Parametrized black hole ringdown and black hole scalarization

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Physics and astrophysics in the era of gravitational wave detection
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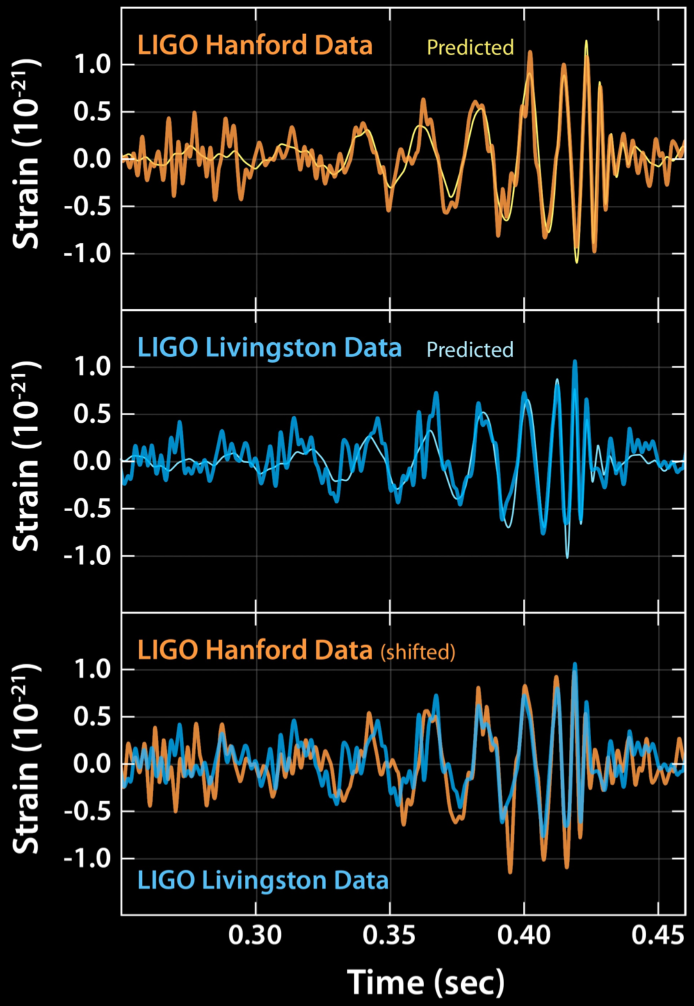
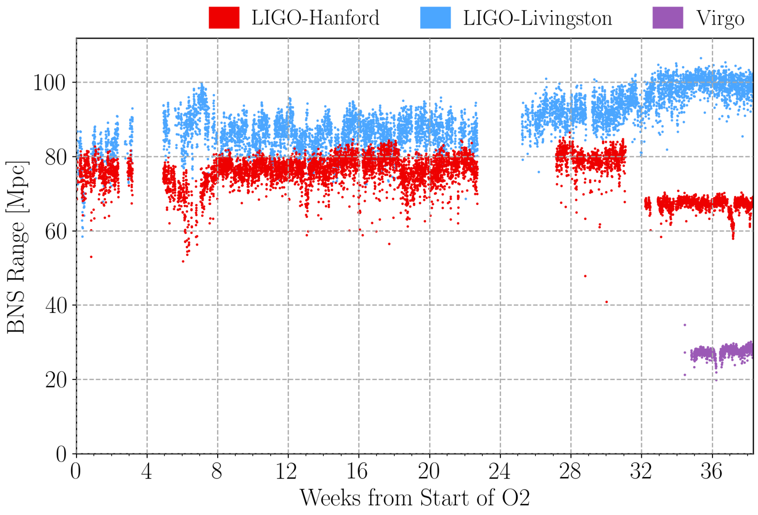


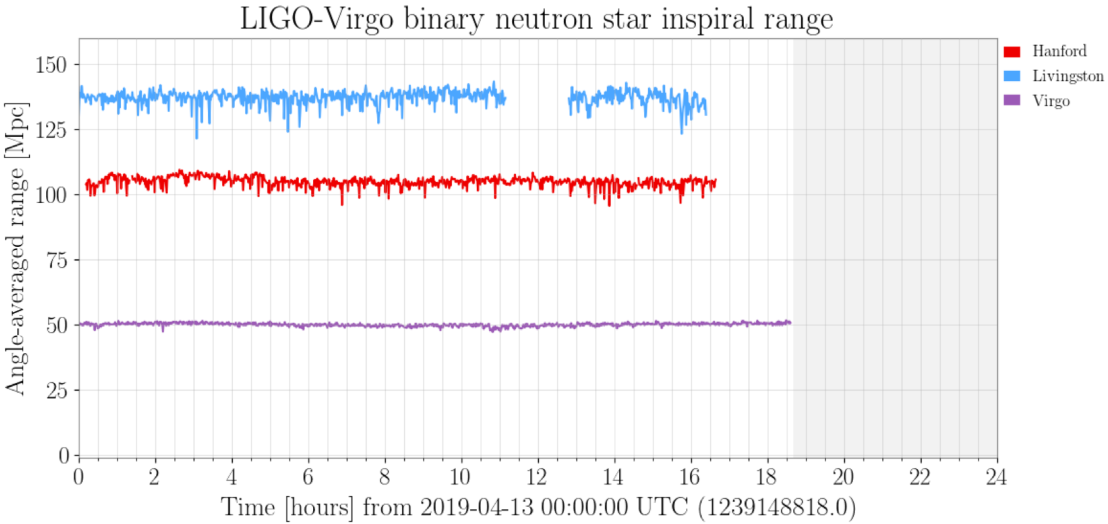
LIGO "testing GR" paper [arXiv:1602.03841]:

"According to the burst analysis, the GW150914 residual is not statistically distinguishable from the instrumental noise recorded in the vicinity of the detection, suggesting that all of the measured power is well represented by the GR prediction for the signal from a BBH merger. […]
We compute the 95% upper bound on the coherent network residual SNR. This upper bound is ≤ 7.3 at 95% confidence, independently of the maximum a posteriori waveform used."

Is there any modification to GR in the (small) residual?

LIGO/Virgo: O1, O2, O3





- **O1: 9/12/2015-1/19/2016: 3 BH-BH**
- **O2: 11/30/2016-8/25/2017: 7 BH-BH + 1 NS-NS**
- **O3: started 4/1/2019: about 20 BH-BH, 1 NS-NS, 1 NS-BH (?)**

https://gracedb.ligo.org/latest/

https://www.gw-openscience.org/detector_status/

How can we go after deviations?

BH-BH binaries **outnumber** NS-NS/NS-BH binaries. **Cleaner** tests of gravity…against what?
Two strategies:

1) **Identify “sensible” theories** with either
   - measurable deviations from GR in LIGO/LISA waveforms or
   - “smoking gun” signatures (superradiance, echoes: Vitor’s and Rafael’s talks)

   Einstein-scalar-Gauss-Bonnet gravity and spontaneously scalarized black hole

2) **Parametrize, then map to specific theories**
   Preferable, but parametrizing small deviations needs a background solution

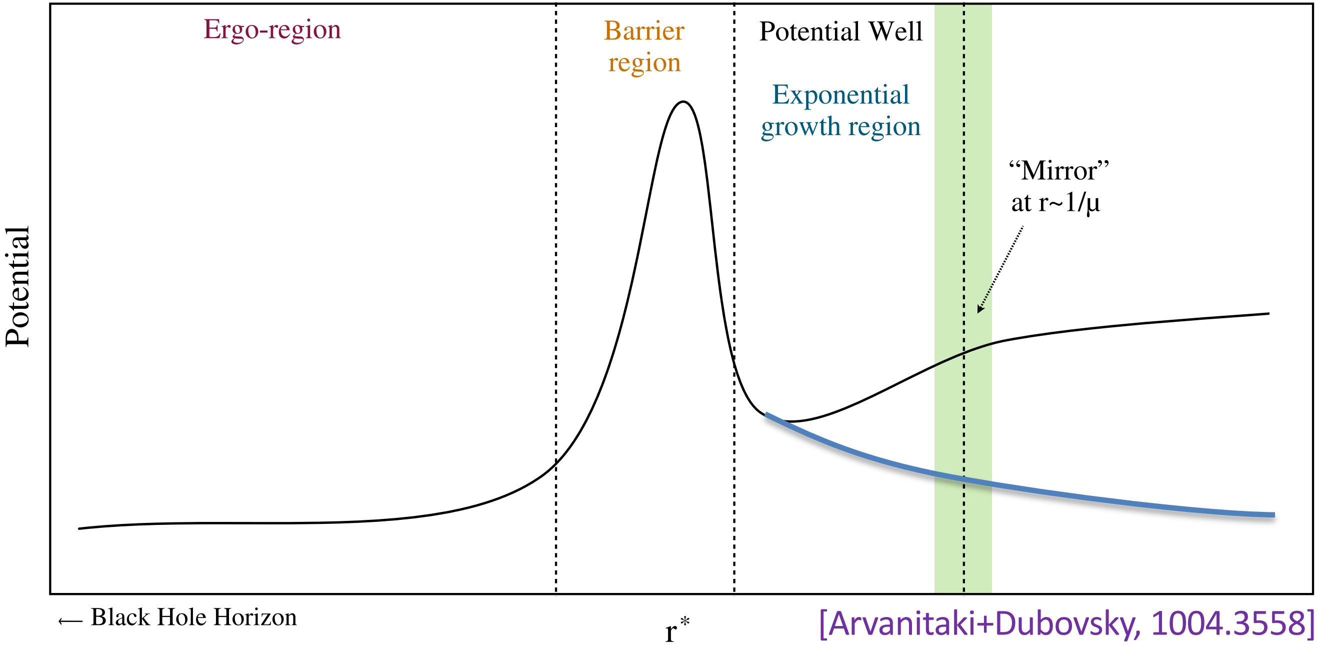
   ppE formalism in the inspiral, where we can use PN theory: Yunes, Yagi, Pretorius…
   Can we **parametrize ringdown** dynamics, where we can use perturbation theory?
   Analogous to inferring nuclear structure from scattering, but harder - coupled system

   General formalism for nonrotating (but otherwise general) parametrized ringdown

Black hole spectroscopy: a null test

Quasinormal (and superradiant) modes

$$\left(\frac{d^2}{dr_*^2} + \omega^2\right)\Phi = V\Phi$$



[Arvanitaki+Dubovsky, 1004.3558]

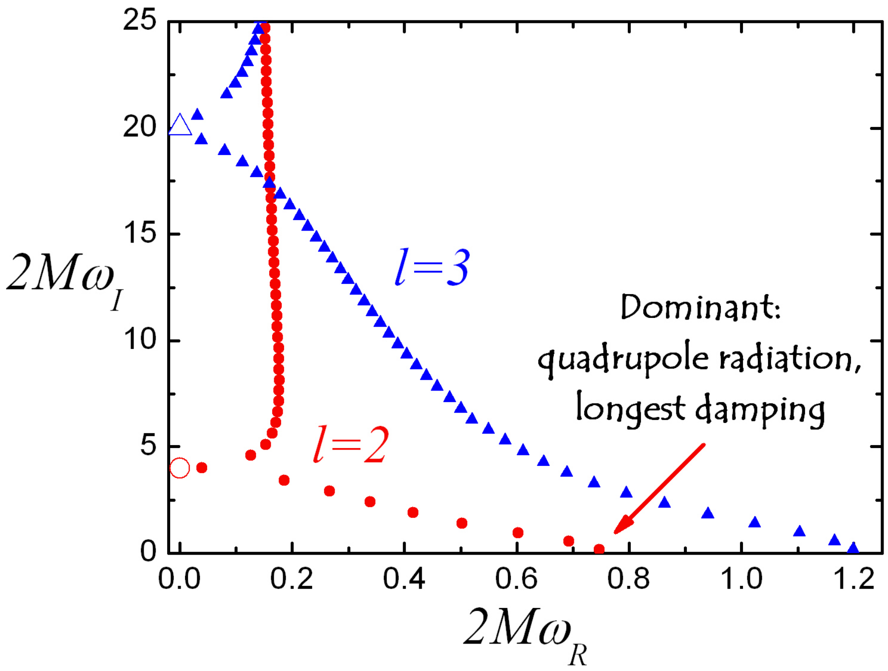
Quasinormal modes:

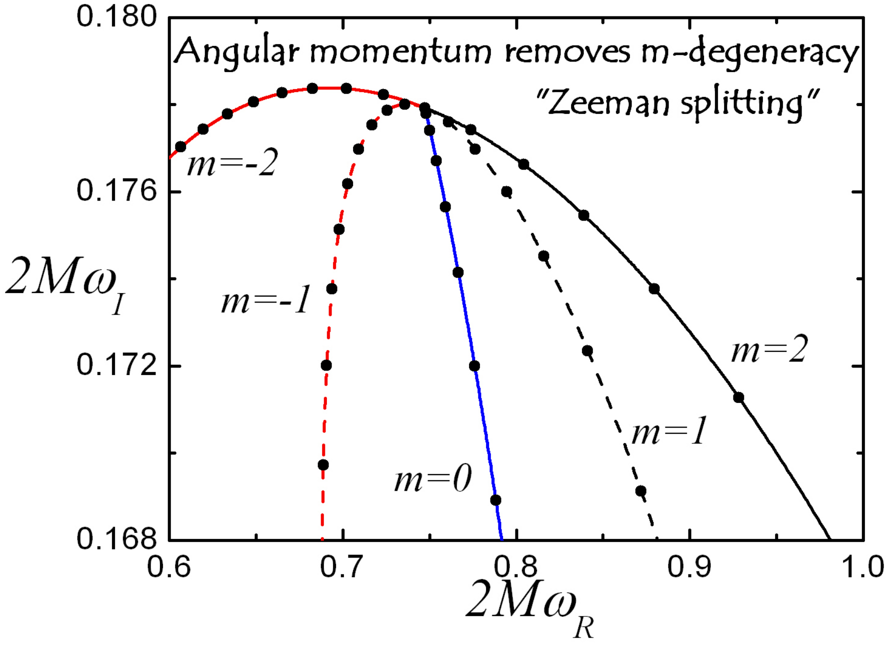
- Ingoing waves at the horizon, outgoing waves at infinity
- Spectrum of damped modes ("ringdown") [EB+, 0905.2975]

Massive scalar field:

- Superradiance: black hole bomb when $0 < \omega < m\Omega_H$ [Press-Teukolsky 1972]
- Hydrogen-like, unstable bound states [Detweiler 1980, Zouros+Eardley, Dolan…]

