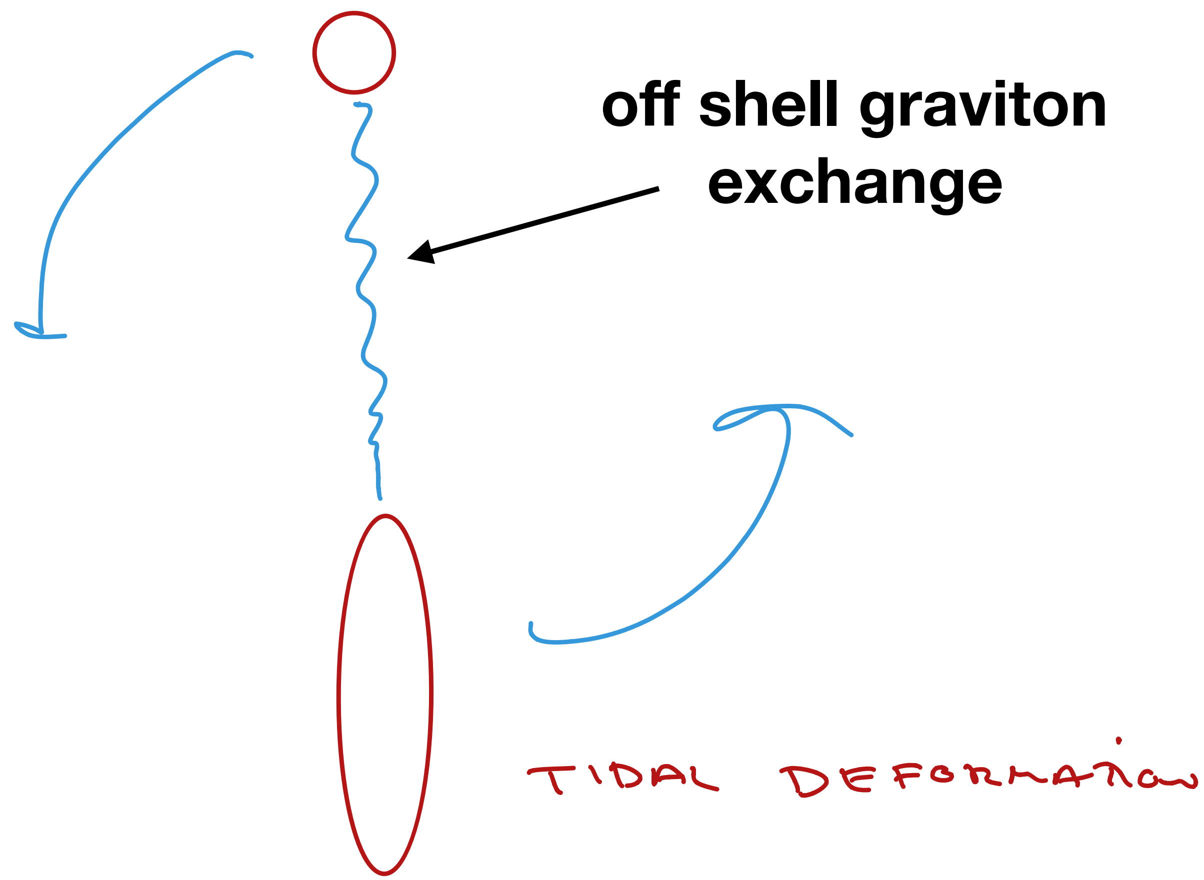
Effective Theory of Black Hole Horizons

(IZR +Walter Goldberger)

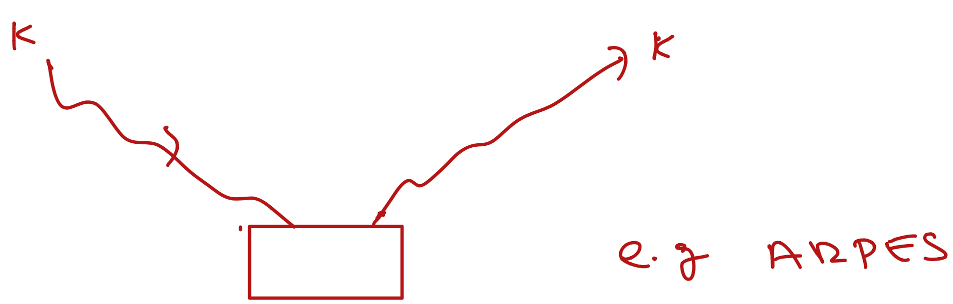
Science of Gravitational Wave Observations

- Cosmological Measurements
- Astrophysical Populations/Production Mechanism
- GR/Beyond GR?
- Internal Structures/Equations of State.

Probing of Internal Structure in Context of Analytic Calculations



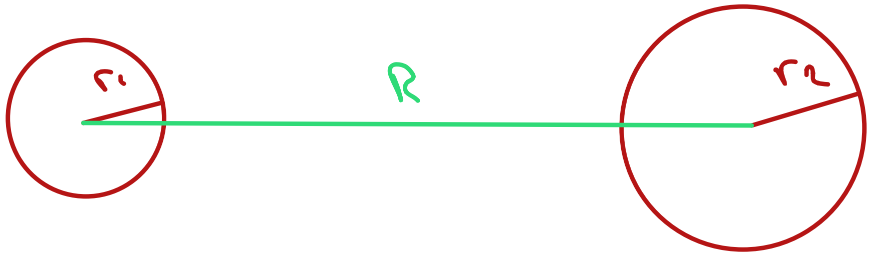
Analogous to Linear Response Theory



Extract Causal Greens Function (e.g. conductivity)

Crucial Distinction is that we dont have access to the direct response field. We only have very indirect information via phase and amplitude of gravitational wave.

Systematic program to extract response function from the wave signal. Effective Field Theory is a useful tool to describe this system.



$R >> r_1 \sim r_2$

Multi-Scale problem is solved by treating one scale at a time in a sequence of course graining

Stages of Coarse Graining

Full Theory: GR (BH)+(QCD neutron star). Einstein-Hilbert+Yang Mills

$$\lambda >> R$$

Point Particle Theory

$$\lambda >> r$$

Composite Object with dynamical set of multipole moments

To maintain analytic controls we will also restrict ourselves to the regime where

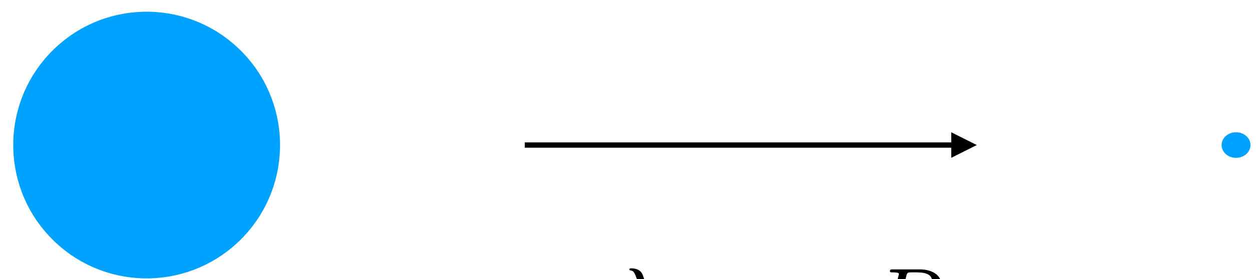
$r_s/r \sim v^2 \ll 1$ **(PN expansion)**

