two unknowns, need frequency at two instants. Result: $M \sim 65$ suns

Use Kepler's law, separation at collision is ~ 500 Km... same using ringdown...

Massive, compact object indeed!

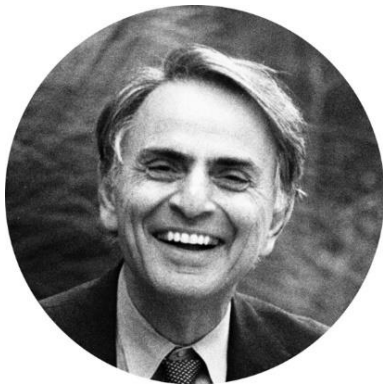
Why is this enough?

BHs are end-point of gravitational collapse, using EoS thought to prevail.

No other massive, dark object has been seen to arise from collapse of known matter.

Why is this not enough?

1. BH exterior is pathology-free, interior is not. Cosmic Censorship extraordinary claim.
2. Quantum effects not fully understood. Non-locality to solve information paradox? Hard-surface to quantize BH area (Bekenstein & Mukhanov 1995); Perhaps classical structures just coarse-grained descriptions of horizonless geometries (“fuzzballs”, Mathur 2005)
3. Tacitly assumed quantum effects at Planck scales. Planck scale could be significantly lower (Arkani-Hamed+ 1998; Giddings & Thomas 2002). Even if not, many orders of magnitude standing, surprises can hide (Bekenstein & Mukhanov 1995).



“Extraordinary claims require extraordinary evidence.”
Carl Sagan

4. *Dark matter exists, and interacts gravitationally. Are there compact DM clumps?*
5. Physics is experimental science. We can test exterior. Aim to quantify evidence for horizons. Similar to quantifying equivalence principle.

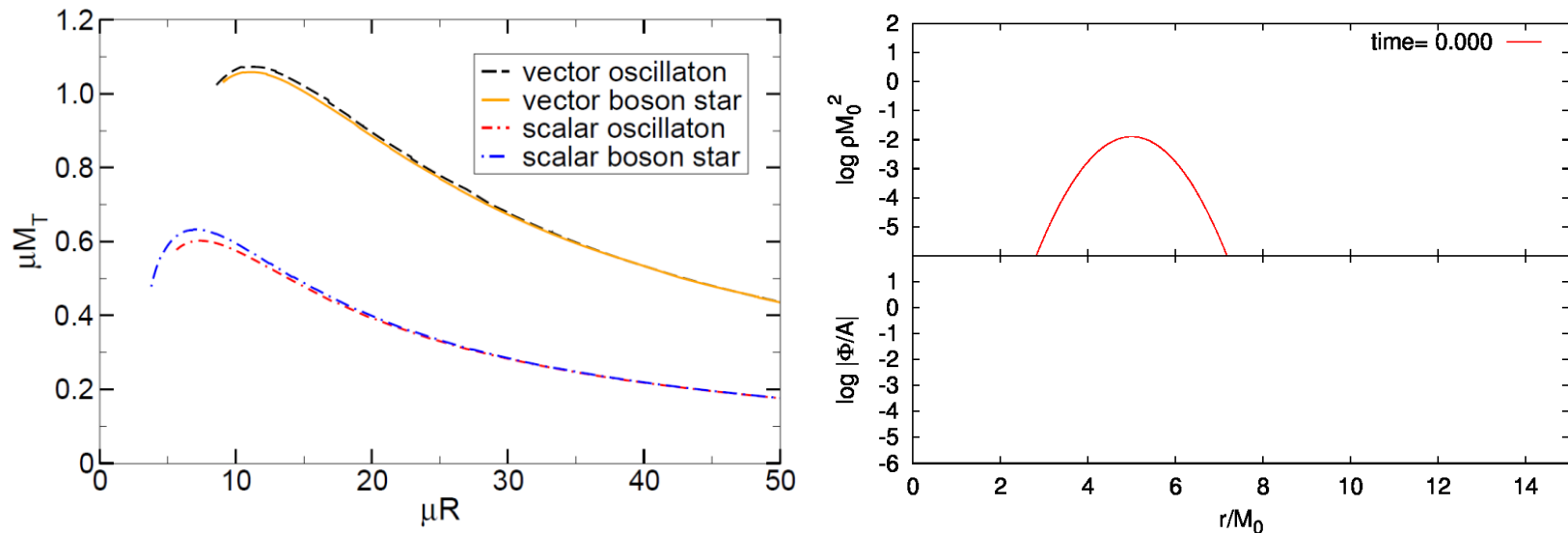
Some challenges

- i. Are there alternatives?
- ii. Do they form dynamically under reasonable conditions?
- iii. Are they stable?
- iv. How do they look like? Is GW or EM signal similar to BHs?
- v. Observationally, how close do we get to horizons?

II. Formation

Boson stars, fermion-boson stars, oscillatons

(Kaup '68; Ruffini, Bonazzolla '69; Colpi+ 1986; Tkachev '91; Okawa+ 2014; Brito+ 2015)



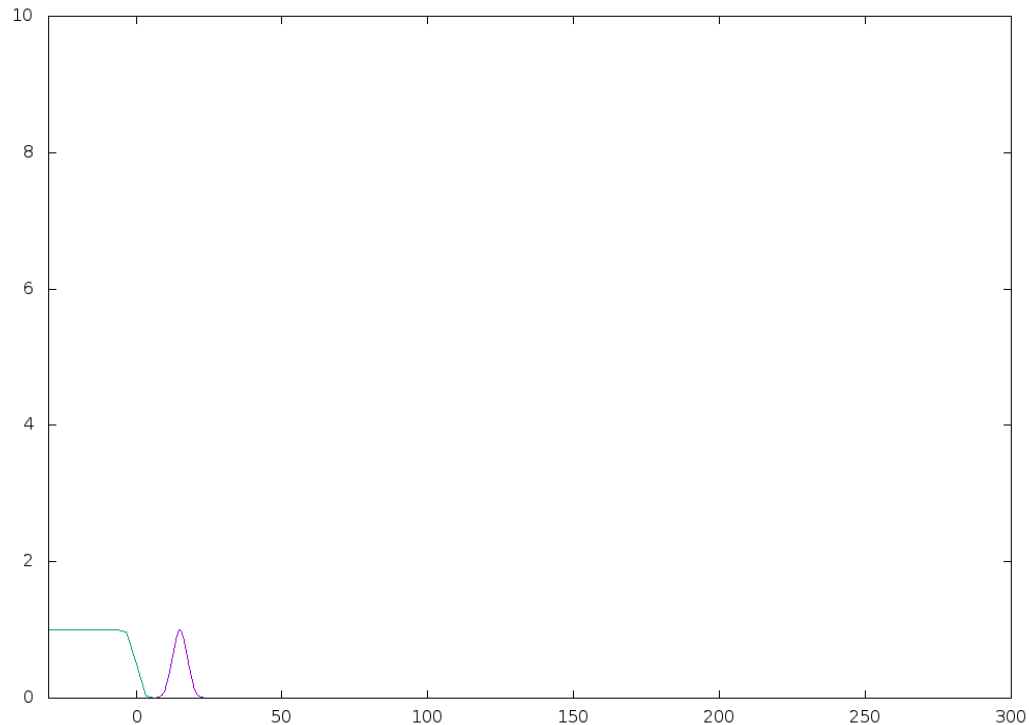
$$\frac{M_{\max}}{M_{\odot}} = 8 \times 10^{-11} \frac{\text{eV}}{m_B c^2}$$

