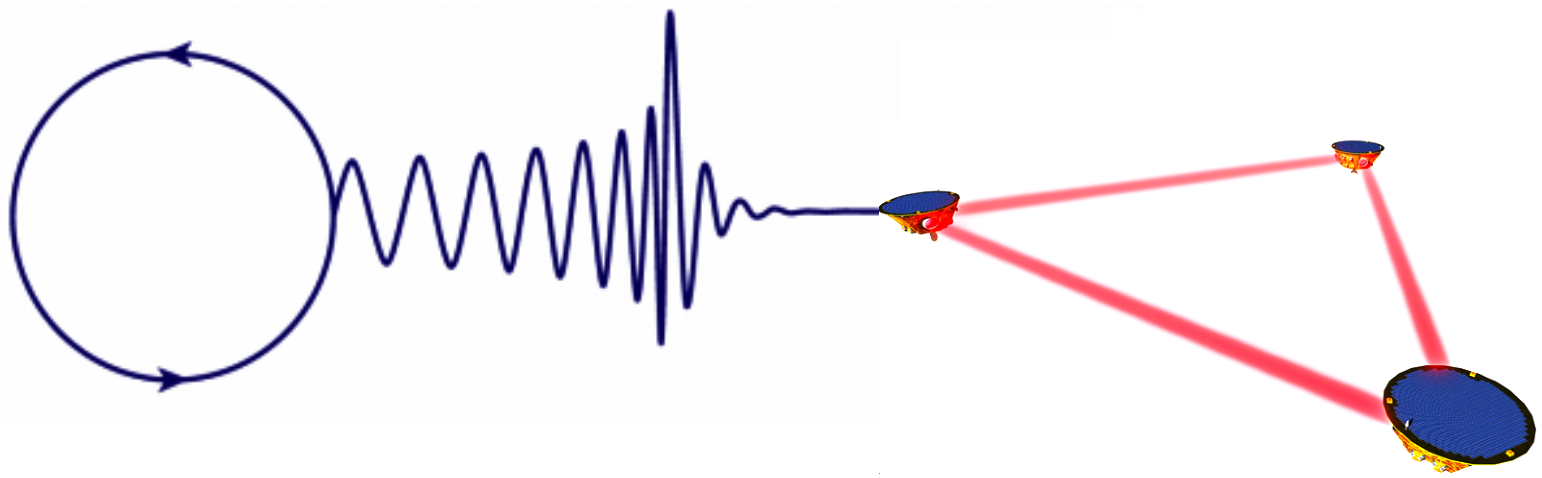
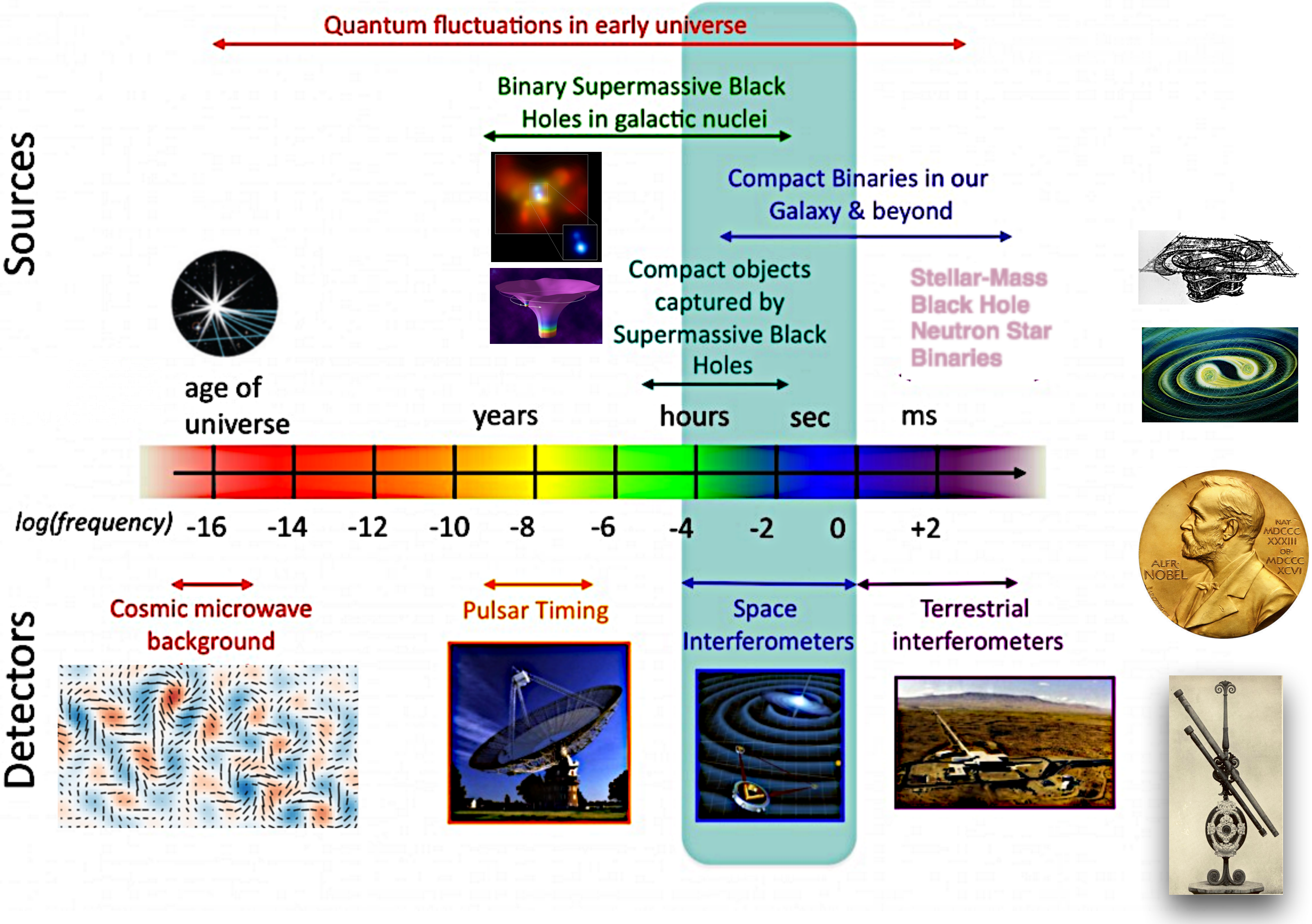
Precision Gravity: LHC to LISA

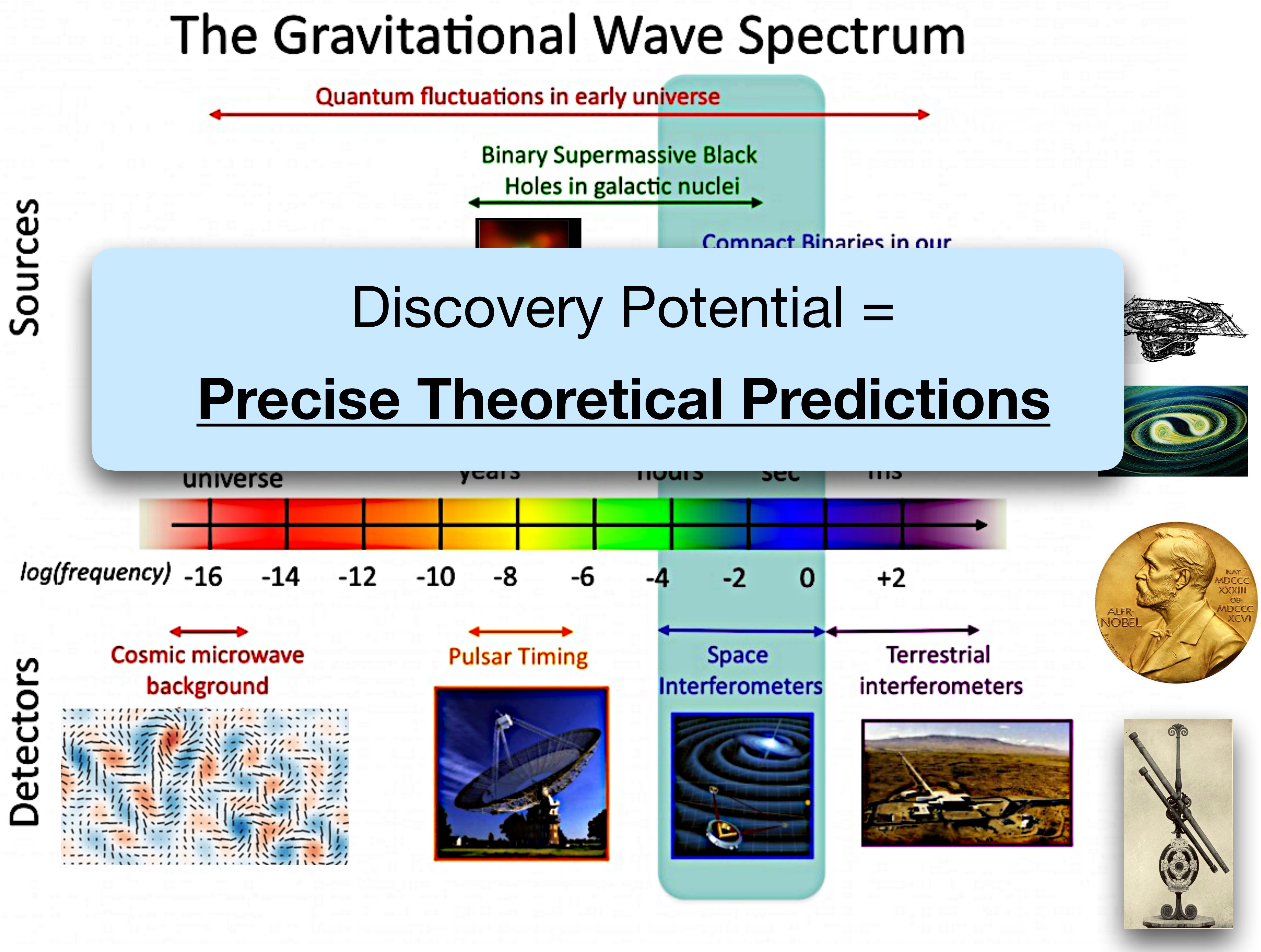


Rafael A. Porto

DESY. ICTP

The Gravitational Wave Spectrum

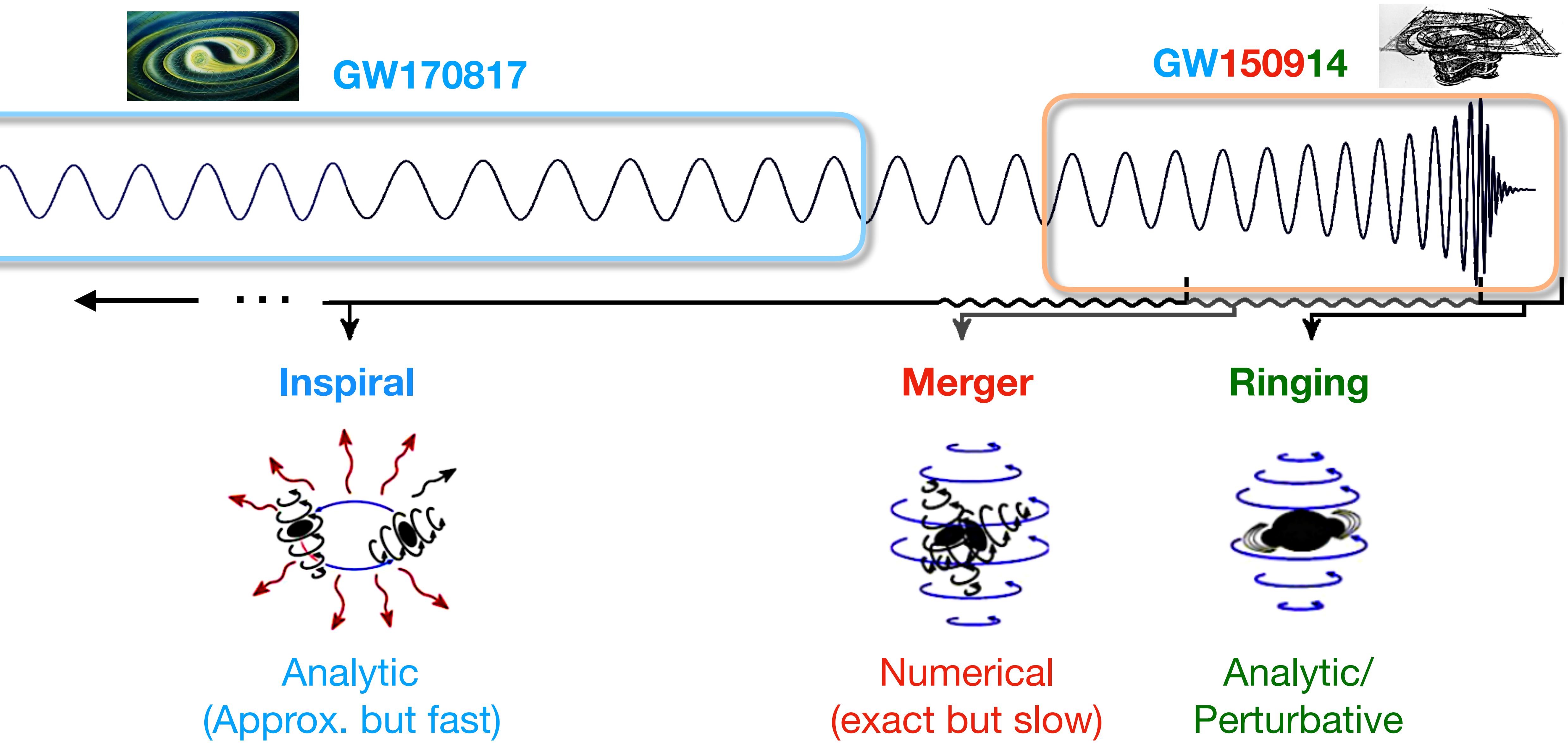




Discovery Potential =

Precise Theoretical Predictions

'GW Precision Data' (**GWPD**) *TM*

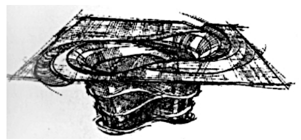


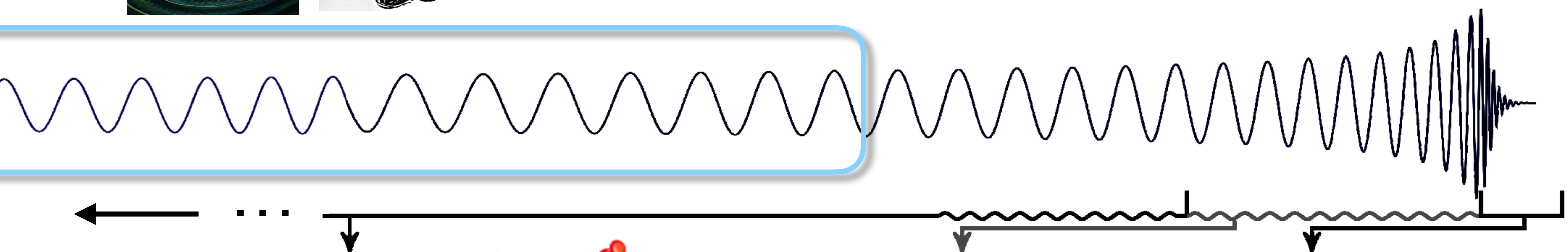
$$R_{im} = \sum_{l} \frac{\partial \Gamma^{l}_{im}}{\partial x_{l}} + \sum_{\rho l} \Gamma^{\rho}_{il} \Gamma^{l}_{m\rho} = -\kappa \left(T_{im} - \frac{1}{2} g_{im} T \right)$$

'GW Precision Data' (**GWPD**) *TM*

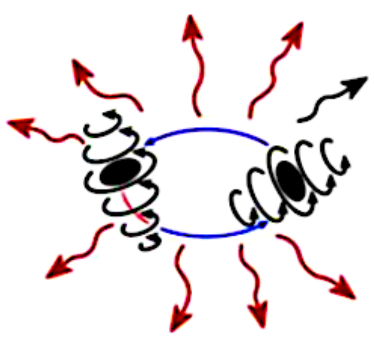
1000+ cycles in band @ Design-Sensitivity

100+ events per year!