Schwarzschild and Kerr quasinormal mode spectrum

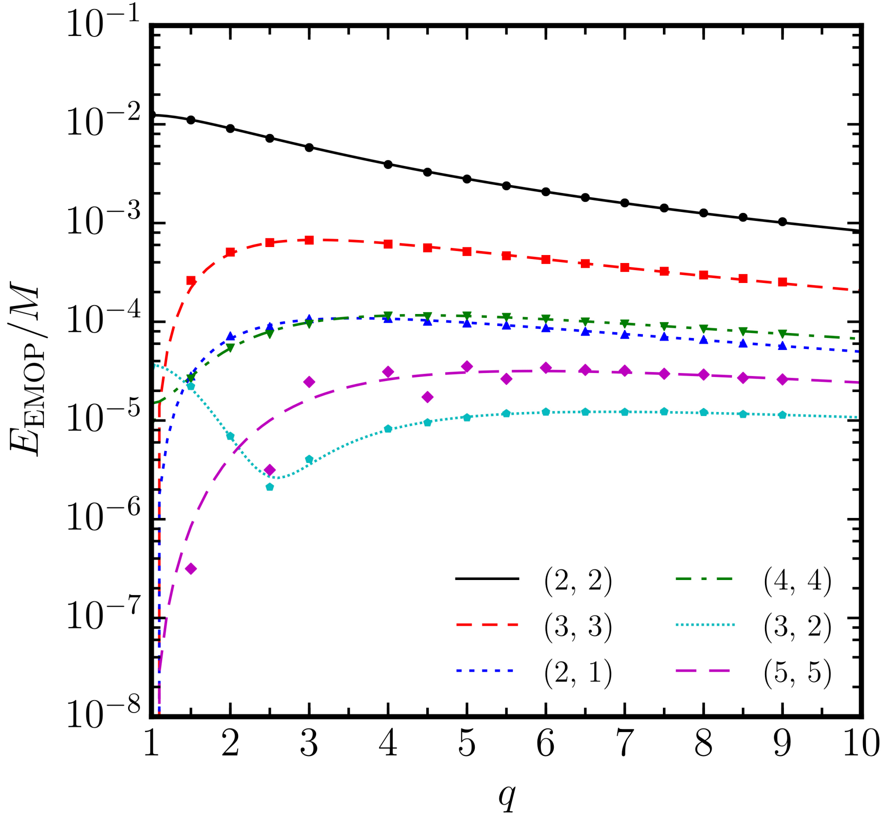


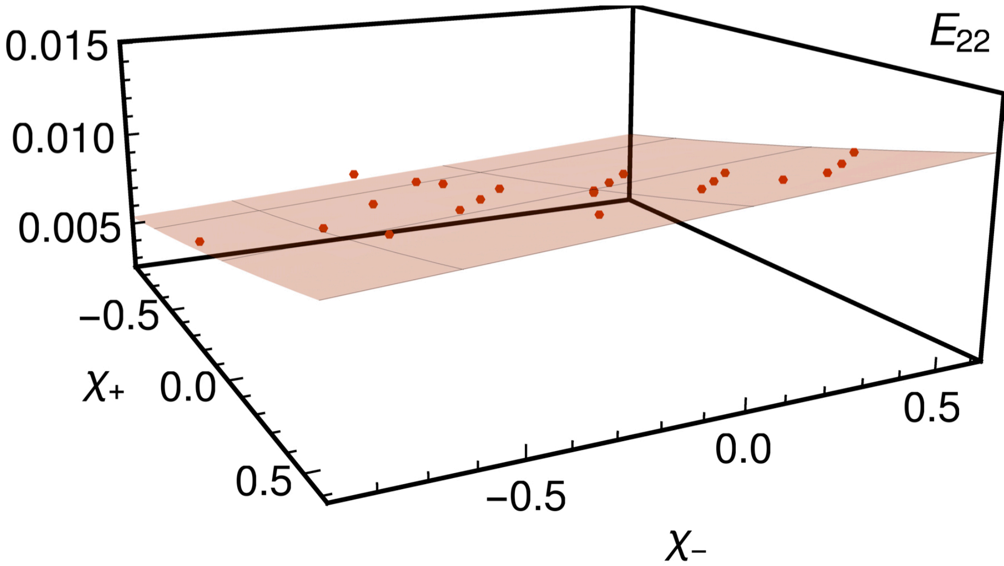


- **One mode** fixes mass and spin – and the whole spectrum!
- **N modes:** N tests of GR dynamics…**if** they can be measured
- Needs SNR>50 or so for a comparable mass, nonspinning binary merger

[Berti-Cardoso-Will, gr-qc/0512160; EB+, gr-qc/0707.1202]

Multi-mode detectability: mass ratio and spin dependence



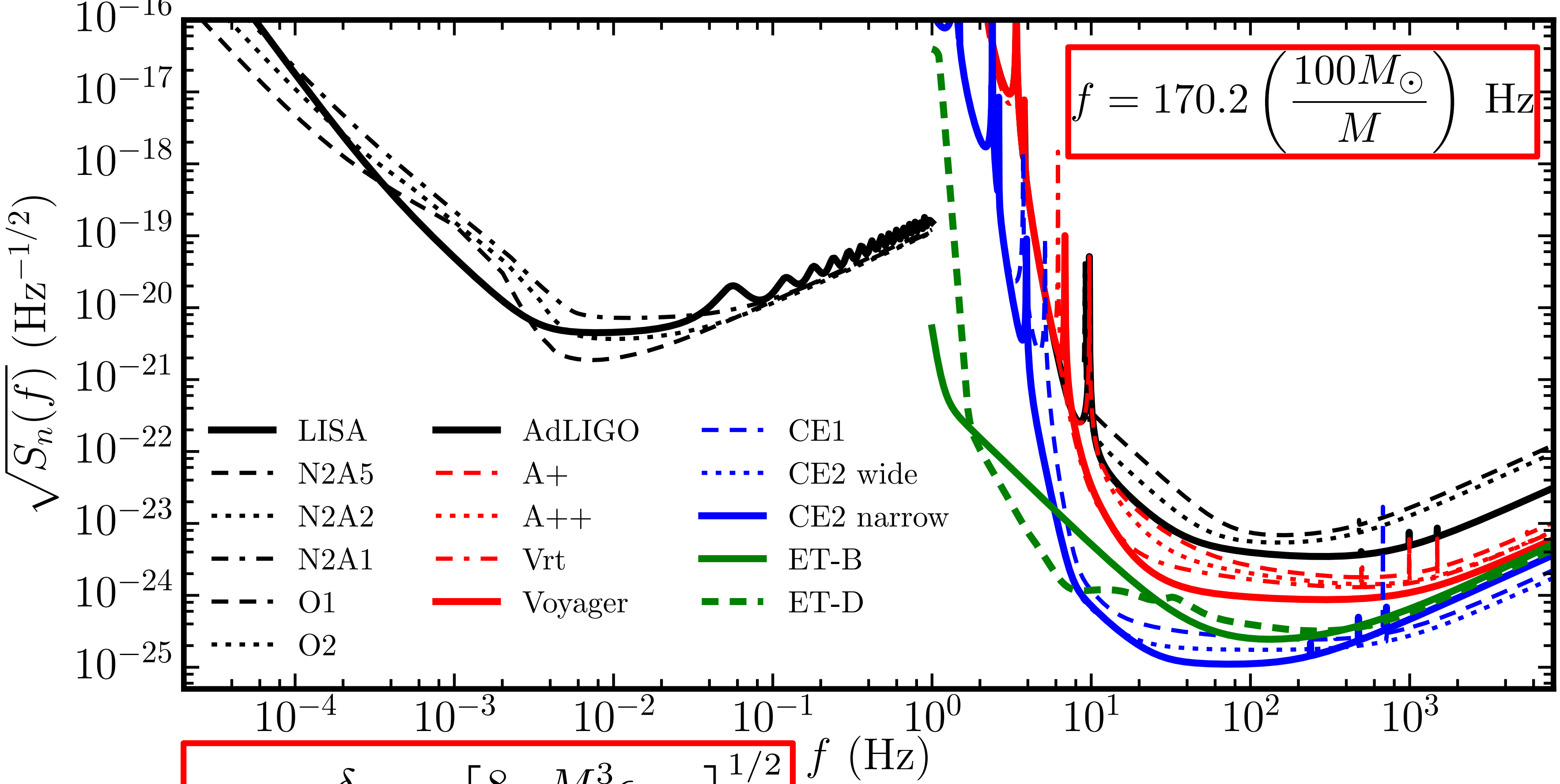


$$\chi_\pm \equiv \frac{m_1\chi_1 \pm m_2\chi_2}{m_1+m_2}$$

Strongest spin dependence: $\ell = 2, m = 1$

[Baibhav+, 1710.02156]

Bridging the mass gap: gravitational wave astronomy in the 2030s

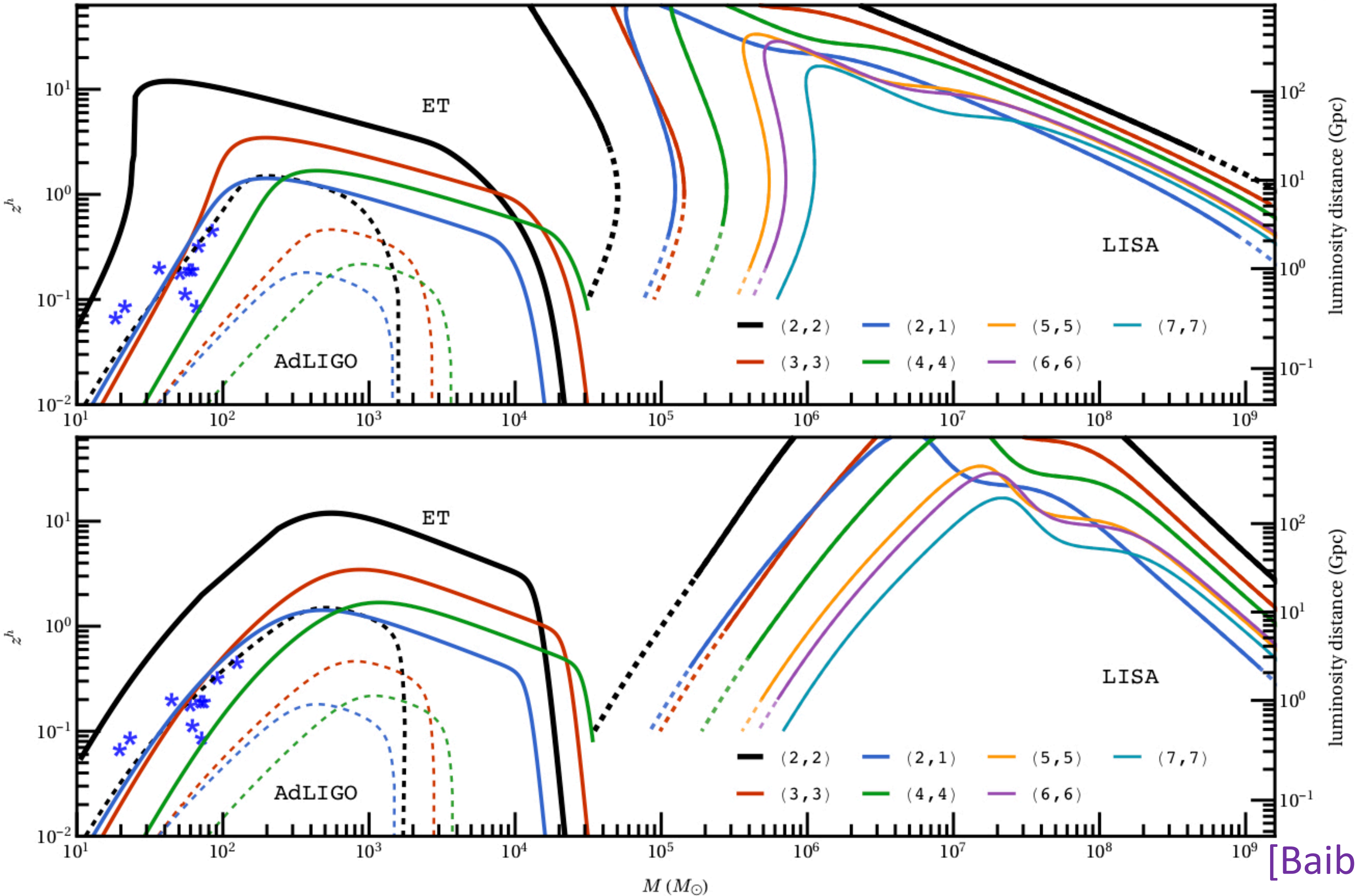


$$f = 170.2 \left(\frac{100 M_\odot}{M}\right) \text{Hz}$$

$$\rho = \frac{\delta_{\rm eq}}{D_{\rm L}\mathcal{F}_{lmn}}\left[\frac{8}{5}\frac{M_z^3\epsilon_{\rm rd}}{S_n(f_{lmn})}\right]^{1/2}$$

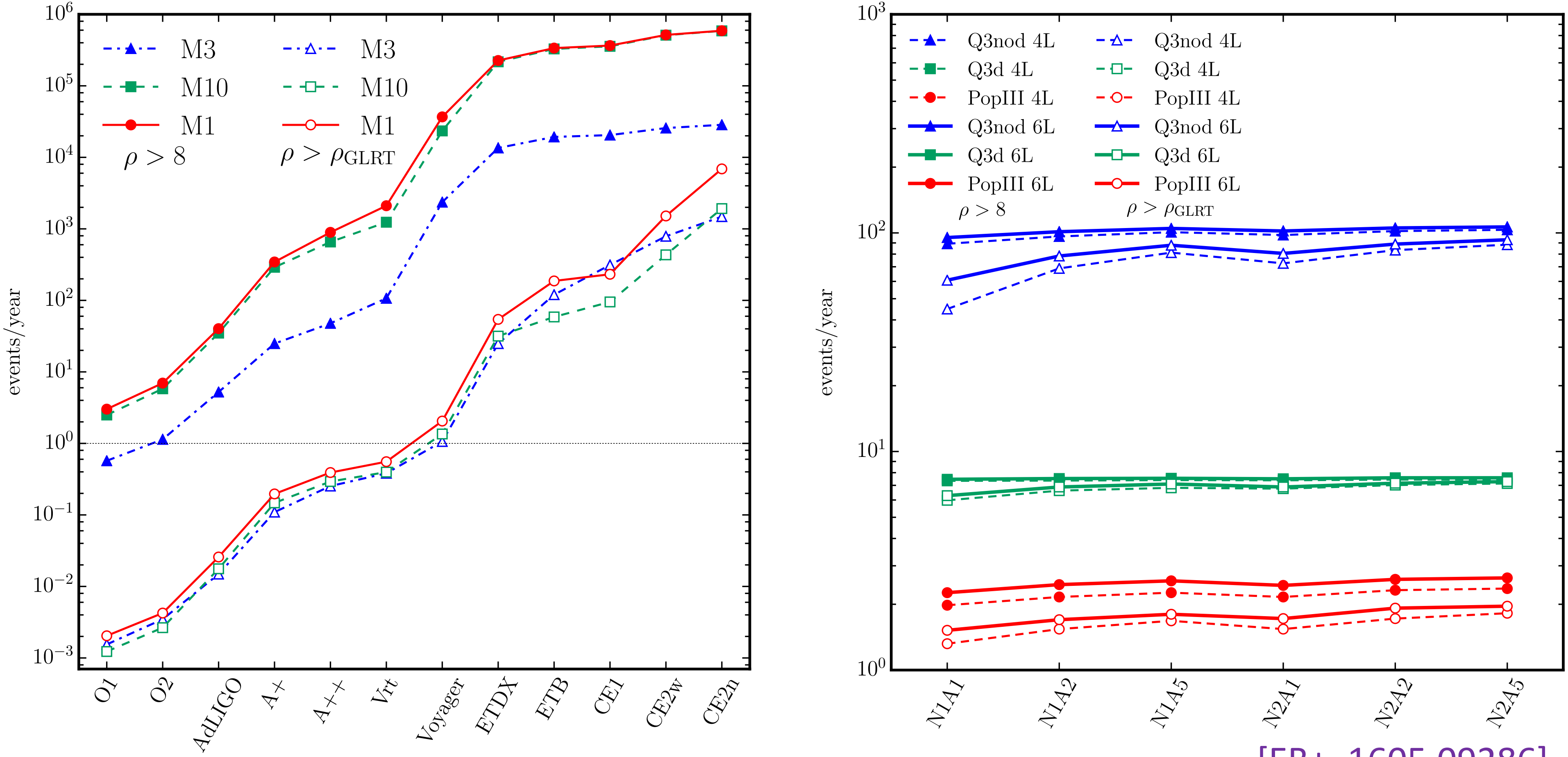
[EB+, 1605.09286]