Sharp Analogy with QCD

	Onia	Binary
Short distances	Weak Coupling Coulomb Phase	Strong coupling
Long Distances	Confinement	Minkowski Space
Non-Linearities	Controlled by $\alpha_s \sim v$	Controlled by v^2
Quantum Effects	Controlled by $\alpha_s \sim v$	Controlled by $(M_{pl} r)^2 \sim \hbar / L$

Allows for Strong Classical gravity

We will be concentrating on the first stage of coarse graining



$$\lambda >> R$$

Point Particle Theory

$$S = \int d^4x\sqrt{g}R + (\int d^4x\sqrt{g}L_{Matter})$$

$$S = \int -md\tau + ...$$

Finite size effects controlled by multipole expansion

Write down all possible terms consistent with symmetries (GCR and RPI), modulo the equations of motion.

$$S = \int C_R R + C_{vvR}\, v^\mu v^\nu R_{\mu\nu} + C_E E^{\mu\nu} E_{\mu\nu} + C_B B^{\mu\nu} B_{\mu\nu} + ...$$

C's are an expansion in 1/rs $\quad E_{\rho\sigma} = v^\mu v^\nu R_{\mu\rho\nu\sigma}$

By performing a field redefinition we can reduce the action to give the leading terms (Birkoffs Theorem)

$$S_{FS} = \int d\tau C_E E^2 + C_B B^2$$

se coefficients correspond to the quadrapole static susceptabilities (Love numbers). In the PN expansion these coefficients scale as v^{10}

This is called the ``effacement theorem" (Damour)

These coefficients are fixed by a ``matching calculation"

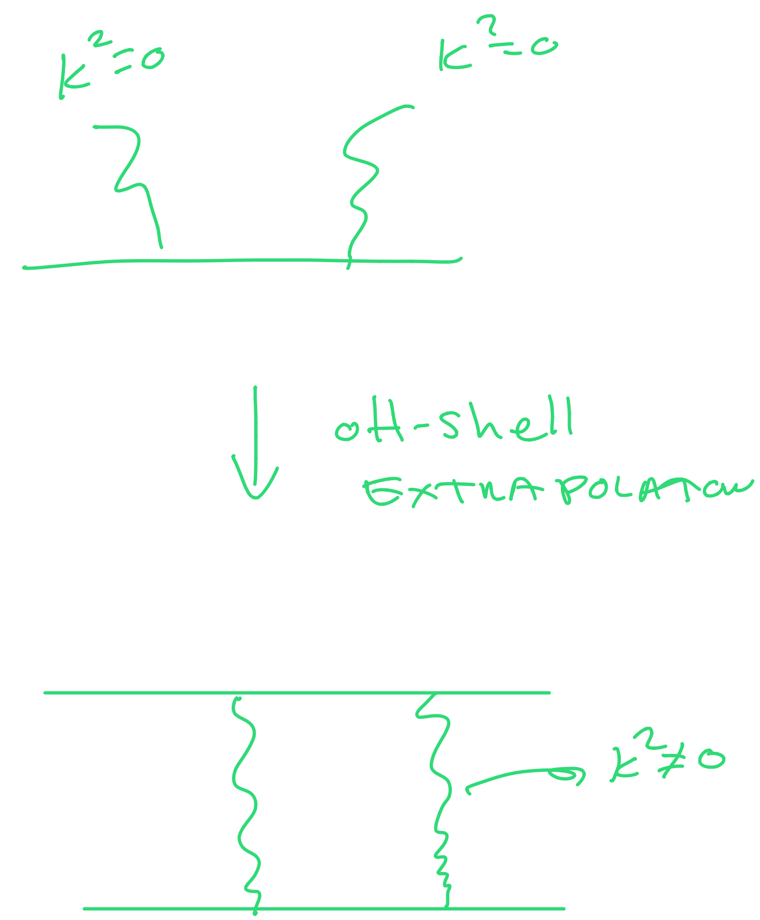
$$h = \langle h \rangle + \delta h \qquad \delta h(x) = \int d^4x G(x-x')\langle h(x') \rangle$$

background field

Static response (time independent background)

The power of the matching procedure lies in the fact that we are free to match any quantity we wish (even if its not gauge invariant, as long as we use the same gauge in the full and EFT). Thus we can match on-shell as that usually simplest. Moreover by working at the level of the Lagrangian, we have an off-shell extrapolation.

e.g.



Thus we can extract response on shell and use it to calculate off-shell quantities (like potential for binaries)

For the case of black holes Love numbers vanish

(Damour and Nagar, Binnington +Poisson, Kol and Smolkin)

Kol /Smolkin: no longer true outside of 4 dimensions

This result implies a fine tuning as defined by Dirac since the Love numbers are allowed by symmetry, so should be order one in units of the Schwarzchild radius.

Consider E+M analogy atomic physics

$$\alpha_E = \frac{1}{2\pi}\sum_n \frac{\langle 0 \mid \mathbf{p_z}(0) \mid n\rangle\langle n \mid \mathbf{p_z}(0) \mid 0\rangle}{E_n - E_0} = 0$$

Each term is positive semi-definate

$$\sum_m \frac{\mid \langle \Omega \mid Q_{ab} \mid m\rangle \mid^2}{E_\Omega - E_m} = 0$$

Gravitational case

This assumes a pure state, which is thus ruled out

$$\sum_{m,n} e^{-\beta(E_n)} \frac{\langle n \mid Q_{ab} \mid m\rangle\langle m \mid Q_{ab} \mid n\rangle}{E_n - E_m} = 0$$

Thermal states allowed but not necessary

Note: Q's do not excited QNM's which are outside the validity of the EFT, they correspond to the long time tails of the QNMs. By necessity the systems is ungapped.

Is there is dual to this theory?

But this is NOT a fine tuning in the sense of ‘tHooft. i.e. it’s is technically natural (e.g. C.C)