Inspiral



Merger

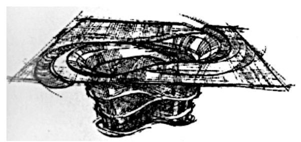
Ringing

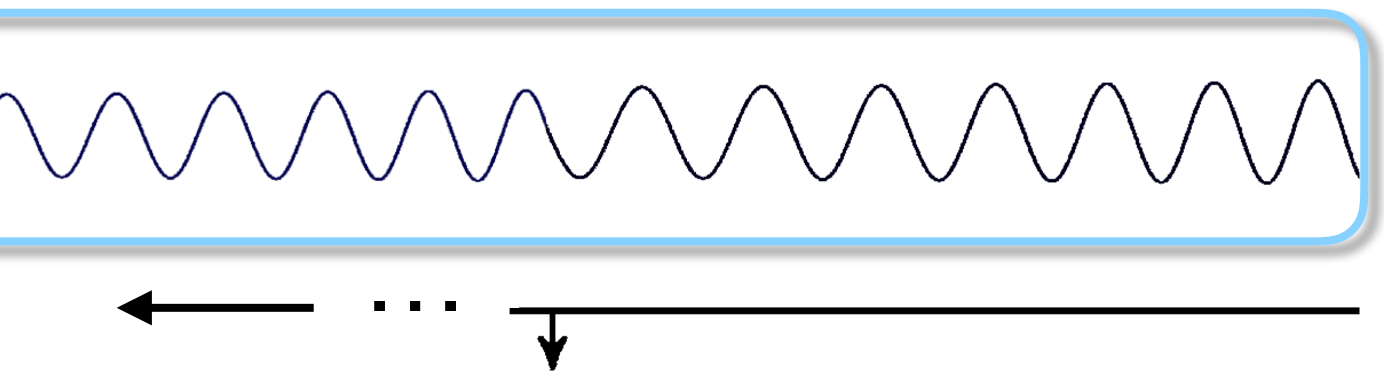
Post-Newtonian

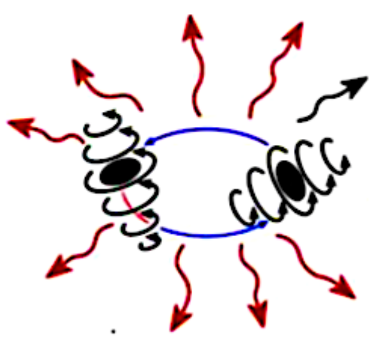
$$Gm \ll r \ll \lambda \sim r/v$$

$$R_{im} = \sum_l \frac{\partial \Gamma^l_{im}}{\partial x_l} + \sum_{\rho l} \Gamma^l_{i\rho} \Gamma^\rho_{ml} = -\varkappa \left(T_{im} - \frac{1}{2} g_{im} T \right)$$

State of the Art





3.5PN order (almost 4PN)

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2}\right\}$$

$$4\pi R^2 \bar{S} = \frac{\varkappa}{40\pi}\left[\sum_{\mu\nu} \dddot{J}_{\mu\nu}^2 - \frac{1}{3}\left(\sum_{\mu} \dddot{J}_{\mu\mu}\right)^2\right]$$

$\nu \sim m_2/m_1$

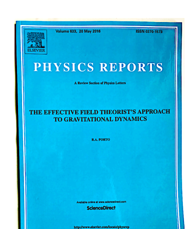
$x \sim (v/c)^2$

The effective field theorist's approach to gravitational dynamics

Physics Reports

Rafael A. Porto

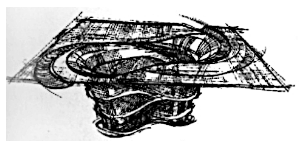
Volume 633, 20 May 2016, Pages 1-104

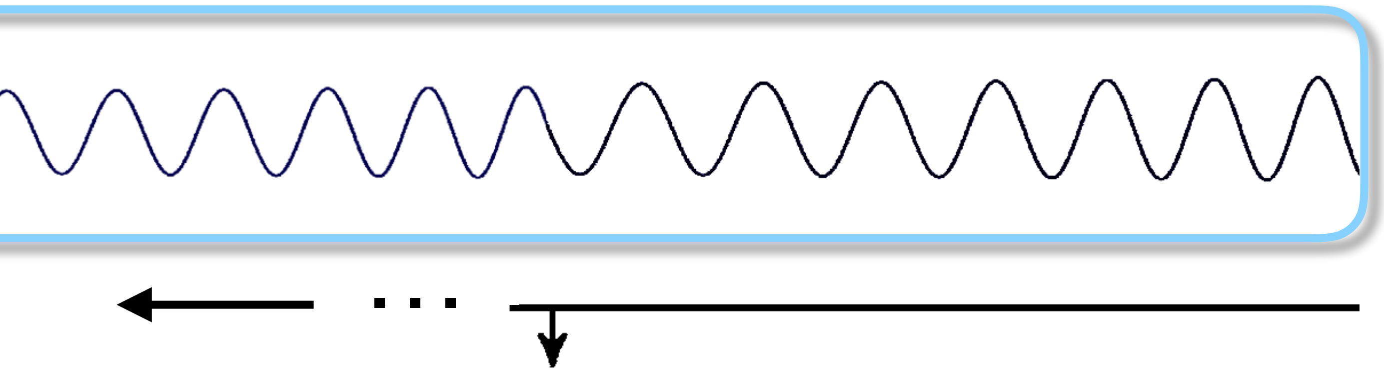


Blanchet, Damour, Faye et al. (Harmonic)
Jaranowski, Schaefer, et al. (ADM)

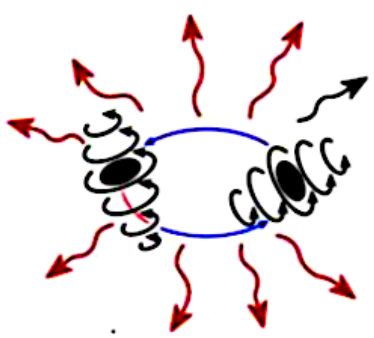
Goldberger, Rothstein, Ross, Foffa, Galley, Leibovich, Sturani, et al.

Ready for the future?

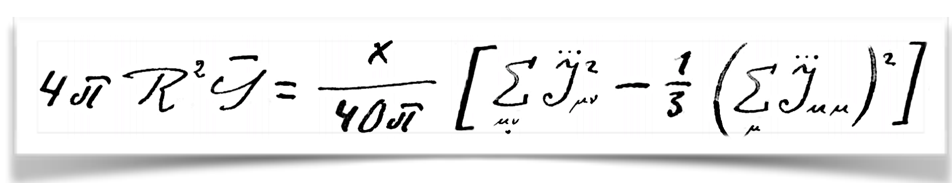




3.5PN order (almost 4PN)



$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2}\right\}$$



$$4\pi R^2 \bar{\mathcal{J}} = \frac{\varkappa}{40\pi}\left[\sum_{\mu\nu} \dddot{J}^2_{\mu\nu} - \frac{1}{3}\left(\sum_{\mu} \dddot{J}_{\mu\mu}\right)^2\right]$$

$\nu \sim m_2/m_1$

$x \sim (v/c)^2$

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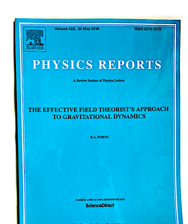


Blanchet, Damour, Faye et al. (Harmonic)
Jaranowski, Schaefer, et al. (ADM)

Goldberger, Rothstein, Ross, Foffa, Galley, Leibovich, Sturani, et al.

ON THE MOTION OF PARTICLES IN GENERAL RELATIVITY THEORY

A. EINSTEIN and L. INFELD

This problem was solved for the first time some ten years ago and the equations of motion for two particles were then deduced [1]. A more general and simplified version of this problem was given shortly thereafter [2].

Mr. Lewison pointed out to us, that from our approximation procedure, it does not follow that the field equations can be solved up to an arbitrarily high approximation. This is indeed true. We believe that the present work not only removes this difficulty, but that it gives a new and deeper insight into the problem of motion. From the logical point of view the present theory is considerably simpler and clearer than the old one. But as always, we must pay for these logical simplifications by prolonging the chain of technical argument.

TABLE OF SURFACE INTEGRALS FOR $\int_6^1 \Lambda_{ms} n_s \, dS$

No.	Expression	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}	a_{17}	Result	Remarks
1	$\overset{1}{m} g_{,s} \dot\eta^s \dot\eta^m$	$-\frac{16}{3}$				$-\frac{4}{3}$		$-\frac{8}{3}$	$-\frac{4}{15}$		$\frac{8}{15}$	$\frac{4}{15}$				$\frac{4}{5}$			-8	$\tilde g_{,s} = -2\overset{2}{m} \frac{\partial \frac{1}{r}}{\partial \eta^s}$
2	$\overset{1}{m} \tilde g \ddot\eta^m$	-2						-4	$-\frac{4}{3}$			$-\frac{29}{3}$	3	$\frac{11}{3}$	$-\frac{5}{3}$	$\frac{2}{3}$	$\frac{32}{3}$	$-\frac{22}{3}$	-8	$\tilde g = -\frac{2\overset{2}{m}}{r}$; $\ddot\eta^m = -\frac{1}{2}\tilde g_{,m}$
3	$\overset{1}{m} \tilde g_{,m} \dot\eta^s \dot\eta^s$	1					$-\frac{4}{3}$	$-\frac{4}{5}$			$\frac{8}{5}$	$\frac{4}{5}$	1	$\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{4}{15}$			2	$\tilde g_{,m} = -2\overset{1}{m}\ddot\eta^m$
4	$\overset{1}{m} \tilde g_{,m} \dot\zeta^s \dot\zeta^s$											2	$\frac{1}{3}$	1	$-\frac{1}{3}$				3	
5	$\overset{1}{m} \tilde g_{,m} \tilde f$	$\frac{4}{3}$				2	$\frac{2}{3}$					$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{2}$	$-\frac{1}{6}$				5	$\tilde g_{,m} \tilde f = -\tilde g \tilde f_{,m}$; $\tilde f = -\frac{2\overset{1}{m}}{r}$
6	$\overset{1}{m}\overset{2}{m} \tilde r_{,00m}$											-2							-2	$\tilde r_{,00m} = (\overset{2}{r}_{,00m})$ for $x^s = \eta^s$ *
7	$\overset{1}{m} \tilde g_{,s} \dot\zeta^m \dot\eta^s$	$\frac{16}{5}$				$\frac{8}{3}$	$\frac{4}{5}$	$\frac{4}{3}$											8	
8	$\overset{1}{m} \tilde g_{,s} \dot\zeta^s \dot\eta^m$	$\frac{16}{5}$					$-\frac{8}{15}$	4	$\frac{4}{3}$	-2									6	
9	$\overset{1}{m} \tilde g_{,m} \dot\eta^s \dot\zeta^s$	$-\frac{32}{15}$		$-\frac{16}{3}$	$-\frac{8}{3}$		$\frac{4}{5}$		$\frac{4}{3}$										-8	
10	$\overset{1}{m} \tilde g_{,s} \dot\zeta^s \dot\zeta^m$	$-\frac{8}{3}$				-4	$-\frac{4}{3}$												-8	

* $\tilde r_{,00m} = \frac{\partial^3 r}{\partial \eta^s \partial \eta^r \partial \eta^m} \dot\zeta^s \dot\zeta^r$, as $\frac{\partial^2 r}{\partial \eta^s \partial \eta^m} \frac{\partial \frac{1}{r}}{\partial \eta^s} = 0.$

THE SECOND POST-NEWTONIAN EQUATIONS OF HYDRODYNAMICS IN GENERAL RELATIVITY

S. CHANDRASEKHAR AND YAVUZ NUTKU
University of Chicago
Received 1969 February 26

$$
\begin{aligned}
\Theta^{\alpha\beta} = {} & \rho v_\alpha v_\beta + p\delta_{\alpha\beta} + \frac{1}{c^2}\left[\rho v_\alpha v_\beta\left(v^2 + 6U + \Pi + \frac{p}{\rho}\right) + 2pU\delta_{\alpha\beta}\right] \\
& + \frac{1}{c^4}\Big\{\rho v_\alpha v_\beta\left[v^4 + 10v^2U + 12U^2 + (v^2+6U)\left(\Pi + \frac{p}{\rho}\right) - 2v_\mu P_\mu - Q_{\sigma\sigma}\right] \\
& + p\left[Q_{\alpha\beta} - (2U^2 + 4\Phi + Q_{\sigma\sigma})\delta_{\alpha\beta}\right]\Big\} \\
& + \frac{1}{16\pi G}\Big[\!\Big[4\frac{\partial U}{\partial x_\alpha}\frac{\partial U}{\partial x_\beta} - 2\delta_{\alpha\beta}\left(\frac{\partial U}{\partial x_\mu}\right)^2 \\
& + \frac{1}{c^2}\Big\{8\left(\frac{\partial U}{\partial x_\alpha}\frac{\partial \Phi}{\partial x_\beta} + \frac{\partial U}{\partial x_\beta}\frac{\partial \Phi}{\partial x_\alpha}\right) + 4\left(\frac{\partial U}{\partial x_\alpha}\frac{\partial P_\beta}{\partial t} + \frac{\partial U}{\partial x_\beta}\frac{\partial P_\alpha}{\partial t}\right) \\
& \quad - \left(\frac{\partial P_\mu}{\partial x_\alpha}\frac{\partial P_\mu}{\partial x_\beta} + \frac{\partial P_\alpha}{\partial x_\mu}\frac{\partial P_\beta}{\partial x_\mu}\right) + \left(\frac{\partial P_\alpha}{\partial x_\mu}\frac{\partial P_\mu}{\partial x_\beta} + \frac{\partial P_\beta}{\partial x_\mu}\frac{\partial P_\mu}{\partial x_\alpha}\right) \\
& \quad + \delta_{\alpha\beta}\left[-8\frac{\partial U}{\partial x_\mu}\frac{\partial \Phi}{\partial x_\mu} - 6\left(\frac{\partial U}{\partial t}\right)^2 - 4\frac{\partial U}{\partial x_\mu}\frac{\partial P_\mu}{\partial t} + \tfrac{1}{2}\frac{\partial P_\nu}{\partial x_\mu}\left(\frac{\partial P_\nu}{\partial x_\mu} - \frac{\partial P_\mu}{\partial x_\nu}\right)\right]\Big\} \\
& + \frac{1}{c^4}\Big\{(-48U^2 + 16\Phi - 4Q_{\sigma\sigma})\frac{\partial U}{\partial x_\alpha}\frac{\partial U}{\partial x_\beta} - 8\frac{\partial \Phi}{\partial x_\alpha}\frac{\partial \Phi}{\partial x_\beta} - 8U\left(\frac{\partial U}{\partial x_\alpha}\frac{\partial \Phi}{\partial x_\beta} + \frac{\partial U}{\partial x_\beta}\frac{\partial \Phi}{\partial x_\alpha}\right) \\
& \quad - \frac{\partial U}{\partial x_\alpha}\frac{\partial}{\partial x_\beta}(P_\lambda^2) - \frac{\partial U}{\partial x_\beta}\frac{\partial}{\partial x_\alpha}(P_\lambda^2) - 2\frac{\partial U}{\partial x_\mu}\frac{\partial}{\partial x_\mu}(P_\alpha P_\beta) \\
& \quad - \left(\frac{\partial P_\alpha}{\partial x_\beta} + \frac{\partial P_\beta}{\partial x_\alpha}\right)\frac{\partial}{\partial t}(U^2 + 2\Phi + \tfrac{1}{2}Q_{\sigma\sigma}) + 4\frac{\partial U}{\partial t}\left(P_\alpha\frac{\partial U}{\partial x_\beta} + P_\beta\frac{\partial U}{\partial x_\alpha}\right) \\
& \quad + 2\frac{\partial U}{\partial x_\mu}\left(P_\alpha\frac{\partial P_\mu}{\partial x_\beta} + P_\beta\frac{\partial P_\mu}{\partial x_\alpha}\right) + 2P_\mu\left(\frac{\partial U}{\partial x_\alpha}\frac{\partial P_\beta}{\partial x_\mu} + \frac{\partial U}{\partial x_\beta}\frac{\partial P_\alpha}{\partial x_\mu}\right) \\
& \quad - \frac{\partial Q_{\sigma\sigma}}{\partial x_\alpha}\left[\tfrac{1}{2}\frac{\partial P_\beta}{\partial t} + \frac{\partial}{\partial x_\beta}(U^2 + 2\Phi)\right] - \frac{\partial Q_{\sigma\sigma}}{\partial x_\beta}\left[\tfrac{1}{2}\frac{\partial P_\alpha}{\partial t} + \frac{\partial}{\partial x_\alpha}(U^2 + 2\Phi)\right] \\
& \quad - \frac{\partial Q_{\alpha\beta}}{\partial x_\mu}\frac{\partial}{\partial x_\mu}\left[3(U^2 + 2\Phi) + Q_{\sigma\sigma}\right] - \frac{\partial U}{\partial t}\frac{\partial Q_{\alpha\beta}}{\partial t} - 2Q_{\alpha\beta}\left(\frac{\partial U}{\partial x_\mu}\right)^2 \\
& \quad + 4\frac{\partial U}{\partial x_\mu}\left(Q_{\alpha\mu}\frac{\partial U}{\partial x_\beta} + Q_{\beta\mu}\frac{\partial U}{\partial x_\alpha}\right) + \left(\frac{\partial Q_{\alpha\mu}}{\partial x_\beta} + \frac{\partial Q_{\beta\mu}}{\partial x_\alpha}\right)\frac{\partial}{\partial x_\mu}(U^2 + 2\Phi + \tfrac{1}{2}Q_{\sigma\sigma}) \\
& \quad + \left(\frac{\partial P_\mu}{\partial x_\alpha} - \frac{\partial P_\alpha}{\partial x_\mu}\right)\left(\frac{\partial Q_{\beta 0}}{\partial x_\mu} - \frac{\partial Q_{\mu 0}}{\partial x_\beta}\right) + \left(\frac{\partial P_\mu}{\partial x_\beta} - \frac{\partial P_\beta}{\partial x_\mu}\right)\left(\frac{\partial Q_{\alpha 0}}{\partial x_\mu} - \frac{\partial Q_{\mu 0}}{\partial x_\alpha}\right) \\
& \quad + \frac{\partial P_\mu}{\partial x_\alpha}\frac{\partial Q_{\beta\mu}}{\partial t} + \frac{\partial P_\mu}{\partial x_\beta}\frac{\partial Q_{\alpha\mu}}{\partial t} - \frac{\partial P_\mu}{\partial t}\frac{\partial Q_{\alpha\beta}}{\partial x_\mu} - \tfrac{1}{4}\frac{\partial Q_{\sigma\sigma}}{\partial x_\alpha}\frac{\partial Q_{\mu\mu}}{\partial x_\beta} + \tfrac{1}{2}\frac{\partial Q_{\mu\nu}}{\partial x_\alpha}\frac{\partial Q_{\mu\nu}}{\partial x_\beta} \\
& \quad + 4\frac{\partial U}{\partial x_\alpha}\left(\frac{\partial Q_{\beta 0}}{\partial t} - \tfrac{1}{2}\frac{\partial Q_{00}}{\partial x_\beta}\right) + 4\frac{\partial U}{\partial x_\beta}\left(\frac{\partial Q_{\alpha 0}}{\partial t} - \tfrac{1}{2}\frac{\partial Q_{00}}{\partial x_\alpha}\right) \qquad (90) \\
& \quad + \frac{\partial Q_{\alpha\mu}}{\partial x_\nu}\frac{\partial Q_{\beta\mu}}{\partial x_\nu} - \left(\frac{\partial Q_{\alpha\mu}}{\partial x_\nu}\frac{\partial Q_{\mu\nu}}{\partial x_\beta} + \frac{\partial Q_{\beta\mu}}{\partial x_\nu}\frac{\partial Q_{\mu\nu}}{\partial x_\alpha}\right)
\end{aligned}
$$