Tests of GR: multi-mode detection and IMBHs at large z



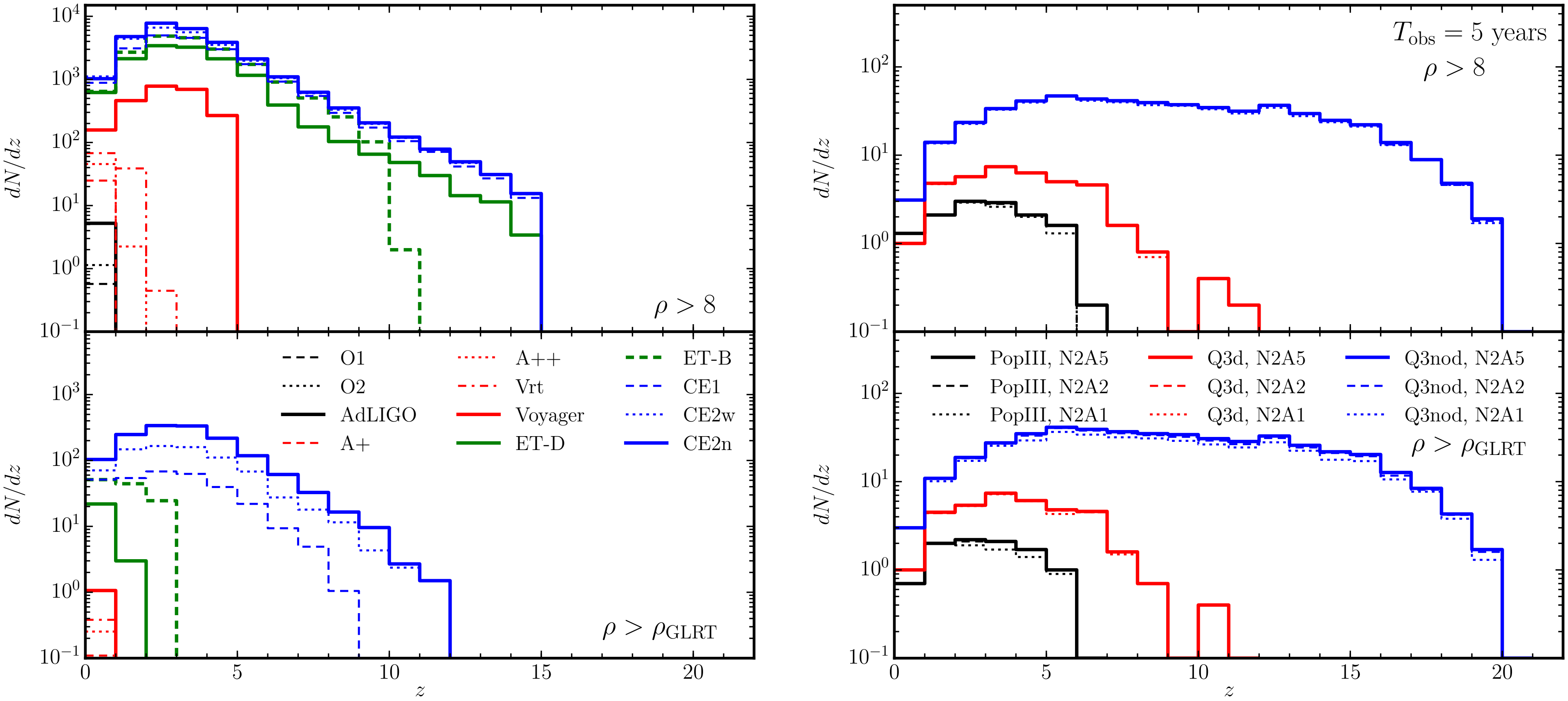
[Baibhav+, 1809.03500]

Earth vs. space-based: ringdown detections and black hole spectroscopy



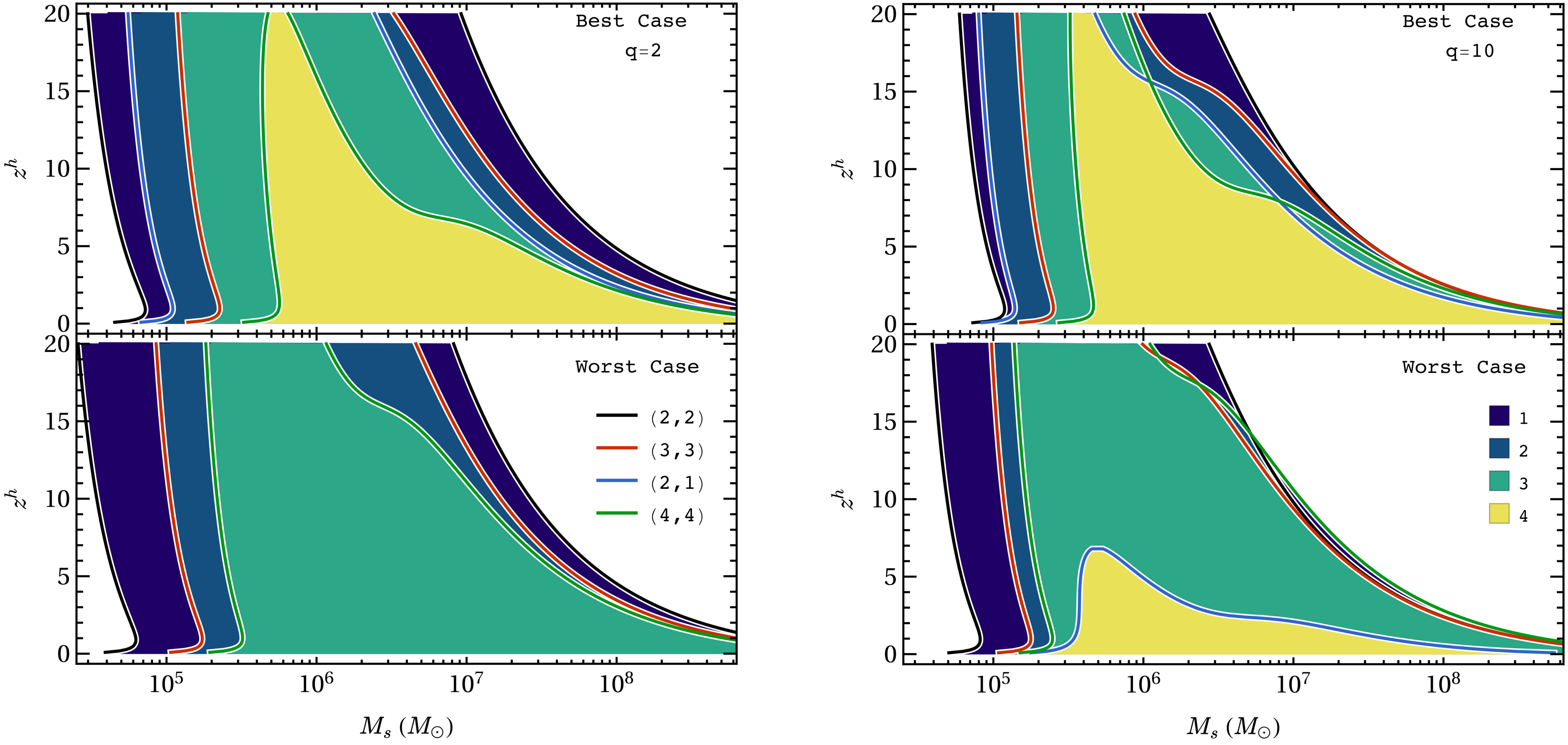
[EB+, 1605.09286]

Earth vs. space-based: redshift distribution



[EB+, 1605.09286]

LISA: multi-mode tests



[Baibhav+EB, 1809.03500]

Which theories of gravity can we test with black hole mergers?

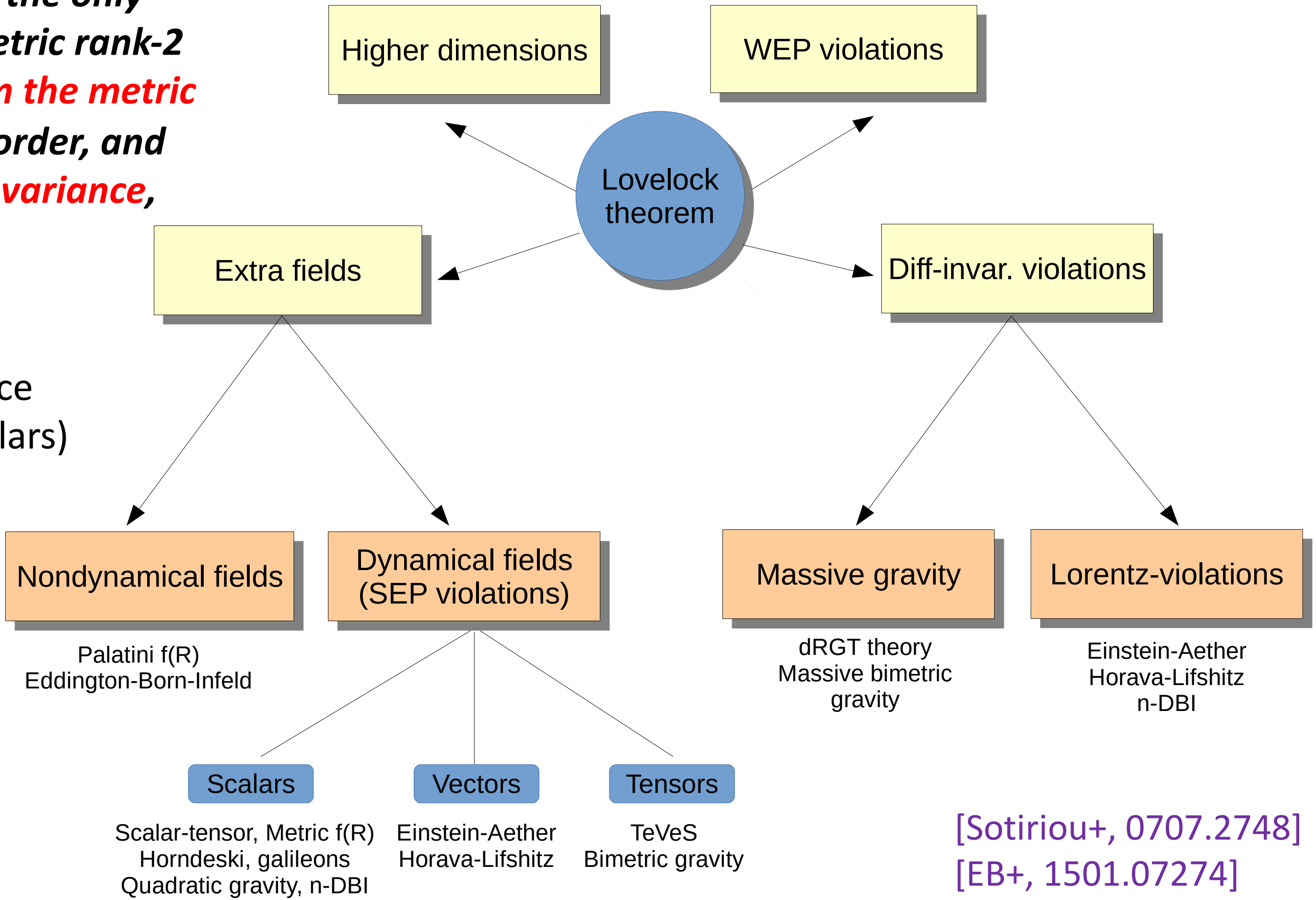
A guiding principle to modified GR: Lovelock's theorem

In four spacetime dimensions the only divergence-free (WEP) symmetric rank-2 tensor constructed solely from the metric and its derivatives up to 2nd order, and preserving diffeomorphism invariance, is the Einstein tensor plus Λ.

Generic modifications introduce additional fields (simplest: scalars)

Minimal requirements:

- Action principle
- Well-posed
- Testable predictions
- Black holes, neutron stars
- Cosmologically viable



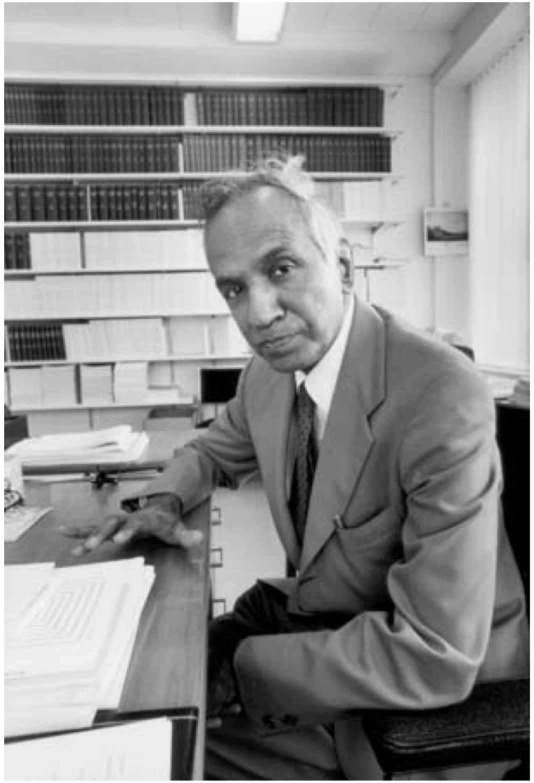
[Sotiriou+, 0707.2748]
[EB+, 1501.07274]

The modified gravity zoo

Theory	Field content	Strong EP	Massless graviton	Lorentz symmetry	Linear $T_{\mu\nu}$	Weak EP	Well-posed?	Weak-field constraints
Extra scalar field								
Scalar-tensor	S	✗	✓	✓	✓	✓	✓ [34]	[35–37]
Multiscalar	S	✗	✓	✓	✓	✓	✓ [38]	[39]
Metric $f(R)$	S	✗	✓	✓	✓	✓	✓ [40, 41]	[42]
Quadratic gravity								
Gauss-Bonnet	S	✗	✓	✓	✓	✓	✓?	[43]
Chern-Simons	P	✗	✓	✓	✓	✓	✗✓? [44]	[45]
Generic	S/P	✗	✓	✓	✓	✓	?	
Horndeski	S	✗	✓	✓	✓	✓	✓?	
Lorentz-violating								
Æ-gravity	SV	✗	✓	✗	✓	✓	✓?	[46–49]
Khronometric/ Hořava-Lifshitz	S	✗	✓	✗	✓	✓	✓?	[48–51]
n-DBI	S	✗	✓	✗	✓	✓	?	none ([52])
Massive gravity								
dRGT/Bimetric	SVT	✗	✗	✓	✓	✓	?	[17]
Galileon	S	✗	✓	✓	✓	✓	✓?	[17, 53]
Nondynamical fields								
Palatini $f(R)$	–	✓	✓	✓	✗	✓	✓	none
Eddington-Born-Infeld	–	✓	✓	✓	✗	✓	?	none
Others, not covered here								
TeVeS	SVT	✗	✓	✓	✓	✓	?	[37]
$f(R)\mathcal{L}_m$	?	✗	✓	✓	✓	✗	?	
$f(T)$	?	✗	✓	✗	✓	✓	?	[54]

[EB+, 1501.07274]

Black holes are simple. Too simple?