Non-Renormalization Theorem

Straight forward to show a CLASSICAL no renormalization theorem

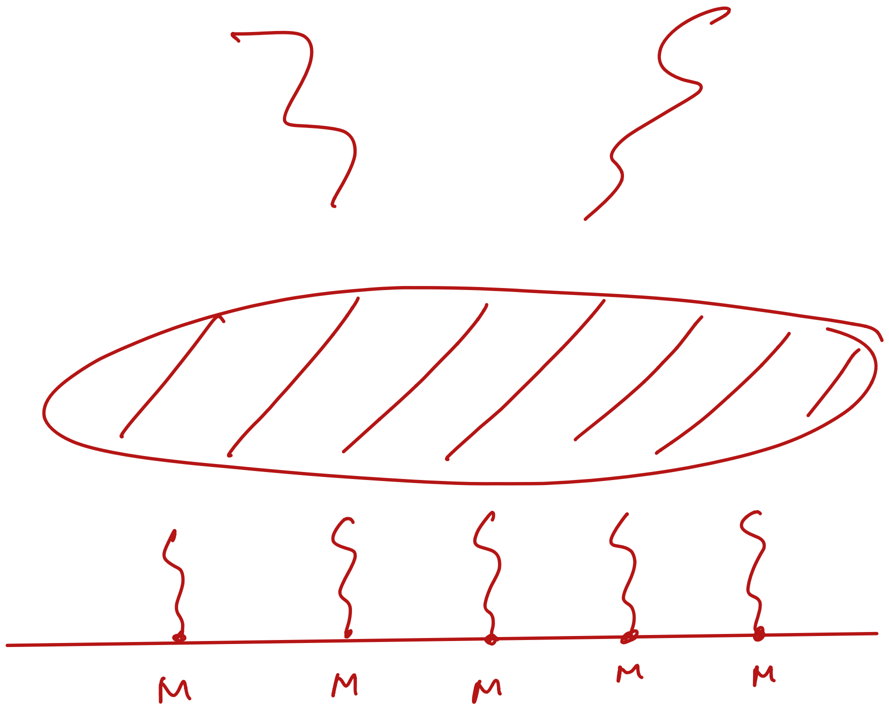
Classical power counting implies that $C_E \sim (r_s^3)(r_s M_{pl})^n$

We can fix n by enforcing that it scale classically: n=2

Consider all Feynman diagrams which have classical scaling and could contribute to the renormalization of the Love number.

Any diagram which renormalizes Love number must have five and only five mass insertions i.e. rs^5.

Generalize to other static susceptibilities



In 4d no way to hook up bulk vertices and get powers of Planck mass needed

No static susceptibility get renormalized

$$L = C_{NS}\dot{E}^2$$

However, non-static cases are renormalized and expected tone non-vsnihsing

This EFT (with local world line couplings) can not be the end of the story, as it leads to conservative propagation. Does not account for absorption

Need to include internal dynamics that can arise from gapless degrees of freedom

Introduce a Hilbert space with a ground state $|\Omega\rangle$ **(Izr/Goldberger)**

The action of which we are ignorant. We do know how the relevant operators couple to gravity

$$S = \int d\tau Q_E^{\mu\nu}(\tau) E_{\mu\nu} + \int d\tau Q_B^{\mu\nu}(\tau) E_{\mu\nu} + \ldots.$$

$Q_{\mu\nu}^{E,B}(\tau)$ **Interpolate for gapless DOF**

Note: at this point we have not specified the nature of the UV theory. i.e. could be NS or BH or something else

What can we say about correlation functions?

$$\langle \Omega \mid Q(t_1).....Q(t_n) \mid \Omega \rangle$$

Concentrate on the two point functions for the moment, in particular let us try to construct the Wightman functions as building blocks

$$A_+ = \int dt \langle \Omega \mid Q(t)Q(0)) \mid \Omega \rangle e^{-i\omega t}$$

$$A_- = \int dt \langle \Omega \mid Q(0)Q(t) \mid \Omega \rangle e^{-i\omega t}$$

Let us for the moment confine ourselves to classical processes where

$$G_{ret} = \int \frac{d\omega'}{2\pi} \frac{A_+(\omega') - A_-(\omega')}{\omega - \omega' + i\epsilon} + C$$

$A_-(\omega) = 0$ **We are considering pure absorption for the moment**

We can extract $A_+(\omega)$ by matching to absorptive cross section

$$\langle 0, k \mid \int dt Q(t) \cdot E(e) \mid 0' \rangle$$

one graviton absorption process

$$\sigma = \sum_\lambda \frac{\omega^3}{8M_{pl}^2} \int dt e^{-i\omega t} \langle 0 | \; Q_{ij}^E(t) Q_{kl}^E(0) \; |0\rangle \epsilon_{ij}^\lambda \epsilon_{kl}^{\lambda\star},$$

Cross section

This cross section (in low energy limit) is given by

$$\sigma_{abs} = \frac{4\pi r_s^6 \omega^4}{45}$$

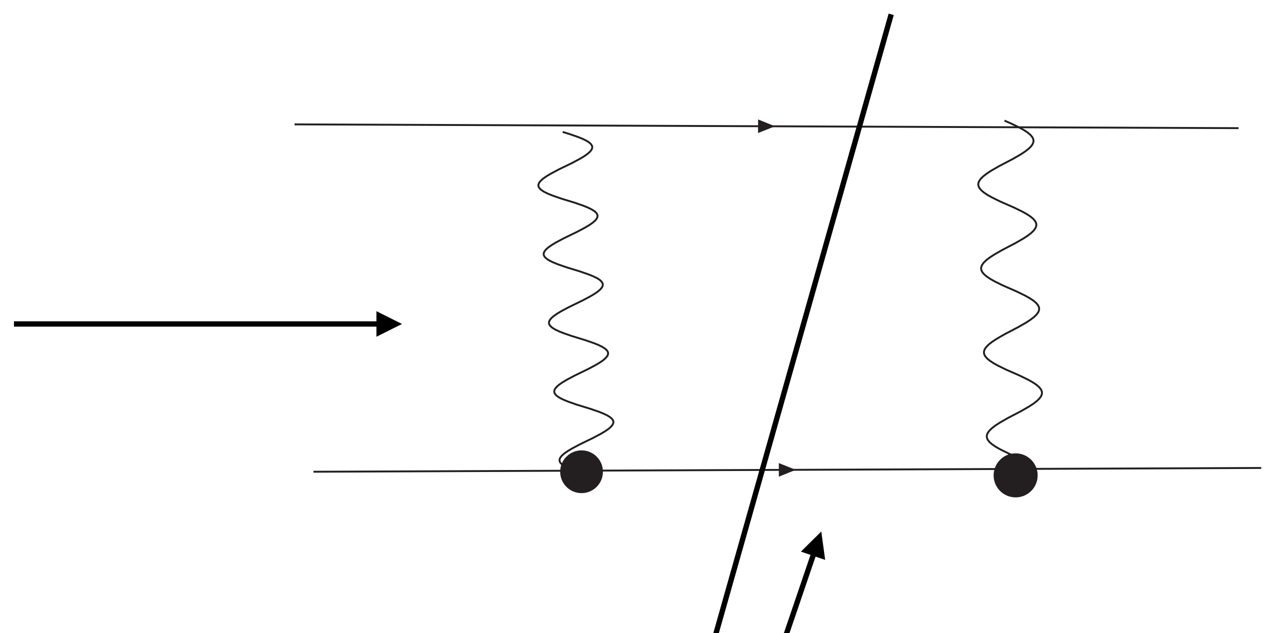
$$\int dt e^{-i\omega t} \langle 0 \mid Q_{ij}^E(t) Q_{kl}^E(0) \mid 0 \rangle \equiv A(\omega)(\frac{2}{3}\delta_{ij}\delta_{kl} - \delta_{il}\delta_{kj} - \delta_{ik}\delta_{lj})$$

$$A(\omega) = \frac{16}{45} m^6 G_N \omega$$

This is an on shell piece of data which we can now use to calculate off-shell

Given the Wightman function we can predict the power loss in a binary inspired due to (internal) dissipative processes

off-shell gravitons



Wightman function

By fluctuation dissipation theorem this can be written as the imaginary part of the potential between the two constituents i.e. the absorptive power loss.