Challenge: repeat for anisotropic stars, wormholes, gravastars, etc

IIIa. Stability of objects with ergoregions

AS flat, horizonless spacetimes with ergoregions are linearly unstable

Friedmann Comm. Math. Phys. 63:243 (1978); Moschidis Comm. Math. Phys. 358: 437 (2016)



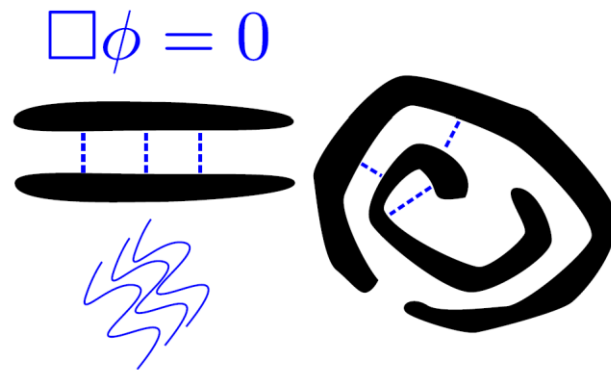
Vicente & Cardoso PRD97:084032 (2018); Brito+ Lect. Notes Phys 906 (2015)

IIIb. Stability of objects with photospheres

Static objects: *No uniform decay estimate with faster than logarithmic decay can hold for axial perturbations of ultracompact objects.*

Keir CQG33: 135009 (2016); Cardoso + PRD90:044069 (2014)

$$\mathcal{E}_{\text{local}}^{(N)}(t) \lesssim \frac{1}{(\log(2+t))^2} \mathcal{E}_{(2)}^{(N)}(0)$$



Burq, Acta Mathematica 180: 1 (1998)

IV a. EM constraints

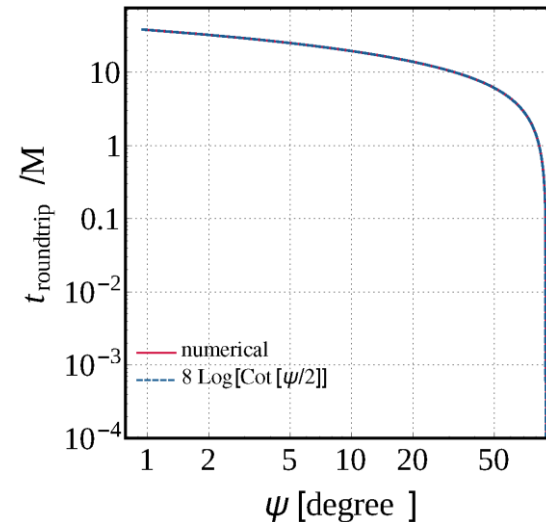
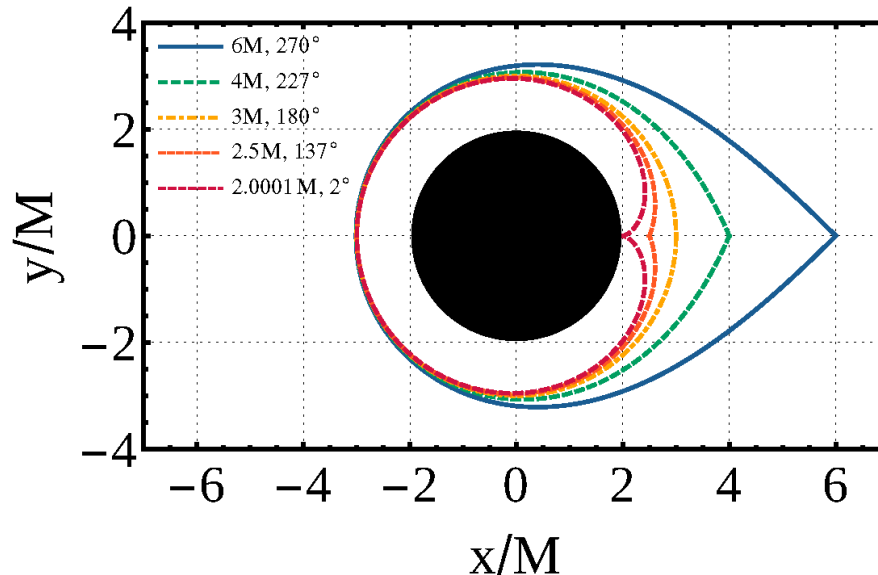
$$r = 2M(1 + \epsilon)$$
$$\epsilon \lesssim 10^{-5}$$
$$\epsilon \lesssim 10^{-35}$$

Absence of transients from tidal disruptions

Dark central spot on SgrA

Carballo-Rúbio, Kumar, PRD97:123012 (2018)