Black holes in GR uniquely described by mass and spin
[Carter, Israel, Hawking, Robinson, 1970s]

"In my entire scientific life, extending over forty-five years, the most shattering experience has been the realization that an exact solution of Einstein's equations of general relativity, discovered by the New Zealand mathematician, Roy Kerr, provides the absolutely exact representation of untold numbers of massive black holes that populate the universe." (S. Chandrasekhar)

Similar "no hair" theorems in modified gravity. Examples:

Brans-Dicke [Hawking, Thorne & Dykla, Chase, Bekenstein]
Multiple scalars [Heusler, gr-qc/9503053]
Bergmann-Wagoner, f(R) [Sotiriou-Faraoni, 1109.6324]
Higher-order curvature [Psaltis+, 0710.4564]
Horndeski [Hui-Nicolis, 1202.1296] – **but loophole: EsGB!**

Dynamical no-hair results in scalar-tensor theories

$$S = \frac{1}{16\pi}\int \sqrt{-g}d^4x\left[\phi R - \frac{\omega(\phi)}{\phi}g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + M(\phi)\right] + \int \mathcal{L}_{\mathrm{M}}(g^{\mu\nu},\Psi)d^4x$$

Orbital period derivative: $\frac{\dot{P}}{P} = -\frac{\mu m}{r^3}\kappa_D(s_1 - s_2)^2 - \frac{8}{5}\frac{\mu m^2}{r^4}\kappa_1$

$$\kappa_D = 2\mathcal{G}\xi\left(\frac{\omega^2 - m_s^2}{\omega^2}\right)^{\frac{3}{2}}\Theta(\omega - m_s)$$

$$\kappa_1 = \mathcal{G}^2\left[12 - 6\xi + \xi\Gamma^2\left(\frac{4\omega^2 - m_s^2}{4\omega^2}\right)^{\frac{5}{2}}\Theta(2\omega - m_s)\right]$$

$$\xi = \frac{1}{2+\omega_{\mathrm{BD}}}$$

$$G = 1 - \xi(s_1 + s_2 - 2s_1 s_2)$$

$$\Gamma = 1 - 2\frac{s_1 m_2 + m_1 s_2}{m}$$

For black hole binaries, $s_1 = s_2 = \frac{1}{2}$ and dipole vanishes identically

Quadrupole: $\Gamma = 0$

Result extended to higher PN orders, BH-NS, and is exact in the large mass ratio limit [Will & Zaglauer 1989; Alsing+, 1112.4903; Mirshekari & Will, 1301.4680; Yunes+, 1112.3351; Bernard 1802.10201, 1812.04169, 1906.10735]

Ways around: matter (but EOS degeneracy), cosmological BCs (but small corrections), or **curvature itself sourcing the scalar field: dCS, EsGB** [Yagi+ 1510.02152]

A loophole in no-hair theorems: Einstein-scalar-Gauss-Bonnet gravity

Horndeski Lagrangian: most general scalar-tensor theory with second-order EOMs

$$S = \sum_{i=2}^{5} \int d^4x \sqrt{-g}\mathcal{L}_i$$

$$G_i = G_i(\phi, X) \qquad \phi^2_{\mu\nu} \equiv \phi_{\mu\nu}\phi^{\mu\nu}$$

$$X = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi \qquad \phi^3_{\mu\nu} \equiv \phi_{\mu\nu}\phi^{\nu\alpha}\phi^\mu_\alpha$$

$$\mathcal{L}_2 = G_2$$
$$\mathcal{L}_3 = -G_3\Box\phi$$
$$\mathcal{L}_4 = G_4 R + G_{4X}\left[(\Box\phi)^2 - \phi^2_{\mu\nu}\right]$$
$$\mathcal{L}_5 = G_5 G_{\mu\nu}\phi^{\mu\nu} - \tfrac{1}{6}G_{5X}\left[(\Box\phi)^3 + 2\phi^3_{\mu\nu} - 3\phi^2_{\mu\nu}\Box\phi\right]$$

Set:

$$G_2 = X + 8f^{(4)}X^2(3 - \ln X)$$
$$G_3 = 4f^{(3)}X(7 - 3\ln X)$$
$$G_4 = \tfrac{1}{2} + 4f^{(2)}X(2 - \ln X)$$
$$G_5 = -4f^{(1)}\ln X$$

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}R + X + f(\phi)\mathcal{G}\right)$$

$$\mathcal{G} \equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

Shift symmetry: invariance under $\phi \to \phi + c$, i.e. $G_i = G_i(X)$

EsGB is **a special case of Horndeski *and* of quadratic gravity**

[Kobayashi+, 1105.5723; Sotiriou+Zhou, 1312.3622; Maselli+, 1508.03044]

Scalarization

Scalar-tensor theory and spontaneous scalarization

- Action (in the Einstein frame):

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g^\star}\left[R^\star - 2g^{\star\mu\nu}\left(\partial_\mu\varphi\right)\left(\partial_\nu\varphi\right) - V(\varphi)\right] + S_M[\Psi, A^2(\varphi)g^\star_{\mu\nu}]$$

- Gravity-matter coupling:

$$\alpha(\varphi) \equiv d(\ln A(\varphi))/d\varphi$$

$$\alpha(\varphi) = \alpha_0 + \beta_0(\varphi - \varphi_0) + \ldots$$

- Field equations:

$$G^\star_{\mu\nu} = 2\left(\partial_\mu\varphi\partial_\nu\varphi - \frac{1}{2}g^\star_{\mu\nu}\partial_\sigma\varphi\partial^\sigma\varphi\right) - \frac{1}{2}g^\star_{\mu\nu}V(\varphi) + 8\pi T^\star_{\mu\nu}\,,$$

$$\Box_{g^\star}\varphi = -4\pi\alpha(\varphi)T^\star + \frac{1}{4}\frac{dV}{d\varphi}\,,$$

[Damour+Esposito-Farese, PRL 70, 2220 (1993); PRD 54, 1474 (1996)]

Scalarization threshold: a back-of-the-envelope argument

$$\Box_{g^*}\varphi = -4\pi\alpha(\varphi)T^* \qquad \alpha(\varphi) = \beta_0\varphi$$

$$-T^* = A^4(\epsilon^* - 3p^*) \sim \frac{3}{4\pi R^2}\frac{m}{R} \quad \text{for} \quad r < R$$

$$\nabla^2\varphi = \mathrm{sign}(\beta_0)\left[\frac{3|\beta_0|(m/R)}{R^2}\right]\varphi = \mathrm{sign}(\beta_0)\kappa^2\varphi$$

$$\beta_0 < 0 \Longrightarrow \varphi_{\mathrm{inside}} = \varphi_{\mathrm{c}}\frac{\sin(\kappa r)}{\kappa r}$$

$$\varphi_{\mathrm{c}} = \frac{\varphi_0}{\cos(\kappa R)} \gg \varphi_0 \qquad \boxed{\kappa R \sim \pi/2}$$

$$\boxed{m/R \sim 0.2 \Longrightarrow \beta \sim -4}$$

[Damour+Esposito-Farese, PRL 70, 2220 (1993)]

No-hair conditions in EsGB

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[R - \frac{1}{2}\nabla_\alpha\varphi\nabla^\alpha\varphi + f(\varphi)\mathscr{G}\right] + S_\mathrm{m}[g_{\mu\nu},\psi]$$

$$\mathscr{G} \equiv R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$$\Box\varphi = -f_{,\varphi}\mathscr{G}$$

$$G_{\mu\nu} = T_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}\partial_\rho\varphi\partial^\rho\varphi + \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi$$

$$- g_{\gamma(\mu}g_{\nu)\delta}\epsilon^{\sigma\delta\rho\chi}\nabla_\tau[{}^*R^{\gamma\tau}{}_{\rho\chi}\nabla_\sigma f]$$

Matter: zero in vacuum

GB contribution $\propto f_{,\varphi}$

Kerr is a solution with constant scalar field if: $\boxed{f_{,\varphi}(\varphi_0) = 0}$

Dilatonic theories $f \sim \exp(\varphi)$ and shift-symmetric theories $f \sim \varphi$ **do not have a GR limit!**

No-hair theorem: in addition, $\boxed{f_{,\varphi\varphi}\mathscr{G} < 0}$

[Silva+, 1711.02080]

A proof, and a heuristic argument

$$\Box\varphi = -f_{,\varphi}\mathscr{G}$$

$$\int_{\mathscr{V}} \mathrm{d}^4x\sqrt{-g}\left[f_{,\varphi}\Box\varphi + f_{,\varphi}^2(\varphi)\mathscr{G}\right] = 0$$

Integrate by parts, divergence theorem:

$$\int_{\mathscr{V}} \mathrm{d}^4x\sqrt{-g}\left[f_{,\varphi\varphi}\nabla^\mu\varphi\nabla_\mu\varphi - f_{,\varphi}^2(\varphi)\mathscr{G}\right] = \int_{\partial\mathscr{V}} \mathrm{d}^3x\sqrt{|h|}f_{,\varphi}n^\mu\nabla_\mu\varphi$$

The RHS vanishes for stationary, asymptotically flat spacetimes; if $\boxed{f_{,\varphi\varphi}\mathscr{G} < 0}$ both terms on the LHS vanish separately, i.e. $\varphi = \varphi_0 = c$

In alternative, linearize the scalar field equation: $[\Box + f_{,\varphi\varphi}(\varphi_0)\mathscr{G}]\delta\varphi = 0$

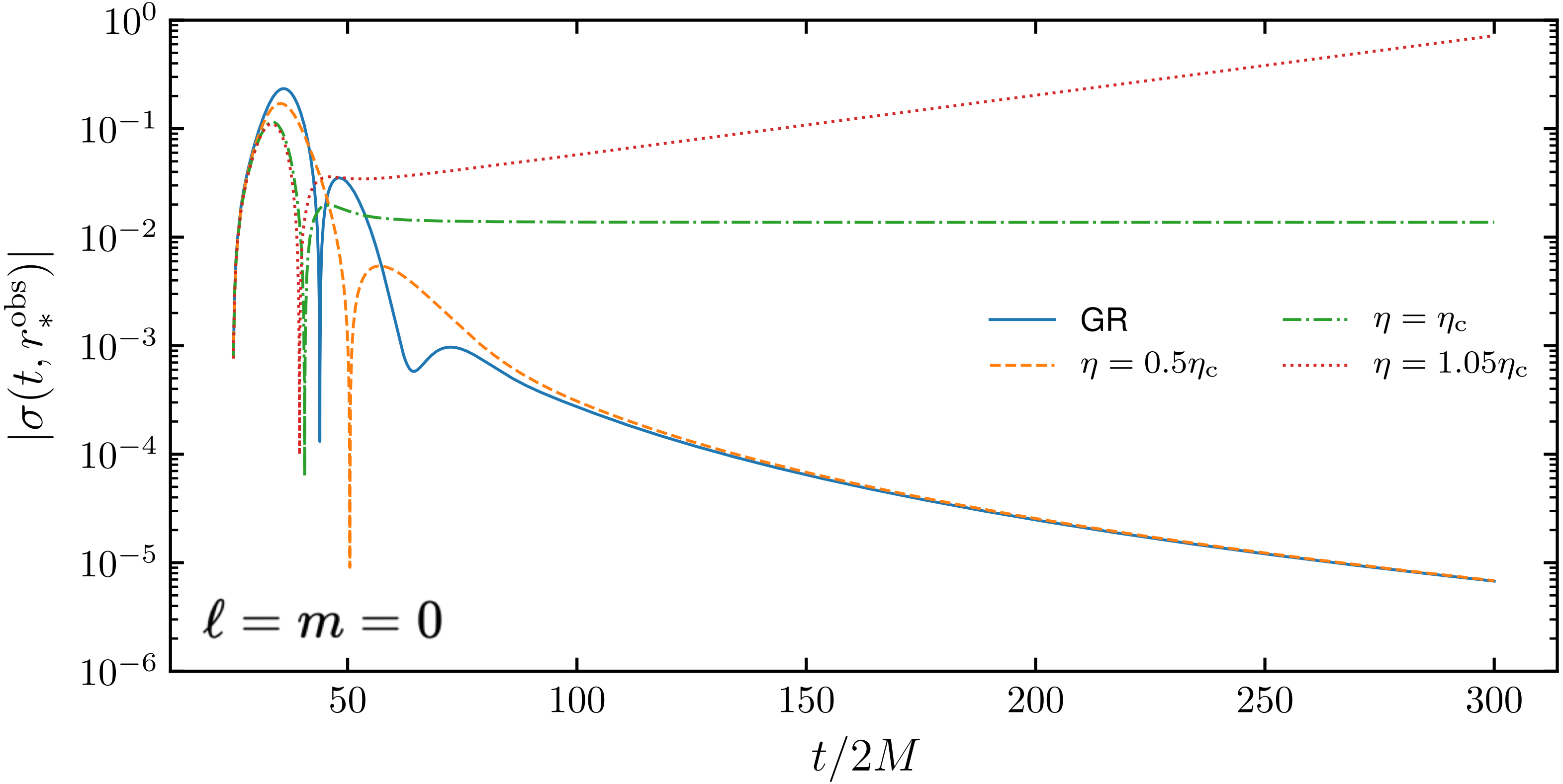
$m_{\mathrm{eff}}^2 = -f_{,\varphi\varphi}\mathscr{G}$ is an effective mass for the perturbation – tachyonic instability?

[Silva+, 1711.02080]

Time evolution of a test scalar field (linearized monopolar perturbations)

$$[\Box + f_{,\varphi\varphi}(\varphi_0)\mathscr{G}]\delta\varphi = 0 \qquad \delta\varphi = \sum_{\ell m} \frac{\sigma_{\ell m}(t,r)}{r} Y_{\ell m}(\theta,\phi) \qquad f = \frac{1}{8}\eta\varphi^2$$

$$\eta_c/M^2 = 2.90$$



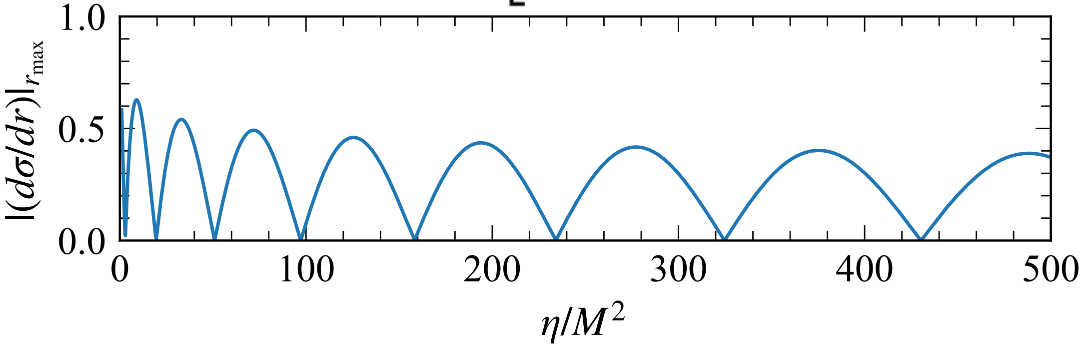
Linearized case: monopolar perturbations, asymptotic flatness

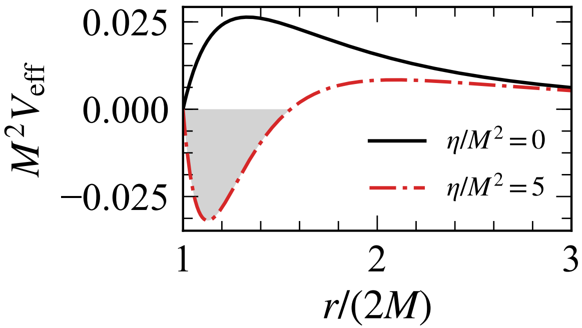
Discrete solutions (scalarization thresholds) with n nodes for $\eta/M^2 = 2.90, 19.50, 50.93\ldots$