The imaginary part of the potential gives the power loss

Only justified to keep leading order cross section

$$\frac{dP_{abs}}{d\omega} = -\frac{1}{T}\frac{G_N}{64\pi^2}\sum_{a\neq b}\frac{\sigma^{(b)}_{abs}(\omega)}{\omega^2}m_a^2|q^{(a)}_{ij}(\omega)|^2$$

$$q^{(a)}_{ij}(t) = \partial^a_i\partial^a_j|\mathbf{x}_{12}(t)|^{-1}\ (a=1,2)$$

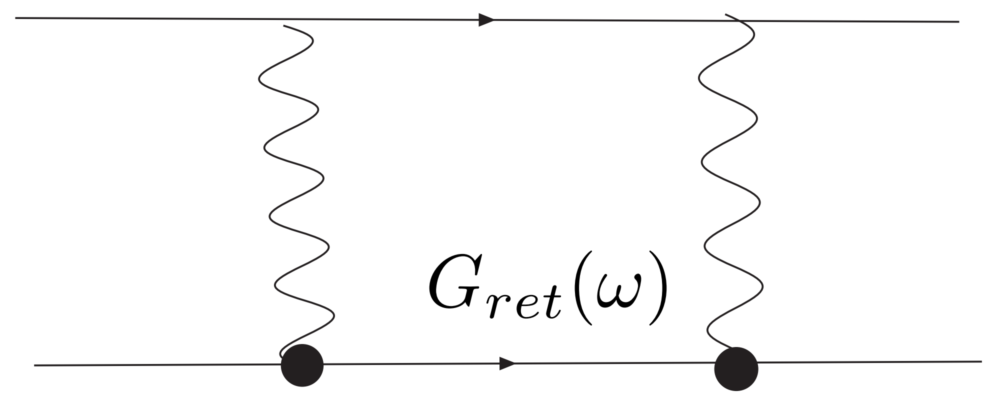
This is a general result that for a neutron star is sensitive to the equation of state. Given an model for the neutron stars low energy modes one can calculate the power loss and its subsequent effect on the wave templates.

For a black hole this reduces to

$$P_{abs} = \frac{32}{5}G_N^7(m_1^6m_2^2 + m_2^6m_1^2)\left\langle\frac{\mathbf{v}^2}{|\mathbf{x}|^8} + 2\frac{(\mathbf{x}\cdot\mathbf{v})^2}{|\mathbf{x}|^{10}}\right\rangle.$$

(Poisson)

What about the real part? The relevant observable is the EOM in which case we must use the in-in formalism



Real part yields the susceptibilities with the zero frequency response being the Love number

$$G_{ret} = \int \frac{d\omega'}{2\pi} \frac{A_+(\omega') - A_-(\omega')}{\omega - \omega' + i\epsilon} + C$$

subtraction/counter-term

One can match onto the retarded greens function directly by calculating the tidal response

$$G_{ret}(0) = 0$$

Open Questions

How do we go beyond leading order in absorption?

What about higher order correlation functions?

What happens quantum mechanically? Hawking Radiation?

What can this EFT teach us about soft radiation from black holes?

Black Hole Soft theorems

What can we say about the black hole S-matrix? i.e. $\langle m \mid n \rangle$

Can we use it to calculate in the interacting theory, i.e. QM cross section for black hole?

Black hole Scattering

- Regge-Wheeler/Zerilli eqs: BH perturbation theory, wish to go beyond classical wave equation.
- Black holes not asymptotic states (quarks)
- ``non-unitary” horizon absorption. (information loss)
- Contains relevant information for constructing classical observable (gravitational wave astronomy)
- What rolls does Hawking radiation play? Is there any way in which it is not Planck suppressed?

Extend to Quantum Mechanics: allow for spontaneous/stimulate emission

We could just choose to ignore particle emission in our classical higher order analysis, but this would seem to contradict power counting

$$\lambda = \omega r_c$$ **Classical**

$$\delta = \omega / M_{pl}$$ **Quantum**

Naively one might think at $$A_-(\omega)/A_+(\omega) \sim \delta$$

However, this would fly in the face of detailed balance:

Einstein Coefficients : A ~ B

$$A_-(\omega)/A_+(\omega) \sim f(\omega)$$

$$E/T_H \sim (\hbar\omega)/(\hbar/r_s)$$

This is consistent with the fact that Hawking give

$$\langle n(\omega) \rangle = \frac{\Gamma(\omega)}{e^{\beta E} - 1}$$

So in the extraction of the Wightman functions we have a power expansion in $r_s\omega$

Note that power counting can not distinguish between quantum gravity effects and classical effects!

None the less in free field theory, if the only observable quantity is the retarded propagator then all of the effects having to do with the state must cancel

$$G_{ret} = Tr(\rho\theta(t)[\phi(t), \phi(0)])$$

C number if phi satisfies linear wave equation

Therefore there must be a Cancellation between + and - Wightman functions in any FREE field theory

e.g free field theory in thermal state $F(\omega) = 1/(e^{\beta|\omega|} - 1)$

$$A_+^T = i(2\pi)\delta(k^2 - m^2)(F(\omega) + \theta(\omega)) \quad A_-^T = i(2\pi)\delta(k^2 - m^2)(F(\omega) + \theta(-\omega))$$

$$G_{ret} = \int \frac{d\omega'}{2\pi} \frac{A_+(\omega') - A_-(\omega')}{\omega - \omega' + i\epsilon} + C \qquad = \frac{1}{k^2 - m^2 + isgn(\omega)\epsilon}$$

Extracting Wightman Functions for positive freq.

2) Extract from (n|m) probabilities (Bekenstein+Meisels, Panafanden+Wald)

1) Use full theory Wightman functions in asymptotic limit (Candelas)

$$\vec{x} \to \vec{x}'$$

$$r \longrightarrow \infty \qquad \langle\Psi|\phi(x)\phi(x')|\Psi\rangle = \langle 0|\phi(x)\phi(x')|0\rangle + \frac{1}{4\pi r^2}\int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega(t-t')}\sum_{\ell}(2\ell+1)F_\ell^{\Psi}(\omega)$$

$$F_\ell^H(\omega) = F_\ell^U(\omega) = \frac{1}{2\omega}\frac{|B_\ell(\omega)|^2}{1-e^{-\beta_H\omega}}$$

Unruh vacuum: no incoming flux from infinity

Hartle/Hawking vacuum: thermal equilibrium at T.

$$F_\ell^B(\omega) \quad = \quad \frac{1}{2\omega}\theta(\omega)|B_\ell(\omega)|^2$$

Boulware vacuum:

Each choice of vacua should lead to the same causal (retarded) Greens function, but differing Wightman functions

Now to match onto EFT utilize Schwinger Keldysh

$$\begin{aligned} G(x,x') &= \langle 0|\phi(x)\phi(x')|0\rangle \\ &\quad + \int d\tau d\tau' G^{2a}(x,x(\tau))G^{1b}(x',x(\tau'))\langle Q_a(\tau)Q_b(\tau')\rangle \\ &\quad + \cdots \end{aligned}$$