$$
\begin{aligned}
& + \delta_{\alpha\beta}\Big[(28U^2 - 8\Phi + 2Q_{\sigma\sigma})\left(\frac{\partial U}{\partial x_\mu}\right)^2 + 8\left(\frac{\partial \Phi}{\partial x_\mu}\right)^2 + 16U\frac{\partial U}{\partial x_\mu}\frac{\partial \Phi}{\partial x_\mu} \qquad (90) \\
& \quad - 2\frac{\partial U}{\partial t}\frac{\partial}{\partial t}(U^2 + 2\Phi) - 4P_\mu\frac{\partial U}{\partial x_\mu}\frac{\partial U}{\partial t} + \frac{\partial U}{\partial x_\mu}\frac{\partial P_\lambda^2}{\partial x_\mu} - 2P_\mu\frac{\partial P_\nu}{\partial x_\mu}\frac{\partial U}{\partial x_\nu} \\
& \quad + 2\frac{\partial Q_{\sigma\sigma}}{\partial x_\mu}\frac{\partial}{\partial x_\mu}(U^2 + 2\Phi) + \frac{\partial Q_{\sigma\sigma}}{\partial x_\mu}\frac{\partial P_\mu}{\partial t} + \frac{\partial Q_{\sigma\sigma}}{\partial t}\frac{\partial U}{\partial t} - 2Q_{\mu\nu}\frac{\partial U}{\partial x_\mu}\frac{\partial U}{\partial x_\nu} \\
& \quad + 2\frac{\partial U}{\partial x_\mu}\left(\frac{\partial Q_{00}}{\partial x_\mu} - 2\frac{\partial Q_{\mu 0}}{\partial t}\right) - \frac{\partial P_\nu}{\partial x_\mu}\left(\frac{\partial Q_{\mu 0}}{\partial x_\nu} - \frac{\partial Q_{\nu 0}}{\partial x_\mu} + \frac{\partial Q_{\mu\nu}}{\partial t}\right) \\
& \quad + \tfrac{1}{4}\left(\frac{\partial Q_{\sigma\sigma}}{\partial x_\mu}\right)^2 - \tfrac{1}{4}\frac{\partial Q_{\mu\nu}}{\partial x_\sigma}\frac{\partial Q_{\mu\nu}}{\partial x_\sigma} + \tfrac{1}{2}\frac{\partial Q_{\sigma\mu}}{\partial x_\nu}\frac{\partial Q_{\sigma\nu}}{\partial x_\mu}\Big]\Big\}\Big]\!\Big] .
\end{aligned}
$$

TABLE 1

INFORMATION ON THE METRIC COEFFICIENTS THAT IS NEEDED IN THE VARIOUS APPROXIMATIONS

EQUATIONS OF MOTION	ORDERS* OF THE METRIC COEFFICIENTS NEEDED		
	$g_{\alpha\beta}$	$g_{0\alpha}$	g_{00}
Newtonian	0	1	2
First post-Newtonian	2(4)†	3	4
Second post-Newtonian	4(6)†	5	6
First radiative corrections	5	6	7

VOLUME 74, NUMBER 18 PHYSICAL REVIEW LETTERS 1 MAY 1995

Gravitational-Radiation Damping of Compact Binary Systems to Second Post-Newtonian Order

Luc Blanchet,[1] Thibault Damour,[2,1] Bala R. Iyer,[3] Clifford M. Will,[4] and Alan G. Wiseman[5]*

$$
\dot{\omega} = \frac{96}{5}\eta m^{5/3}\omega^{11/3}\left[1 - \left(\frac{743}{336} + \frac{11}{4}\eta\right)(m\omega)^{2/3} + (4\pi - \beta)(m\omega) + \left(\frac{34\,103}{18\,144} + \frac{13\,661}{2016}\eta + \frac{59}{18}\eta^2 + \sigma\right)(m\omega)^{4/3}\right]
$$

Gravitational Radiation from Inspiralling Compact Binaries Completed at the Third Post-Newtonian Order

(W/OUT SPIN)

Luc Blanchet,[1] Thibault Damour,[2] Gilles Esposito-Farèse,[1] and Bala R. Iyer[3]
[1]*GReCO, FRE 2435-CNRS, Institut d'Astrophysique de Paris, 98bis boulevard Arago, F-75014 Paris, France*
[2]*Institut des Hautes Études Scientifiques, 35 route de Chartres, F-91440 Bures-sur-Yvette, France*
[3]*Raman Research Institute, Bangalore 560 080, India*
(Received 3 June 2004; published 26 August 2004)

Three previously introduced ambiguity parameters, coming from the Hadamard self-field regularization of the 3PN source-type mass quadrupole moment, are consistently determined by means of dimensional regularization,

Third post-Newtonian higher order ADM Hamilton dynamics for two-body point-mass systems

Piotr Jaranowski*
Institute of Physics, Białystok University, Lipowa 41, 15-424 Białystok, Poland

Gerhard Schäfer†
Theoretisch-Physikalisches Institut, Friedrich-Schiller-Universität, Max-Wien-Platz 1, 07743 Jena, Germany
(Received 17 December 1997; published 8 May 1998)

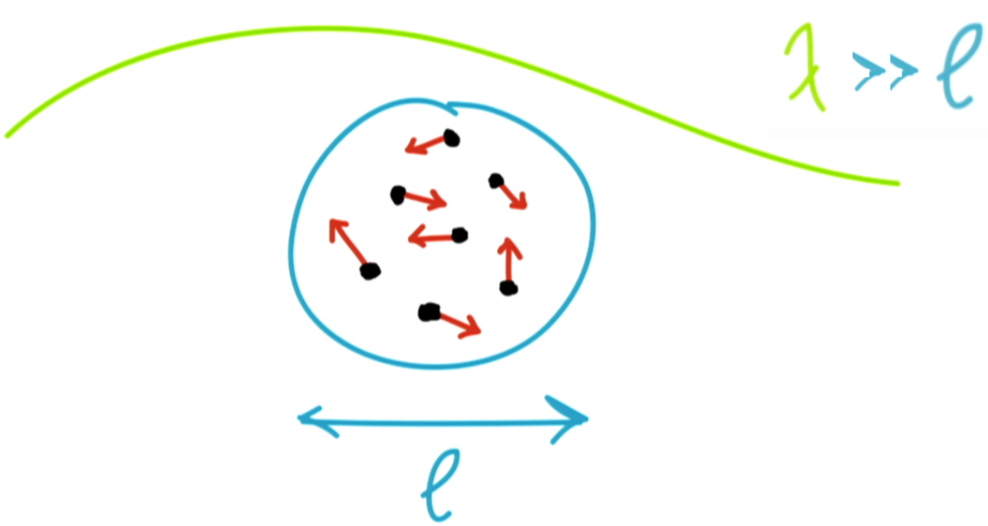
We conjecture that the ambiguity has its origin in the zero extension of the bodies. We started with point-like bodies but the formalism reacted in such a manner that the Schwarzschild radii of the bodies got introduced.

The (UV) divergences are due to the 'point-particle' limit

However, they are unphysical at this order (More soon…)

Goldberger Rothstein (2004)
RAP (2005)

(Classical) EFT



point-particle theory

$$S_{\rm pp} = -\int d\tau \left(m + \frac{1}{2}\omega_\mu^{ab} S_{ab} v^\mu + \cdots \right)$$

localized source introduces **UV** divergences
in a non-linear theory (General Relativity)

Strings and other distributional sources in general relativity

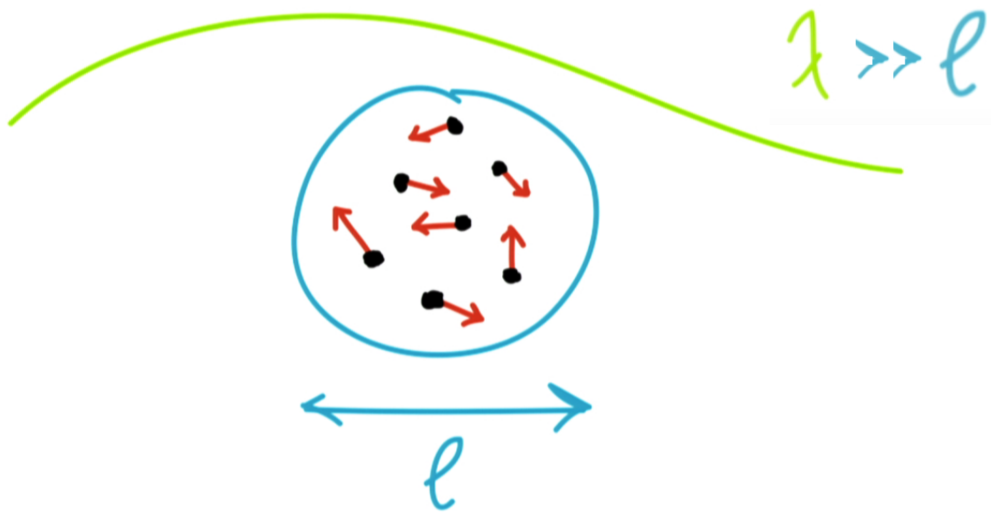
Robert Geroch and Jennie Traschen
Phys. Rev. D **36**, 1017 – Published 15 August 1987

—but neither point particles nor strings—can be described by metrics in this class.

The ‘extra’ terms in the EFT renormalize the theory!

Goldberger Rothstein (2004)
RAP (2005)

(Classical) EFT



point-particle theory

$$S_{\rm pp} = -\int d\tau \left(m + \frac{1}{2}\omega_\mu^{ab} S_{ab} v^\mu + \cdots \right)$$

$$\cdots = c_R R + c_V R_{\mu\nu} v^\mu v^\nu + \cdots + \frac{1}{2} Q^{ij} E_{ij} + \cdots$$

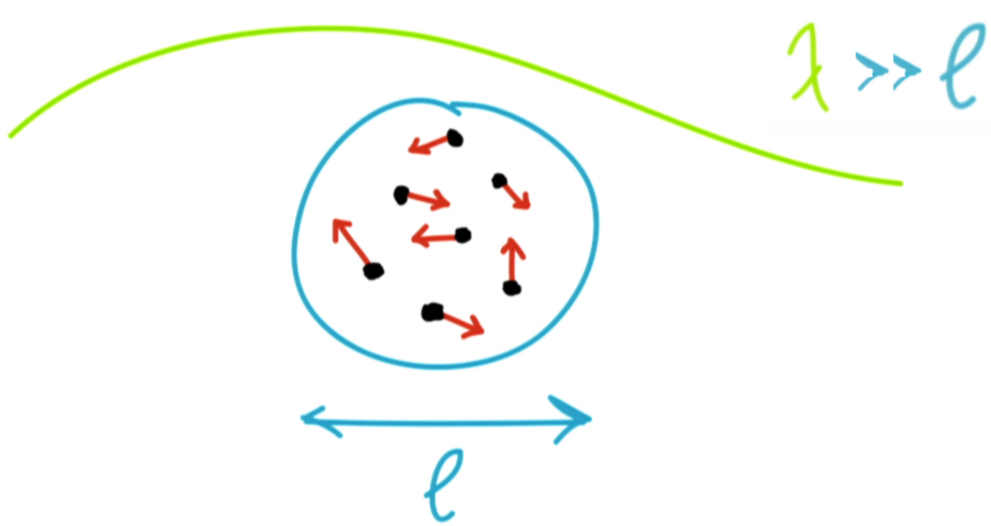
Removed by field redefinitions (but needed to absorb unphysical UV poles due to pp approx.)

Finite Size Effects

Electric component of Weyl tensor

Goldberger Rothstein (2004)
RAP (2005)

(Classical) EFT

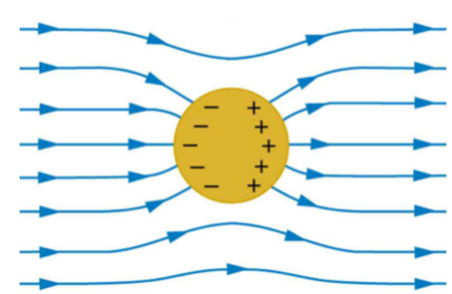


point-particle theory

$$S_{\rm pp} = -\int d\tau \left(m + \frac{1}{2}\omega_\mu^{ab} S_{ab} v^\mu + \cdots \right)$$

$$Q_{ij} = C_E E_{ij}$$

('Susceptibility')



$$c_E E_{ij}^2$$

(gauge invariant and unambiguous!)

Effacement Theorem

The first finite size effect
— for non-spinning spherically
symmetric (in isolation) —
starts at **MR^4**
(**5th** Post-Newtonian order)

Damour (80's)

Goldberger Rothstein (2004)
RAP (2005)

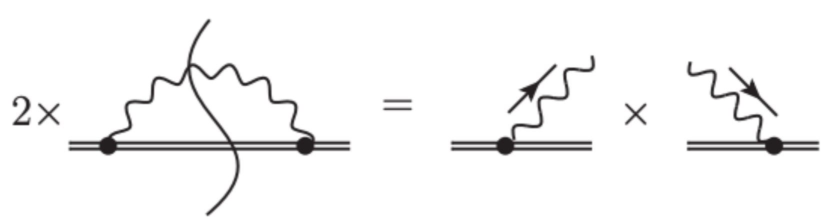
(Classical) EFT

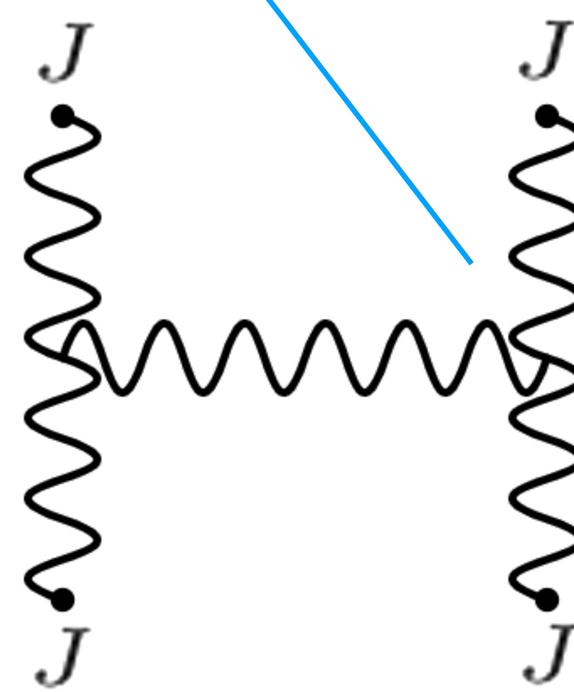
$$e^{iW} = \int Dh \, e^{i(S_{\rm EH}[h] + S_{\rm pp}[h, x_a])}$$

$$\underbrace{{\rm Re}\, W[x_a]}_{\text{binding}} + i \underbrace{{\rm Im}\, W[x_a]}_{\text{radiation}}$$

↑

Classical optical theorem with Feynman b.c.