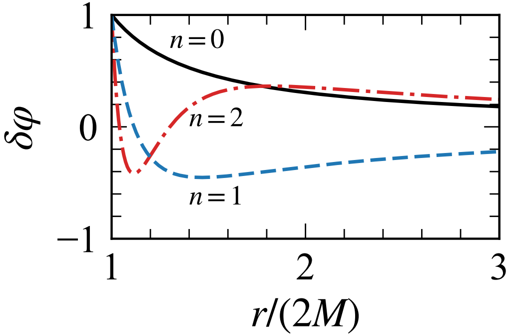
$$-\frac{\partial^2\sigma}{\partial t^2}+\frac{\partial^2\sigma}{\partial r_*^2}=V_{\rm eff}\sigma$$

$$ds^2=-a(r)\mathrm{d}t^2+b(r)\mathrm{d}r^2+r^2\mathrm{d}\Omega \qquad \mathrm{d}r/\mathrm{d}r_*=\sqrt{a/b}$$

$$V_{\rm eff}\equiv a\left[\frac{\ell(\ell+1)}{r^2}+\frac{1}{2ra}\frac{\mathrm{d}\left(ab^{-1}\right)}{\mathrm{d}r}-\frac{\eta\mathscr{G}}{4}\right]$$





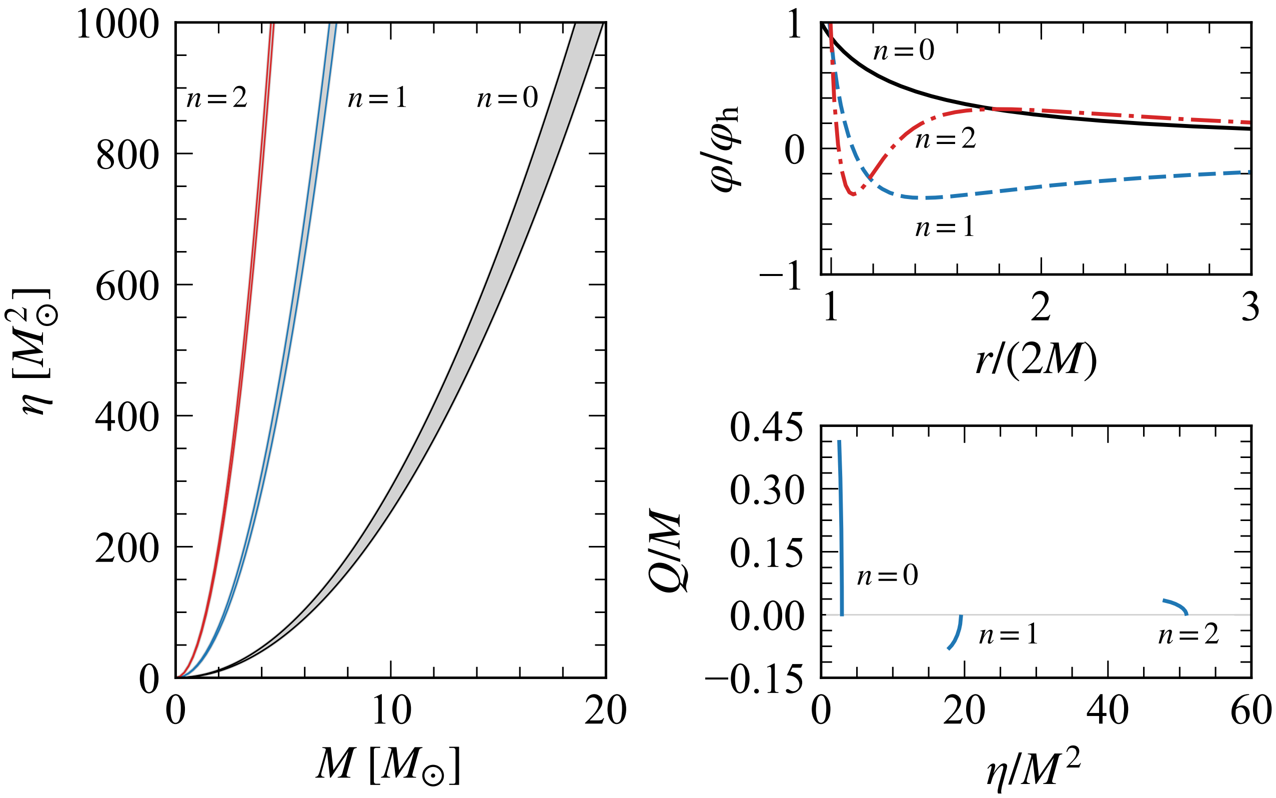


Full nonlinear scalarized solutions: band structure

For given η, solutions differ from GR in finite black hole **mass bands**:

$$\eta/M^2 = [2.73,\ 2.90], [17.89,\ 19.50], [47.90,\ 50.93] \ldots$$

Top of band: threshold, bottom: no solutions (GB repulsive - naked singularity?)



EsGB: black hole scalarization and other solutions

Einstein-dilaton-Gauss-Bonnet: $f = e^{\alpha\varphi}$
Kerr not a solution, minimum BH mass

[Mignemi-Stewart 93, Kanti+ 96, Pani-Cardoso 09, Yunes-Stein 11…]

Shift-symmetric Gauss-Bonnet: $f = a\varphi$
Kerr not a solution!

[Sotiriou-Zhou 14, Barausse-Yagi 15, Benkel+ 16…]

Minimal scalarization model: $f = \frac{1}{8}\eta\varphi^2$

[Silva+ 1711.02080]

Nonminimal scalarization model: $f = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right)$

[Doneva+ 1711.01187]

Polynomial, inverse polynomial, logarithmic…

[Antoniou+ 1711.03390/07431; Brihaye-Ducobu 1812.07438…]

Radial instability of the minimal scalarization model

Are the solutions stable under radial perturbations?

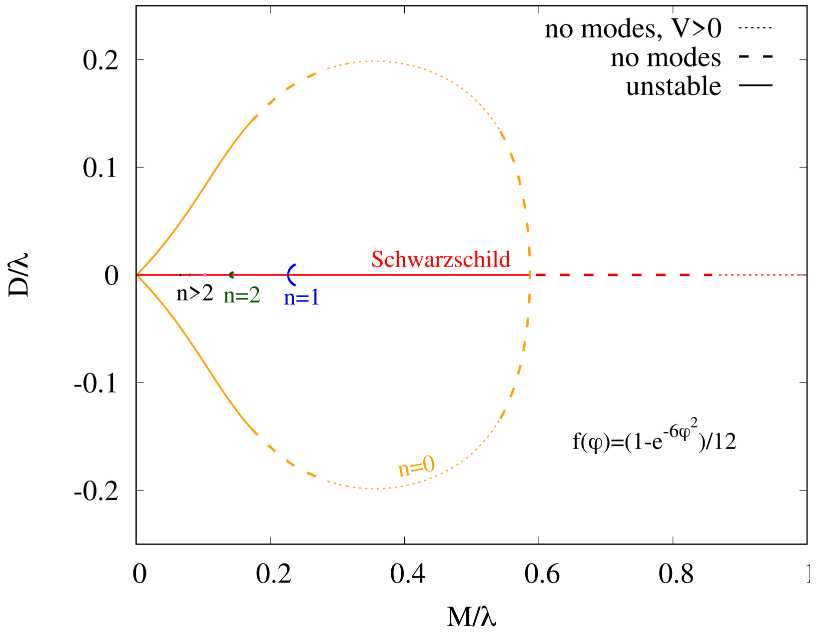
Dashed/dotted: stable (no unstable modes, or positive potential); solid: unstable

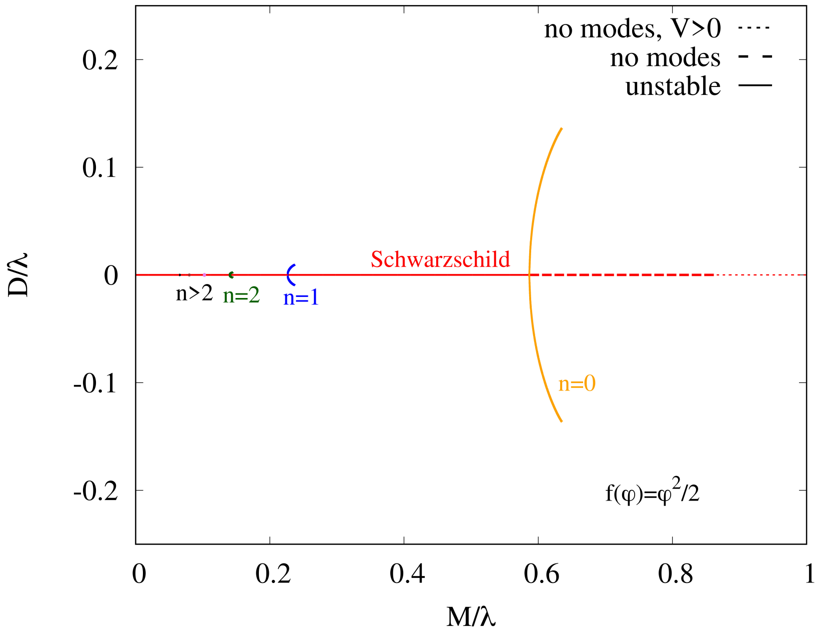
$M > M_{\rm thr} = 0.587\eta^{1/2}$: Schwarzschild is stable; threshold is coupling-independent

Intermediate mass: nodeless scalarized BHs with exponential coupling are stable

No BHs are stable below a certain mass (as in EdGB)

(scalar charge Q)





[Blazquez-Salcedo+, 1805.05755]

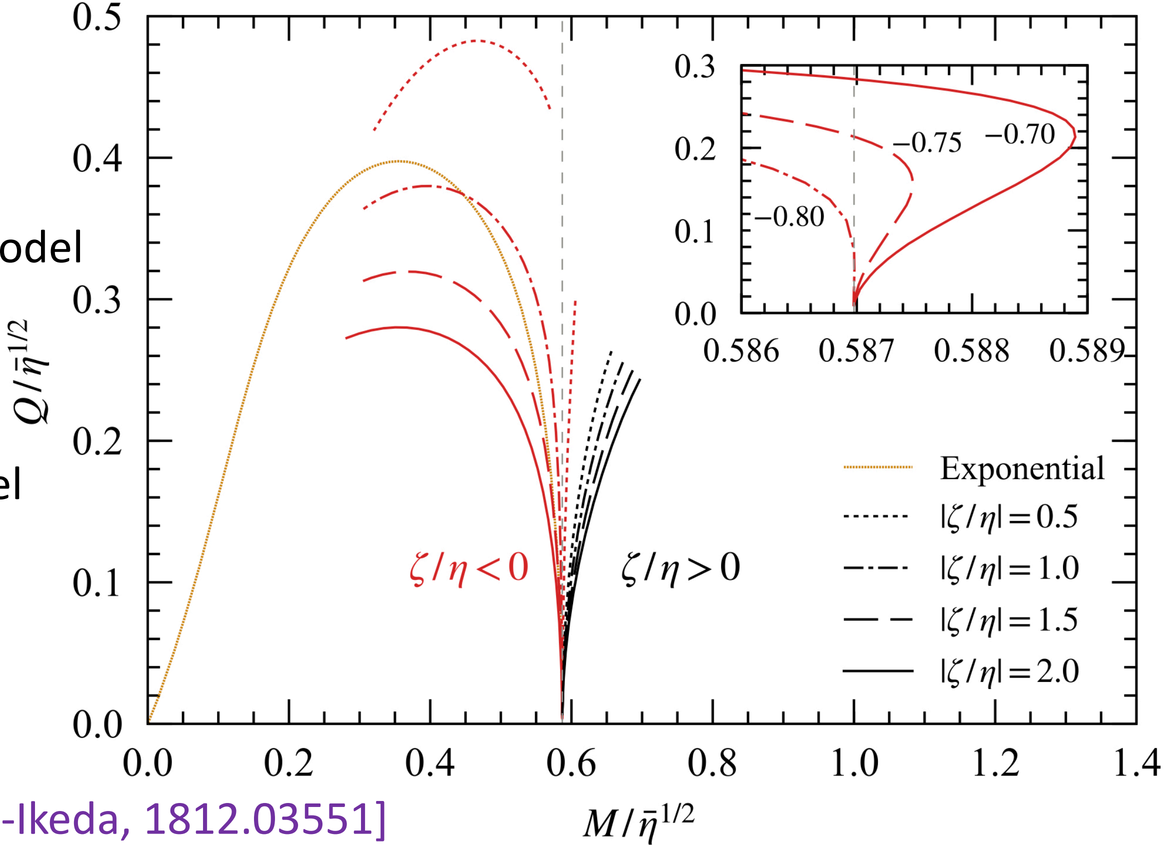
Stabilizing the solutions: quartic coupling is enough

$$f(\varphi) = \frac{1}{8}\eta\varphi^2 + \frac{1}{16}\zeta\varphi^4$$

Vertical line: scalarization threshold
Left: radially stable
Right: radially unstable
Small charge Q:
exponential = quartic $\zeta/\eta = 1.5$ model

Key:

- The scalar EOM is linear in φ in the minimal scalarization model
- Backreaction is sufficient to quench the instability
- From an EFT viewpoint, this is **not** the most natural choice



[Macedo+, 1812.05590; Minamitsuji-Ikeda, 1812.03551]