Where $$G^{a,b}(x,x') = \begin{pmatrix} D_F(x-x') & -W(x'-x) \\ -W(x-x') & D_D(x-x') \end{pmatrix}$$

$$\langle Q_a(\tau)Q_b(\tau')\rangle = \begin{pmatrix} \langle TQ(\tau)Q(\tau')\rangle & \langle Q(\tau')Q(\tau)\rangle \\ \langle Q(\tau)Q(\tau')\rangle & \langle T^*Q(\tau)Q(\tau')\rangle \end{pmatrix}$$

Taking the coincident point limit as well as as taking r to infinty leaves

$$\frac{1}{(4\pi r)^2}\int \frac{d\omega}{2\pi} e^{-i\omega(t-t')}\left[A_+(\omega) + \theta(\omega)(A_+(\omega) - A_+(-\omega))\right]$$

Matching to the full theory result we find

$$A_+^U(\omega) = \frac{\omega\sigma_{abs}(|\omega|)}{e^{\beta_H\omega}-1}\left[2e^{\beta_H\omega}\theta(-\omega) + \theta(\omega)(1+e^{\beta_H\omega})\right]$$

$$A_-^U(\omega) = -\frac{\omega\sigma_{abs}(|\ \omega\ |)}{e^{-\beta_H\omega}-1}\left[2e^{-\beta_H\omega}\theta(\omega) + \theta(-\omega)(1+e^{\beta_H\omega})\right]$$

It is interesting to see how Hawking cancels in the retarded propagator

$$= A_{+}^{U}(\omega) - A_{-}^{U}(\omega) = \omega\sigma_{abs}(|\omega|)\left[\theta(\omega) - \theta(-\omega)\right]$$

No trace of Hawking radiation remains! This had to be the case, otherwise the effects of Hawking radiation would not be Planck Suppressed at LIGO.

Note that this result only holds for free field theory, beyond which there is no a priori reason that the state effects (i.e. Hawking radiation) can not show up that is NOT Planck suppressed. However, physical arguments would indicate that some cancellations must still occur. Calculation is technically challenging.

How do we include higher orders to match high n-pnt functions?

Probability of having m incoming and n out going modes (same partial wave and frequency)

Bekenstein+Meirels

$$p(m\,|\,n) = \frac{(e^x - 1)e^{xn}\,\Gamma^{m+n}}{(e^x - 1 + \Gamma)^{n+m+1}} \sum_{k=0}^{\min(n,m)} \frac{(-1)^k (m+n-k)!}{k!(n-k)!(m-k)!} \times \left[1 - 2\frac{1-\Gamma}{\Gamma^2}(\cosh x - 1)\right]^k, \quad (18)$$

Γ **Grey Body Factor** $\qquad x = r_s \omega$

Utilizes only Hawking result detailed balance and principle of maximal entropy, and agrees with Walds curved space calculation

We can extract Wightman functions directly from these results including higher order correlators

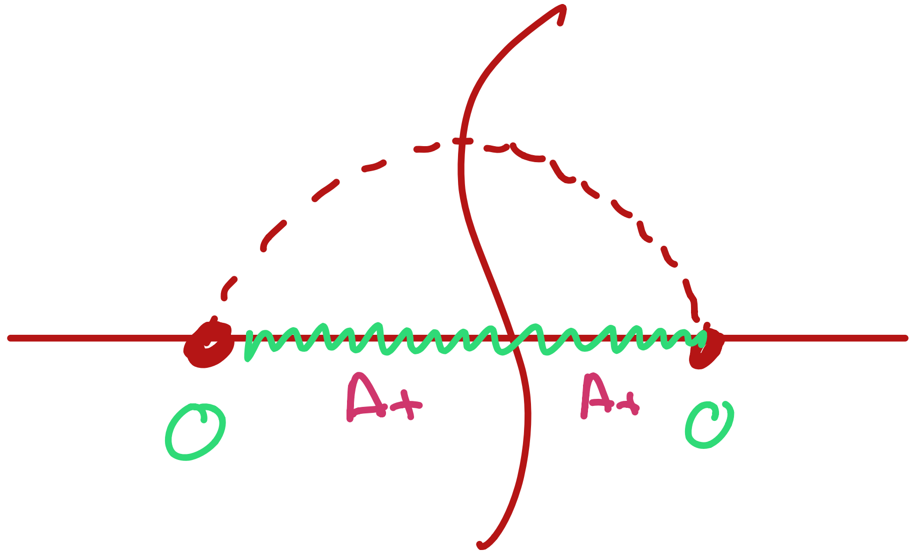
$$P^{full}(1|0) = \Gamma \frac{(e^x - 1)}{(e^x - 1 + \Gamma)^2} \qquad P(0|1)^{full} = e^x P^{full}(1|0)$$

$$\Gamma \sim a\omega^2 + b\omega^3 + \ldots.$$

(For a scalar grey body factor)

Leading order matching in EFT

$$S = \int dt \phi O \sim \lambda^{\sigma}$$



Determine sigma by matching at LO

$$\begin{aligned} P^{EFT}(0|1) &= \frac{\omega}{2\pi} A_{+}(\omega) \\ P^{EFT}(1|0) &= \frac{\omega}{2\pi} A_{+}(-\omega) \end{aligned}$$

$$\Gamma^{0} \sim \omega^{2} \sigma^{abs}_{l=0} \sim \omega^{2} r_s^{2}$$

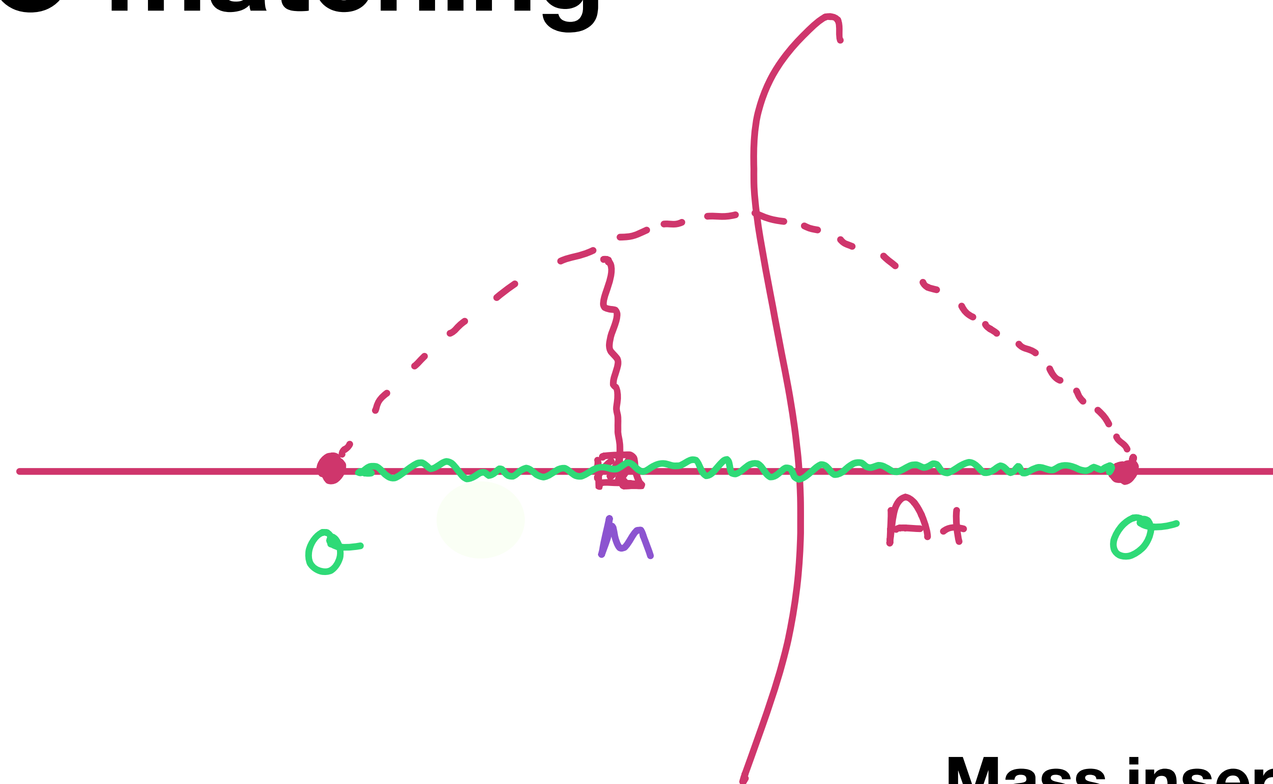
$$\begin{aligned} P^{full}(1|0) &= \frac{\Gamma^{(0)}(\omega)}{x} - \frac{1}{2}\Gamma^{(0)} - \frac{2(\Gamma^{(0)})^2}{x^2} + \frac{\Gamma^{(1)}}{x} ... \\ P^{full}(0|1) &= \frac{\Gamma^{(0)}(\omega)}{x} + \frac{1}{2}\Gamma^{(0)} - \frac{2(\Gamma^{(0)})^2}{x^2} + \frac{\Gamma^{(1)}}{x} ... \end{aligned}$$