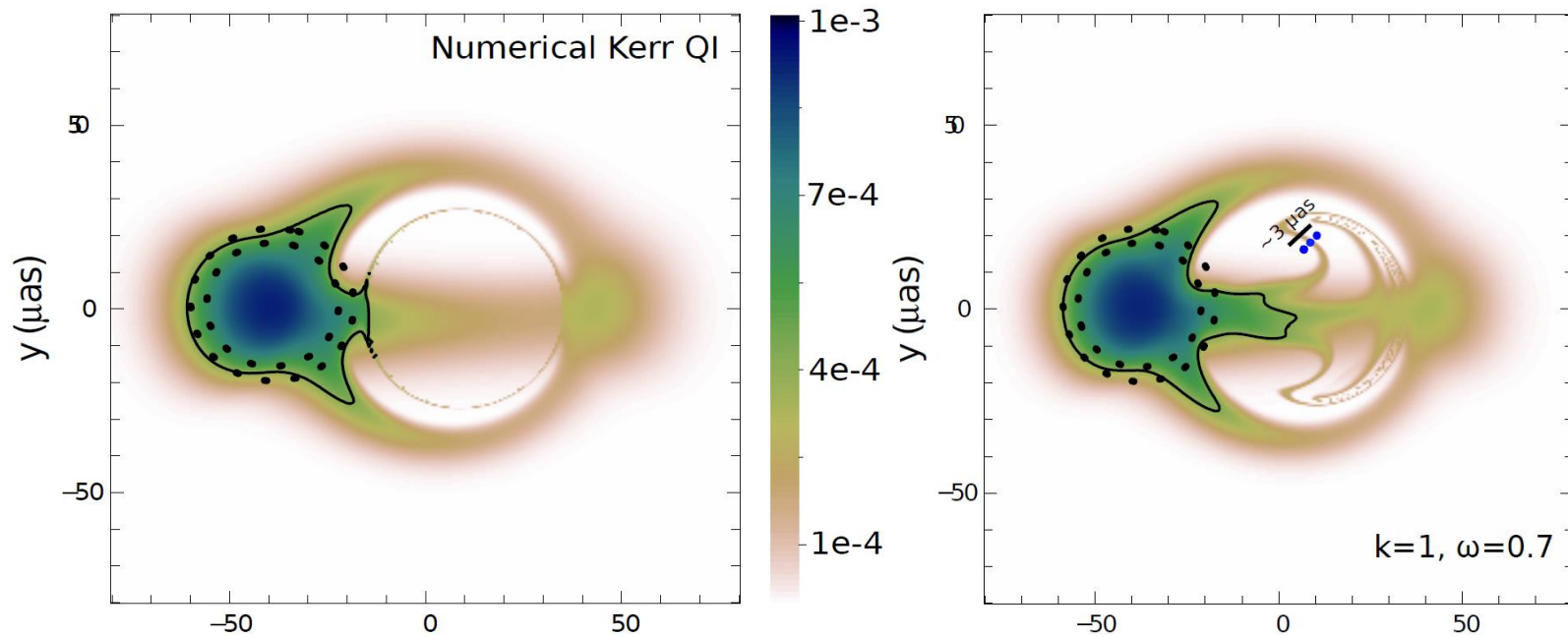
Broderick, Narayan CQG24:659 (2007)



Lensing has to be properly included, as well as emission into other channels

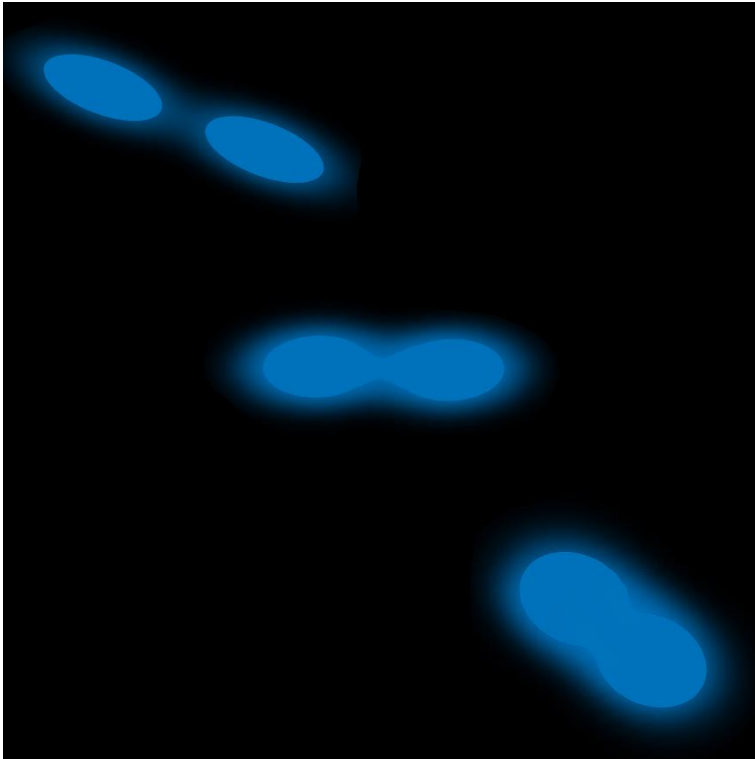
Abramowicz, Kluzniak, Lasota 2002; Cardoso, Pani Nature Astronomy 1 (2017)

shadows



Vincent+ CQG 33:105015 (2016)

IV b. GW signal



Nature of inspiralling objects is encoded

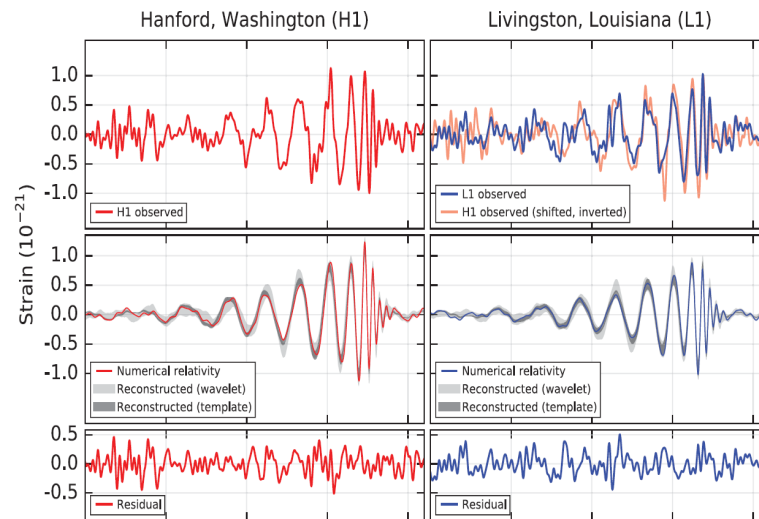
(i) in way they respond to own field
(multipolar structure)

(ii) in way they respond when acted upon
by external field of companion – through
their tidal Love numbers (TLNs), and

