Gravitational waves & exploring possible deviations in strong field/dynamical scenarios

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outline

- GWs provide important opportunities to explore gravity in rather ‘controlled/clean’ scenarios
- Some comments on null GR tests vs search for deviations
- Examples, implications and challenges

"Using a term like nonlinear science is like referring to the bulk of zoology as the study of non-elephant animals." —Stanislaw Ulam

Eg in GR:

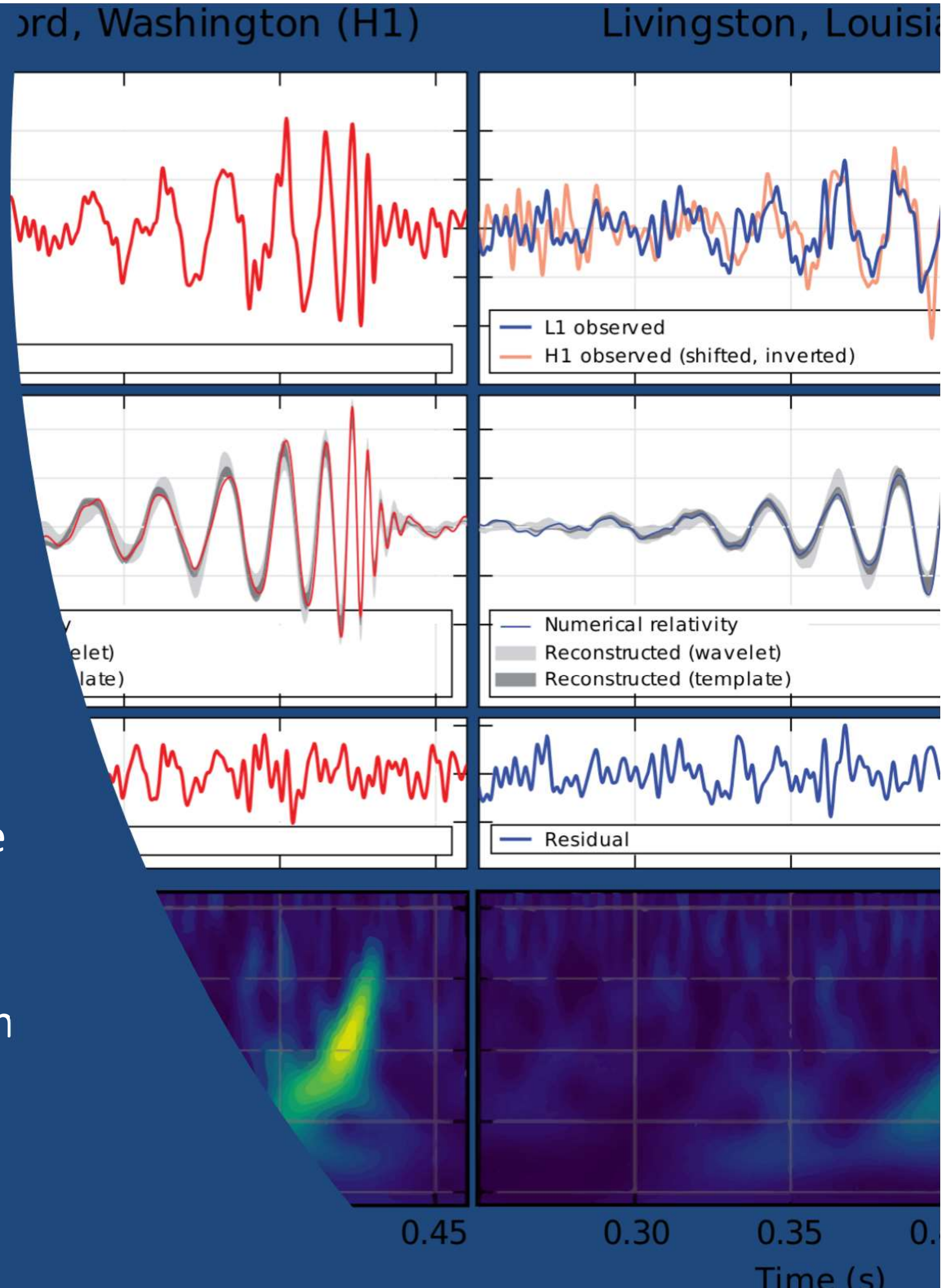
- *QNMs in BHs, modal stability*, non-linear stability
- singularity theorems vs stability of Minkowski

Sandbox rules:

- – GW150914 happened!
- – Do GWs behave as in GR?
- – Digging for ‘what else’ and what it means

- Surprises in GR, with other objects or in other theories.
  - What are ideal corners for exploring ‘extremes’?
  - How to dig as deep as possible in testing whichever option?

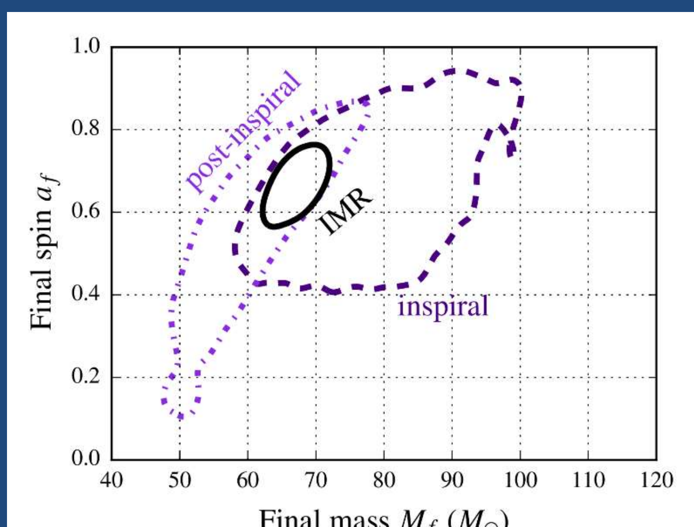
- For extensions of GR, far less is known
  - Can such knowledge be obtained?