Matching with wave packets

$$A_{+}^{LO}(\omega) = A_{-}^{LO}(\omega) = (2\pi)\frac{\Gamma^{(0)}(\omega)}{r_s \omega^2} \sim \lambda/\omega$$

NLO matching

At NLO in the EFT we have to subtract mass insertions. note higher partial wave don't mix in not relevant for s wave matching.



Imaginary part diverges but cancels in sum over cuts

Mass insertion

$$
\begin{aligned}
P^{EFT}(1|0)_{NLO} &= \frac{\omega}{2\pi}(A_+^{NL0} + A_+^{LO} r_s \omega) \\
P^{EFT}(0|1)_{NLO} &= \frac{\omega}{2\pi}(A_-^{NL0} + A_-^{LO} r_s \omega)
\end{aligned}
$$

Note mass insertion cancel in the retarded propagator, this had to be the case since its part of the state (geometry). As such, if were only interested in classical physics we can ignore mass insertions in the matching.

NLO Full Theory **NLO EFT**

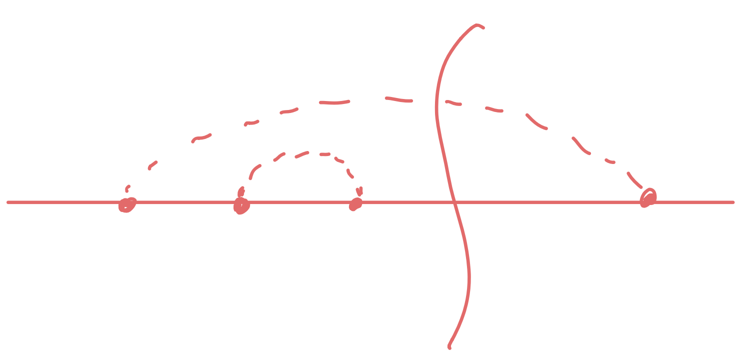
$$-\frac{\Gamma^{(0)}}{2} - \frac{2(\Gamma^{(0)})^2}{x^2} + \frac{\Gamma^{(1)}}{x} = (2\pi)^2\omega(A_+^{NL0} + \frac{1}{(2\pi)^2}\frac{\Gamma^{(0)}(\omega)}{r_s\omega^2} \times r_s\omega)$$

$$= \frac{1}{2\pi}\omega(A_+^{NL0}) + \Gamma^{(0)}$$

$$A_+^{NLO}(\omega) = \frac{2\pi}{\omega}[-\frac{3}{2}\Gamma^{(0)} - \frac{2(\Gamma^{(0)})^2}{x^2} + \frac{\Gamma^{(1)}}{x}]$$

$$A_-^{NLO}(\omega) = \frac{2\pi}{\omega}\left[-\frac{1}{2}\Gamma^{(0)} + \frac{\Gamma^{(1)}}{x} - \frac{2(\Gamma^{(0)})^2}{x^2}\right]$$

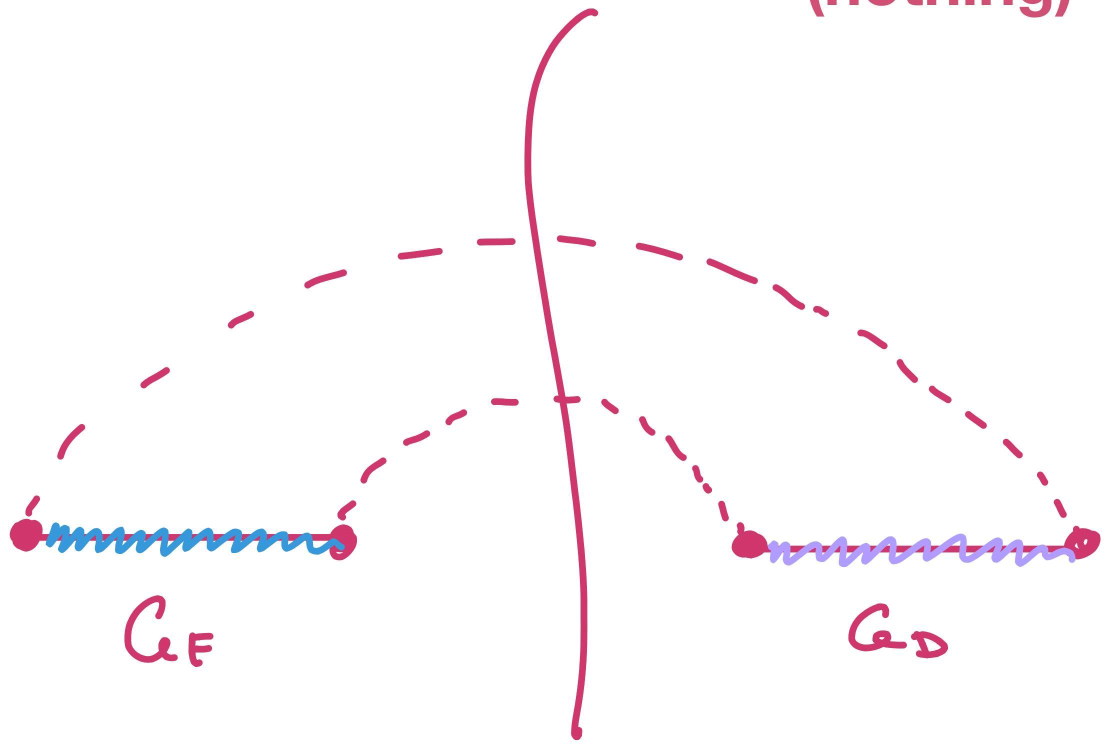
In addition at this order we can also have more insertions of the composite operator