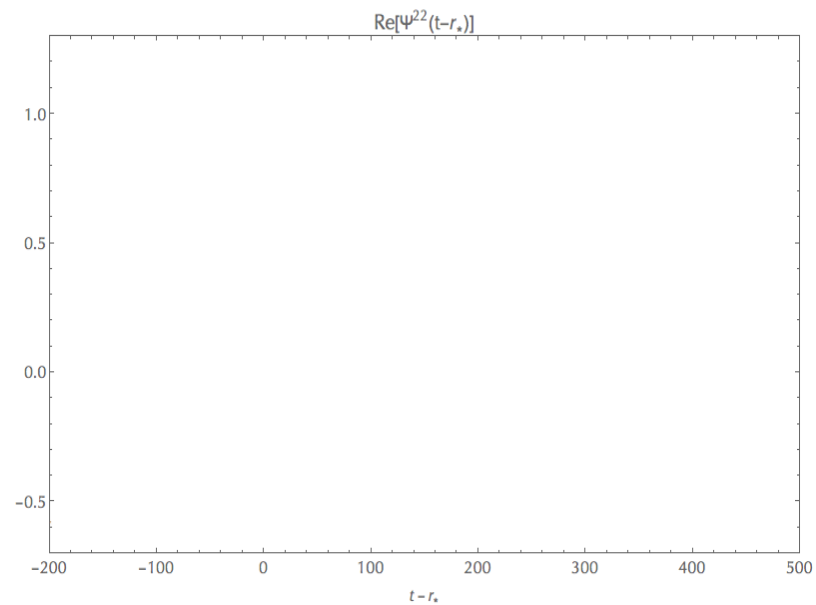
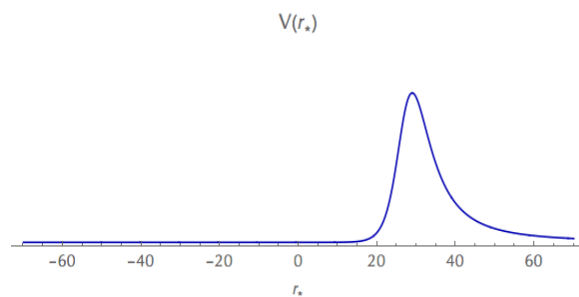
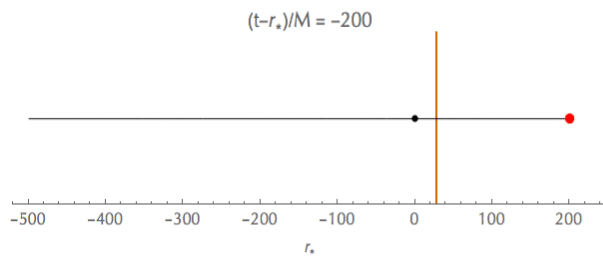
(iii) on amount of radiation absorbed, i.e.,
tidal heating

$$\tilde{h}(f) = \mathcal{A}(f)e^{i(\psi_{\text{PP}} + \psi_{\text{TH}} + \psi_{\text{TD}})}$$

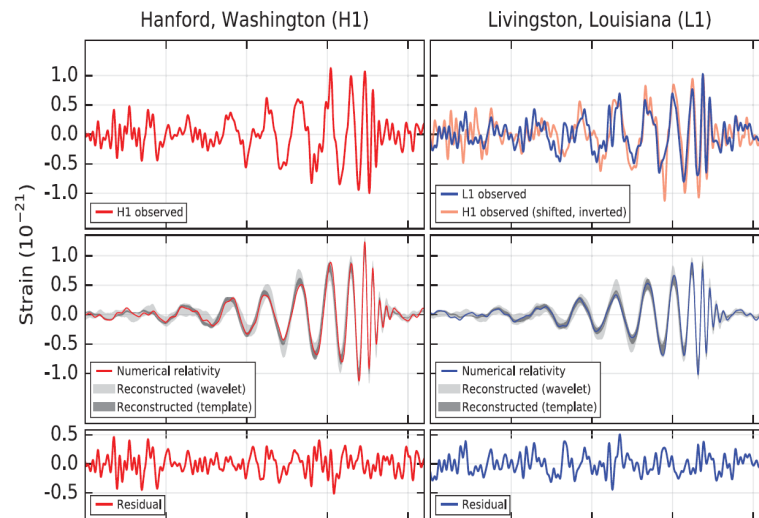
Post-merger



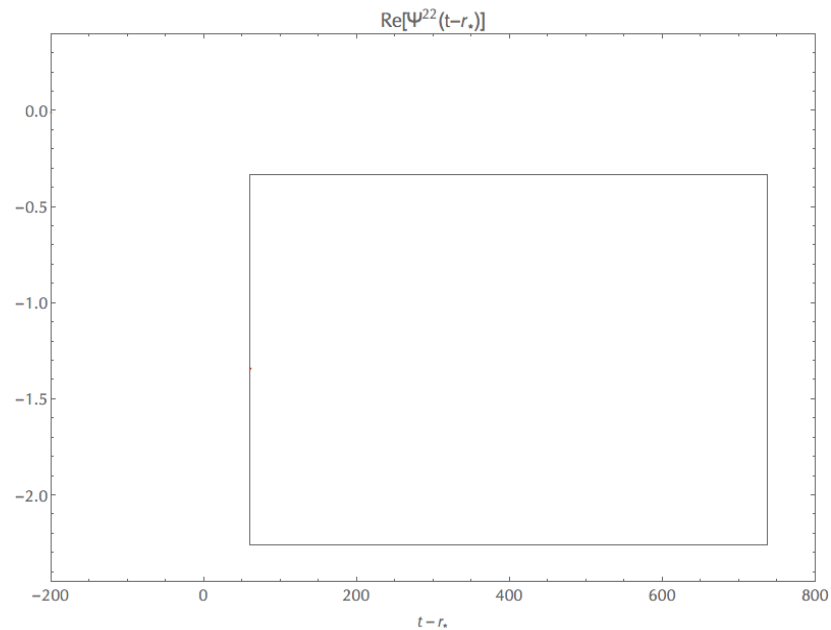
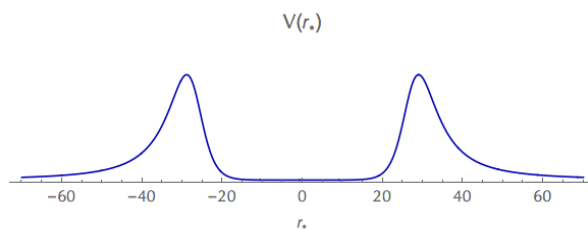
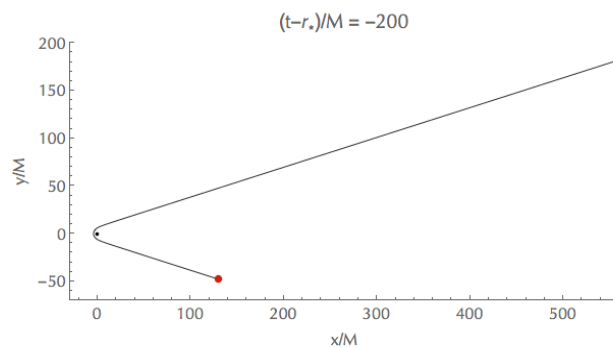
$$\mathcal{E} = 1.5, \mathcal{J} = 0$$



