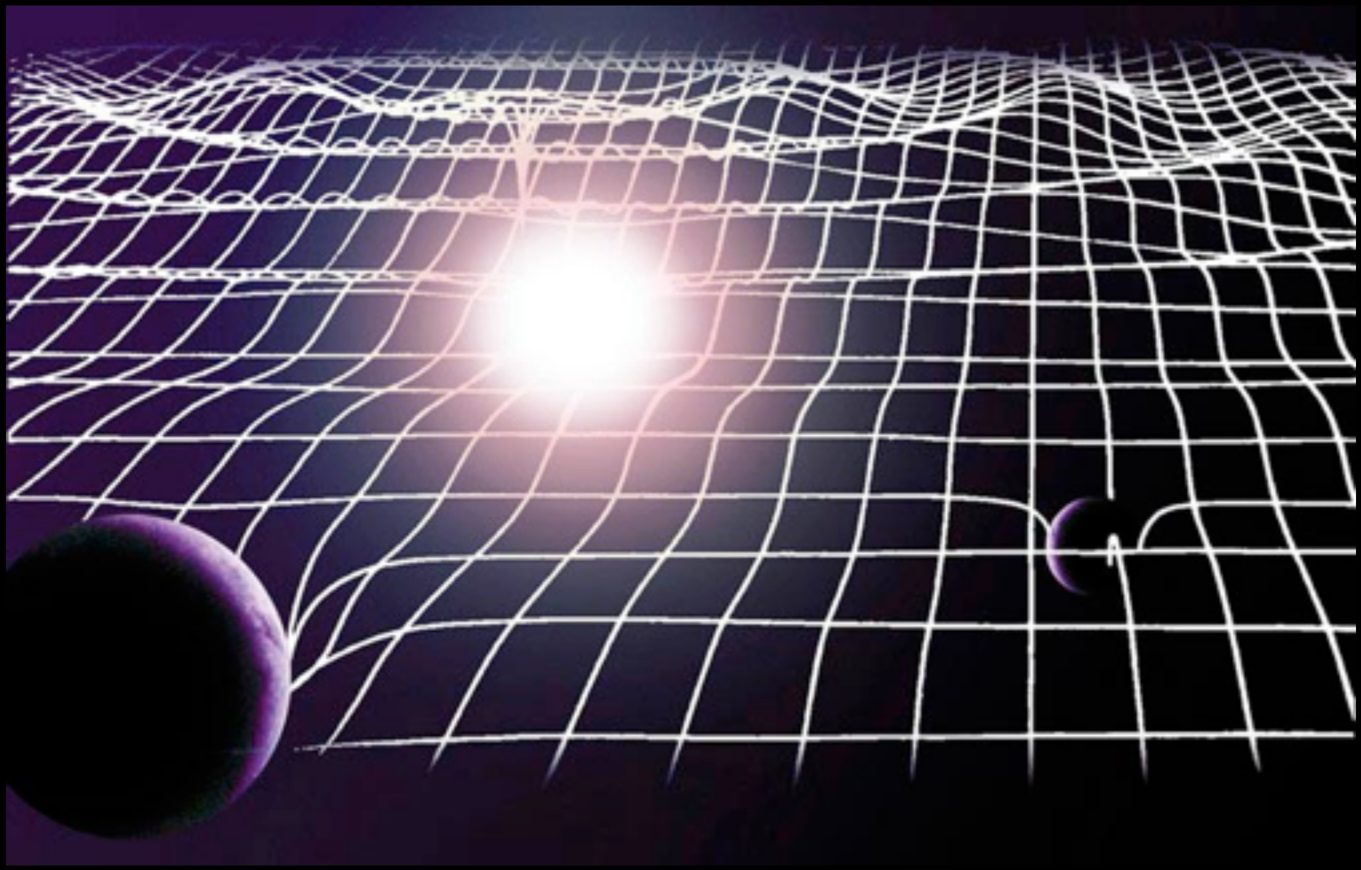
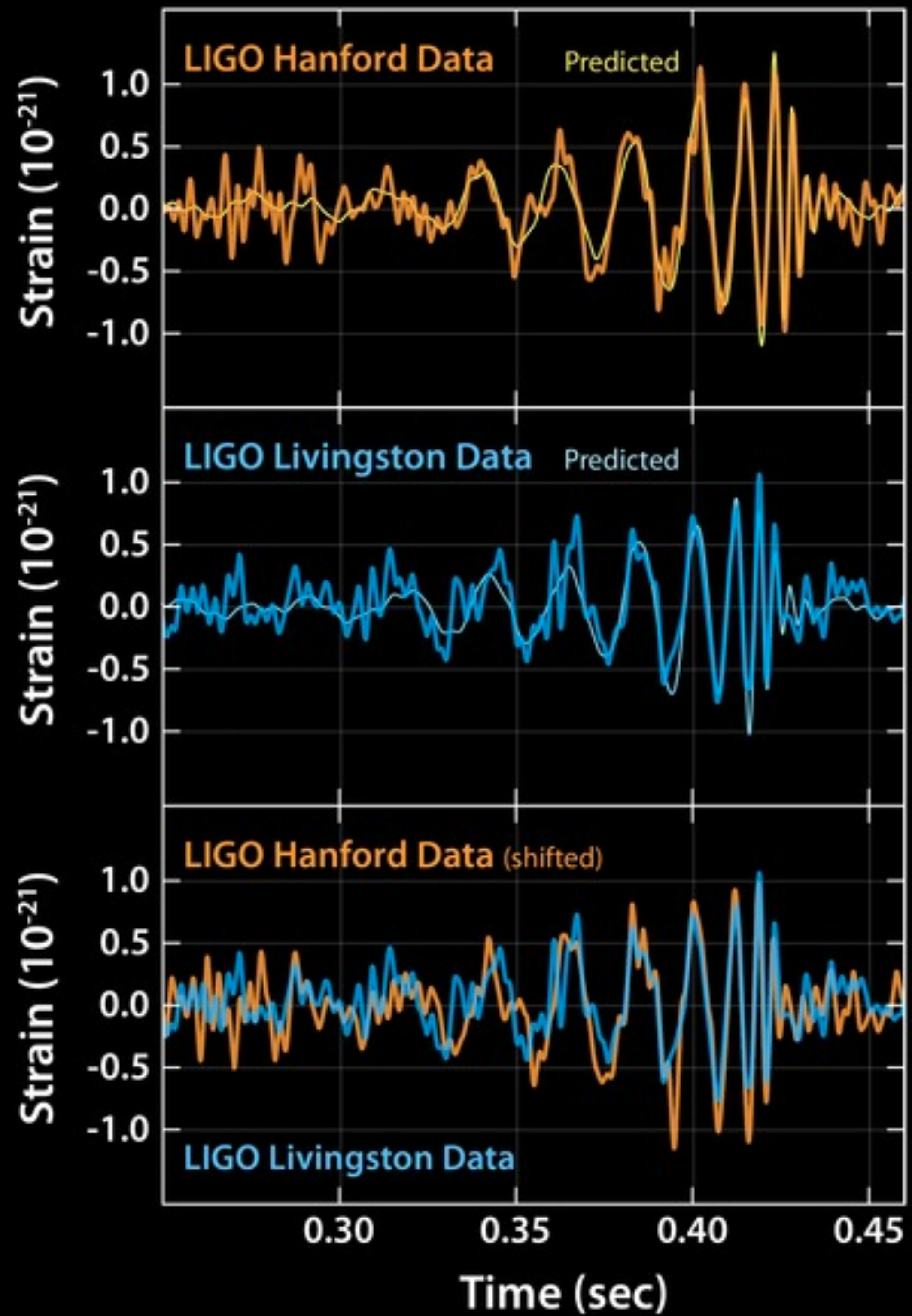
Dynamical Formation of Black Hole Mergers