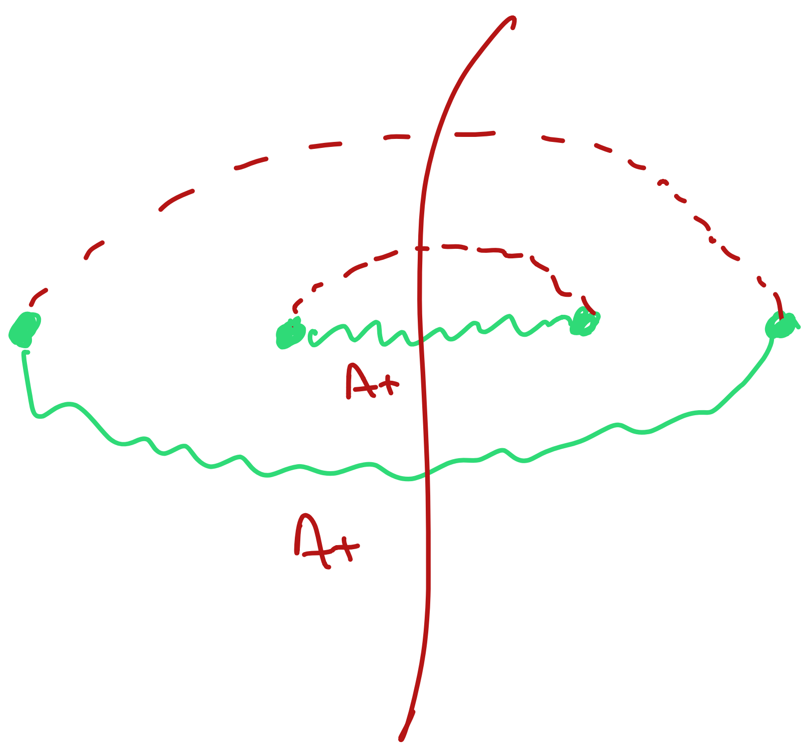
necessitates $\langle OOO \rangle$

Matching P(2|0) to study non Gaussianities allows us to extract four point function

Subtract Gaussian piece from the full theory, see whats left over (nothing)



vanishes by momentum conservation

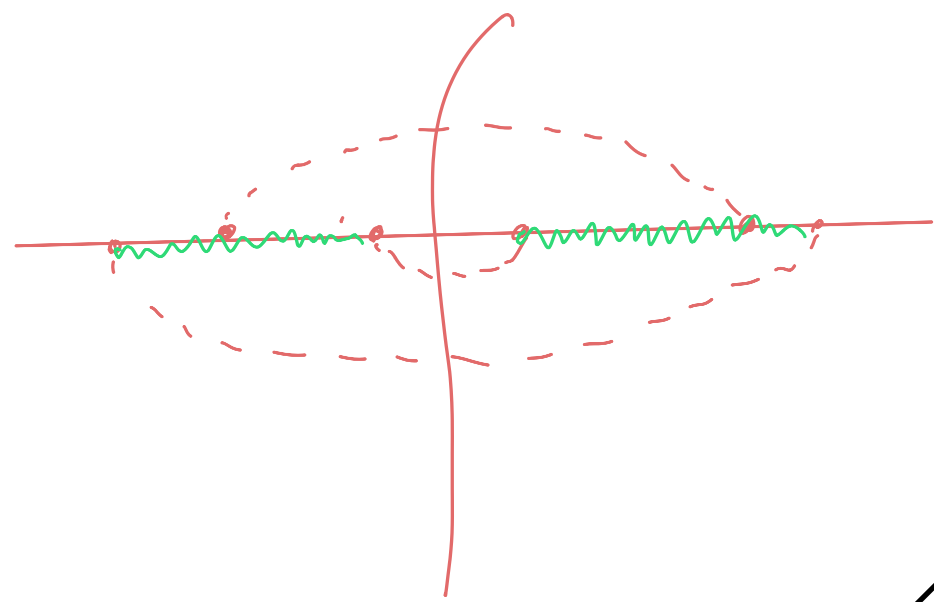


$$= p_1 p_2 A_+(p_1) A_+(p_2) (2\pi)^2$$

$$= \Gamma_0^2 / (r_s^2 \omega_1 \omega_2) = P(2|0)$$

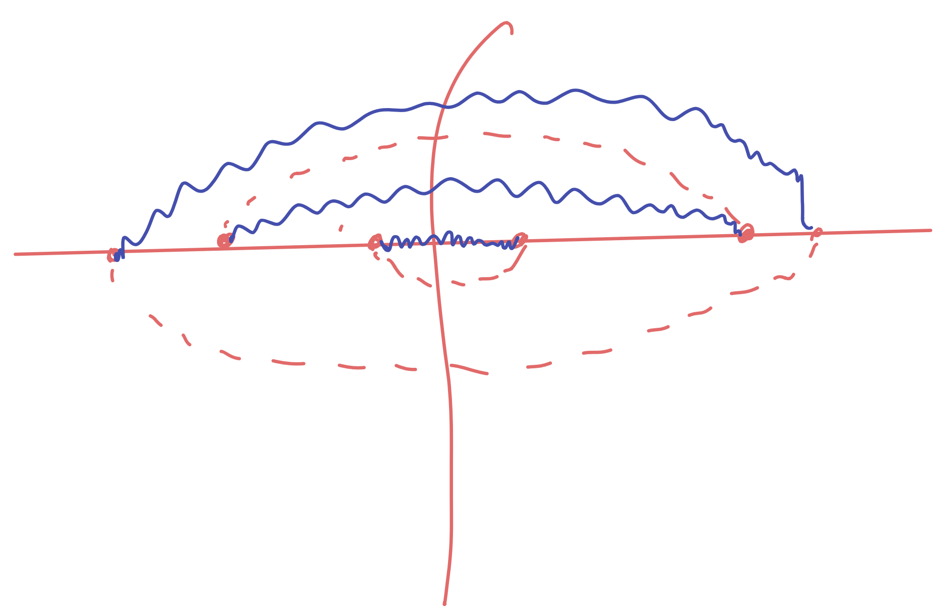
In general P(0,n)=P(0,1)^n !!