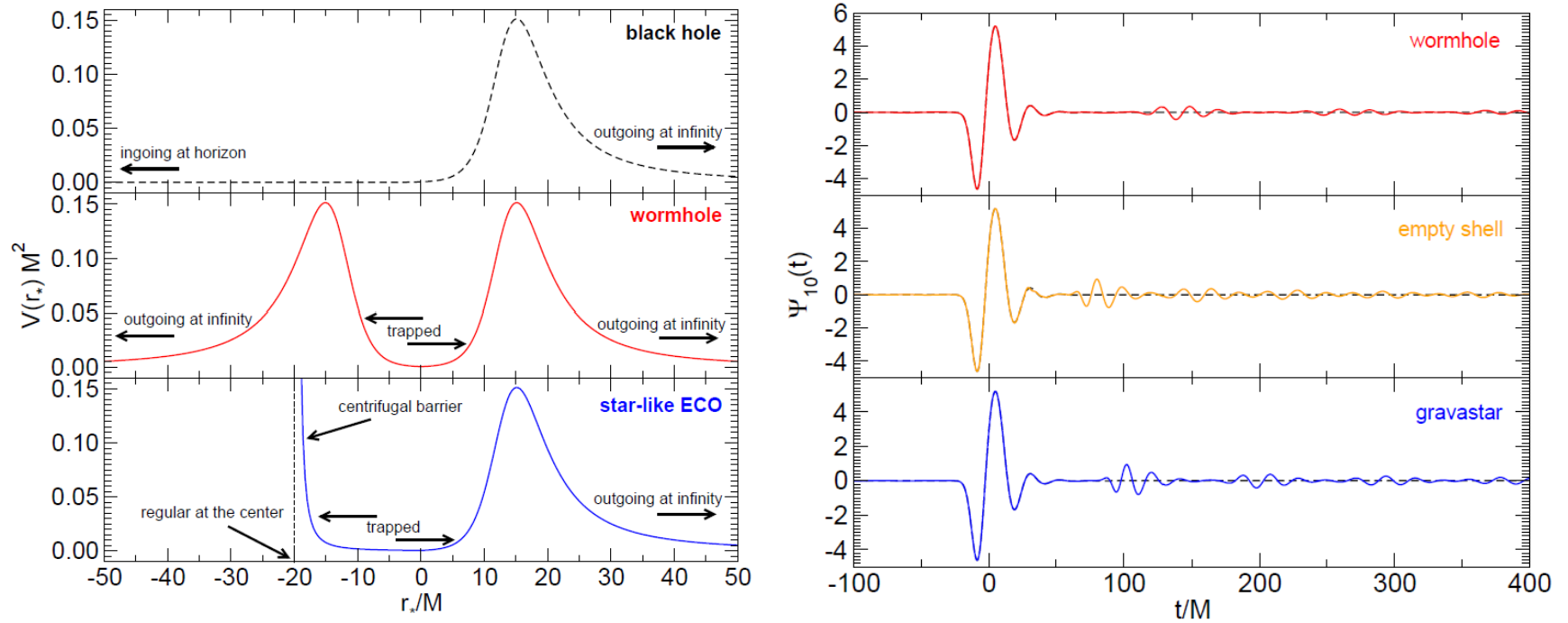
Scattering echoes



$$\mathcal{E} = 1.5, r_{\min}=4.3M, r_0-2M = 10^{-6}M$$



Echoes



Cardoso + PRL116:171101 (2016); Nature Astronomy 1: 2017; Living Reviews in Relativity 22:1 (2019)

The evidence for black holes

Cardoso and Pani, Living Reviews in Relativity 22: 1 (2019); *arXiv:1904.05363*

	Constraints		Source	
	$\epsilon(\lesssim)$	$\frac{\nu}{\nu_\infty}(\gtrsim)$		
1a.	$\mathcal{O}(1)$	$\mathcal{O}(1)$	Sgr A* & M87	ISCO and light ring Merger frequency
1b.	0.74	1.5	GW150914	
2.	$\mathcal{O}(0.01)$	$\mathcal{O}(10)$	GW150914	Ringdown consistency
4.	10^{-14}	10^7	Sgr A*	Low relative luminosity of SgrA
6.	10^{-47}	10^{23}	GW150914	Absence of echoes
7*.	$e^{-10^4/\zeta}$	$e^{5000/\zeta}$	EMRIs	Projected constraints on spin-induced quadrupole and TLNs

Plenty of caveats, but enormous potential

Open questions

Why are photosphere modes not in spectrum?

What is amplitude of QNM excitation, are power-law tails excited to observable levels?

What are EM signals of ultracompact objects?