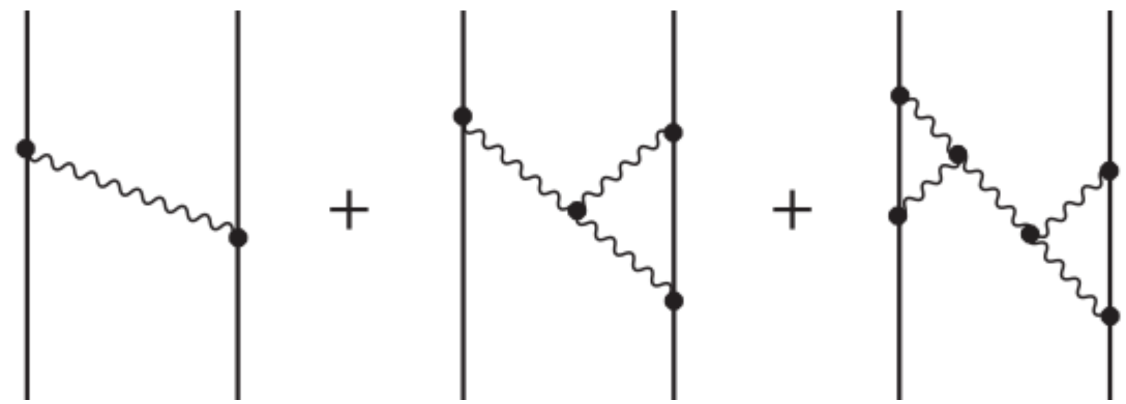


Fully relativistic

Goldberger Rothstein (2004)
RAP (2005)

(Classical) EFT

$$e^{iW} = \int Dh\, e^{i(S_{\rm EH}[h]+S_{\rm pp}[h,x_a])}$$



$$S_{\rm red}(T) = \frac{1}{2} T\, G\, T + V_3(G\,T, G\,T, G\,T) + \ldots$$

e.g. Duff (70's);
Damour et al. (90's)

Classical Electrodynamics in Terms of Direct Interparticle Action[1]

JOHN ARCHIBALD WHEELER AND RICHARD PHILLIPS FEYNMAN[2]
Princeton University, Princeton, New Jersey

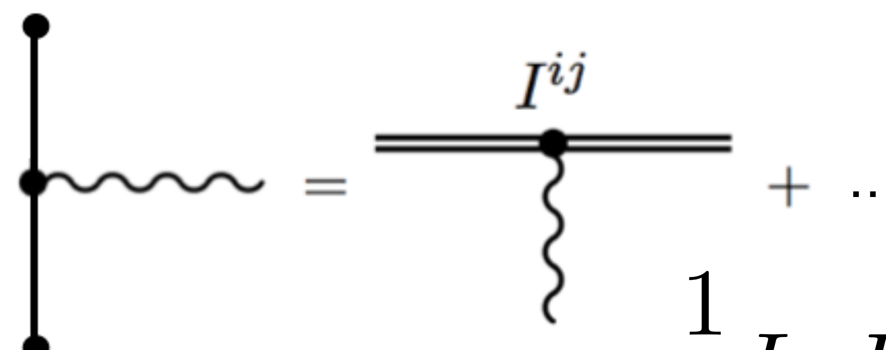
$$J = -\sum_a m_a c \int (-da_\mu da^\mu)^{\frac{1}{2}} + \sum_{a<b} (e_a e_b / c) \times \int\int \delta(ab_\mu ab^\mu)(da_\nu db^\nu) = \text{extremum.} \quad (1)$$

Goldberger Rothstein (2004)
RAP (2005)

NRGR

$$e^{iW} = \int Dh\, e^{i(S_{\rm EH}[h] + S_{\rm pp}[h, x_a])}$$

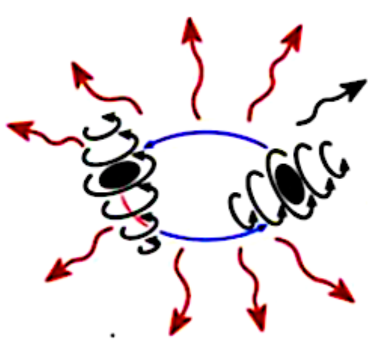
$$\underbrace{\mathrm{Re}\, W[x_a]}_{\text{binding}} + i\, \underbrace{\mathrm{Im}\, W[x_a]}_{\text{radiation}}$$



$$\frac{1}{2} I_{ij} E^{ij}$$

binary

potential & radiation



$$Gm \ll r \ll \lambda \sim r/v$$

$$h_{\mu\nu} = \eta_{\mu\nu} + H_{\mu\nu} + \bar{h}_{\mu\nu}$$

$(1/r, v/r)$ **(off-shell binding)**

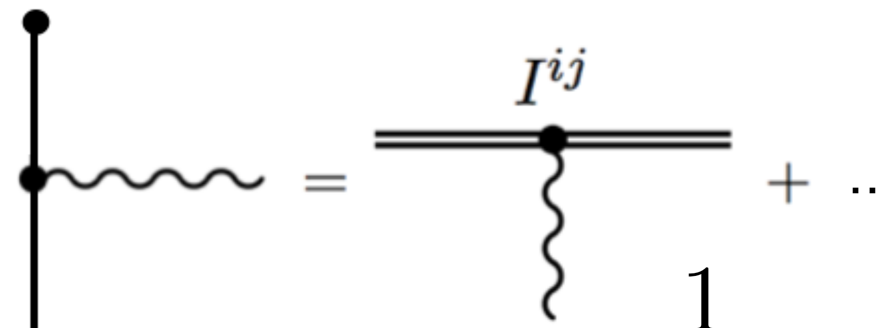
$(v/r, v/r)$ **(emission)**

Goldberger Rothstein (2004)
RAP (2005)

NRGR

$$e^{iW} = \int Dh\, e^{i(S_{\rm EH}[h]+S_{\rm pp}[h,x_a])}$$

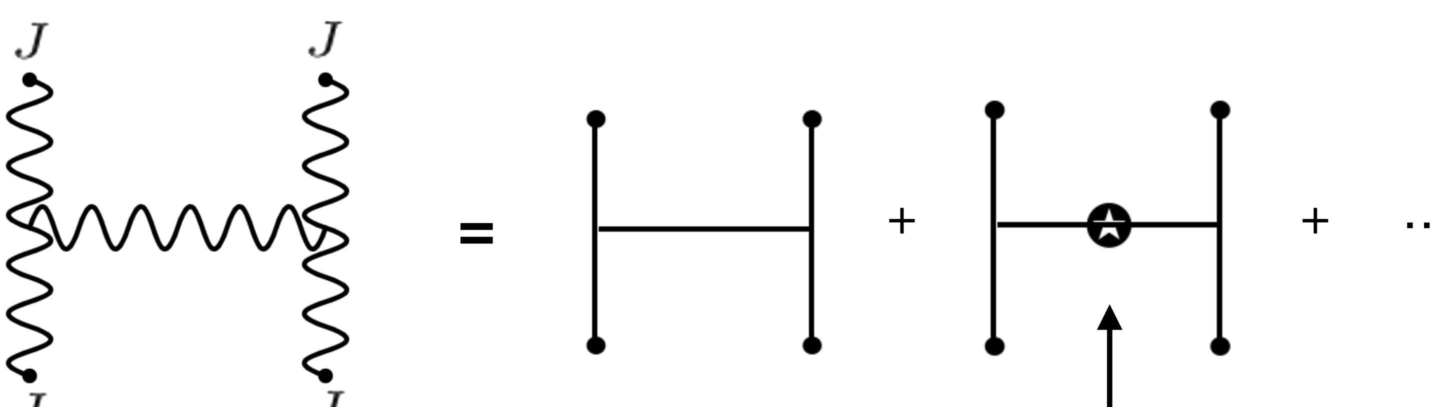
$$\underbrace{\mathrm{Re}\, W[x_a]}_{\text{binding}} + i\, \underbrace{\mathrm{Im}\, W[x_a]}_{\text{radiation}}$$



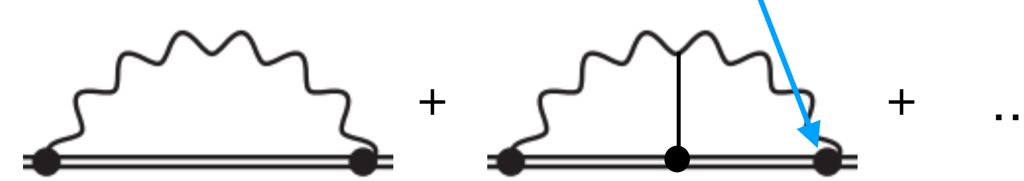
potential & radiation

$$\frac{1}{2} I_{ij} E^{ij}$$

binary



$$\frac{1}{p_0^2 - p^2} = \frac{1}{p^2}\left(-1 + p_0^2/p^2 + \cdots\right)$$



Includes both dissipative & conservative!

NRGR contributions to State-of-the-art

* **General Relativity and Gravitation: A Centennial Perspective**

Chapter 6: **Sources of Gravitational Waves: Theory and Observations**

Alessandra Buonanno and B.S. Sathyaprakash

Spin effects

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \underbrace{\cdots + [\cdots]x^{7/2}}\right\}$$

* the EFT approach has extended the knowledge of the conservative dynamics and multipole moments to high PN orders [134–145].

[134] **Porto, R. A.** 2006. *Phys. Rev. D*, **73**, 104031.
[135] **Porto, R. A.**, Rothstein, I. Z. 2006. *Phys. Rev. Lett.*, **97**, 021101.
[136] Kol, B., Smolkin, M. 2008. *Class. Quant. Grav.*, **25**, 145011.
[137] **Porto, R. A.**, Rothstein, I. Z. 2008. *Phys. Rev. D*, **78**, 044013.
[138] **Porto, R. A.**, Rothstein, I. Z. 2008. *Phys. Rev. D*, **78**, 044012.
[139] **Porto, R. A.**, Ross, A., Rothstein, I. Z. 2011. *JCAP*, **1103**, 009.
[140] **Porto, R. A.** 2010. *Class. Quant. Grav.*, **27**, 205001.
[141] Levi, M. 2010. *Phys. Rev. D*, **82**, 104004.
[142] Levi, M. 2012. *Phys. Rev. D*, **85**, 064043.
[143] Hergt, S., Steinhoff, J., Schaefer, G. 2012. *Annals Phys.*, **327**, 1494–1537.
[144] Hergt, S., Steinhoff, J., Schaefer, G. 2014. *J.Phys.Conf.Ser.*, **484**, 012018.
[145] **Porto, R. A.**, Ross, A., Rothstein, I. Z. 2012. *JCAP*, **1209**, 028.

Latest in PN: Binding energy to N^4LO (no spin)

$$E^{4\mathrm{PN}} = -\frac{\mu c^2 x}{2}\Big\{1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2$$
$$+ \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)x^3$$
$$+ \left(-\frac{3969}{128} + \left[-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_\mathrm{E} + \frac{448}{15}\ln(16x)\right]\nu\right.$$
$$\left. + \left[-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right]\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\right)x^4\Big\}$$

$\nu \sim m_2/m_1$

$x \sim (v/c)^2$

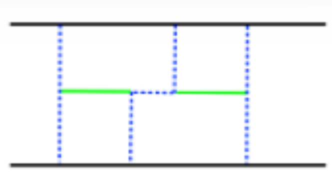
Damour Jaranowski Schafer (2014)
Blanchet et al. (2018)

Galley RAP Leibovich Ross (2016)
Foffa Sturani Mastrolia Sturm (2016)
RAP Rothstein (2017)
RAP (2017)
Foffa Sturani (2019)
Foffa RAP Sturani Rothstein (2019)

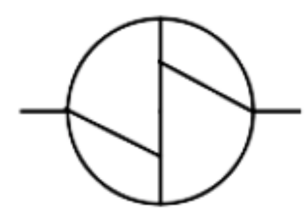
EFT re-derivation to 3PN:
Gilmore and Ross (2PN)
Foffa and Sturani (3PN)

NRGR: Reduced to Feynman integrals

$$E^{\text{4PN}} = -\frac{\mu c^2 x}{2}\Bigg\{1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2$$
$$+ \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)x^3$$
$$+ \left(-\frac{3969}{128} + \left[-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\ln(16x)\right]\nu\right.$$
$$\left. + \left[-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right]\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\right)x^4\Bigg\}$$



$$= -2\,\mathrm{i}\,(8\pi G_N)^5\left(\frac{(d-2)}{(d-1)}\,m_1 m_2\right)^3$$



$$[N_{49}]$$

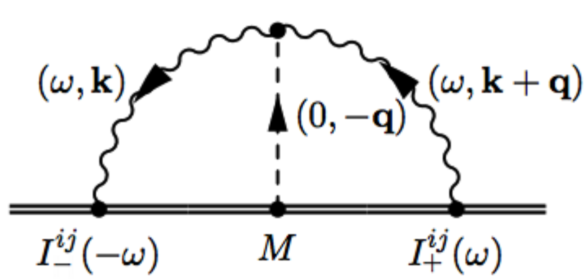
$$[N_{49}] \equiv \int_{k_1,k_2,k_3,k_4} \frac{N_{49}}{k_1^2\; p_2^2\; k_3^2\; p_4^2\; k_{12}^2\; k_{13}^2\; k_{23}^2\; k_{24}^2\; k_{34}^2},$$

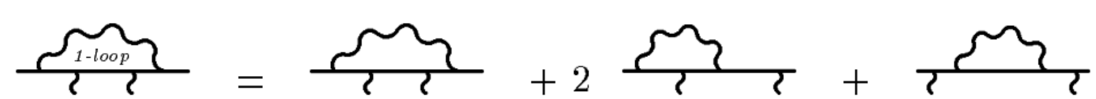
Foffa Sturani Mastrolia Sturm (2016,2019)

$$\mathcal{V}^{(5\text{PN})}_{\text{static}} = \frac{5}{16}\frac{G_N^6 m_1^6 m_2}{r^6} + \frac{91}{6}\frac{G_N^6 m_1^5 m_2^2}{r^6} + \frac{653}{6}\frac{G_N^6 m_1^4 m_2^3}{r^6} + (m_1 \leftrightarrow m_2)$$

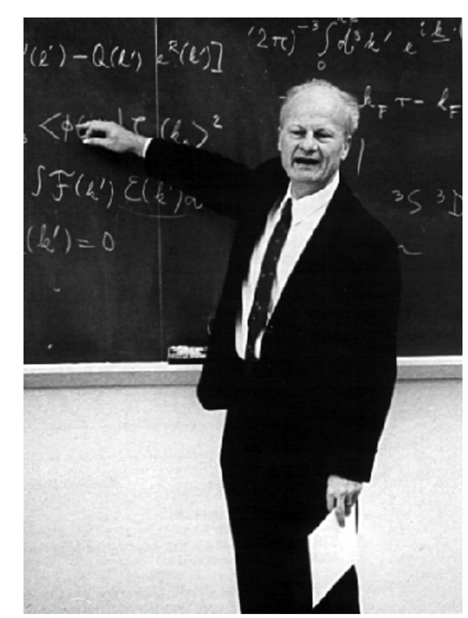
IR log from ‘radiation/soft’ modes!

$$E^{4\mathrm{PN}} = -\frac{\mu c^2 x}{2}\left\{1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2 \right.$$
$$+ \left(-\frac{675}{64} + \left[\frac{34445}{576} - \frac{205}{96}\pi^2\right]\nu - \frac{155}{96}\nu^2 - \frac{35}{5184}\nu^3\right)x^3$$
$$+ \left(-\frac{3969}{128} + \left[-\frac{123671}{5760} + \frac{9037}{1536}\pi^2 + \frac{896}{15}\gamma_E + \frac{448}{15}\ln(16x)\right]\nu \right.$$
$$\left.\left. + \left[-\frac{498449}{3456} + \frac{3157}{576}\pi^2\right]\nu^2 + \frac{301}{1728}\nu^3 + \frac{77}{31104}\nu^4\right)x^4\right\}$$





Galley RAP Leibovich Ross (2016)