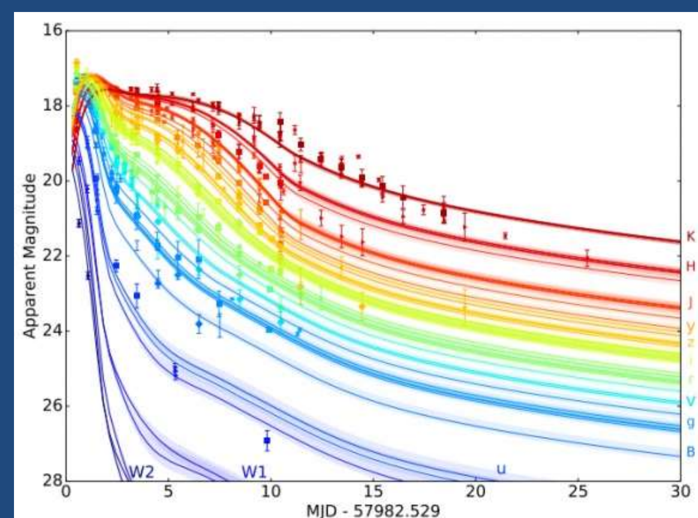


ANATOMY

INSPIRAL MERGER POST-MERGER

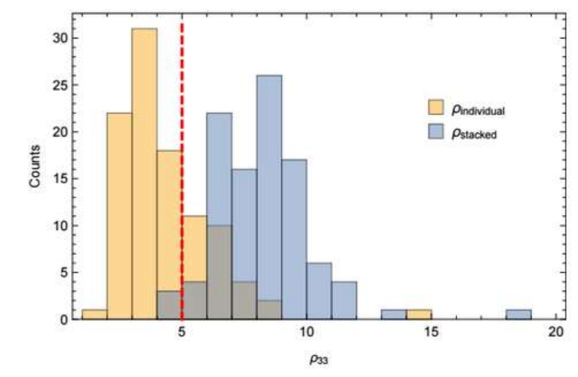
- Inspiral parameters → (M_f , {a_f , Ω_J}, {δm_i})





Hunting for deviations?

- Stress-test implications of General Relativity: e.g. full I-M-R; decay predictions of (predicted) BHs.
  - High SNR required: wait for golden signals/better detectors, or exploit 'mappable' predictions and stack multiple events [Yang etal]
  - Understand full zoo of possibilities. What maps?
  - If not exactly as in GR (for BH,NS)? Residuals are a way to go, but in a suboptimal way



- Beyond BHs/NSs, still lots to understand on dynamics of alternatives

- Beyond GR, gain insights of possible departures from relevant (?) theories in the non-linear regime
  - Too many options! Constraints? Restrict to 2nd order eqns (to avoid Ostrogradsky's ghosts?
  - Approach: treat extensions to GR within an EFT philosophy $\mathcal{L} = \mathcal{R} + ??$

Why any particular: ‘beyond Einstein theory’ for compact object mergers? Or beyond standard compact objects?

- Critique of this line of work could be that since there is no need yet for modified gravity for {BH,NS} mergers. No ‘evidence’ for anytahing other than BHs/NSs. Are we simply inventing problems to solve?

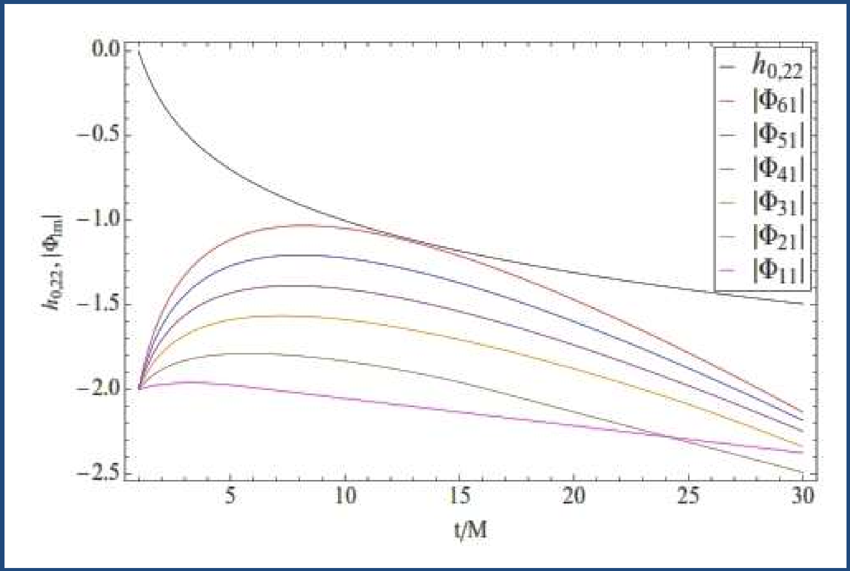
- On the other hand, one motivation is of theoretical interest even if GR turns to be correct up to any level GW observations can probe *:*

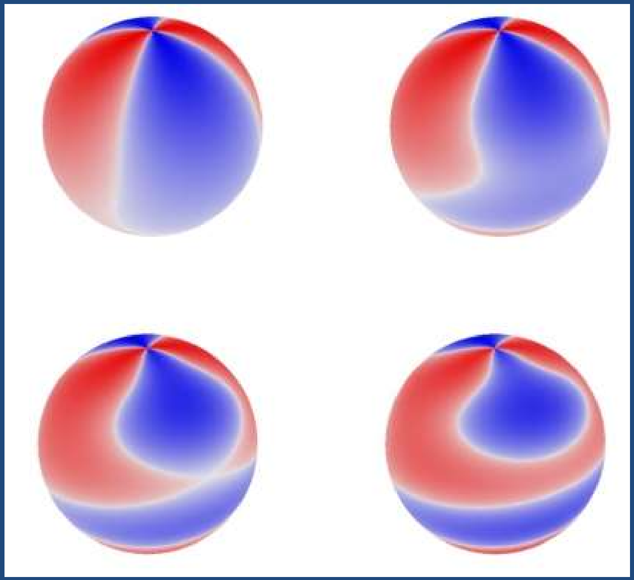
  To learn what mergers events could conceivably look like in a generic metric theory of gravity that has {BH,NS} solutions. Or, what other objects might lurke out there (or deviate BHs/NSs)

- *Last,* GR is built upon ‘fundamental’ principles, testing basic tenants has been a fruitful enterprise in physics throughout history

Potential 'surprises' already in GR

- After-merger ringdown in (highly spinning) binary black holes → non-linearities 'linger' as decay rate is slower and can significantly alter late-time behavior expectations [Yang-Zimmerman-LL '15]