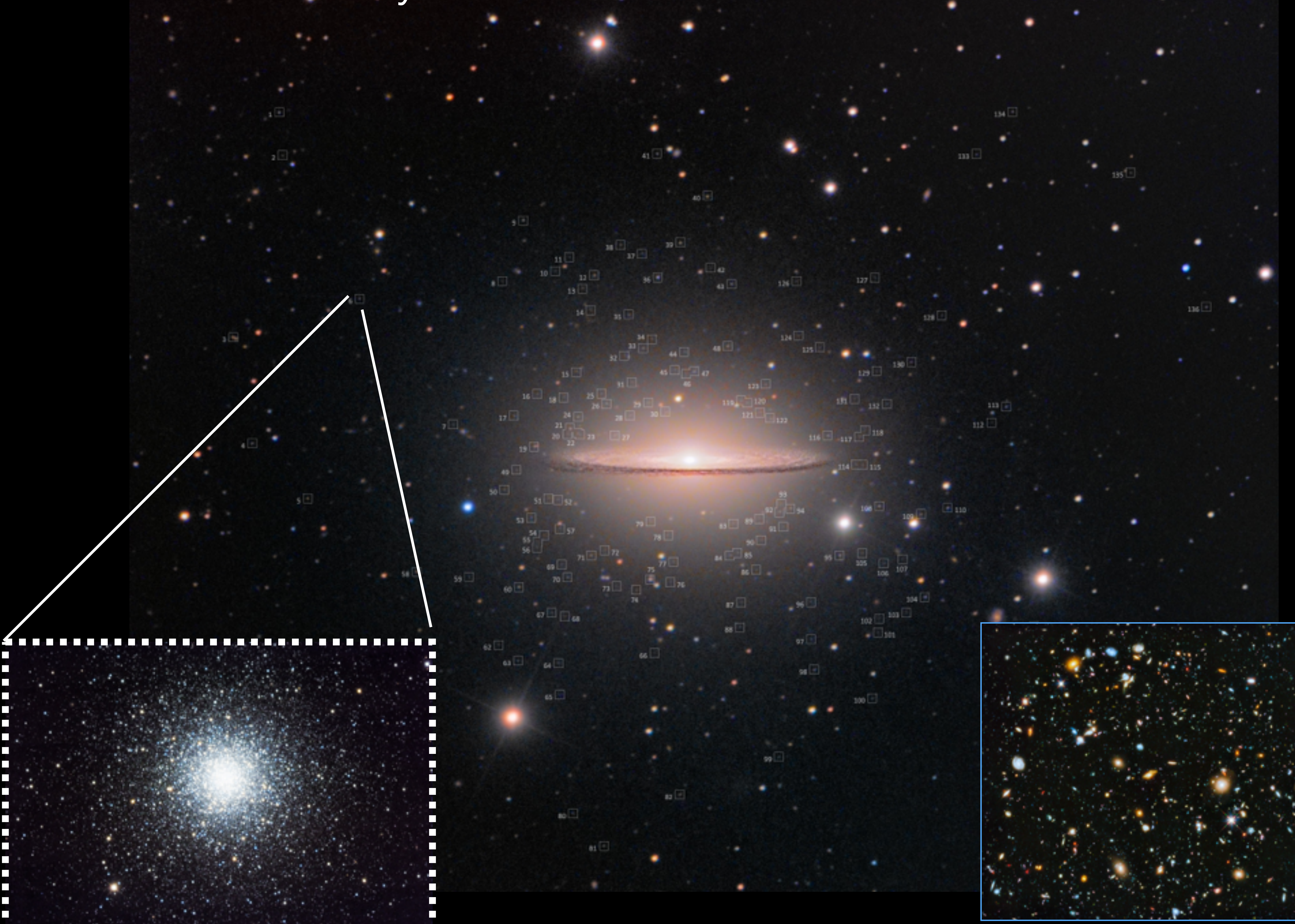
Johan Samsing
Princeton University



This is what we observe:

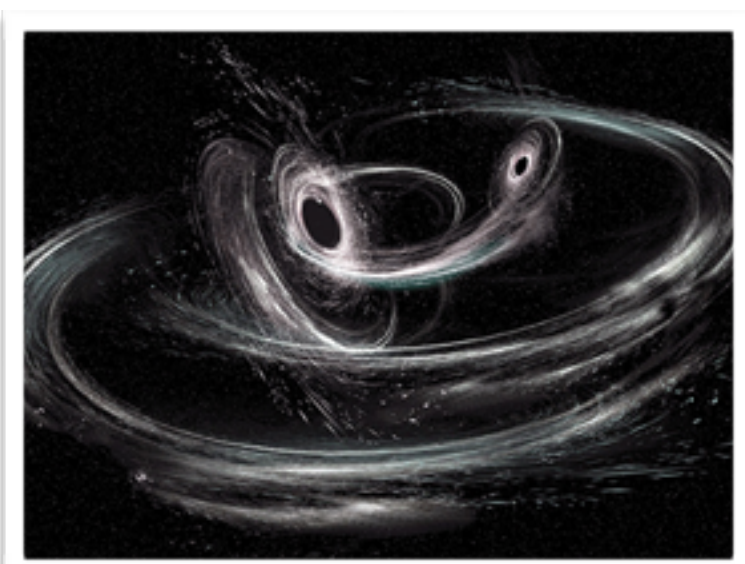


But where do they come from?

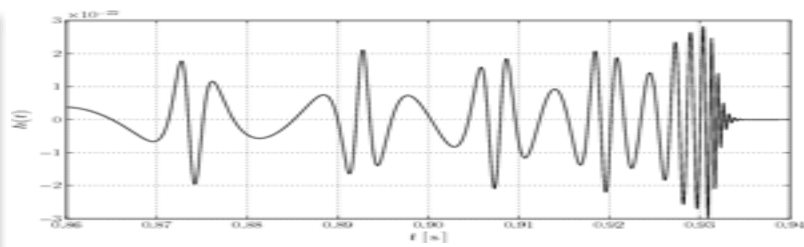


What is the origin of BBH mergers?