PHYSICAL REVIEW D **96**, 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

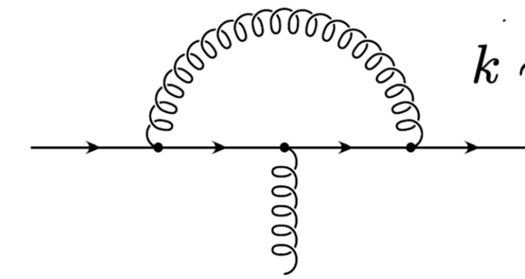
Rafael A. Porto

PHYSICAL REVIEW D **96,** 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

Rafael A. Porto

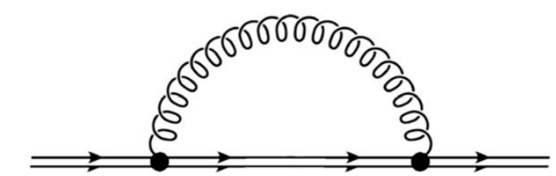
Computation in NRQED:



$$(\delta E_{n,\ell})_{c_V} = \frac{e^2}{8m_e^2} c_V |\psi_{n,\ell}(\boldsymbol{x}=0)|^2 = \frac{4\alpha_e^2}{3m_e^2}\left(-\frac{1}{\epsilon_{\rm IR}} + \log\frac{m_e}{\mu}\right)|\psi_{n,\ell}(\boldsymbol{x}=0)|^2.$$

$$-ie\bar{u}(p_1)\left[F_1(q^2)\gamma^\mu + \frac{i}{2m_e}F_2(q^2)\sigma^{\mu\nu}q_\nu\right]u(p_2)\,,$$

$$(\delta E_{n,\ell})_{US} = \frac{2\alpha_e}{3\pi}\left[e^2\left(\frac{1}{\epsilon_{\rm UV}} + \frac{5}{6}\right)\frac{|\psi_{n,\ell}(\boldsymbol{x}=0)|^2}{2m_e^2} - \sum_{m\neq n,\ell}\left\langle n,\ell\left|\frac{\boldsymbol{p}}{m_e}\right|m,\ell\right\rangle^2 (E_m - E_n)\log\frac{2|E_n - E_m|}{\mu}\right]$$



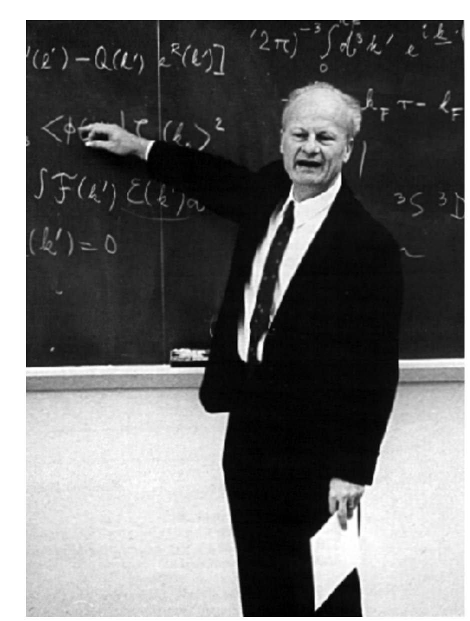
$(p^0_{US}, \boldsymbol{p}_{US}) \sim (m_e v^2, m_e v^2)$

H. A. Bethe, The electromagnetic shift of energy levels, Phys. Rev. **72**, 339 (1947).

F. J. Dyson, The electromagnetic shift of energy levels, Phys. Rev. **73**, 617 (1948).

J. B. French and V. F. Weisskopf, The electromagnetic shift of energy levels, Phys. Rev. **75**, 1240 (1949).

N. M. Kroll and W. E. Lamb, On the self-energy of a bound electron, Phys. Rev. **75**, 388 (1949).



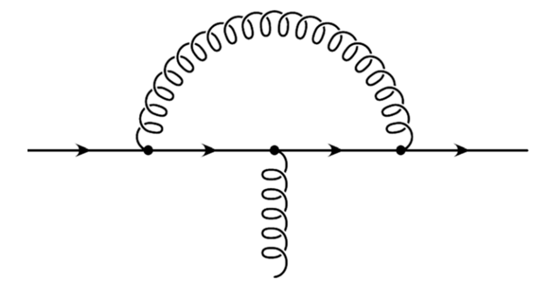
PHYSICAL REVIEW D **96**, 024063 (2017)

Lamb shift and the gravitational binding energy for binary black holes

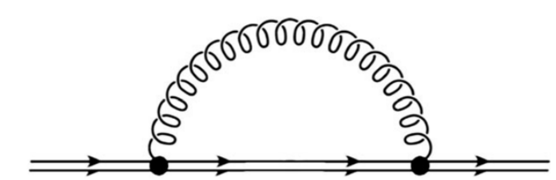
Rafael A. Porto

Computation in NRQED:

$$(\delta E_{n,\ell})_{c_V} = \frac{e^2}{8m_e^2} c_V |\psi_{n,\ell}(\boldsymbol{x}=0)|^2 = \frac{4\alpha_e^2}{3m_e^2}\left(-\frac{1}{\epsilon_{\rm IR}} + \log\frac{m_e}{\mu}\right)|\psi_{n,\ell}(\boldsymbol{x}=0)|^2.$$



$$(\delta E_{n,\ell})_{US} = \frac{2\alpha_e}{3\pi}\left[e^2\left(\frac{1}{\epsilon_{\rm UV}} + \frac{5}{6}\right)\frac{|\psi_{n,\ell}(\boldsymbol{x}=0)|^2}{2m_e^2} - \sum_{m\neq n,\ell}\left\langle n,\ell\left|\frac{\boldsymbol{p}}{m_e}\right|m,\ell\right\rangle^2 (E_m - E_n)\log\frac{2|E_n - E_m|}{\mu}\right]$$

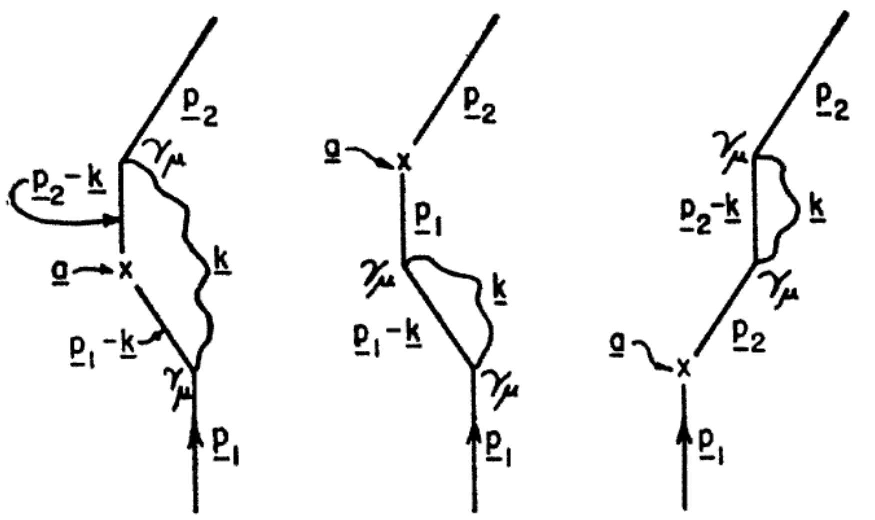


Adding up ‘near’ and ‘far’ zone contributions:

$$\begin{aligned}\delta E_{n,\ell} &= (\delta E_{n,\ell})_{US} + (\delta E_{n,\ell})_{c_V} + \cdots \\ &= \frac{2\alpha_e}{3\pi}\left[\frac{5}{6}e^2\frac{|\psi_{n,\ell}(\boldsymbol{x}=0)|^2}{2m_e^2} - \sum_{m\neq n,\ell}\left\langle n,\ell\left|\frac{\boldsymbol{p}}{m_e}\right|m,\ell\right\rangle^2 (E_m - E_n)\underbrace{\log\frac{2|E_n - E_m|}{m_e}}_{\log\alpha_e}\right] + \\ &\quad + \frac{4\alpha_e^2}{3m_e^2}\left(\frac{1}{\epsilon_{\rm UV}} - \frac{1}{\epsilon_{\rm IR}}\right)|\psi_{n,\ell}(\boldsymbol{x}=0)|^2 .\end{aligned}$$

Bethe log

$$\Delta H = \frac{e^2}{2\pi\hbar c}\left\{ -\frac{\hbar e i}{2\mu c}\beta\alpha\cdot\nabla\varphi + \frac{2\hbar^2 e}{3\mu^2 c^2}(\nabla^2\varphi)\right.$$
$$\left.\times\left(\ln\frac{\mu c}{2\hbar k_{\min}}+\frac{5}{8}\right)\right.$$



a. Eq. 12 b. Eq. 13 c. Eq. 14

Space-Time Approach to Quantum Electrodynamics

R. P. Feynman

Department of Physics, Cornell University, Ithaca, New York

(Received May 9, 1949)

Lamb shift as interpreted in more detail in B.[13]

[13] That the result given in B in Eq. (19) was in error was repeatedly pointed out to the author, in private communication, by V. F. Weisskopf and J. B. French, as their calculation, completed simultaneously with the author's early in 1948, gave a different result. French has finally shown that although the expression for the radiationless scattering B, Eq. (18) or (24) above is correct, it was incorrectly joined onto Bethe's non-relativistic result. He shows that the relation $\ln 2k_{\max}-1=\ln\lambda_{\min}$ used by the author should have been $\ln 2k_{\max}-5/6=\ln\lambda_{\min}$. This results in adding a term $-(1/6)$ to the logarithm in B, Eq. (19) so that the result now agrees with that of J. B. French and V. F. Weisskopf,

$$\begin{aligned}\delta E_{n,\ell} &= (\delta E_{n,\ell})_{US} + (\delta E_{n,\ell})_{cV} + \cdots \\ &= \frac{2\alpha_e}{3\pi}\left[\frac{5}{6}e^2\frac{|\psi_{n,\ell}(\boldsymbol{x}=0)|^2}{2m_e^2} - \sum_{m\neq n,\ell}\left\langle n,\ell\left|\frac{\boldsymbol{p}}{m_e}\right|m,\ell\right\rangle^2 (E_m-E_n)\log\frac{2|E_n-E_m|}{m_e}\right] + \\ &\quad + \frac{4\alpha_e^2}{3m_e^2}\left(\frac{1}{\epsilon_{\rm UV}}-\frac{1}{\epsilon_{\rm IR}}\right)|\psi_{n,\ell}(\boldsymbol{x}=0)|^2 .\end{aligned}$$

$\log\alpha_e$

Bethe log

PHYSICAL REVIEW D **89**, 064058 (2014)

Nonlocal-in-time action for the fourth post-Newtonian conservative dynamics of two-body systems

T. Damour, P. Jaranowski, and G. Schäfer,

This completion is obtained by resolving the infra-red ambiguity which had blocked a previous 4PN calculation [P. Jaranowski and G. Schäfer, Phys. Rev. D **87**, 081503(R) (2013)] by taking into account the 4PN breakdown of the usual near-zone expansion due to infinite-range tail-transported temporal correlations

$$H_{4\mathrm{PN}}^{\text{near-zone (s)}}[\mathbf{x}_a, \mathbf{p}_a] = H_{4\mathrm{PN}}^{\mathrm{loc0}}[\mathbf{x}_a, \mathbf{p}_a] + F[\mathbf{x}_a, \mathbf{p}_a]\left(\ln\frac{r_{12}}{s} + C\right)$$

still contains an unknown constant C, entering Eq. (3.7). To determine it analytically we need a calculation which fully takes into account the transition between the near zone and the wave zone,

These spurious (IR) divergences are due to the split into regions (More soon…)

fixed by using results outside of PN

$$C = -\frac{1681}{1536}$$

PHYSICAL REVIEW D **93**, 084037 (2016)

Fokker action of nonspinning compact binaries at the fourth post-Newtonian approximation

Laura Bernard,[1,*] Luc Blanchet,[1,†] Alejandro Bohé,[2,‡] Guillaume Faye,[1,§] and Sylvain Marsat[3,4,]

we find that it differs from the recently published result derived within the ADM Hamiltonian formulation of general relativity [T. Damour, P. Jaranowski, and G. Schäfer, Phys. Rev. D **89**, 064058 (2014)]. More work is needed to understand this discrepancy.

PHYSICAL REVIEW D **93**, 084014 (2016)

Conservative dynamics of two-body systems at the fourth post-Newtonian approximation of general relativity

T. Damour, P. Jaranowski, and G. Schäfer,

(iii) several claims in a recent harmonic-coordinates Fokker-action computation [L. Bernard *et al.*, arXiv:1512.02876v2 [gr-qc]] are incorrect, but can be corrected by the addition of a couple of *ambiguity parameters* linked to subtleties in the regularization of infrared and ultraviolet

VII. SUGGESTION FOR ADDING MORE IR AMBIGUITY PARAMETERS IN REF. [21]

$$(a,b,c)^{\mathrm{new}}_{\mathrm{B^3FM}} = (a,b,c)_{\mathrm{B^3FM}} + \Delta C\,\frac{16}{15}\,(-11,12,0).$$

“But as always, we must pay for these logical simplifications by prolonging the chain of technical argument”

PHYSICAL REVIEW D **96**, 024062 (2017)

Apparent ambiguities in the post-Newtonian expansion for binary systems

Rafael A. Porto[1] and Ira Z. Rothstein[2]

$$V^{(n)}_{\rm pot}(\boldsymbol{q}) = \frac{A^{(n)}_{\rm pot}}{\epsilon_{\rm UV}} + \frac{B^{(n)}_{\rm pot}}{\epsilon_{\rm IR}} + f^{(n)}_{\rm pot}(\boldsymbol{q}, c_i^{(n)}, \mu) \qquad V^{(n)}_{\rm rad}(\omega) = \frac{A^{(n)}_{\rm rad}}{\epsilon_{\rm UV}} + f^{(n)}_{\rm rad}(\omega, \mu)\,,$$

- **Near zone UV removed by "counter-term":** $c_i^{\rm c.t.} \propto 1/\epsilon_{\rm UV}$
- **Spurious IR/UV divergences cancel out (after zero-bin):***

$$B^{(n)}_{\rm pot} = -A^{(n)}_{\rm rad}$$

$$V^{(n)}_{\rm pot} - V^{(n)}_{\rm zero-bin} + V^{(n)}_{\rm rad} \to V^{(n)}_{\rm tot}(\boldsymbol{q}, \omega, c_i^{(n)}(\mu), \mu)$$

VII. SUGGESTION FOR ADDING MORE IR AMBIGUITY PARAMETERS IN REF. [21]

$$(a,b,c)^{\rm new}_{\rm B^3FM} = (a,b,c)_{\rm B^3FM} + \Delta C\,\frac{16}{15}(-11,12,0).$$

T. Damour, P. Jaranowski, and G. Schäfer,

*** Zero-bin subtraction. In dim. reg. at 4PN:** $\epsilon_{\rm IR} \to \epsilon_{\rm UV}$

A. V. Manohar and I. W. Stewart, The zero-bin and mode factorization in quantum field theory, Phys. Rev. D **76**, 074002 (2007).

PHYSICAL REVIEW D **96**, 024062 (2017)

Apparent ambiguities in the post-Newtonian expansion for binary systems

?