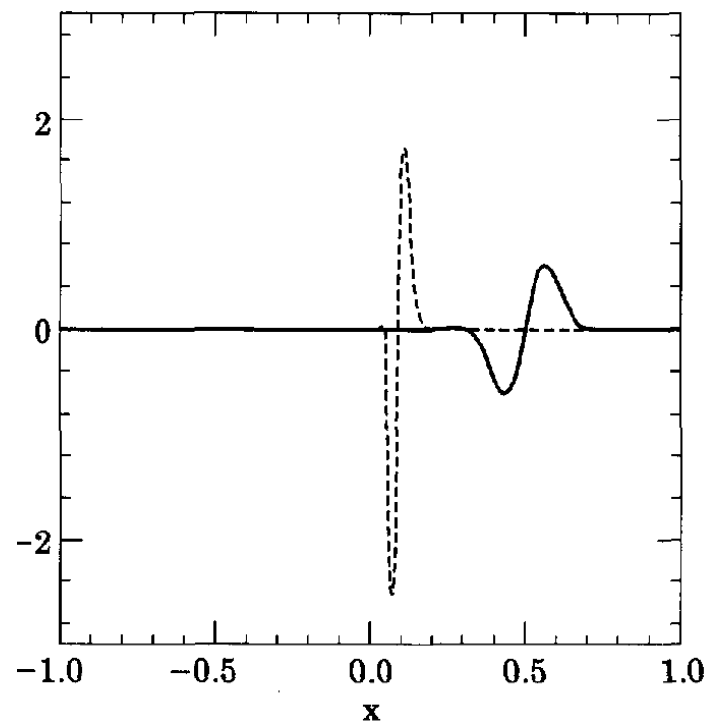
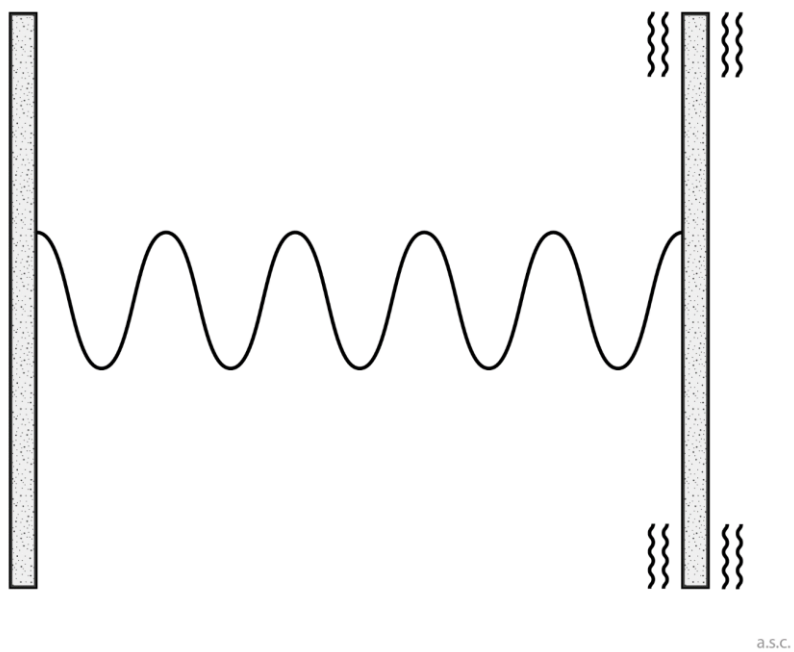
Can we build “reasonable” ultracompact objects?

What are generic consequences of resolving singularities?
Is any of this affecting horizons?

Energy extraction from black hole binaries?

Superradiance, if BHs spin; Ergoregions in binaries; Slingshot extraction

Parametric resonance? Fermi-like?



Cooper IEEE Trans. Ant. Propag. 1993

Conclusions: exciting times!

Gravitational wave astronomy *can* become a precision discipline, mapping compact objects throughout the entire visible universe.

Black holes remain the most outstanding object in the universe. BH spectroscopy will allow to test GR and provide strong evidence for the presence of horizons... improved sensitivity pushes putative surface closer to horizon, like probing short-distance structure with accelerators. BHs can play the role of perfect laboratories for particle physics, or high energy physics

“The excitement of the next generation of astronomical facilities is not in the old questions which will be answered, but in the new questions that they will raise.”

K. I. Kellermann + “The exploration of the unknown”

Thank you



GWs and dark matter I

DM is not a strong-field phenomenon, but GW observations may reveal a more “mundane” explanation in terms of heavy BHs

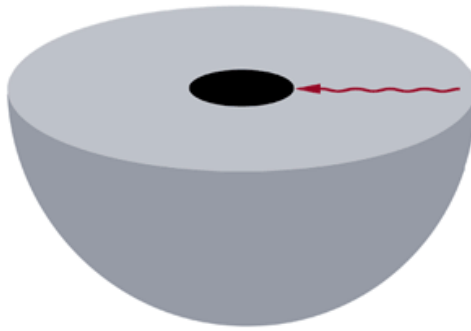
Bird + PRL116:201301 (2016)

II

Inspiral occurs in DM rich environment and may modify inspiral rate, given dense-enough media: accretion and drag play important role.

Eda + PRL110:221101 (2013); Macedo + ApJ774:48 (2013)

Bombs and superradiant instabilities



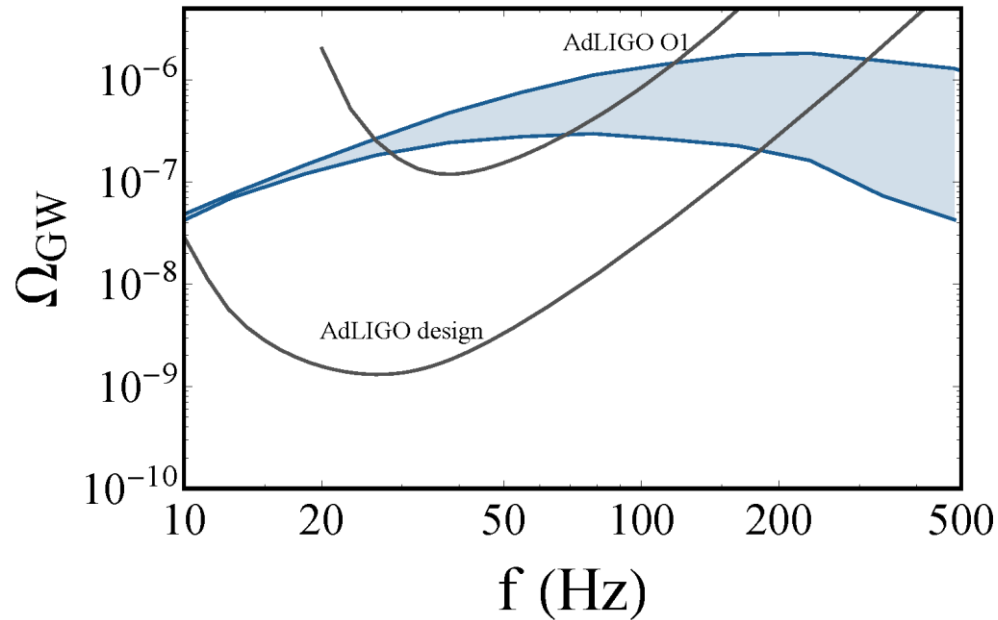
© A.S. Dybko

$$\tau \sim 100 \left(\frac{10^6 M_\odot}{M} \right)^8 \left(\frac{10^{-16} \text{eV}}{\mu} \right)^9 \text{ seconds}$$