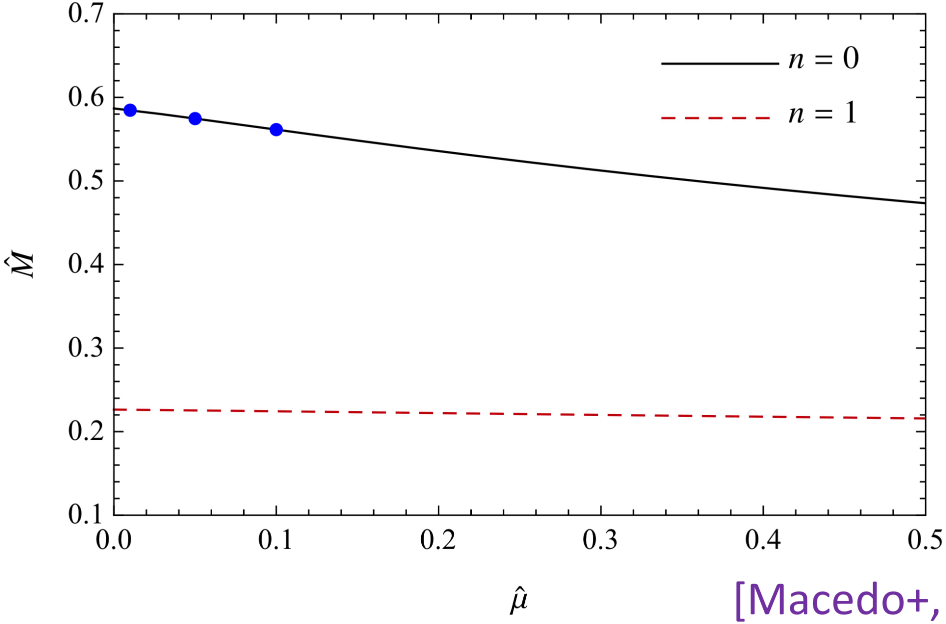
A better EFT-motivated choice

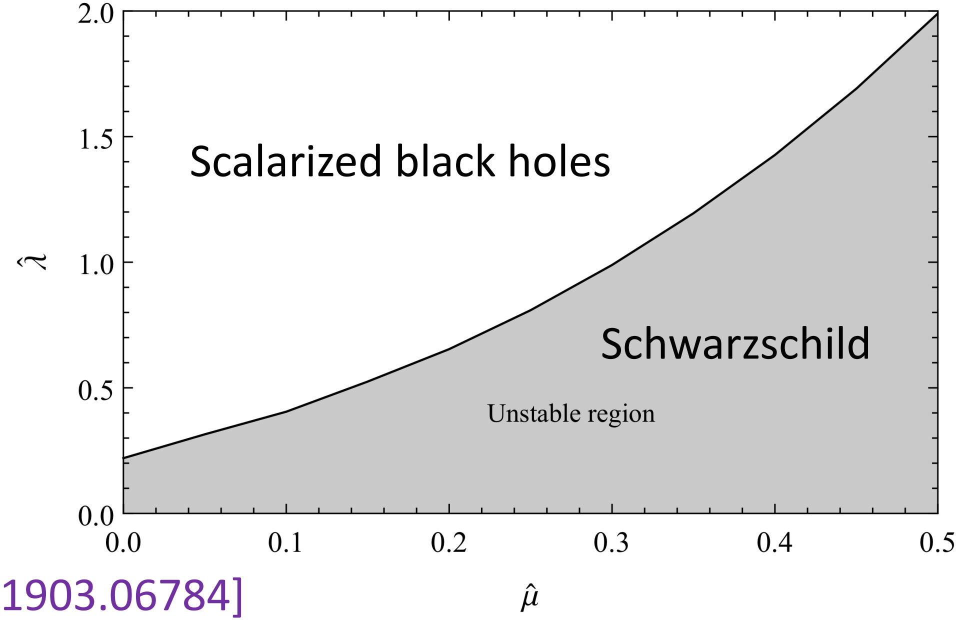
$$S = \frac{1}{2}\int d^4x\sqrt{-g}\left[M_{\rm Pl}^2 R - \frac{1}{2}\nabla_a\phi\nabla^a\phi - \frac{1}{2}\mu^2\phi^2 - \frac{1}{2}\lambda\phi^4 + \frac{\phi^2}{2\mathcal{M}^2}\mathcal{G}\right]$$

$$\Box\varphi = V_{\text{eff},\varphi}(\varphi)$$

$$V_{\text{eff}}(\varphi) = \frac{1}{2}\left(\mu^2 - \frac{\eta}{4}\mathcal{G}\right)\varphi^2 + \frac{1}{2}\lambda\varphi^4$$

$$m_{\text{eff}}^2 = \mu^2 - \frac{\eta}{4}\mathcal{G}$$

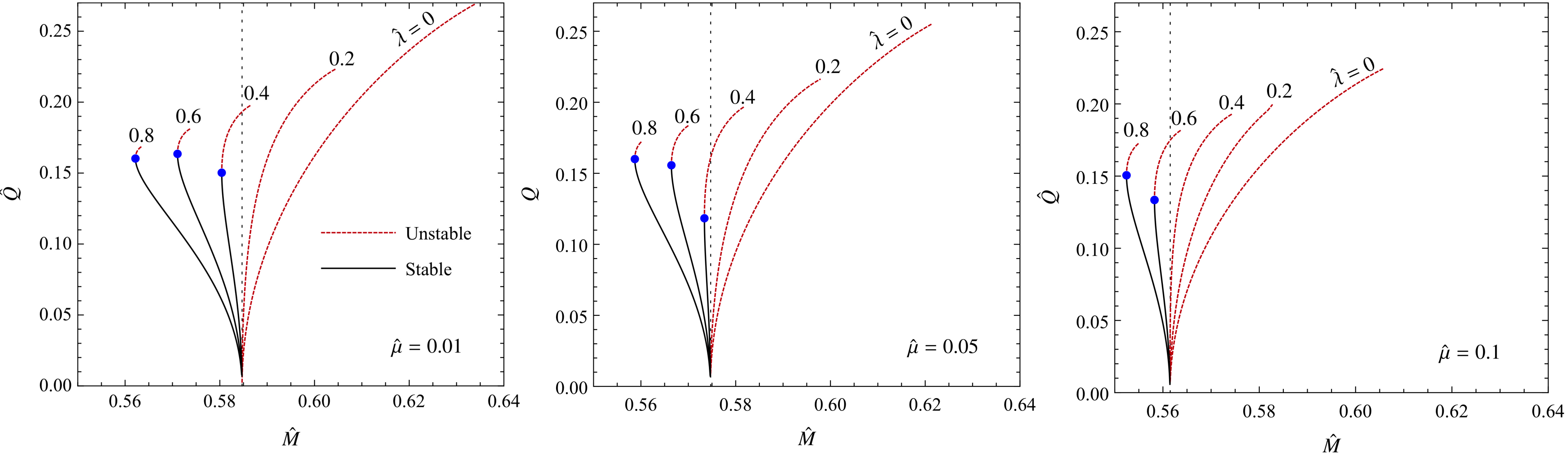




[Macedo+, 1903.06784]

A better EFT-motivated choice

$$S = \frac{1}{2}\int \mathrm{d}^4x\sqrt{-g}\left[M_{\mathrm{Pl}}^2 R - \frac{1}{2}\nabla_a\phi\nabla^a\phi - \frac{1}{2}\mu^2\phi^2 - \frac{1}{2}\lambda\phi^4 + \frac{\phi^2}{2\mathcal{M}^2}\mathcal{G}\right]$$

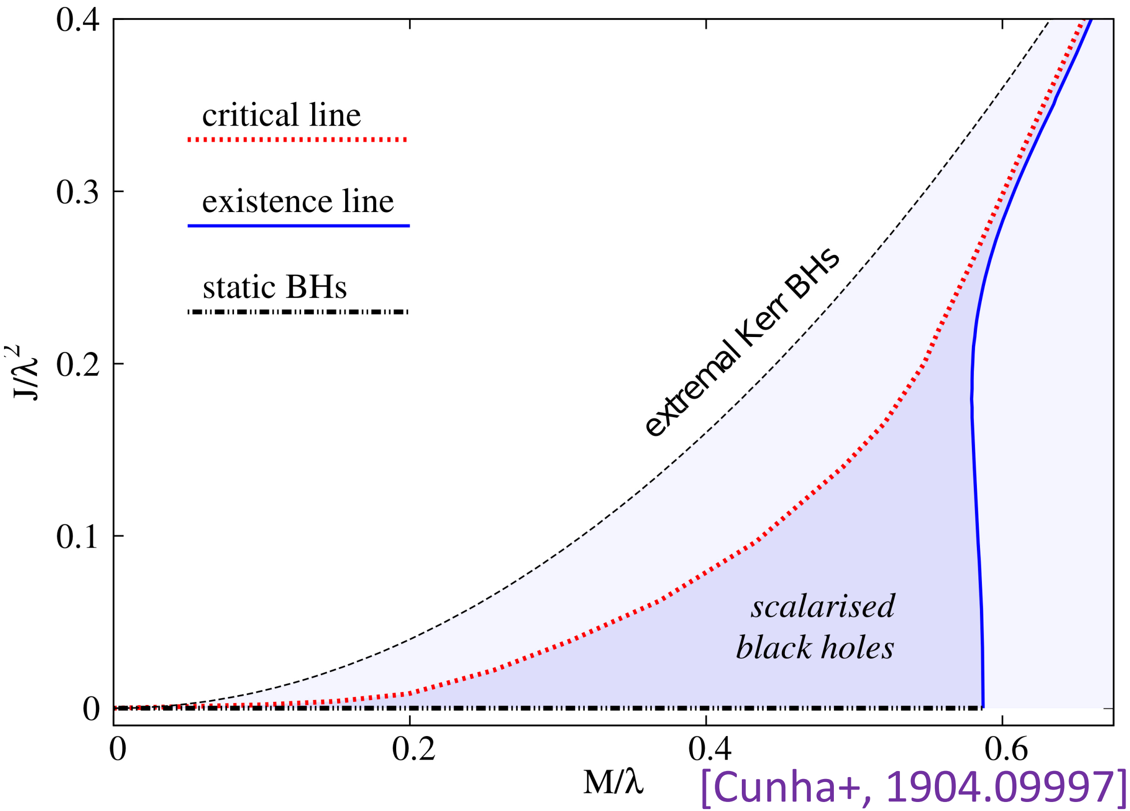


Blue dots: marginal radial stability – minimum mass, maximum charge for any given theory

What about spin?

$$f = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right)$$

- For each spin J, scalarized BHs exist between two critical mass values (lowest bound: zero in static limit)
- Scalarized black holes are entropically favored
- Spin reduces difference between scalarized/unscalarized solutions: differences nearly unmeasurable for $j \geq 0.5$ (LIGO range!)
- Coupling dependence?



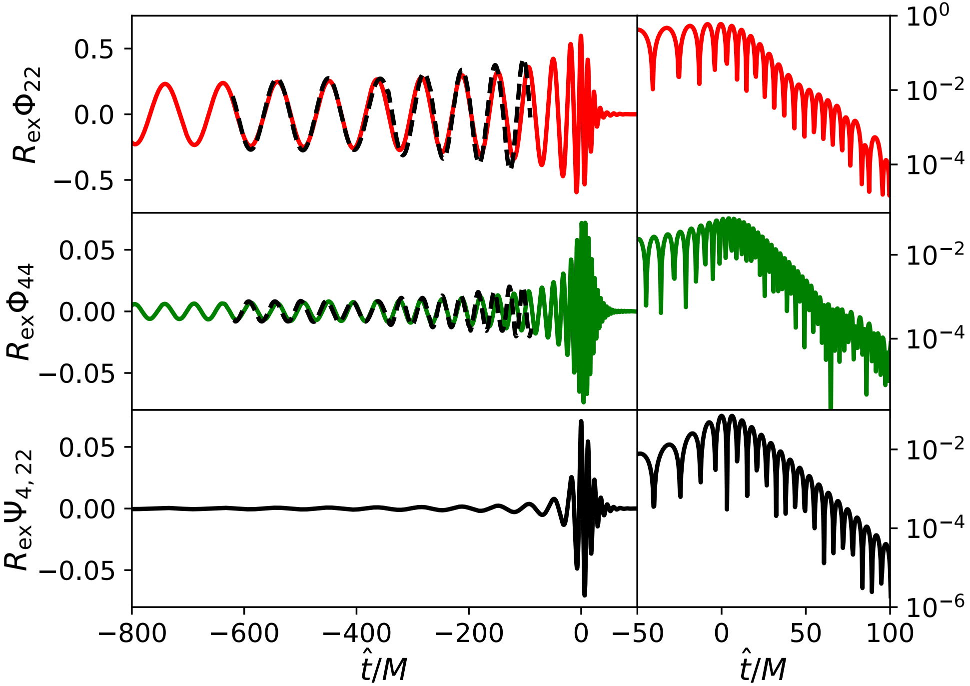
[Cunha+, 1904.09997]

Binaries in Einstein-scalar-Gauss-Bonnet

- Post-Newtonian calculations in the weak coupling limit:
  [Yagi+, 1110.5990]
  Higher-order in coupling, generic EsGB:
  [Julié+, in preparation]

- Dynamical scalarization:
  [Khalil+, 1906.08161]

- Numerical simulations (weak coupling limit):
  Scalar waveforms
  Scalar-led QNMs + gravitational-led QNMs
  [Witek+, 1810.05177]



- Related work in dynamical Chern-Simons (weak coupling limit):
  Scalar and gravitational waveforms
  [Okounkova+ 1705.07924, 1906.08789]

Black hole thermodynamics, skeletonization and the two-body Lagrangian

- Analytical solutions for generic coupling functions (up to fourth order in the EsGB coupling constant) satisfy the first law of black hole thermodynamics

- Skeletonization à la Eardley: slow inspiral means that black holes evolve with **constant Wald entropy** and a nontrivial asymptotic value of the scalar field

- Padé resummation seems to correctly predict **poles** in coupling of black holes to scalar fields

- EsGB-induced corrections to the (conservative) two-body Lagrangian:
formally 1PN contribution, but for small coupling, effectively a 3PN term
"Miraculously", no regularization is needed to solve the EOMs at 1PN: "simple" Fock integral

- Apply to dynamical black hole scalarization?

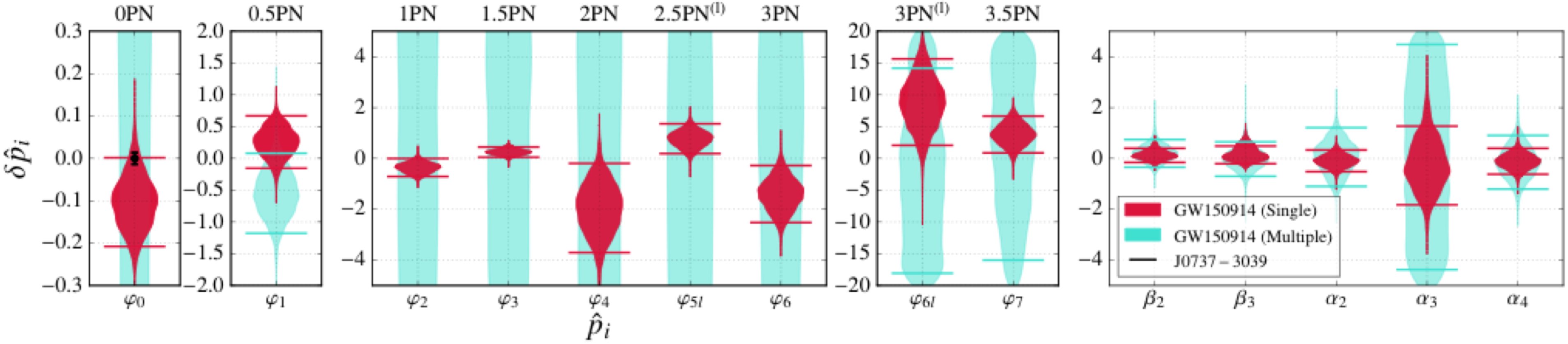
- Can we find analytic solutions and skeletonize scalarized black holes?

[Julié+EB, in preparation]

Parametrized ringdown

Inspiral: GR solution known, parametrized post-Einstein

$$\tilde{h}(f) = \tilde{A}_{\rm GR}(f)\left[1 + \alpha_{\rm ppE}\, v(f)^a\right] e^{i\Psi_{\rm GR}(f) + i\beta_{\rm ppE}\, v(f)^b}$$



[Yunes-Pretorius+, 0909.3328]

Mapping to theories – can we do the same for ringdown?