Rafael A. Porto[1] and Ira Z. Rothstein[2]

$$V_{\rm pot}^{(n)}(\boldsymbol{q}) = \frac{A_{\rm pot}^{(n)}}{\epsilon_{\rm UV}} + \frac{-A_{\rm rad}^{(n)}}{\epsilon_{\rm IR}} + f_{\rm pot}^{(n)}(\boldsymbol{q}, c_i^{(n)}, \mu) \qquad V_{\rm rad}^{(n)}(\omega) = \frac{A_{\rm rad}^{(n)}}{\epsilon_{\rm UV}} + f_{\rm rad}^{(n)}(\omega, \mu)\,,$$

$$-\int dt\, V_{\rm rad}^{\rm 4PN} = \frac{2G_N^2 M}{5} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi}\, \omega^6\, I_-^{ij}(-\omega) I_+^{ij}(\omega) \left[-\frac{1}{(d-4)_{\rm UV}} - \gamma_E + \log\pi - \log\frac{\omega^2}{\mu^2} + \frac{41}{30} + i\pi\,{\rm sign}(\omega) \right].$$

Lamb Shift!
"log+5/6"

dissipative term

PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

PHYSICAL REVIEW D **96**, 024062 (2017)

Apparent ambiguities in the post-Newtonian expansion for binary systems

Rafael A. Porto[1] and Ira Z. Rothstein[2]

$$V^{(n)}_{\rm pot}(\boldsymbol{q}) = \frac{A^{(n)}_{\rm pot}}{\epsilon_{\rm UV}} + \frac{B^{(n)}_{\rm pot}}{\epsilon_{\rm IR}} + f^{(n)}_{\rm pot}(\boldsymbol{q}, c^{(n)}_i, \mu) \qquad V^{(n)}_{\rm rad}(\omega) = \frac{A^{(n)}_{\rm rad}}{\epsilon_{\rm UV}} + f^{(n)}_{\rm rad}(\omega,\mu)\,,$$

$$-\int dt\, V^{\rm 4PN}_{\rm rad} = \frac{2G_N^2 M}{5}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}\,\omega^6\, I^{ij}_-(-\omega) I^{ij}_+(\omega)\left[-\frac{1}{(d-4)_{\rm UV}} - \gamma_E + \log\pi - \log\frac{\omega^2}{\mu^2} + \frac{41}{30} + i\pi\,\mathrm{sign}(\omega)\right].$$

Lamb Shift!
"log+5/6"

dissipative term

PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

PHYSICAL REVIEW D **97**, 044023 (2018)

Ambiguity-free completion of the equations of motion of compact binary systems at the fourth post-Newtonian order

Tanguy Marchand,[1,2,*] Laura Bernard,[3,†] Luc Blanchet,[1,‡] and Guillaume Faye[1,§]

Remarkably, the value $\kappa = \frac{41}{60}$ we have obtained in our result for the tail agrees with the result found by Galley *et al* [10]

$$V^{(n)}_{\rm pot}(\boldsymbol{q}) = \frac{A^{(n)}_{\rm pot}}{\epsilon_{\rm UV}} + \frac{B^{(n)}_{\rm pot}}{\epsilon_{\rm IR}} + f^{(n)}_{\rm pot}(\boldsymbol{q}, c^{(n)}_i, \mu) \qquad V^{(n)}_{\rm rad}(\omega) = \frac{A^{(n)}_{\rm rad}}{\epsilon_{\rm UV}} + f^{(n)}_{\rm rad}(\omega, \mu)\,,$$

Extra Regulator

Only afterwards do we apply the limit $\varepsilon \to 0$ and look for the presence of poles $1/\varepsilon$. This regularization will be called the "$\varepsilon\eta$" regularization.

$$\boldsymbol{\xi}_1 = \frac{11}{3}\frac{G^2 m_1^2}{c^6}\left[\frac{1}{\varepsilon} - 2\ln\left(\frac{\overline{q}^{1/2} r'_1}{\ell_0}\right) - \frac{327}{1540}\right] \boldsymbol{a}^{(d)}_{1,\,\rm N} + \frac{1}{c^8}\boldsymbol{\xi}_{1,\,\rm 4PN}\,,$$

$$\frac{1}{\epsilon}\left(A_{\rm pot} + B_{\rm pot} + A_{\rm rad}\right)$$

does not cancel

Short- and long-distance poles & unphysical scales removed by world-line redef. iff combined

PHYSICAL REVIEW D **97**, 044023 (2018)

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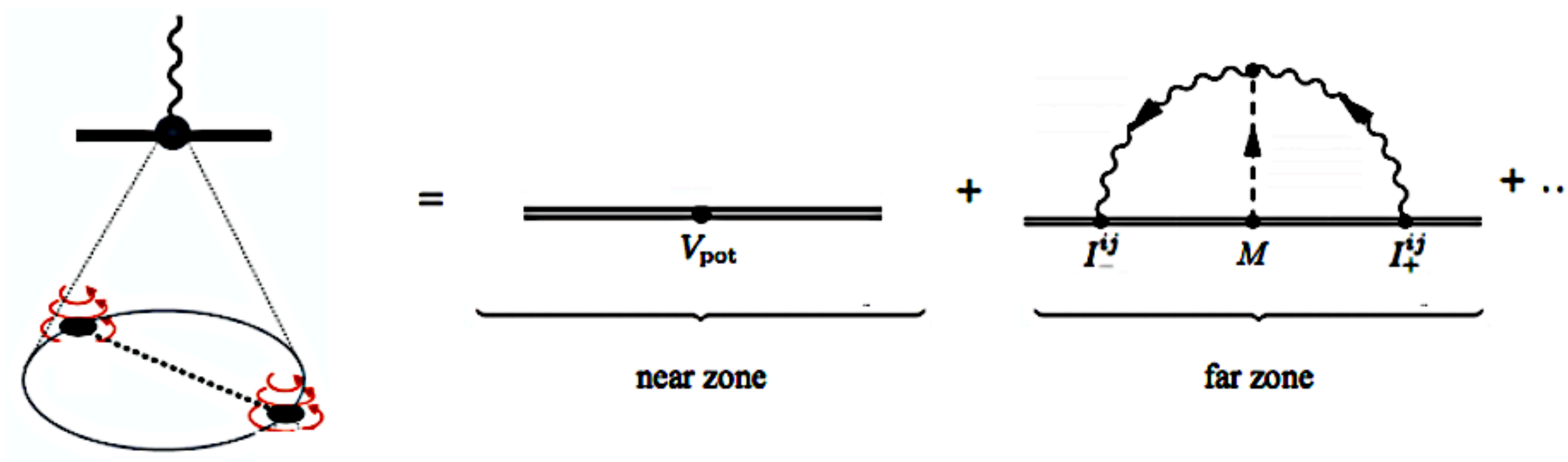
Tanguy Marchand,[1,2,*] Laura Bernard,[3,†] Luc Blanchet,[1,‡] and Guillaume Faye[1,§]

PHYSICAL REVIEW D **100**, 024048 (2019)

Conservative dynamics of binary systems to fourth post-Newtonian order in the EFT approach. II. Renormalized Lagrangian

Stefano Foffa,[1] Rafael A. Porto,[2,3] Ira Rothstein,[4] and Riccardo Sturani[5]

$$V^{(n)}_{\rm pot} - V^{(n)}_{\rm zero-bin} + V^{(n)}_{\rm rad} \to V^{(n)}_{\rm tot}(\boldsymbol{q}, \omega, c_i^{(n)}(\mu), \mu)$$



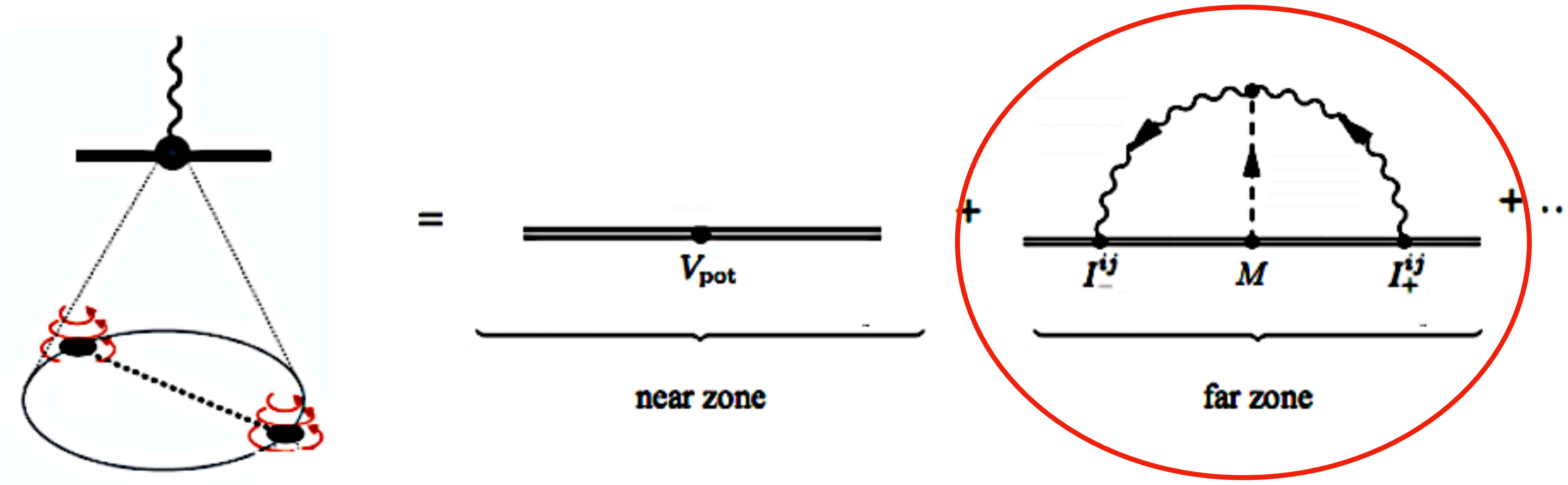
NO AMBIGUITIES NOR ADDITIONAL REGULATORS

IR/UV SPURIOUS POLES CANCEL OUT

PHYSICAL REVIEW D **96,** 024062 (2017)

Apparent ambiguities in the post-Newtonian expansion for binary systems

Rafael A. Porto[1] and Ira Z. Rothstein[2]



PHYSICAL REVIEW D **93,** 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

IR/UV cancelation There are no ambiguities!

$$\mu \frac{d}{d\mu} V_{\rm ren}(\mu) = \frac{2G_N^2 M}{5} I^{ij(3)}(t) I^{ij(3)}(t)$$

Universal log in binding energy

$$E_{\rm log} = -2G_N^2 M \langle I^{ij(3)}(t) I^{ij(3)}(t) \rangle \log v$$

PHYSICAL REVIEW D **96,** 024062 (2017)

Apparent ambiguities in the post-Newtonian expansion for binary systems

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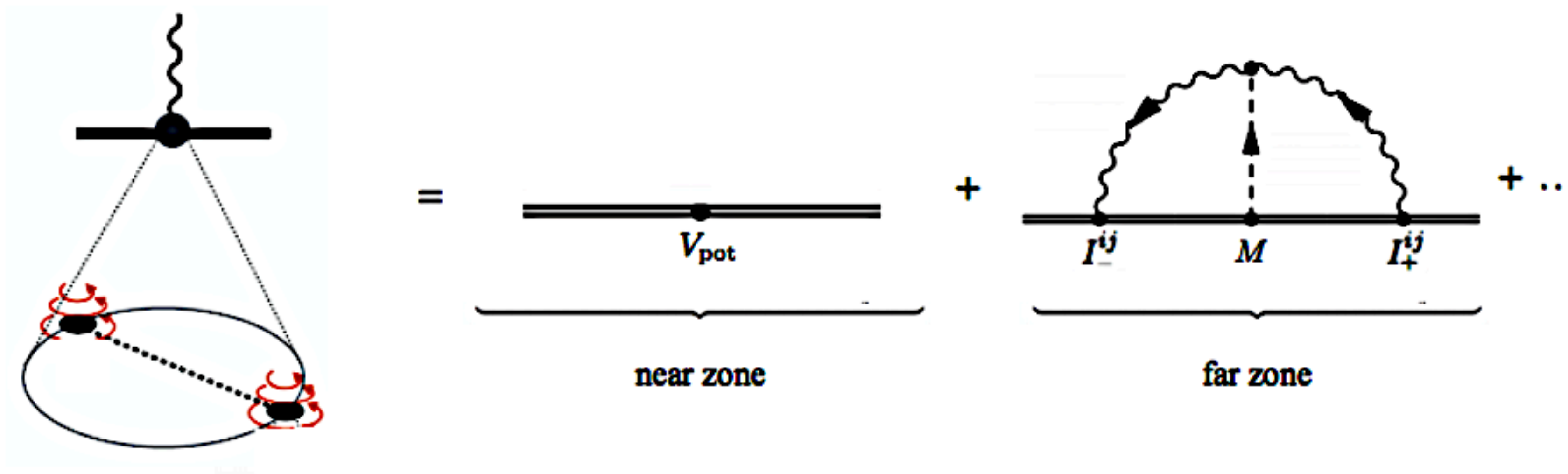


PHYSICAL REVIEW D **100,** 024048 (2019)

Conservative dynamics of binary systems to fourth post-Newtonian order in the EFT approach. II. Renormalized Lagrangian

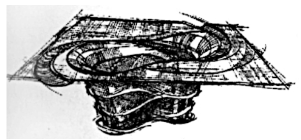
Stefano Foffa,[1] Rafael A. Porto,[2,3] Ira Rothstein,[4] and Riccardo Sturani[5]

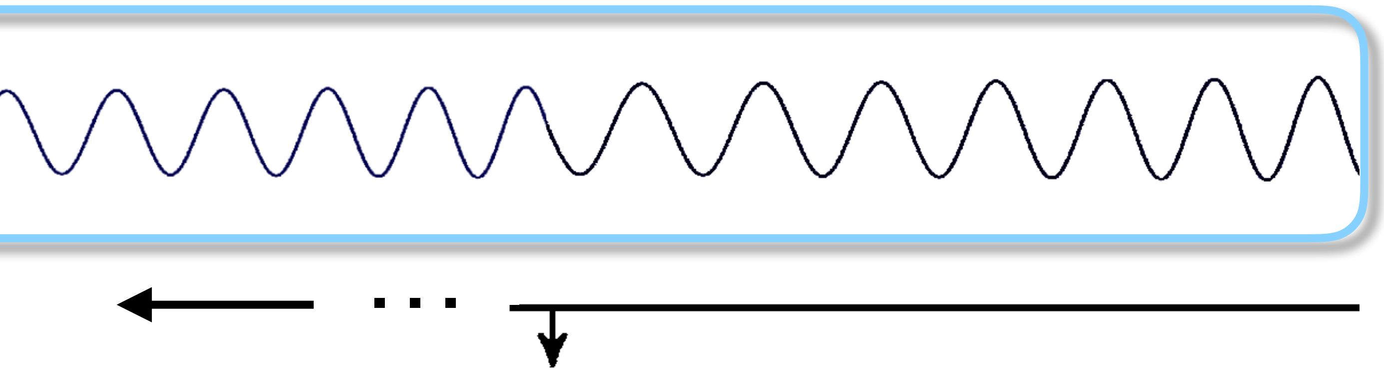
$$\underbrace{\left(\mathcal{L}^{\rm UV\,(near+self)}_{n\rm PN} + \mathcal{L}^{\rm c.t.\,(near)}_{n\rm PN}\right)}_{\text{near zone renormalization (via counter-terms)}} + \underbrace{\left(\mathcal{L}^{\rm UV\,(IR\,near+self-ZB)}_{n\rm PN} + \mathcal{L}^{\rm UV\,(far)}_{n\rm PN}\right)}_{\text{cancelation of near/far IR/UV spurious poles*}} \to \text{finite}\,,$$

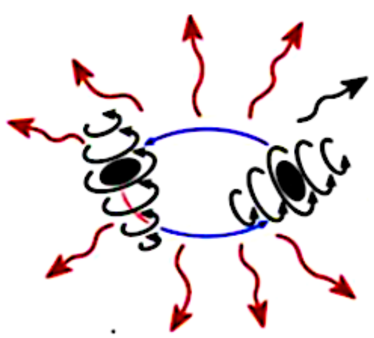
near zone renormalization (via counter-terms)

cancelation of near/far IR/UV spurious poles*

Ready for the future?





3.5PN order (almost 4PN)

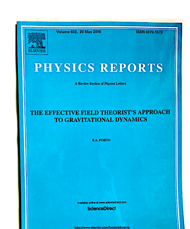
$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2}\right\}$$

The effective field theorist's approach to gravitational dynamics

Rafael A. Porto

Physics Reports

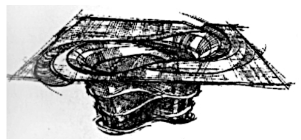
Volume 633, 20 May 2016, Pages 1-104

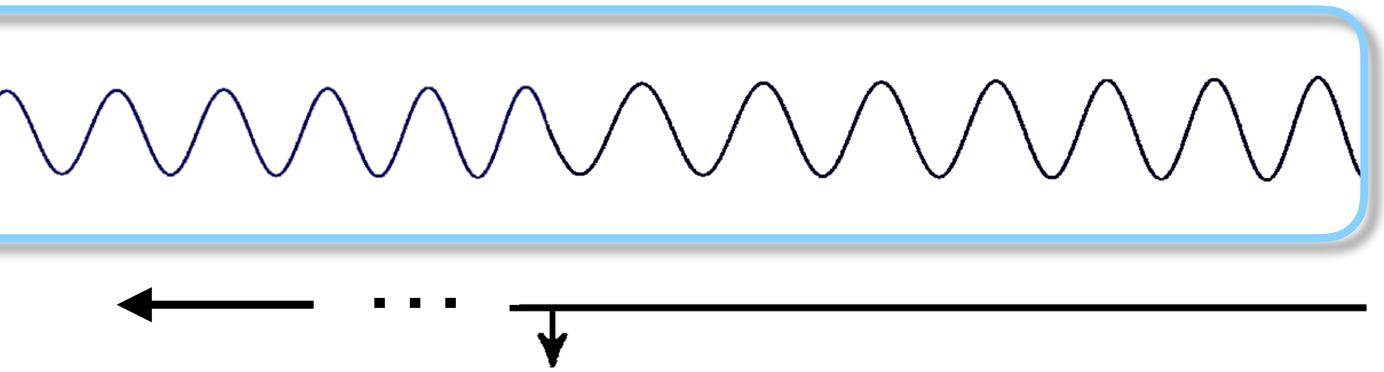


Blanchet, Damour, Faye et al. (harmonic)
Jaranowski, Schaefer, et al. (ADM)

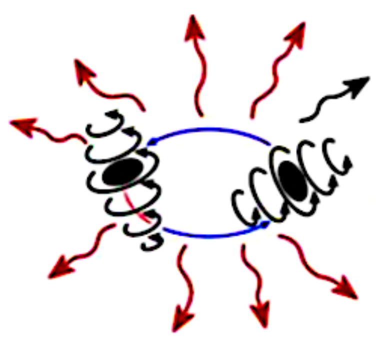
Goldberger, Rothstein, Ross, Foffa, Galley, Leibovich, Sturani, et al.