Massive “states” around Kerr are linearly unstable

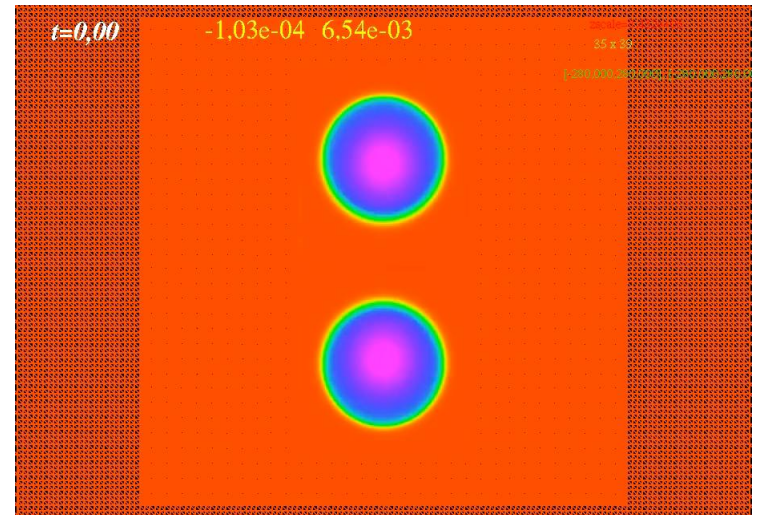
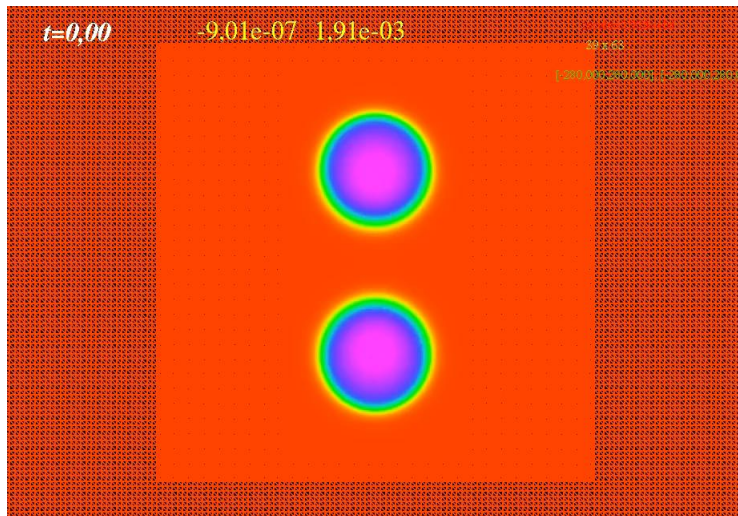
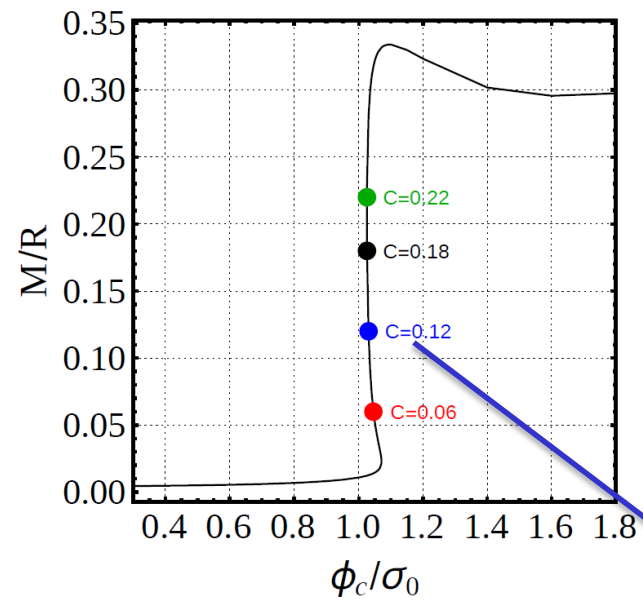
See review Brito, Cardoso, Pani, Lect. Notes Phys. 906: 1 (2015); arXiv:1501.06570

Stochastic background of GWs

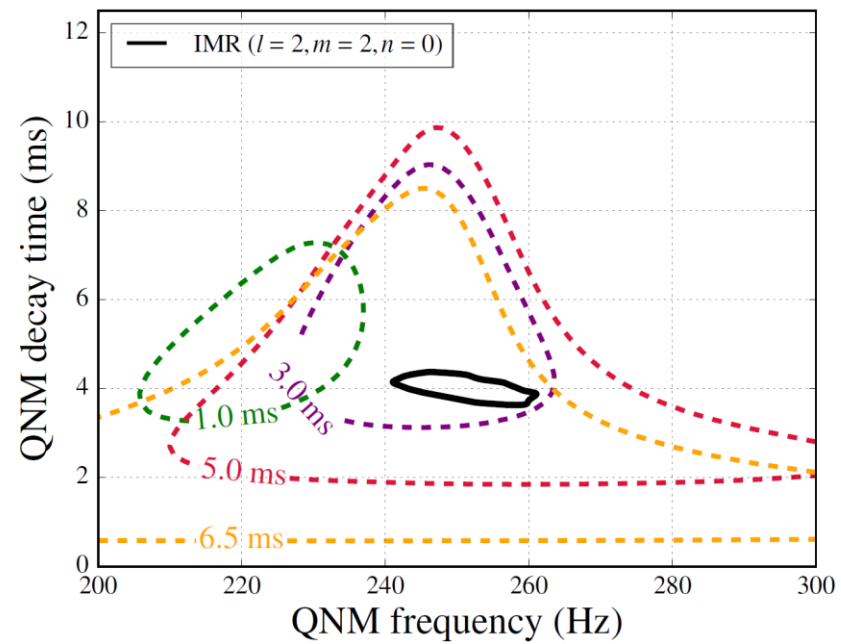
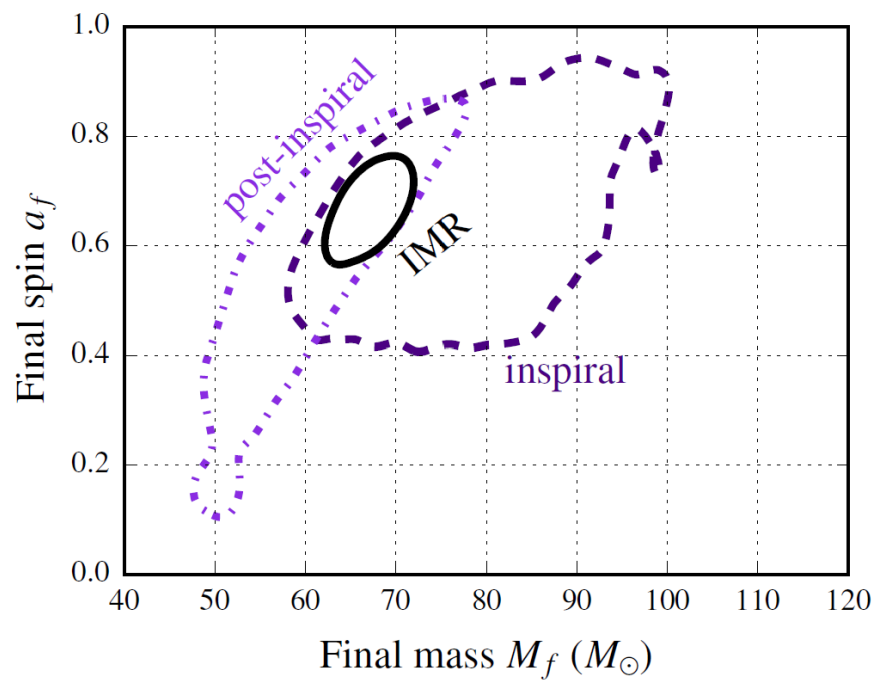