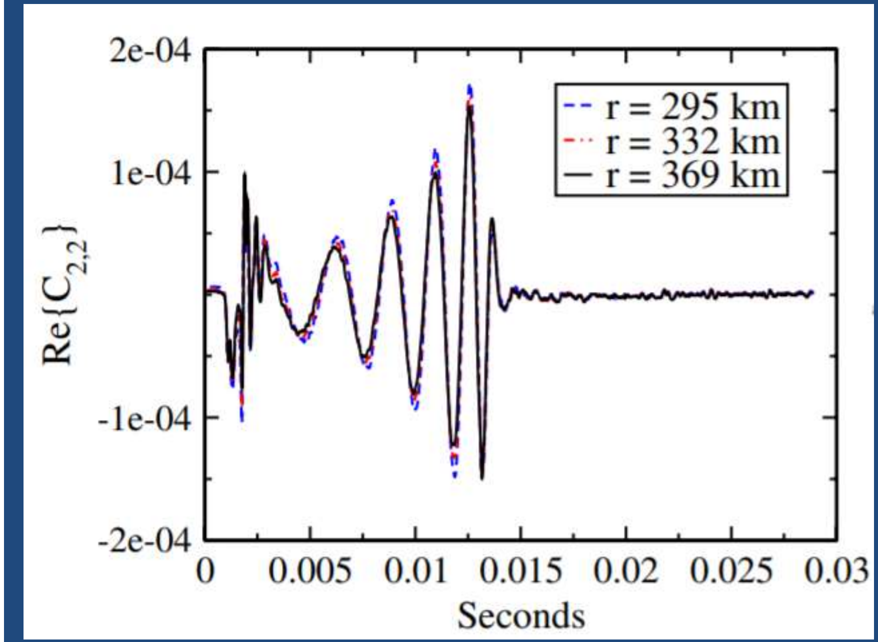


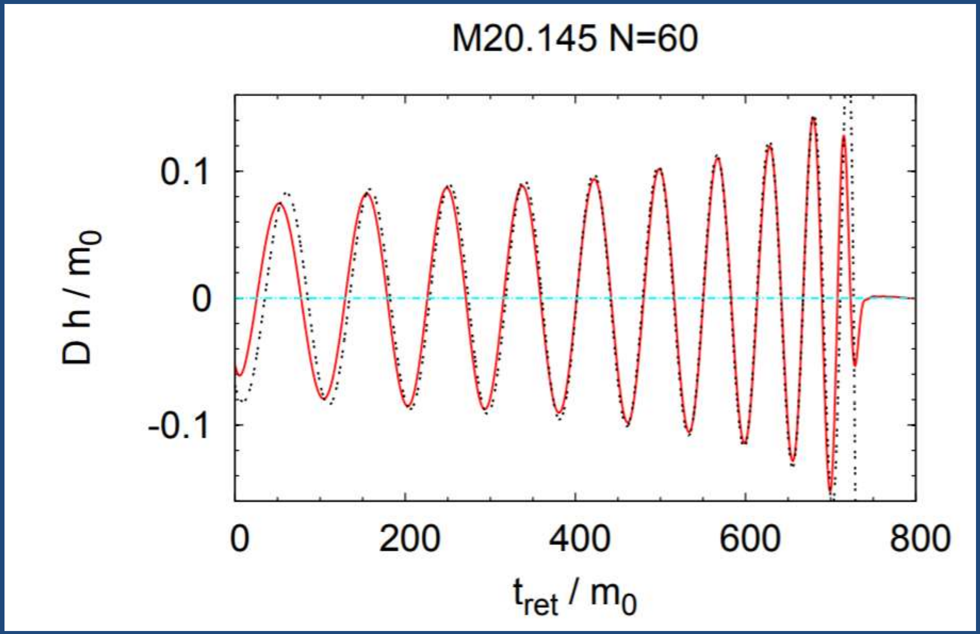


- *Connection with high Reynold's number behavior in hydrodynamics*

Potential 'surprises' already in GR

- After-merger *ringdown in particular cases* of BH-NS mergers: final BH will not exhibit naïve exponential decay

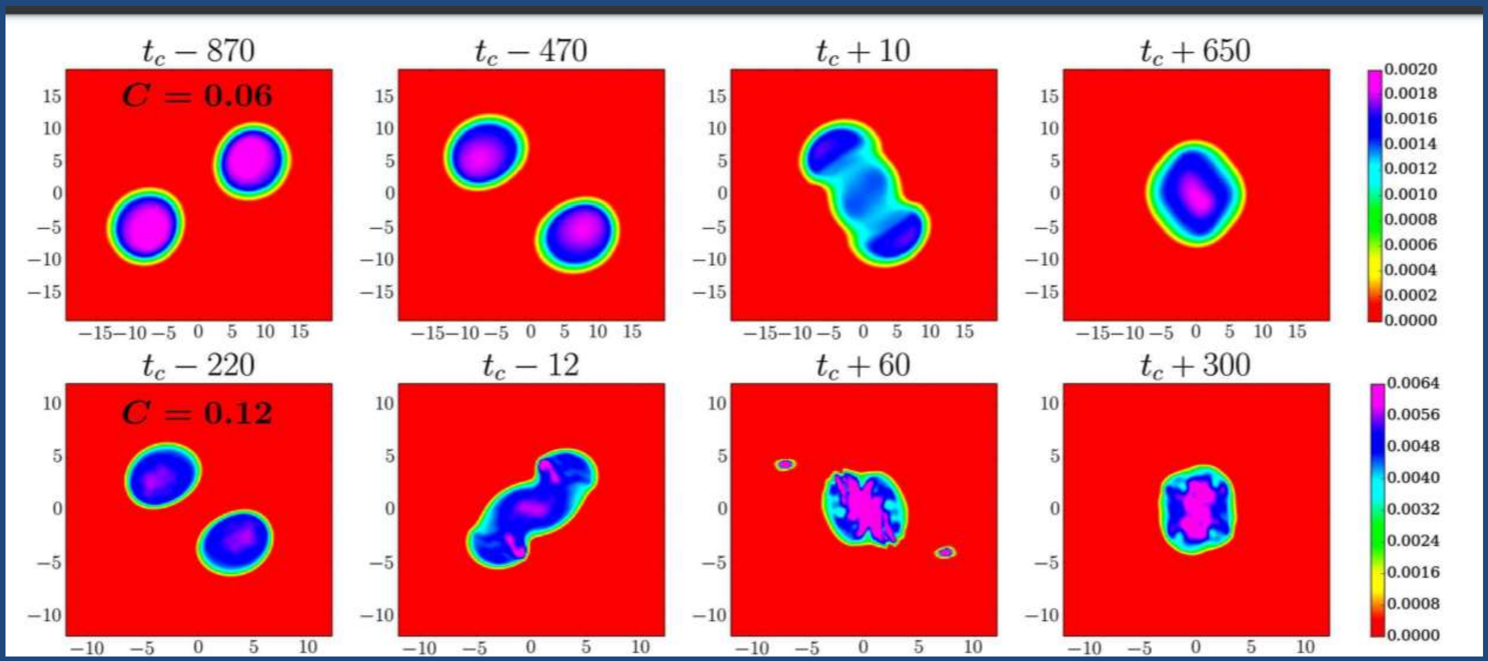




QNM can be 'squashed' by incoherent emission driven by accretion.