Table 2 Mapping of ppE parameters to those in each theory for a black hole binary

Theory	$\beta_{\rm ppE}$	b
Scalar–tensor [36, 179, 180]	$-\frac{5}{1792}\dot{\phi}^2\eta^{2/5}\left(m_1 s_1^{\rm ST} - m_2 s_2^{\rm ST}\right)^2$	-7
EdGB, D^{2}GB [23]	$-\frac{5}{7168}\zeta_{\rm GB}\frac{\left(m_1^2 s_2^{\rm GB} - m_2^2 s_1^{\rm GB}\right)^2}{m^4\eta^{18/5}}$	-7
dCS [181]	$\frac{1549225}{11812864}\frac{\zeta_{\rm CS}}{\eta^{14/5}}\left[\left(1-\frac{231808}{61969}\eta\right)\chi_s^2+\left(1-\frac{16068}{61969}\eta\right)\chi_a^2-2\delta_m\chi_s\chi_a\right]$	-1
EA [182]	$-\frac{3}{128}\left[\left(1-\frac{c_{14}}{2}\right)\left(\frac{1}{w_2^{\rm Æ}}+\frac{2c_{14}c_+^2}{(c_++c_--c_-c_+)^2 w_1^{\rm Æ}}+\frac{3c_{14}}{2w_0^{\rm Æ}(2-c_{14})}\right)-1\right]$	-5
Khronometric [182]	$-\frac{3}{128}\left[(1-\beta_{\rm KG})\left(\frac{1}{w_2^{\rm KG}}\frac{3\beta_{\rm KG}}{2w_0^{\rm KG}(1-\beta_{\rm KG})}\right)-1\right]$	-5
Extra dimension [183]	$\frac{25}{851968}\left(\frac{dm}{dt}\right)\frac{3-26\eta+34\eta^2}{\eta^{2/5}(1-2\eta)}$	-13
Varying G [151]	$-\frac{25}{65536}\dot{G}\mathcal{M}$	-13
Mod. disp. rel. [184]	$\frac{\pi^{2-\alpha_{\rm MDR}}}{(1-\alpha_{\rm MDR})}\frac{D_{\alpha_{\rm MDR}}}{\lambda_{\mathbb{A}}^{2-\alpha_{\rm MDR}}}\frac{\mathcal{M}^{1-\alpha_{\rm MDR}}}{(1+z)^{1-\alpha_{\rm MDR}}}$	$3(\alpha_{\rm MDR}-1)$

Scalar, electromagnetic and gravitational perturbations in GR

Gravitational perturbations of a Schwarzschild BH: Regge-Wheeler/Zerilli equations

$$f\frac{d}{dr}\left(f\frac{d\Phi}{dr}\right)+\left[\omega^2-fV_\pm\right]\Phi=0 \qquad f=1-\frac{r_H}{r}$$

Isospectrality: the odd/even potentials

$$V_-=\frac{\ell(\ell+1)}{r^2}-\frac{3r_H}{r^3}$$

$$V_+=\frac{9\lambda r_H^2 r+3\lambda^2 r_H r^2+\lambda^2(\lambda+2)r^3+9r_H^3}{r^3(\lambda r+3r_H)^2}$$

have the same quasinormal mode spectrum [Chandrasekhar-Detweiler 1975]

Scalar, electromagnetic and (odd) gravitational perturbations:

$$V_s=\frac{\ell(\ell+1)}{r^2}+(1-s^2)\frac{r_H}{r^3}$$

[e.g. EB+, 0905.2975]

Generic (but decoupled) corrections to GR potentials

Modifications to the gravity sector and/or beyond Standard Model physics: expect

- small modifications to the functional form of the potentials – parametrize!
- coupling between the wave equations (more later)

$$V = V_\pm + \delta V_\pm \qquad \delta V_\pm = \frac{1}{r_H^2}\sum_{j=0}^{\infty}\alpha_j^\pm\left(\frac{r_H}{r}\right)^j \qquad \omega_{\rm QNM}^\pm = \omega_0^\pm + \sum_{j=0}^{\infty}\alpha_j^\pm e_j^\pm$$

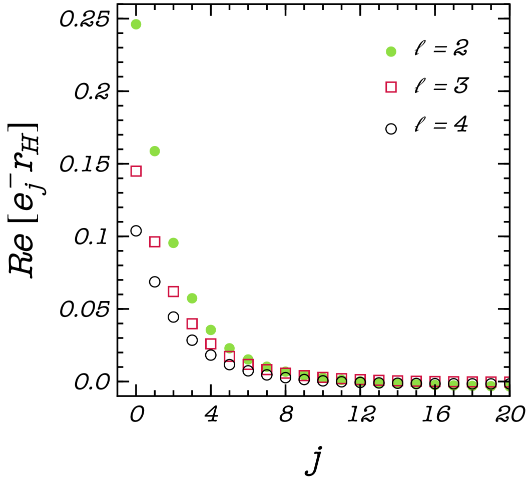
$$V = V_s + \delta V_s \qquad \delta V_s = \frac{1}{r_H^2}\sum_{j=0}^{\infty}\beta_j^s\left(\frac{r_H}{r}\right)^j \qquad \omega_{\rm QNM}^s = \omega_0^s + \sum_{j=0}^{\infty}\beta_j^s d_j^s$$

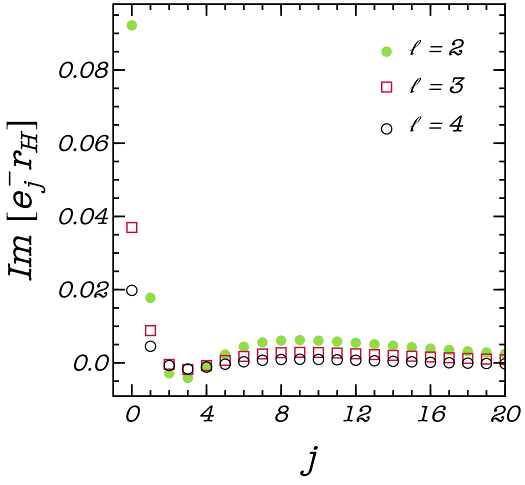
Maximum of $f(r)\alpha_j^\pm\left(\frac{r_H}{r}\right)^j$ is $\alpha_j^\pm\frac{(1+1/j)^{-j}}{j+1}$, so corrections are small if:

$$(\alpha_j^\pm, \beta_j^s) \ll (1+1/j)^j(j+1)$$

[Cardoso+, 1901.01265]

Correction coefficients and their asymptotic behavior



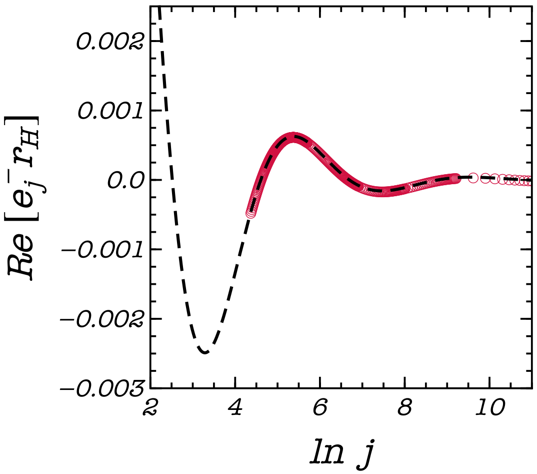


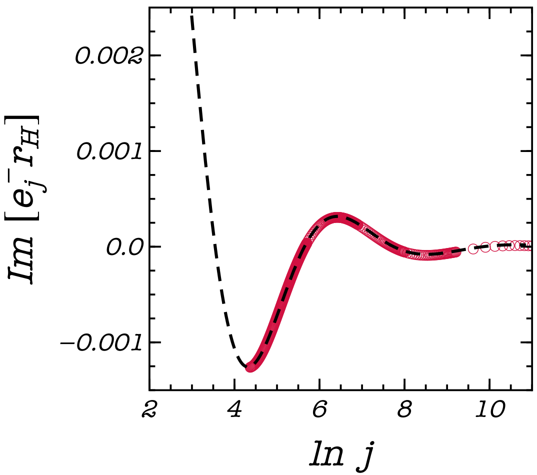
QNM frequency correction coefficients by direct integration [Pani, 1305.6759]

Asymptotics:

$$e_j \propto \frac{j^{2i\omega_0 r_H}}{j} = \frac{e^{2ir_H\omega_R \ln j}}{j^{1+2r_H\omega_I}}$$

Damped oscillatory behavior for large j





Fitting the numerics by

$$e_j \sim \frac{\kappa}{j^{\beta}} \sin(\gamma \ln j + \zeta)$$

confirms this.

[Pani, 1305.6759]

Generic isospectrality breaking

Isospectrality follows from the existence of a "superpotential" such that:

$$fV_{\pm} = W_0^2 \mp f\frac{dW_0}{dr} - \frac{\lambda^2(\lambda+2)^2}{36r_H^2} \qquad W_0 = \frac{3r_H(r_H - r)}{r^2(3r_H + \lambda r)} - \frac{\lambda(\lambda+2)}{6r_H}$$

Perturb to find conditions for isospectrality to hold:

$$2\frac{d\delta W}{dr} = \delta V_- - \delta V_+ \qquad 4\frac{W_0}{f}\delta W = \delta V_+ + \delta V_-$$

Preserving isospectrality needs fine tuning!

$$\alpha_0^+ = \alpha_0^-$$
$$\alpha_1^+ = \alpha_1^-$$
$$\alpha_2^+ = \alpha_2^- + \frac{6\left(\alpha_0^- - \alpha_1^-\right)}{\lambda(\lambda+2)}$$

[Chandrasekhar-Detweiler 1975]

Example 1: EFT

EFT corrections quartic in the curvature lead to a modified Regge-Wheeler equation:

$$\frac{d^2\Psi_-}{dr_\star^2} + \left[\omega^2 - f(V_- + \delta V_-)\right]\Psi_- = 0$$

$$\delta V_- = \epsilon_2 \frac{18(\ell+2)(\ell+1)(\ell-1)r_H^8}{r^{10}}$$

Trivially read off the correction coefficient: $\alpha_{10}^- = 18(\ell+2)(\ell+1)(\ell-1)\epsilon_2$

Plug into $\omega_{\mathrm{QNM}}^\pm = \omega_0^\pm + \sum_{j=0}^{\infty} \alpha_j^\pm e_j^\pm$

to find

$$r_H\omega = r_H\omega_0 + (0.0663354 + 0.117439\mathrm{i})\epsilon_2(\ell-1)(\ell+1)(\ell+2)$$

in agreement with numerical integrations.

[Cardoso+, 1808.08962]

Example 2: Reissner-Nordström

Odd gravitational perturbations of Reissner-Nordström satisfy

$$f_{\rm RN}\frac{d}{dr}\left(f_{\rm RN}\frac{d\Phi}{dr}\right)+\left(\omega^2-f_{\rm RN}V_{\rm RN}\right)\Phi=0 \qquad f_{\rm RN}=1-\frac{2M}{r}+\frac{Q^2}{r^2}$$

A simple change of variables brings the wave equation in our "canonical" form, with

$$V_{\rm RN}=\frac{\ell(\ell+1)}{r^2}+\frac{4r_Hr_-}{r^4}-\frac{3(r_H+r_-)}{2r^3}-\frac{\left[4(\ell-1)(\ell+2)r_Hr_-+\frac{9}{4}(r_H+r_-)^2\right]^{1/2}}{r^3}$$

for small charge.

Read off coefficients to find:

$$\omega_{\rm QNM}=\left(1-\frac{r_-}{r_H}\right)\left(\frac{2\Omega_0}{r_H}+e_0\alpha_0^-+e_3\alpha_3^-+e_4\alpha_4^-\right)$$
$$=\frac{\Omega_0}{M}+\frac{(0.0258177-0.002824i)Q^2}{M^3}$$

TABLE II. Relative percentage errors on the real and imaginary parts of the QNMs for RN BHs, as a function of the charge-to-mass ratio Q/M.

Q/M	Δ_R	Δ_I
0.00	0%	0%
0.05	0.11%	0.042%
0.10	0.43%	0.17%
0.20	1.7%	0.66%
0.30	3.8%	1.5%
0.40	6.8%	2.6%
0.50	11%	4.2%

Example 3: Klein-Gordon in slowly rotating Kerr

At linear order in the spin parameter:

$$f\frac{d}{dr}\left(f\frac{d}{dr}\right)\Phi + \left(\omega^2 - fV_0 - \frac{4amM\omega}{r^3}\right)\Phi = 0$$

i.e.

$$f\frac{d}{dr}\left(f\frac{d}{dr}\right)\Phi + \left[\left(\omega - \frac{am}{r_H^2}\right)^2 - f\left(V_0 - \frac{2am\omega}{r_H^2} - \frac{2am\omega}{r_H^2}\frac{r_H}{r} - \frac{2am\omega}{r_H^2}\left(\frac{r_H}{r}\right)^2\right)\right]\Phi = 0$$

Correction coefficients to the scalar wave equation:

$$\beta_0^0 = \beta_1^0 = \beta_2^0 = -2am\omega_0^0$$

$$\omega_{\mathrm{QNM}} = \omega_0^0 + \frac{am}{r_H^2} - 2am\omega_0^0\left(d_0^0 + d_1^0 + d_2^0\right)$$

TABLE III. Relative percentage errors in the real and imaginary parts of the QNM frequencies for scalar perturbations around a slowly spinning black hole, as a function of the BH angular momentum a/M.

a/M	Δ_R	Δ_I
0	0%	0%
10^{-4}	0.0050%	0.83%
10^{-3}	0.049%	5.1%
10^{-2}	0.49%	34%

Coupled perturbations

We really want to solve the coupled $N \times N$ system

$$f\frac{d}{dr}\left(f\frac{d\mathbf{\Phi}}{dr}\right) + \left[\omega^2 - f\mathbf{V}\right]\mathbf{\Phi} = 0 \qquad \mathbf{\Phi} = \{\Phi_i\}\ (i = 1, \ldots, N)$$

$$\mathbf{V}(r) = V_{ij}(r)$$

where each matrix element is perturbed:

$$V_{ij} = V_{ij}^{\mathrm{GR}} + \delta V_{ij} \qquad \delta V_{ij} = \frac{1}{r_H^2}\sum_{k=0}^{\infty} \alpha_{ij}^{(k)} \left(\frac{r_H}{r}\right)^k$$

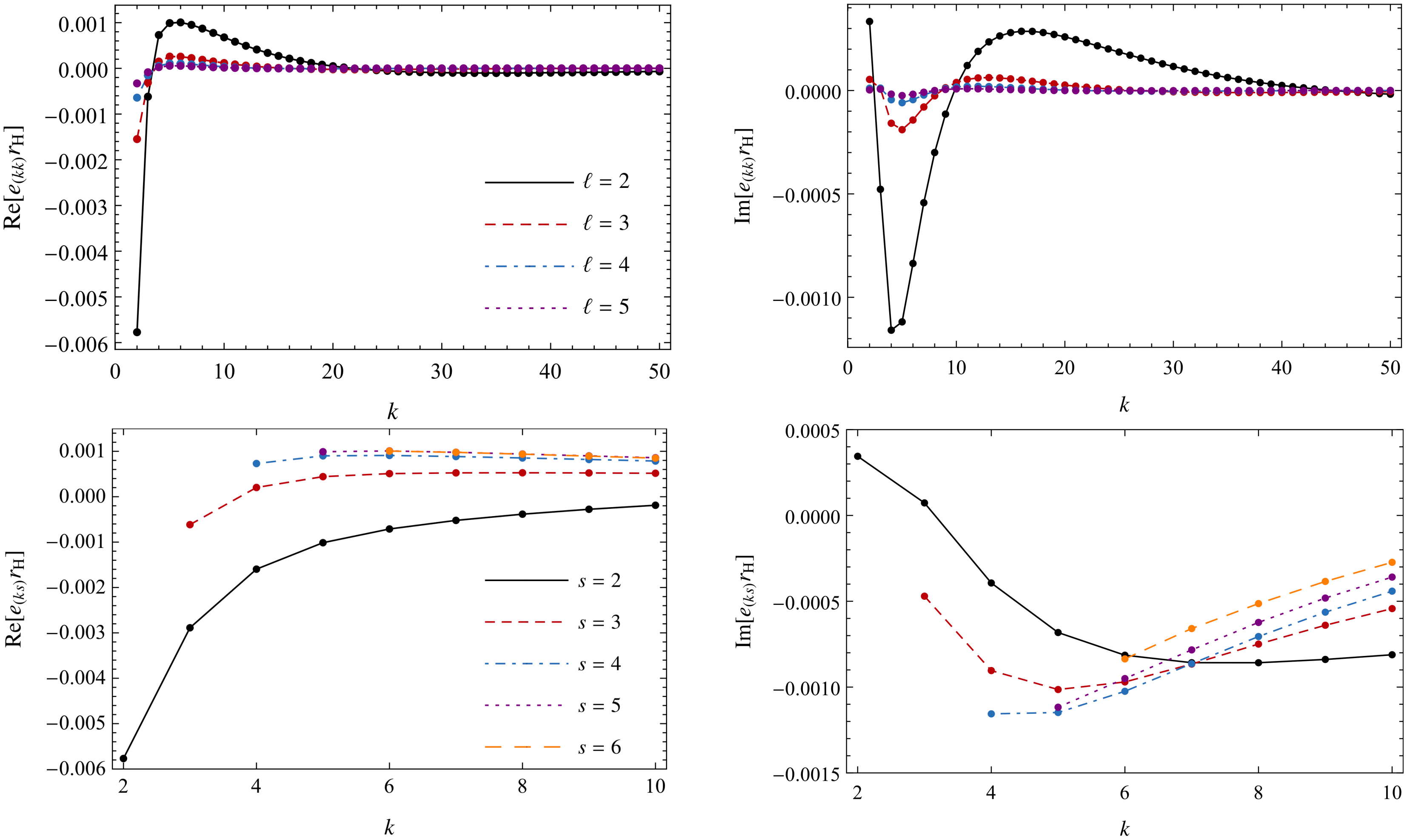
If the background spectra are nondegenerate, coupling will induce **quadratic** corrections. Allow α to depend on ω. We need