Theoretical uncertainties dominate over planned empirical reach





Not Good Enough

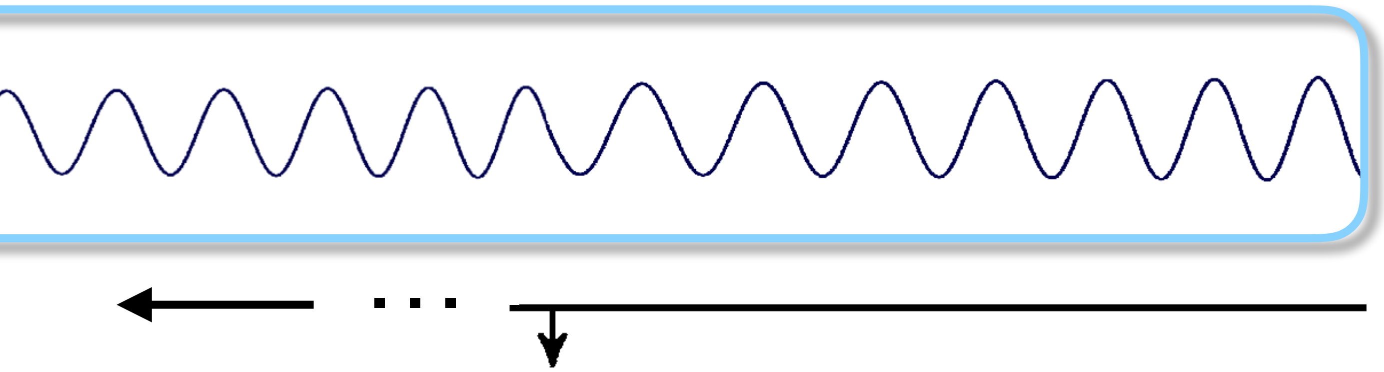


$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \overbrace{\cdots + [\cdots]x^{7/2}}\right\}$$

- **Gravitational-wave experiments on ground and in space** require **more accurate waveform** models: new **theoretical challenges** and **opportunities**.

A. Buonanno (QCD meets Gravity 18')

We haven't reached the analytic precision to distinguish between compact bodies!



'New Physics'
Threshold

$$Q_{ij} = C_E E_{ij}$$

$$C_E \sim R^5 \longrightarrow \left(\frac{R}{r}\right)^5 \sim v^{10}$$

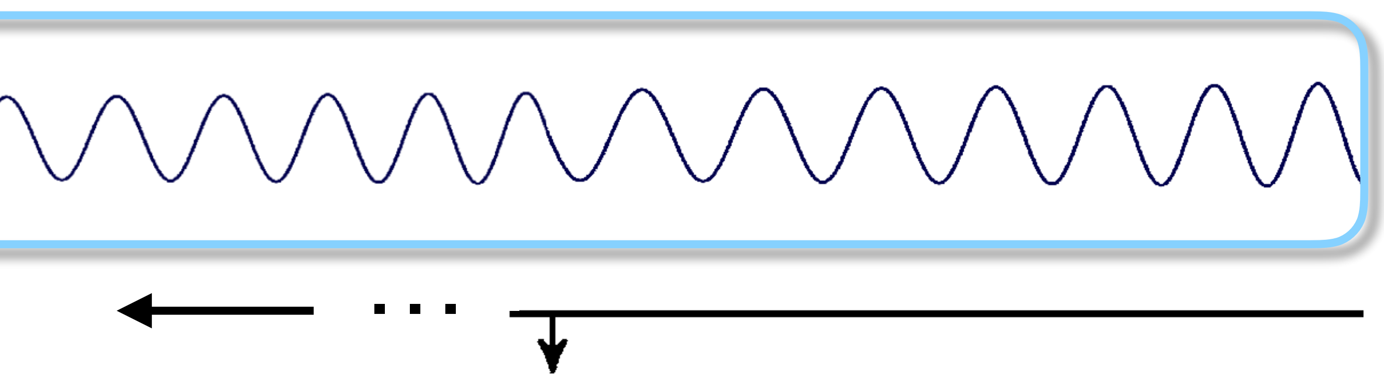
compact objects

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\}$$

N⁵LO N^5LO
5PN

$$\Psi(v) = \Psi_{\rm PP}(v) + \Psi_{\rm tidal}(v)$$

We haven't reached the analytic precision to distinguish between compact bodies!



'New Physics'
Threshold

$$C^{\rm bh}_{E(B)}(\mu) = 0$$

(vanishes at all scales!)

$$\frac{\dot\omega}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\}$$

N^5LO
5PN

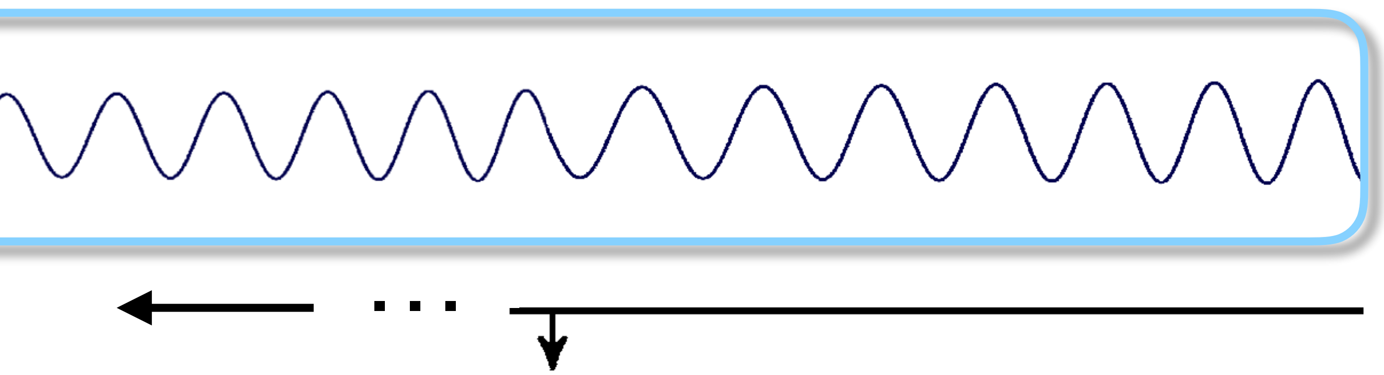
$$\Psi(v) = \Psi_{\rm PP}(v) +$$

Fortschr. Phys. 64, No. 10, 723–729 (2016) / DOI 10.1002/prop.201600064

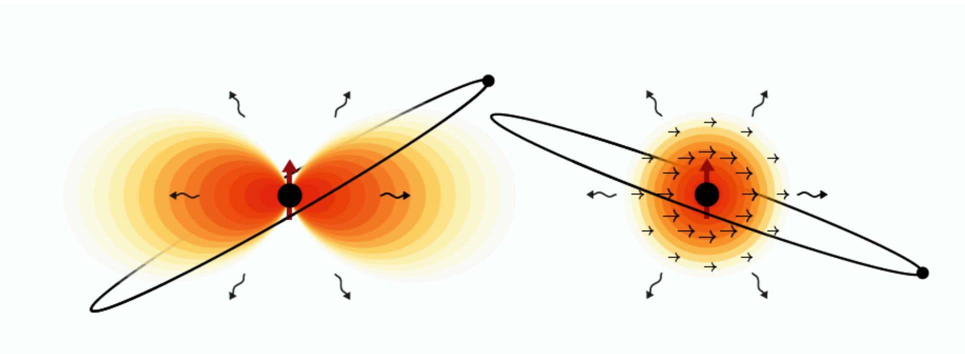
The tune of love and the *nature(ness)* of spacetime

*Rafael A. Porto**

We haven't reached the analytic precision to distinguish between compact bodies!



'New Physics'
Threshold



Probing ultralight bosons with binary black holes

Daniel Baumann, Horng Sheng Chia, and Rafael A. Porto

Phys. Rev. D 99, 044001 (2019)

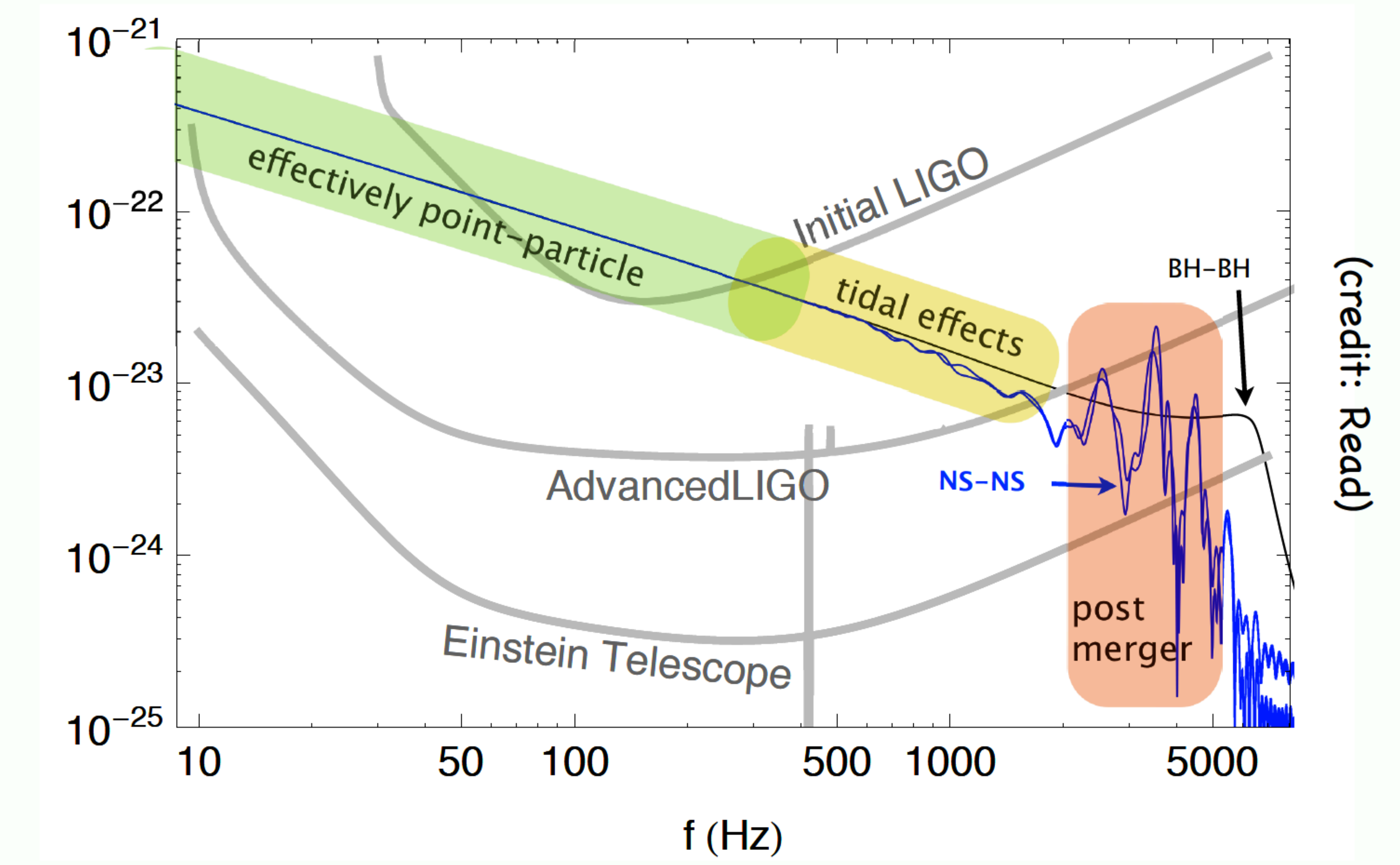
Published February 4, 2019

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\left\{1 + \cdots + [\cdots]x^{7/2} + \mathcal{O}(x^4) + \mathcal{O}(x^5)\right\}$$

N^5LO
5PN

$$\Psi(v) = \Psi_{\rm PP}(v) + \Psi_{\rm tidal}(v)$$

Black Holes Could Reveal New Ultralight Particles



"Waveforms will be far more complex and carry more information than expected. Improved modeling will be needed for extracting the GW's information" 1993

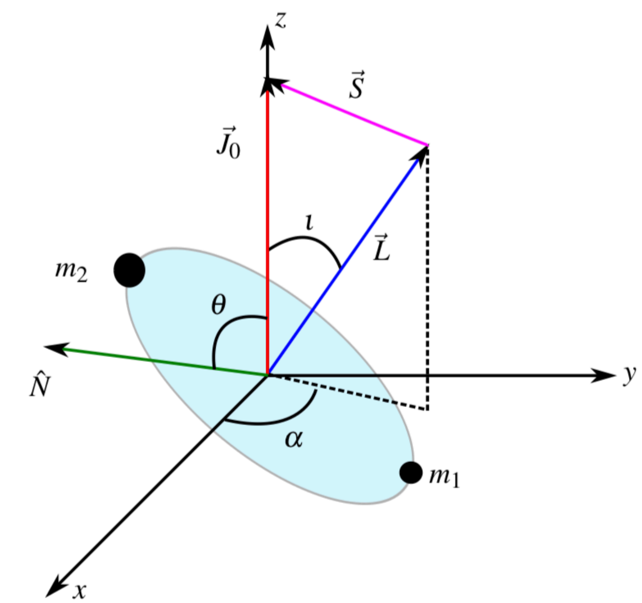
Kip Thorne 'The last 3 minutes' paper

20+ years prior to first detection!

The last three minutes: Issues in gravitational-wave measurements of coalescing compact binaries

Curt Cutler, Theocharis A. Apostolatos, Lars Bildsten, Lee Smauel Finn, Eanna E. Flanagan, Daniel Kennefick, Dragoljub M. Markovic, Amos Ori, Eric Poisson, Gerald Jay Sussman, and Kip S. Thorne
Phys. Rev. Lett. **70**, 2984 – Published 17 May 1993

$$\frac{d\mathcal{N}_{\mathrm{cyc}}}{d\ln f} = \frac{5}{96\pi}\frac{1}{\mu M^{2/3}(\pi f)^{5/3}}\left\{1 + \left(\frac{743}{336} + \frac{11}{4}\frac{\mu}{M}\right)x - [4\pi + \mathrm{S.O.}]x^{1.5} + [\mathrm{S.S.}]x^2 + O(x^{2.5})\right\}.$$



Knowledge at the time

PHYSICAL REVIEW D **100**, 024048 (2019)

Conservative dynamics of binary systems to fourth post-Newtonian order in the EFT approach. II. Renormalized Lagrangian

Stefano Foffa,[1] Rafael A. Porto,[2,3] Ira Rothstein,[4] and Riccardo Sturani[5]

$$\frac{d\mathcal{N}_{\rm cyc}}{d\ln f} = \frac{5}{96\pi}\frac{1}{\mu M^{2/3}(\pi f)^{5/3}}\left\{1+\left(\frac{743}{336}+\frac{11}{4}\frac{\mu}{M}\right)x\right.$$
$$\left. -[4\pi + \text{S.O.}]x^{1.5} + [\text{S.S.}]x^2 + [\text{S.O.}]x^{2.5} + [\text{S.S.}]x^3 + O(x^4)\right.$$

"… + Log + 41/30"

(No ambiguities)

PHYSICAL REVIEW D **93**, 124010 (2016)

Tail effect in gravitational radiation reaction: Time nonlocality and renormalization group evolution

Chad R. Galley,[1] Adam K. Leibovich,[2] Rafael A. Porto,[3] and Andreas Ross[4]

PHYSICAL REVIEW D **96**, 024062 (2017)

Apparent ambiguities in the post-Newtonian expansion for binary systems

Rafael A. Porto[1] and Ira Z. Rothstein[2]

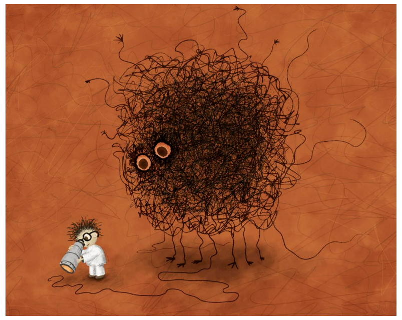
Damour Jaranowski Schafer (…, 2013-2018)
Blanchet Faye Iyer, et al. (…, 2015-2018)

Are we ready?