Potential 'surprises' already in GR

- After-merger ringdown waveforms implying post-merger object considerable angular momentum 'missing'!



- But I cheated…boson stars collided here…BUT… they can mimic BBHs

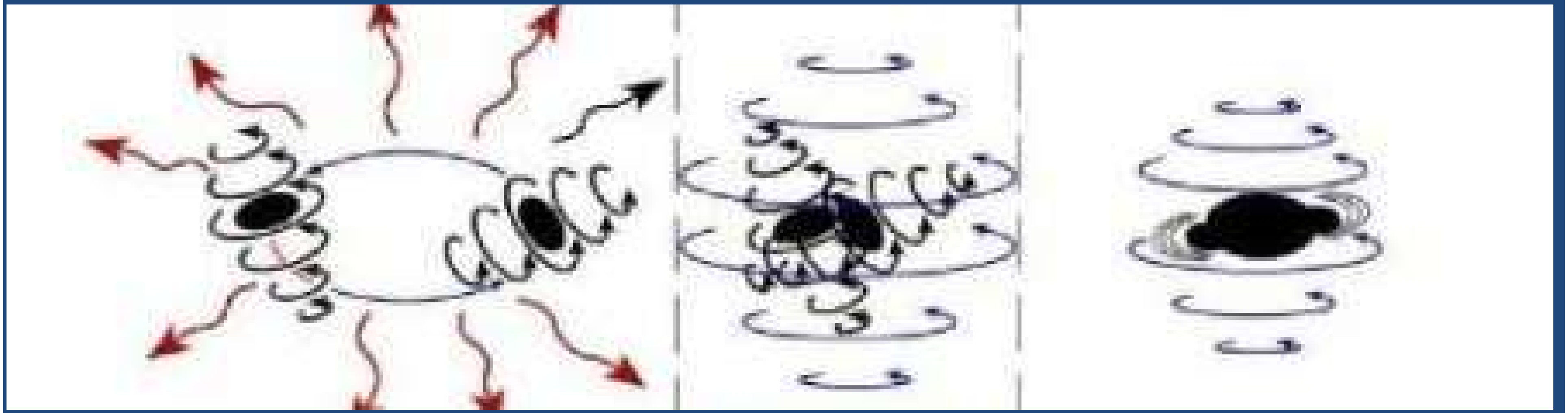
Let’s pause…

- Already at the GR level, with standard compact objects $\rightarrow$ waveforms might be slightly less ‘standard’

- What else can happen if we depart from the traditional theory/objects?
  - But where to go? What are interesting *and* viable options?

Back to merger anatomy (2000's déjà vu)

INSPIRAL MERGER POST-MERGER



Beyond standards…

- COs: Beyond BHs & NSs, only ‘boson stars’ are sufficiently well defined alternative compact objects [though there is a long-list of ‘wishful alternatives’]

- Grav theory: Beyond (a broad set of) Scalar-Tensor; Scalar-Vector-Tensor theories, the rest are interestingly motivated but often not known to be *‘viable’*
  - 2nd order theories, Horndenski family → free of Ostrogradski ghosts
  - Should one consider going beyond 2nd order? Natural to expect curvature corrections

Beyond GR ?

- Restricting to theories known to allow for well-posed problems. I.e. those where one can show $|u(T)| \leq ae^{bt}|u(0)|$
- Few options known to be amenable to well defined initial (boundary) value problems. Examples: Scalar-Vector-Tensor theories.

Scalar-Tensor (ST) {many incarnations}

$$S = \int d^4x \frac{\sqrt{-g}}{2\kappa}\left[\phi R - \frac{\omega(\phi)}{\phi}\partial_\mu\phi\partial^\mu\phi\right] + S_M[g_{\mu\nu},\psi]$$

$$S^{\mathrm{E}} = \int d^4x\,\sqrt{-g^{\mathrm{E}}}\left(\frac{M_{\mathrm{Pl}}^2}{2}R^{\mathrm{E}} - \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - V(\varphi)\right) + S^{\mathrm{E,\,M}}[g^{\mathrm{E}}_{\mu\nu}\,\phi(\varphi)^{-1},\psi]$$

Scalar-Vector-Tensor (EMD)

$$S = \int d^4x\sqrt{-\tilde{g}}\,e^{-2\phi}\left[R + \Lambda + 4(\nabla\phi)^2 - F^2 - \frac{H^2}{12}\right]$$

Exploring beyond 'a' theory…

- e.g. Scalar tensor

$$G^E_{\mu\nu} = \kappa\left(T^{\varphi}_{\mu\nu} + T^E_{\mu\nu}\right),$$

$$\Box^E \varphi = \frac{1}{2}\frac{\mathrm{d}\log\phi}{\mathrm{d}\varphi} T_E\,,$$

$$\nabla^E_\mu T^{\mu\nu}_E = -\frac{1}{2} T_E \frac{\mathrm{d}\log\phi}{\mathrm{d}\varphi} g^{\mu\nu}_E \partial_\mu \varphi\,,$$

where

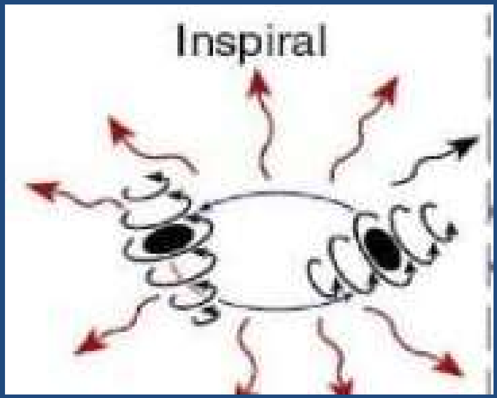
$$T^{\mu\nu}_E = \frac{2}{\sqrt{-g^E}}\frac{\delta S_M}{\delta g^E_{\mu\nu}} \quad \text{and}$$