Blue bands bracket population models, from optimistic to pessimistic

Barausse+ CQG35:20LT01 (2018)



Palenzuela+ PRD96:104058(2017)

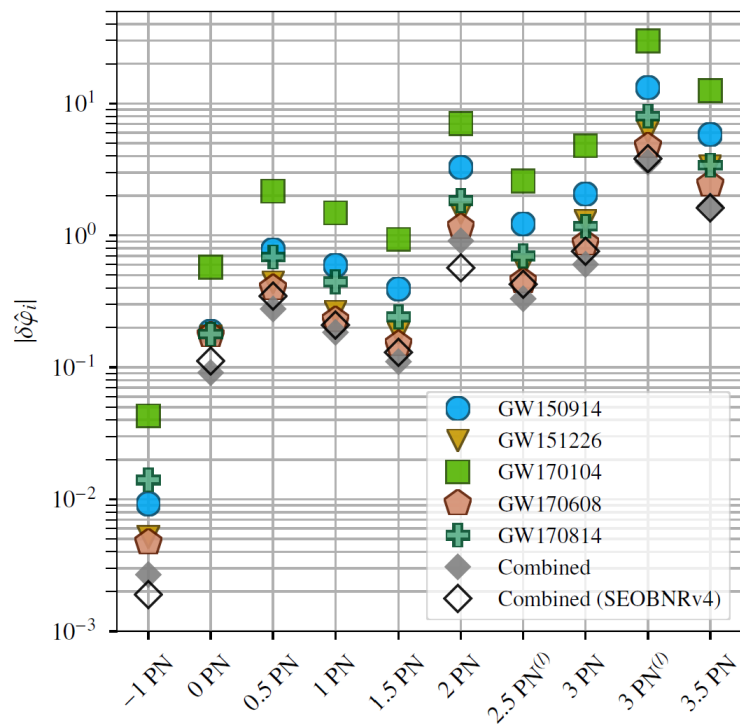


Parametrized tests

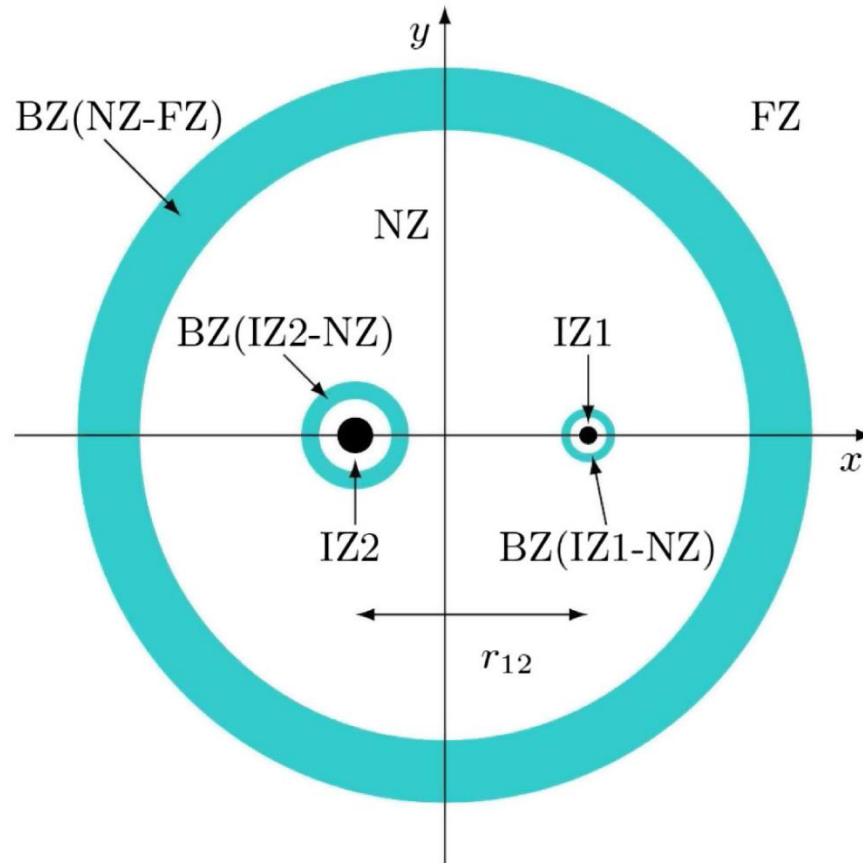
$$h(f, \text{pars}) = A(f, \text{pars}) e^{i\Psi(f, \text{pars})}$$

$$\Psi_{SPA} = \frac{3}{128(\pi \mathcal{M} f)^{5/2}} (1 + \alpha_{1PN} x + \dots + \alpha_{5PN} x^5 + \dots)$$

$$x = (\pi M f)^{2/3}, \quad M = m_1 + m_2, \quad \nu = m_1 m_2 / M^2, \quad \mathcal{M} = \nu^{3/5} M$$



Gravitational molecules: a real BH binary



Mundim+ PRD89: 084008 (2014); Bernard + PRD100: 044002 (2019)