- quadratic corrections in α, besides the linear diagonal terms $d_{(k)}^{ii}$
- coupling-induced corrections

$$\omega \approx \omega_0 + \alpha_{ij}^{(k)} d_{(k)}^{ij} + \alpha_{ij}^{(k)} \alpha_{pq}^{\prime(s)} d_{(k)}^{ij} d_{(s)}^{pq} + \frac{1}{2}\alpha_{ij}^{(k)}\alpha_{pq}^{(s)} e_{(ks)}^{ijpq} \qquad \text{(Einstein summation)}$$

[McManus+, 1906.05155]

Correction coefficients



The degenerate case

Degenerate spectra (e.g. **even/odd gravitational perturbations**) need special care. Why?

$$\left(\frac{d^2}{dr_*^2}+\omega^2-fV_0\right)\phi_1+\alpha Z\phi_2=0$$
$$\left(\frac{d^2}{dr_*^2}+\omega^2-fV_0\right)\phi_2+\alpha Z\phi_1=0$$

Diagonalize:

$$\phi_1=(\phi_++\phi_-)/2$$
$$\phi_2=(\phi_+-\phi_-)/2$$

$$\left(\frac{d^2}{dr_*^2}+\omega^2-fV_0+\alpha Z\right)\phi_+=0$$
$$\left(\frac{d^2}{dr_*^2}+\omega^2-fV_0-\alpha Z\right)\phi_-=0$$

Corrections are linear in α

Use degenerate perturbation theory:

$$\omega=\omega_0+\epsilon\omega_1 \qquad \omega_1=\frac{\delta V_{++}+\delta V_{--}\pm\sqrt{(\delta V_{++}-\delta V_{--})^2+4\delta V_{+-}\delta V_{-+}}}{2}$$

$$\delta V_{\pm\pm}=\sum_{k=0}^{\infty}\alpha_{\pm\pm}^{(k)}\left\langle\omega_0,\pm\left|f(r)\frac{r_H^{k-2}}{r^k}\right|\omega_0,\pm\right\rangle=\sum_{k=0}^{\infty}\alpha_{\pm\pm}^{(k)}\delta V_{\pm\pm}^{(k)}$$

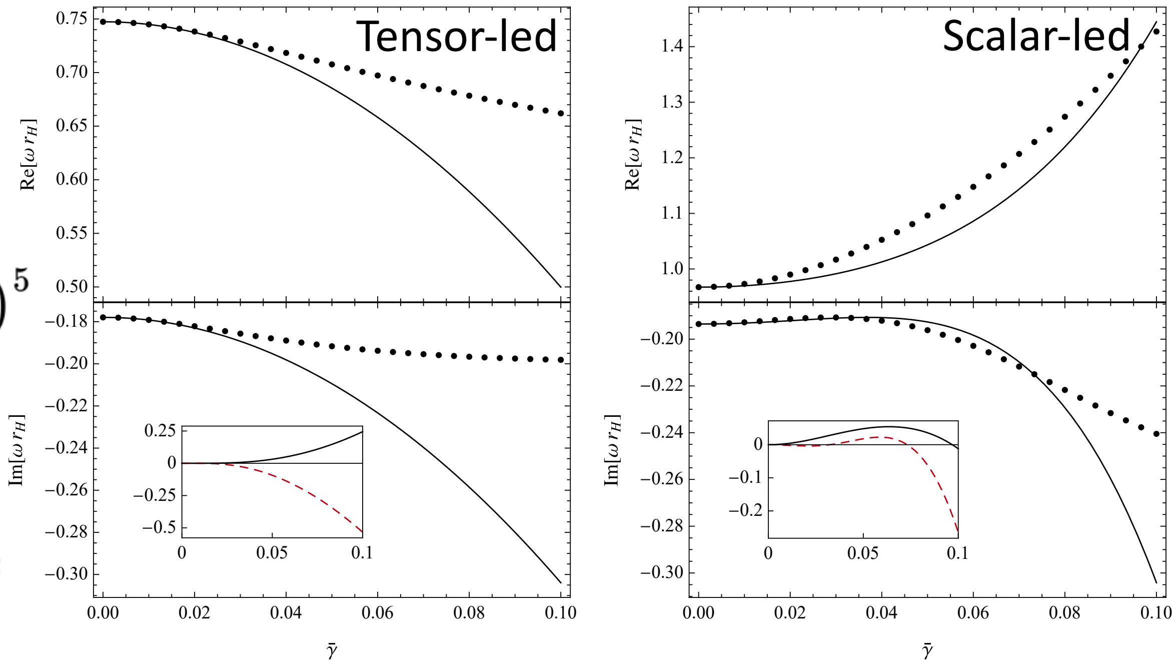
Example 1: scalar/odd gravitational in dynamical Chern-Simons

Spectra are nondegenerate
The perturbed potentials read:

$$V_{11} = V_{-}$$

$$V_{12} = V_{21} = \frac{1}{r_H^2}\frac{12}{\sqrt{\beta}r_H^2}\sqrt{\pi\frac{(\ell+2)!}{(\ell-2)!}}\left(\frac{r_H}{r}\right)^5$$

$$V_{22} = V_{s=0} + \frac{1}{r_H^2}\frac{144\pi\ell(\ell+1)}{\beta r_H^4}\left(\frac{r_H}{r}\right)^8$$



Corrected frequencies:

$$\omega = \omega_0 + e^{1221}_{(55)}\left(12\bar{\gamma}\sqrt{\pi\frac{(\ell+2)!}{(\ell-2)!}}\right)^2$$

$$\omega = \omega_0 + 2d_{(8)}144\pi\ell(\ell+1)\bar{\gamma}^2 + e_{(88)}\left[144\pi\ell(\ell+1)\bar{\gamma}^2\right]^2 + e^{1221}_{(55)}\left(12\bar{\gamma}\sqrt{\pi\frac{(\ell+2)!}{(\ell-2)!}}\right)^2$$

[Cardoso-Gualtieri, 0907.5008; Molina+, 1004.4007]

Example 2: scalar-led perturbations in Horndeski

The scalar-led perturbation is related to background coupling functions in the Horndeski Lagrangian:

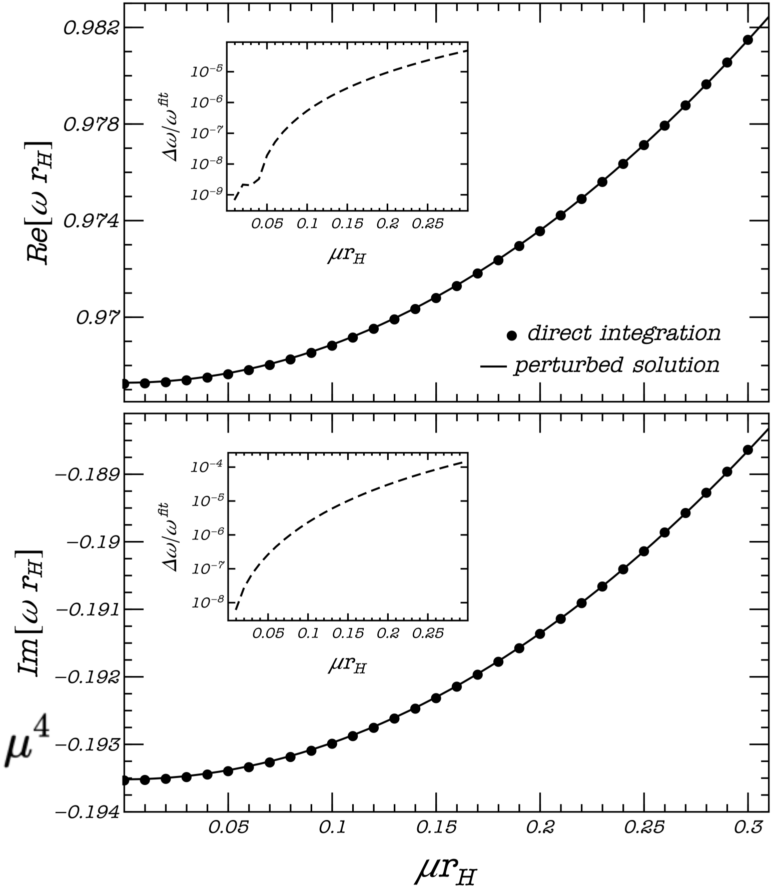
$$\frac{d^2\phi}{dr_*^2} + \left[\omega^2 - f\left(V_{s=0} + \mu^2 + \frac{\ell(\ell+1)}{r^2} f\Gamma\right)\right]\phi = 0$$

$$\mu^2 = \frac{-\overline{G}_{2\phi\phi}}{3\overline{G}_{4\phi}^2 + \overline{G}_{2X} - 2\overline{G}_{3\phi}}$$

$$\Gamma = \frac{8\overline{G}_{4X}}{3\overline{G}_{4\phi}^2 + \overline{G}_{2X} - 2\overline{G}_{3\phi}}$$

Corrected frequencies read (can set $\Gamma = 0$):

$$\omega \approx \omega_0 + d_{(0)}\mu^2 + \left[d_{(2)} + r_H d_{(3)}\right]\ell(\ell+1)\Gamma + \frac{1}{2}e_{(00)}\mu^4$$
$$+ \frac{1}{2}\left[e_{(22)} + 2r_H e_{(23)} + r_H^2 e_{(33)}\right]\left[\ell(\ell+1)\Gamma\right]^2$$
$$+ \left[e_{(02)} + r_H e_{(03)}\right]\ell(\ell+1)\mu^2\Gamma$$



[Tattersall+, 1711.01992]

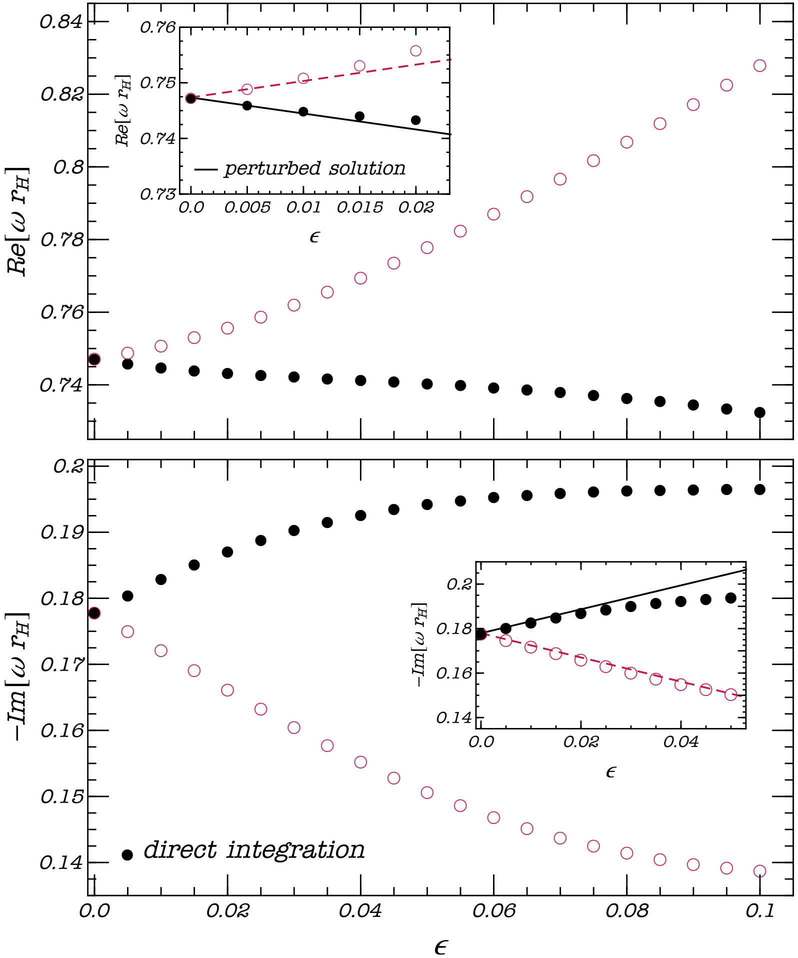
Example 3: odd/even gravitational coupling in EFT (degenerate)

The quartic-in-curvature EFT leads to a degenerate perturbed eigenvalue problem:

$$V_{11} = V_+$$
$$V_{22} = V_-$$
$$V_{12} = V_{21} = \epsilon V(r)$$

where off-diagonal perturbations are given in [Cardoso+, 1808.08962]

Direct integration vs. degenerate parametrization: good agreement, but quadratic corrections could be useful



Black hole scalarization: a summary

- EsGB: subclass of Horndeski theory that evades no-hair theorems
- Scalarized solution exist, are radially stable (as long as backreaction is included), can differ sensibly from GR
- Stable scalarized solutions are well motivated in EFT
- Scalarized solution become close to GR for spins of interest to LIGO remnant, at least for exponential couplings [Cunha+ 1904.09997]
- BHBs produce dipolar radiation [Yagi+ 1510.02152; Julié+, in preparation]
- Binaries have been simulated in the weak-coupling limit [Witek+ 1810.05177]
- Open issues with well posedness in the strong-coupling limit [Papallo-Reall, Ripley-Pretorius, Bernard+…]

Parametrized ringdown: a summary

Modifications to the gravity sector and/or beyond Standard Model physics:

- small modifications to the potentials
- coupling between the (matrix-valued) wave equations

We parametrized modifications by power laws, then computed perturbed QNMs for:

- linear corrections to diagonal terms [Cardoso+, 1901.01265]
- quadratic corrections + coupling [McManus+, 1906.05155]

The formalism is very general!
Examples:

- EFT, Reissner-Nordström, Klein-Gordon in Kerr for slow rotation
- scalar/odd gravitational dCS, scalar-led Horndeski, odd/even gravitational EFT

Needed generalizations:

- higher-order corrections (in particular, in degenerate coupled case)
- **rotation – LIGO/Virgo remnants have spins 0.7 or so!**