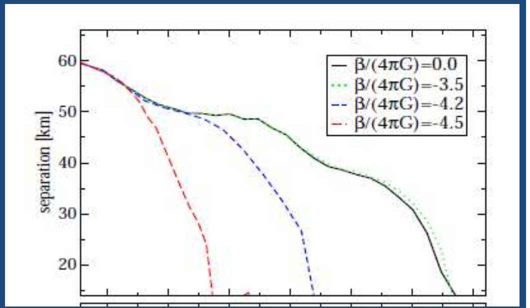
$$T^{\varphi}_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - \frac{g^E_{\mu\nu}}{2} g^{\alpha\beta}_E \partial_\alpha\varphi\partial_\beta\varphi$$

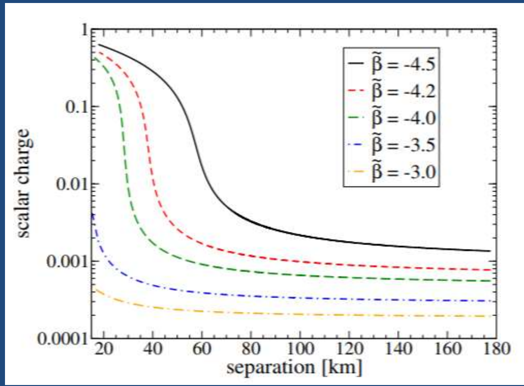
Inspiral

- Inspiral: → in GR, 'internal structure' can (to a degree) be encoded in tidal effects through PN approach

- Beyond GR? New phenomena depending on the theory: dipolar radn, phase transition (G->G_{new}), energy loss in other channels [e.g. 3PN vs 5PN internal structure effects]
  - Behavior can be captured with PN or PPE like approaches, but must take into account effects need not be monotonic in freqn
  - Transition to merger and connection with final state?



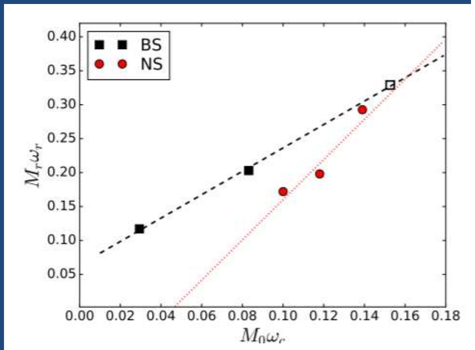




[Barausse+,Palenzuela+]

post-merger

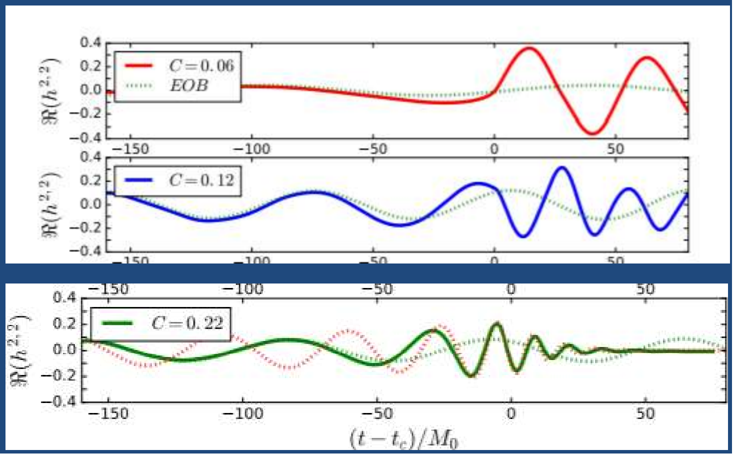
- 'expected' behavior (*within a quasi-circular assumption*): BH QNM, 'merged' object radiating through QNM, or 'bar' (+further modes)
  - Main properties *in each scenario*, are roughly predictable from individual objects [though not not quite which scenario in long timescales]



  - In NS it is reasonably clear what to expect

[LL,+ '16, '18, Palenzuela+ '18]

  - In BS → if a BS : no angular mom (?!)
    radiates through BSQNM
    if a BH: (M,a) can be estimated
    radiates through BHQNM



But this is not exhaustive of 2nd order theories

Horndeski theories

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}(\Sigma_{i=1}^{5}\mathcal{L}_i) \tag{1}$$

where,

$$\mathcal{L}_1 = R + X - V(\phi)\,, \tag{2}$$
$$\mathcal{L}_2 = \mathcal{G}_2(\phi, X)\,, \tag{3}$$
$$\mathcal{L}_3 = \mathcal{G}_3(\phi, X)\Box\phi\,, \tag{4}$$
$$,\mathcal{L}_4 = \mathcal{G}_4(\phi, X)R + \partial_X\mathcal{G}_4(\phi, X)\delta^{ac}_{bd}\nabla_a\nabla^b\phi\nabla_c\nabla^d\phi\,, \tag{5}$$
$$\mathcal{L}_5 = \mathcal{G}_5(\phi, X)G_{ab}\nabla^a\nabla^b\phi - \frac{1}{6}\partial_X\mathcal{G}_5(\phi, X)\delta^{ace}_{bdf}\nabla_a\nabla^b\phi\nabla_c\nabla^d\phi\nabla_e\nabla^g\phi\,. \tag{6}$$

with $X = -1/2\nabla_a\phi\nabla^a\phi$, G_{ab} the Einstein tensor, $\mathcal{G}_i$ are functions of the scalars $\{\phi, X\}$, V is a potential and $\delta^{b_1..b_n}_{a_1..a_n}$ is the generalised Kronecker delta symbol.