Consider P(0,3) to probe three point function



=0 since result is saturated by

$$\langle OO \rangle^3$$



BH soft theorem for Photons/Gravitons

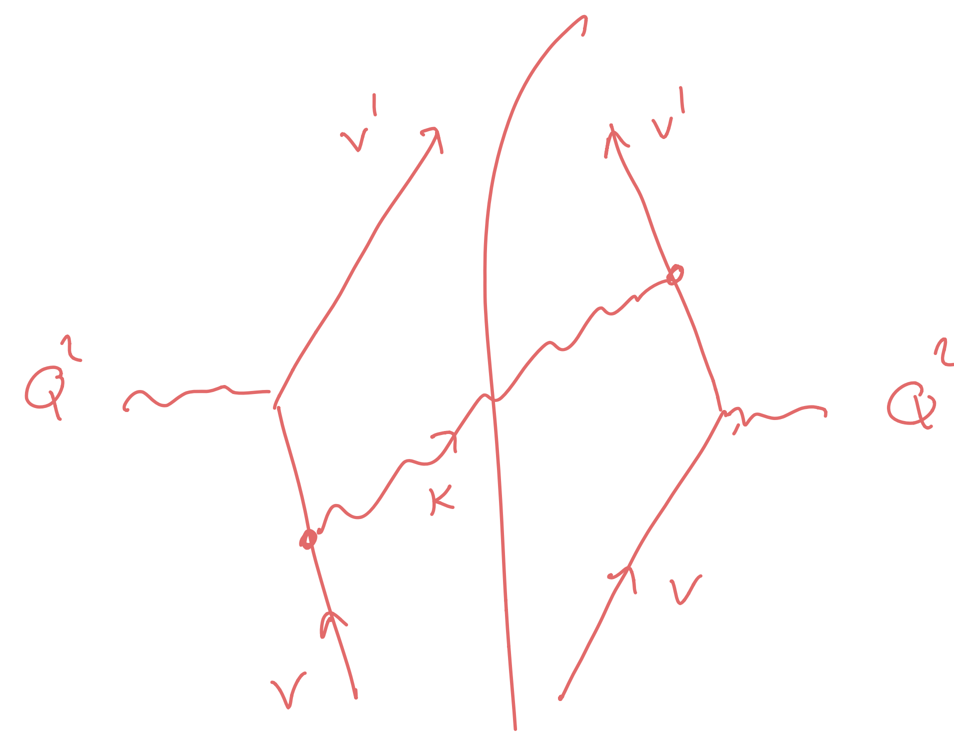
What effect does a collision have on Hawking radiation?

$$M = HS(k, v_i)$$

Canonical Soft Factor

$$S = \prod_i \frac{v_i \cdot \epsilon}{v_i \cdot k}$$

Is there an analogous factorization with Universal soft function ?



$$k \ll Q \ll r_s^{-1}$$

Confines of EFT

$$x^\mu(\lambda) = v^\mu \lambda \qquad \lambda < 0$$
$$x^\mu(\lambda) = v'^\mu \lambda \qquad \lambda > 0.$$

$$iM = H(Q) \int d\lambda \langle X, \epsilon^\lambda(l) \mid (p_I(\lambda) \cdot E^I(x(\lambda)) \mid \Omega \rangle$$

Factorization follows the usual proof (valid up to power of k/Q)

Contrary to the canonical soft theorems, this theorem will hold at the level of the amplitude squared

$$\int \frac{[d^3k]}{2k} \sum_{\lambda} \sum_{\Omega'} \mid M \mid^2 = \int \frac{[d^3k]}{2k} \int \frac{[d\omega]T(\omega)}{(\omega - v\cdot k + i\epsilon)(v'\cdot k - \omega - i\epsilon)} \mid H(k) \mid^2 v_\rho F^{\rho\mu}(k) v'_\tau F^{\tau\nu}(k)^\star \Lambda^I_\mu(v) \Lambda_{I\nu}(v')$$

+ trivial Hawking radiation off of external legs

As before we may extract the Wightman function from Bekenstein full theory calculation

$$\int d\lambda \langle \Omega \mid p_I(\lambda) p_J(0) \mid \Omega \rangle e^{-i\omega\lambda} = \delta_{IJ} T(\omega)$$

Performing the omega integral

$$S = (2\pi)(v'\cdot k)T(v'\cdot k) = (2\pi)(v\cdot k)T(v\cdot k) + O(q/M)$$

Sub-Leading Corrections to Dissipative Power Loss (LIGO phenomenology)

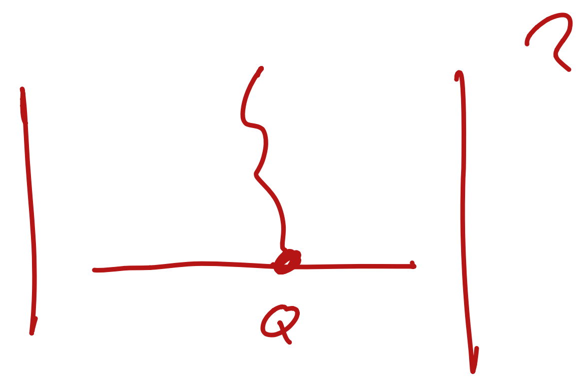
$+ \ldots$

Pure retarded (no Hawking radiation) generalizes our old result (spin important phenomenologically)

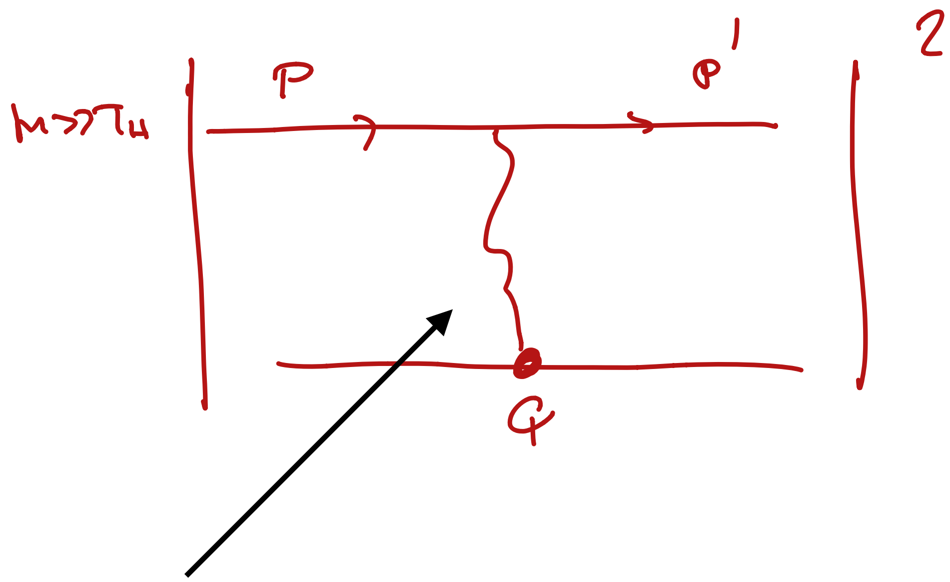
No Obstruction to working to arbitrary order, as all of the necessary correlators are known.

Effects Off-Shell Hawking Radiation

$P(0 \mid 1)$ **Is an on shell process that lets us extract** $W_{\pm}(\omega)$



Heavy particle not produced in Hawking radiation



off-shell Hawking quanta

Conclusions

- Internal (UV) physics rife with discovery potential
- Response Theory in the context of EFT allows for systematic predictions for effect of internal structure on templates.
- BH EFT shows remarkable surprises (?). Can match to extract horizon correlators which can then be used to calculate off shell quantities
- Quantum gravity effects from Hawking radiation do not cancel in all observables (but do for classical response function). Not suppressed by E/Mpl.
- Many open questions about this theory. What are the relevant DOF? Scattering off of Hawking radiation, quantum mechanically.