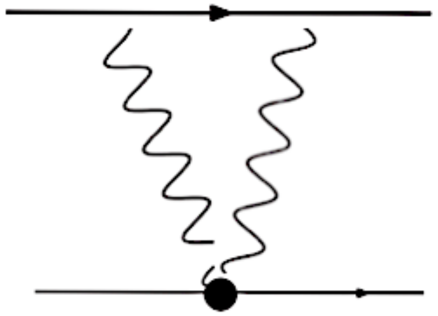
$$\frac{d\mathcal{N}_{\rm cyc}}{d\ln f} = \frac{5}{96\pi}\frac{1}{\mu M^{2/3}(\pi f)^{5/3}}\left\{1+\left(\frac{743}{336}+\frac{11}{4}\frac{\mu}{M}\right)x\right.$$
$$\left. -[4\pi + \text{S.O.}]x^{1.5} + [\text{S.S.}]x^2 + [\text{S.O.}]x^{2.5} + [\text{S.S.}]x^3 + O(x^4) + O(x^5)\right\}.$$

GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral

Gravitational-wave observations alone are able to measure the masses of the two objects and set a lower limit on their compactness, but the results presented here do not exclude objects more compact than neutron stars such as quark stars, black holes, or more exotic objects [57–61].



Non-trivial
operator at 5PN



$c_E E_{ij}^2$

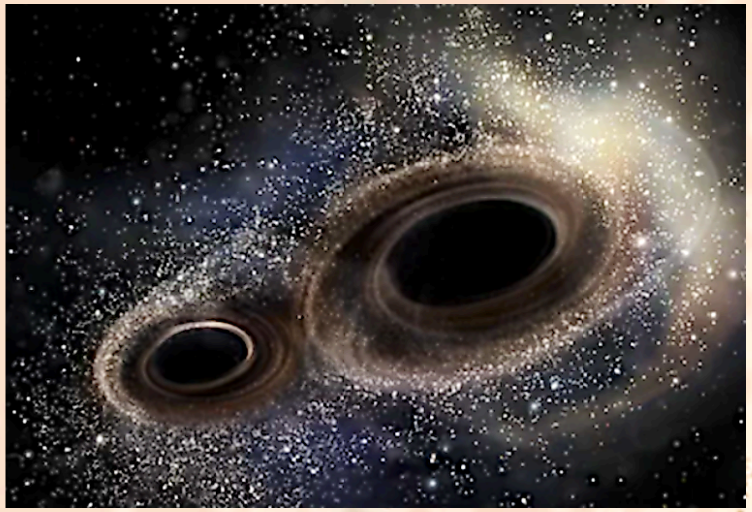
Videnskab

no.203.078 01.01.203X

EinsTein Reloaded!

New era of foundational investigations established through GWPD.

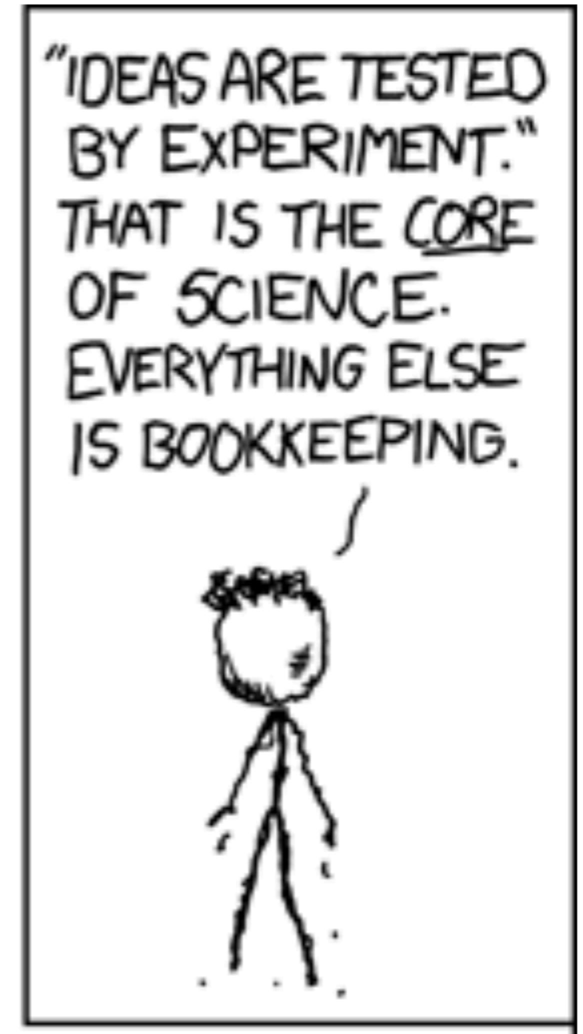
$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\nu x^{5/2}\Big\{1 + \cdots + [\cdots]x^{7/2} + [\cdots]x^4 + [\cdots]x^5\Big\}$$



New particles discovered!
New objects found!
Neutron stars unveiled!

Experts Clash Over Project To Detect Gravity Wave

Physicists say device could help them fathom black holes, but others fault its price.



Extra Slides

"New directions in science are launched by new tools much more often than by new concepts. The effect of a concept-driven revolution is to explain old things in a new way. The effect of a tool-driven revolution is to discover new things that have to be explained"

Freeman Dyson, "Imagined Worlds"

QUANTUM THEORY OF GRAVITATION*

By R. P. Feynman

(Received July 3, 1963)

Møller: May I, as a non-expert, ask you a very simple and perhaps foolish question. Is this theory really Einstein's theory of gravitation in the sense that if you would have here many gravitons the equations would go over into the usual field equations of Einstein?

Feynman: Absolutely.

[...] gravitational radiation when two stars — excuse me, two particles — go by each other, to any order you want (not for stars, then they have to be particles of specified properties; because obviously the rate of radiation of the gravity depends on the give of the starstides are produced). If you do a real problem with real physical things in in then I'm sure we have the right method that belongs to the gravity theory. There's no question about that.

5PN threshold!

Conservative dynamics of binary systems to fourth Post-Newtonian order in the EFT approach II: Renormalized Lagrangian

Stefano Foffa,[1] Rafael A. Porto,[2,3] Ira Rothstein,[4] and Riccardo Sturani[5]

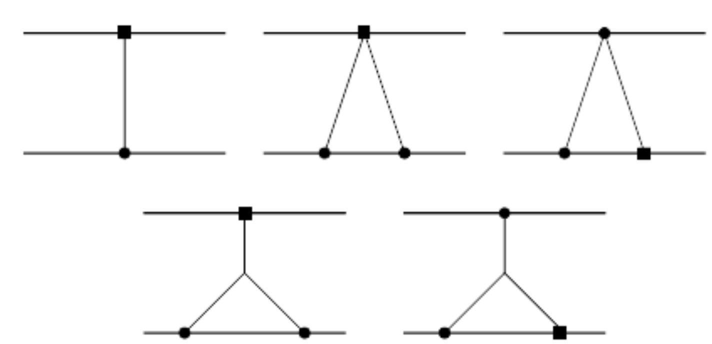
$$\left(\mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(near+self)}} + \mathcal{L}_{n\mathrm{PN}}^{\mathrm{c.t.\,(near)}}\right) + \left(\mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(IR\,near+self-ZB)}} + \mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(far)}}\right) \to \text{finite}\,,$$

$$S_{\rm pp}[x^\alpha_a(\tau_a)] = \sum_a \int d\tau_a \left(-m_a + \sum_i c_i \mathcal{O}_i\left[x^\alpha_a(\tau_a), \dot{x}^\alpha_a(\tau_a), \cdots ; g_{\mu\nu}, \partial_\beta g_{\mu\nu}, \cdots\right]\right)$$

diff invariance + RPI (in dim. reg.)

Effective action to 4PN order:

$$\begin{aligned} S_{\rm pp}[x^\alpha_a(\tau_a)] \;=\; & \sum_a \int d\tau_a \Bigg[-m_a + \left(c^{(a)}_{a\dot{v},\,\rm ren}(\mu) - \frac{11}{3}\frac{G^2 m_a^2}{\epsilon_{\rm UV}}\right) g_{\mu\nu} a^\mu_a \dot{v}^\nu_a \\ & + \left(c^{(a)}_{V,\,\rm ren}(\mu) + \frac{G^2 m_a^2}{\epsilon_{\rm UV}}\right) R_{\mu\nu} v^\mu_a v^\nu_a\Bigg]\,. \end{aligned}$$



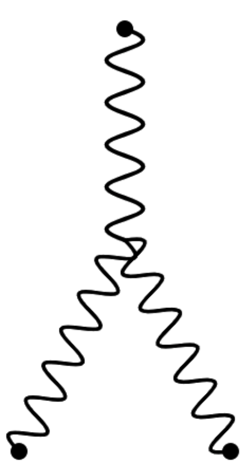
The operators beyond minimal coupling can be **removed by field-redefinitions** until 5PN (no spin)

No renormalization scheme-dependence (no UV ambiguities)

Conservative dynamics of binary systems to fourth Post-Newtonian order in the EFT approach II: Renormalized Lagrangian

Stefano Foffa,[1] Rafael A. Porto,[2,3] Ira Rothstein,[4] and Riccardo Sturani[5]

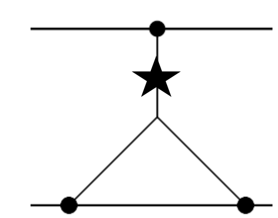
$$\left(\mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(near+self)}} + \mathcal{L}_{n\mathrm{PN}}^{\mathrm{c.t.\,(near)}}\right) + \left(\mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(IR\,near+self-ZB)}} + \mathcal{L}_{n\mathrm{PN}}^{\mathrm{UV\,(far)}}\right) \to \mathrm{finite}\,,$$



Potential $\quad \boldsymbol{a}_1^i \boldsymbol{a}_2^j \int dt \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{\boldsymbol{p}^i \boldsymbol{p}^j}{|\boldsymbol{p}|^5} e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}_1(t)-\boldsymbol{x}_2(t))}$

$$\frac{1}{p_0^2 - p^2} = \frac{1}{p^2}\left(-1 + p_0^2/p^2 + \cdots\right)$$

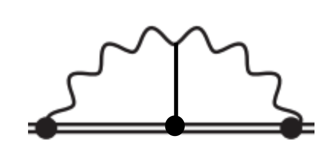
★



IR divergent

cancel out!*

Radiation $\quad \int \frac{d\omega}{\pi} \int \frac{d^3\boldsymbol{p}}{(2\pi)^3} \frac{d^3\boldsymbol{k}}{(2\pi)^3} \boldsymbol{x}_1^i(\omega)\boldsymbol{x}_1^j(\omega) \frac{\boldsymbol{k}^i\boldsymbol{k}^j}{\boldsymbol{p}^2 k^2 (p+k)^2}$



UV divergent

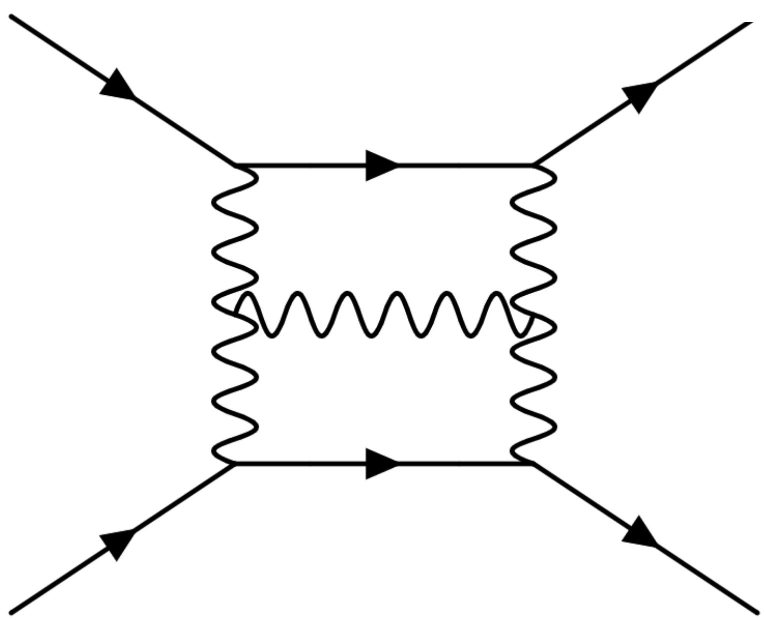
***Zero-bin subtraction (scale-less integrals)**

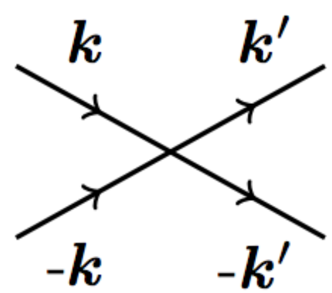
$$I_{\mathrm{ZB}}[n_1, n_2] = \int_{\boldsymbol{k}} \frac{1}{[\boldsymbol{k}^2]^{n_1}[\boldsymbol{p}^2]^{n_2}} \xrightarrow{(n_1=3/2,\,n_2=1/2)} |\boldsymbol{p}|^{-1} \int_{\boldsymbol{k}} \frac{1}{\boldsymbol{k}^3} = \frac{i}{16\pi|\boldsymbol{p}|}\left(\frac{1}{\epsilon_{\mathrm{UV}}} - \frac{1}{\epsilon_{\mathrm{IR}}}\right)$$

Neill Rothstein (2013)
Cheung Rothstein Solon (2017)
Bern et al. (2019)
...

Amplitudes

$$\int D[\mathrm{matter}]\, Dh\, e^{i(S_{\mathrm{EH}}[h]+S[h,\mathrm{matter}])}$$

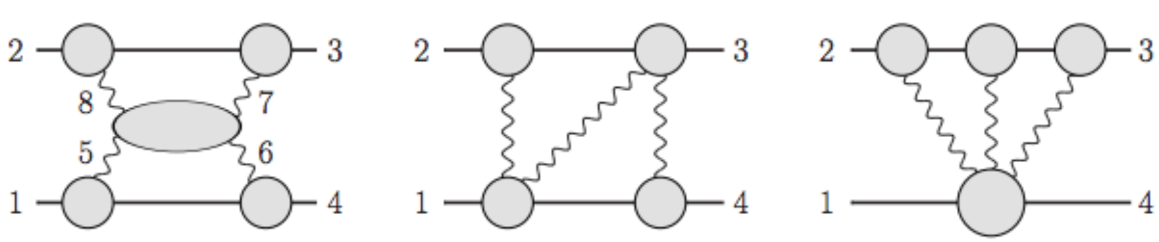




$$= -iV(\boldsymbol{k},\boldsymbol{k}') = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

3PM (G^3) — Subset of full 2PN

Bern et al. (2019)



PRECISION GRAVITY: FROM THE LHC TO LISA

26 August - 20 September 2019

MIAPP Munich Institute for Astro- and Particle Physics

John Joseph Carrasco, Ilya Mandel, Donal O'Connell, Rafael Porto, Fabian Schmidt

'That's nice, but what can you do with it?'

$$V_1(\mathbf{r}) = -\frac{Gm_1m_2}{r^2}\left[1+\left(4+\frac{3m_1}{2m_2}+\frac{3m_2}{2m_1}\right)\frac{\mathbf{P}^2}{m_1m_2c^2}\right]$$
$$+G\left(1+\frac{3m_2}{4m_1}\right)\frac{\hbar\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{r}\times\mathbf{P})}{c^2r^3}$$
$$+G\left(1+\frac{3m_1}{4m_2}\right)\frac{\hbar\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{r}\times\mathbf{P})}{c^2r^3}$$
$$+\frac{G\hbar^2}{4c^2r^3}\left(\frac{3(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{r})(\boldsymbol{\sigma}^{(2)}\cdot\mathbf{r})}{r^2}-\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\right)$$
$$+\frac{4\pi G\hbar^2}{c^2}\left(1+\frac{3m_2}{8m_1}+\frac{3m_1}{8m_2}\right)\delta(\mathbf{r})$$
$$+\frac{2\pi G\hbar^2}{3c^2}(\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})\delta(\mathbf{r}).$$

Derivation of the Equations of Motion of a Gyroscope from the Quantum Theory of Gravitation

B. M. Barker
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803

AND

R. F. O'Connell
Institute of Theoretical Astronomy, University of Cambridge, Cambridge, England
and
Department of Physics and Astronomy, Louisiana State University, Baton Rouge, Louisiana 70803

(Received 1 July 1970)

PHYSICAL REVIEW D **73**, 104031 (2006)

Post-Newtonian corrections to the motion of spinning bodies in nonrelativistic general relativity

Rafael A. Porto

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Calculation of the First Nonlinear Contribution to the General-Relativistic Spin-Spin Interaction for Binary Systems

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