Horndeski 'family' of theories:

- 2^{nd} order EOM, 'natural' to include cosmological constant, accommodate bouncing cosmology scenarios, etc.
- Recently shown *via Harmonic gauge* → ill posed problems unless L_4=0=L_5 *at linear level, (and* argued L_3=0 needed at non-linear level) for local well posedness [Papallo-Real 1705.04370]

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[(1+\mathcal{G}_4(\phi))R + X - V(\phi) + \mathcal{G}_2(\phi, X)\right] \ ,$$

Non-linear regime: The 'look' of the theory can be simplified by a conformal transformation…

- Metric: Einstein eqns with a *sane* RHS
- Scalar: $Box(\phi) = F\ \partial\phi\ \partial\phi\ \partial\partial\phi + S$

- → the field propagates according to a different metric that depends on its gradients
  - For 'weak data' the eqn satisfies the 'null condition' (Klainerman) which together with results on Straus' conjecture on V → global solutions

Beyond this case, can we expect good behavior?

- If shocks arise → uniqueness is lost -> thus well posedness
- Before that, *the character of the equation changes*!
- Does not always happens inside a BH?
- Not necessarily 'beyond EFT applicability' simple estimates

[Bernard+ '1904.12866, Ripley+'18-19, Garfinkle…]

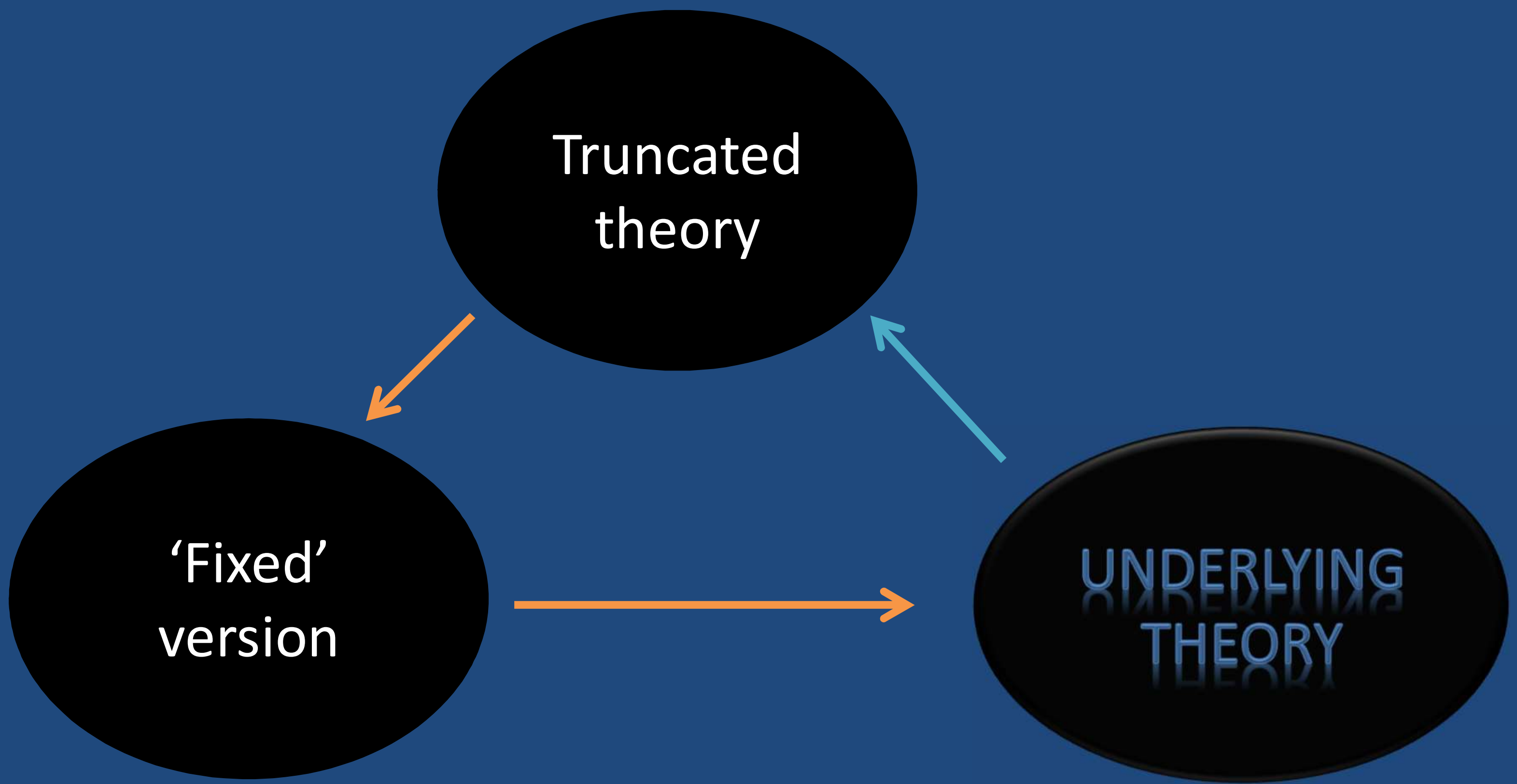
Let’s pause II

- So even ‘insisting’ on 2nd order, we might face strong obstacles

- Perhaps, problems arise within a family of initial conditions and not with others
  - If so, we should understand which ones [tough!]

- Perhaps problems are an ‘artifact’ of truncation → but this line of thinking will bring back higher derivatives (generic from an EFT point of view)

- Schematically, we face:



- Many alternatives motivated [often involving curvature corrections with/without extra d.o.f, nonlocalities, etc]

$$G_{ab} = kO(g_{ab}, \theta)\,;\, L(\theta) = \Omega(g_{ab}, \theta)$$

- *Do BHs differ from those in GR ?*
- *Are they stable?*
- *What is their dynamical behavior?*
- *Relaxation to equilibrium? (what is the final state?)*

How to probe what could take place? Given that potential problems are identified/guessed?

Some (random!) examples

- EinsteinAether theory [Jacobson,Horava]. Do away with Lorentzian invariance.

- BHs in this theory are different from those in GR [Barausse-Sotiriou+]

$$S_{æ} = -\frac{1}{16\pi G_{æ}} \int \Big(R + \frac{1}{3} c_\theta \theta^2 + c_\sigma \sigma_{\mu\nu}\sigma^{\mu\nu} + c_\omega \omega_{\mu\nu}\omega^{\mu\nu} + c_a a_\mu a^\mu \Big) \sqrt{-g}\, d^4x$$

- *noncommutative gravity (quadratic gravity)…*

$$Box(h) = \lambda\, Box(Box(h))$$

$$S \equiv \int d^4x\sqrt{-g}\{\kappa R + \alpha_1 f_1(\vartheta)R^2 + \alpha_2 f_2(\vartheta)R_{ab}R^{ab}$$
$$+ \alpha_3 f_3(\vartheta)R_{abcd}R^{abcd} + \alpha_4 f_4(\vartheta)R_{abcd}{}^{*}R^{abcd}$$
$$- \frac{\beta}{2}\left[\nabla_a\vartheta\nabla^a\vartheta + 2V(\vartheta)\right] + \mathcal{L}_{\rm mat}\}.$$

- *Dynamical Chern-Simons…*

$$Box(h) + \lambda\,(\partial\phi\,\partial^3 h + \partial^2 h) = 0$$
$$Box(\phi) + \lambda\,(\partial^2 h\,\partial^2 h) = 0$$

$$S \equiv \int d^4x\sqrt{-g}\left(\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\vartheta)^2 - \frac{m_{\rm pl}}{8}\ell^2\vartheta\,{}^*RR\right)$$

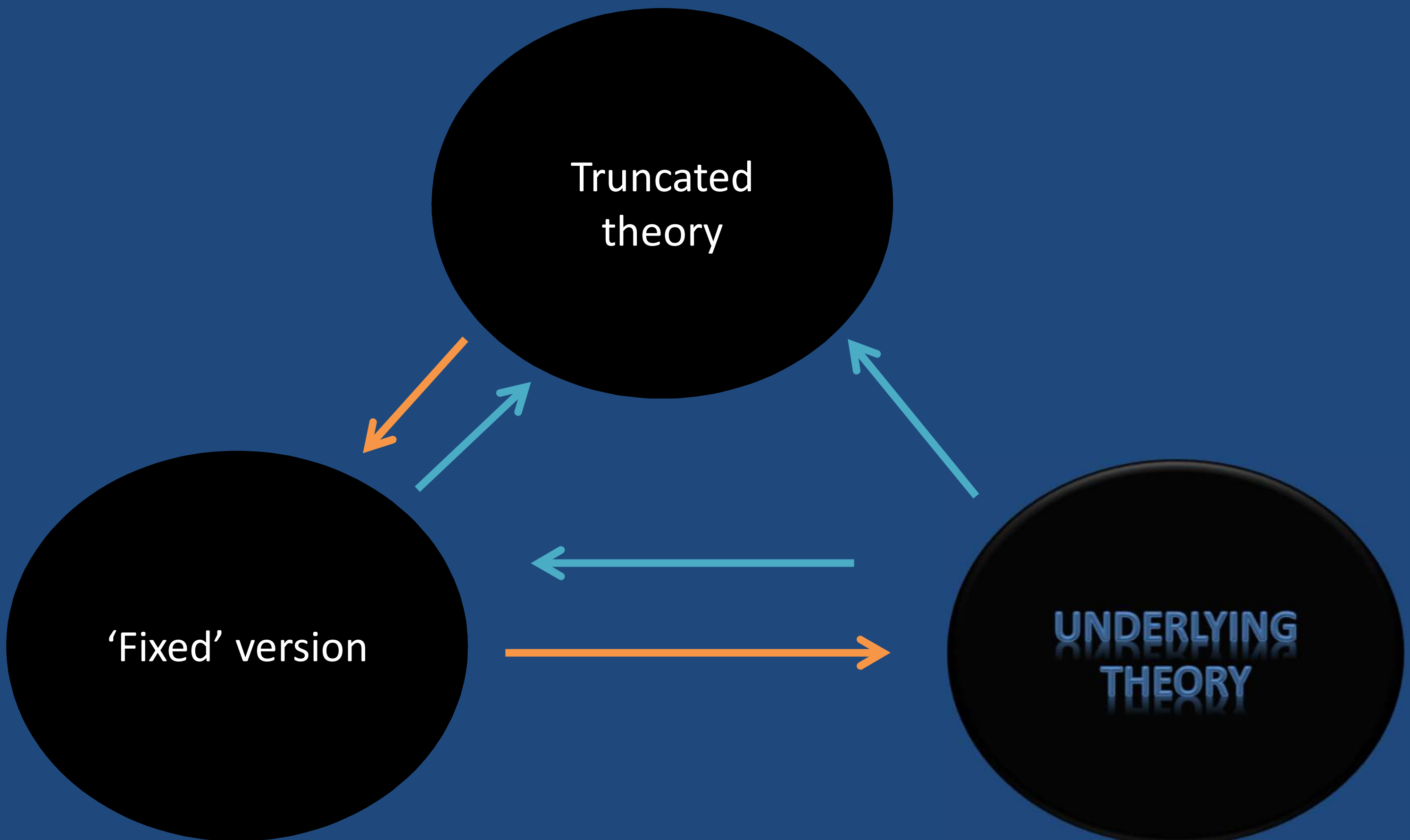
- In some of these extensions BHs have been constructed….

  - Generically, not known if BH’s identified are stable
  - Generically, not known if BH’s can be dynamically formed

  - unknown QNM spectra

  - Traditionally hoped that full dynamical behavior could be worked out
  - Traditionally hoped that linear solutions do carry messages from full theory (especially in cosmology)
  - [*HOPE is the traditional word here]*

To begin answering possible questions, we need to find a way to reconcile the following picture



Strategies?

- ‘Iteration/perturbation’

$$B(g^*) = 0 \ \rightarrow B(g) = S(g^*)$$

  – Rinse and repeat: but during what time frame?
  – Perturbative hierarchy [e.g. Okounkova+ ‘17]
  – justified? can one guarantee the fidelity of the solution obtained?

$$S \equiv \int d^4x\sqrt{-g}\left(\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\vartheta)^2 - \frac{m_{\rm pl}}{8}\ell^2\vartheta\,{}^*RR\right)$$

Example : dCS.

- 3rd order EOMs (surely → ill posed problems!)
- Taming/probing it?
- → perturbative expansion (again), carried out to 1st non-trivial orders in corrections to GR soln). Head-on, spinning BHs considered. Non-trivial departures found in, 'gravity' QNMs

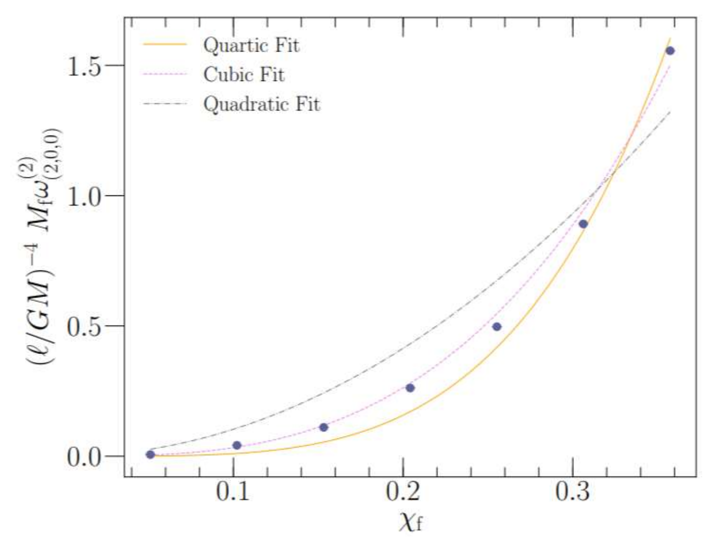


FIG. 10. Leading-order dCS correction $\omega^{(2)}_{(2,0,0)}$ to the QNM frequency $\omega^{(0)}_{(2,0,0)}$, plotted as a function of dimensionless spin $\chi_{\rm f}$ of the final background black hole. Error bars (smaller than the plotted points) are computed by considering $\omega^{(2)}$ for numerical simulations with different resolutions (cf. IV C). We see that $\omega^{(2)}$ increases as a power law with spin. Note that these large values of $\omega^{(2)}(\ell/GM)^{-4}$ must be multiplied by a small, appropriate value of $(\ell/GM)^4$ to have physical meaning.

[Okounkova+, 1906.08789]

- Is this behavior robust?
- (in general, not expected [Allwright+ 1808.08797] ,*BUT* in special regimes/theories it might)

Taking hydrodynamics as 'inspiration'

- E.g. Incompressible : $v v_t + v\,\partial v = \varepsilon\,\eta\,\partial^2\,v$

- Solution? Introduce $v = v0 + \varepsilon\,v1 + \ \ldots$

-> hierarchy of eqns, around $v0$ =const:

$$v1_t + v1\,\partial v0 = 0$$
$$v2_t + v1\partial v1 + v0\partial v2 = \ \eta\partial^2 v1$$

At order 1, no turbulence

At order 2, 'secular' growth through 'passive' source terms

How deep must one go to capture the true behavior?

How deep must one go to obtain reliable predictions?

Option 2. Let's stay in hydro for an example…

- Modify the system of eqns, in an ad-hoc manner to control higher gradients and prevent wild runaway to the UV
- E.g. Israel-Stewart formultion of viscous relativistic hydrodynamics: $T = T^{pf}$ + gradient terms
  - Define $\Pi =$ (shear/bulk)$_{ab}$ + Grad(shear/bulk..)$_{ab}$ as new and independent variable
  - Force an eqn on Π such that $\Pi \sim$ (shear/bulk)$_{ab}$ to leading order always
  - $\tau\, \Pi_{,t} = - \Pi +$ (shear/bulk)$_{ab}$ …. [Geroch, details shouldn't matter]

So, mathematically under control. How about physically? What justifies its use?

- Motivated by this, consider modifying gravity eqns such that
  - Modification resolves issues leading to ill-posedness at non-linear levels
  - Provides for a natural way to assess if such method faithfully captures the sought-after behavior. Ie. *sensitivity to external params*
  - Convenient for numerical implementation

  - One such method [Cayuso+ 1706.07421]

‘Modification’

$$B(g) = F\,;F_t = -\gamma(F - S(g))$$

  - Modifies the system of equations to ‘fix’ problems
  - Introduces a new timescale γ, with which one can assess the fidelity of the solution obtained
  - Tested with toy ‘wave eqn’ models [Cayuso+ 1706.07421]
  - Tested with EFT reduction of complex scalar field with spont symmetry breaking [Allwright+ 1808.07897]

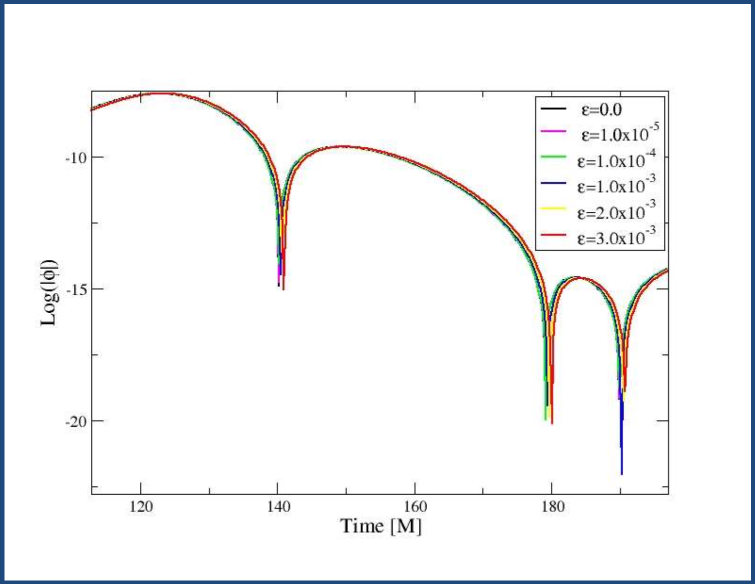
- Application in Gravity [Cayuso R+ (ongoing)]

[Endlich,Gorbenko,Huang,Senatore]

$$S_{\rm eff} = \int d^4x\sqrt{-g}\,2M_{\rm pl}^2\left(R - \frac{\mathcal{C}^2}{\Lambda^6} - \frac{\tilde{\mathcal{C}}^2}{\tilde{\Lambda}^6} - \frac{\tilde{\mathcal{C}}\mathcal{C}}{\Lambda_-^6}\right)$$

$$\mathcal{C} \equiv R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}\,, \quad \tilde{\mathcal{C}} \equiv R_{\alpha\beta\gamma\delta}\tilde{R}^{\alpha\beta\gamma\delta}\,,$$

EOMS -> G_{ab} [g/L^2] ~ F(g^3/L^8)



Final words

- Exciting times ahead, main efforts will continue to enable *exciting and robust* results.

- Bold strategies can help extract golden nuggets in shorter time frames

- Limited knowledge of ‘what else’ requires venturing (far) from ‘safe zones’ but pay off could be important

- Homing in on options to answer key questions to extensions to GR. Impossible to check them all, but sound strategies imply proposers can check them. For the rest: how to choose which ones?