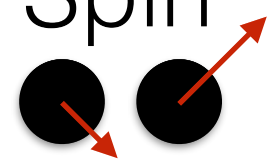
Clusters



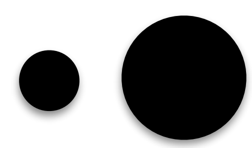
parameters



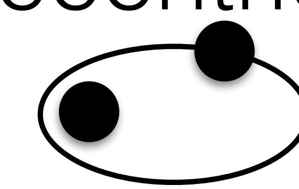
Spin



Mass



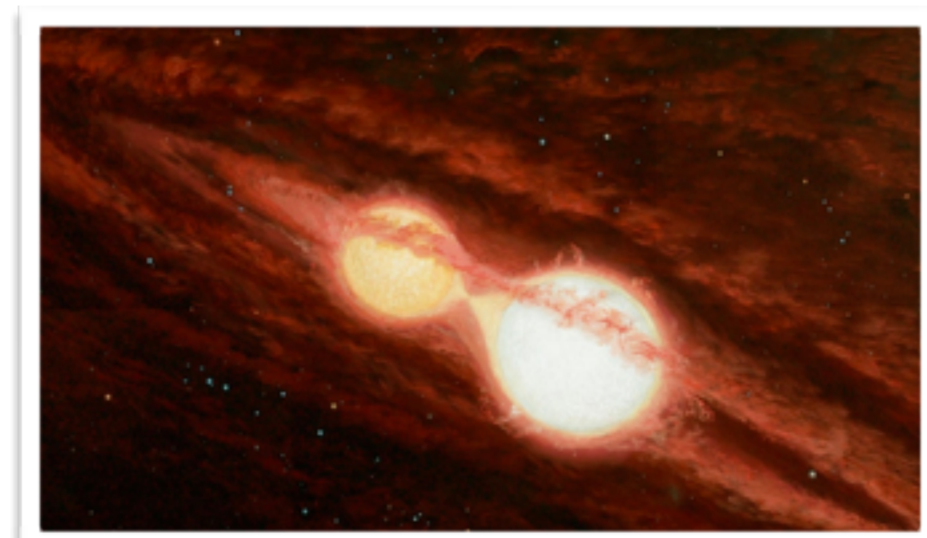
Eccentricity

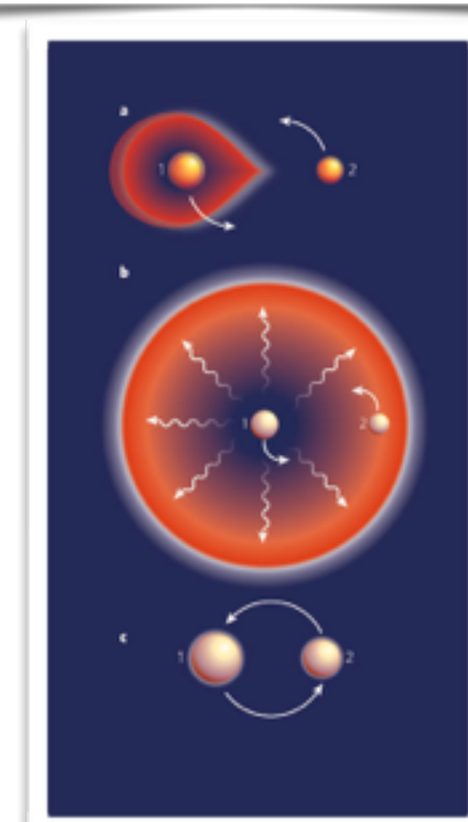


Rates

Location

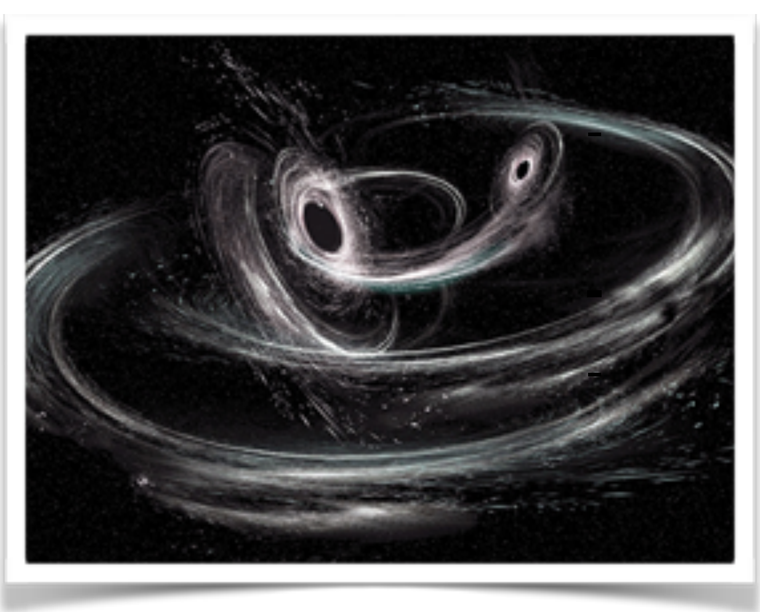
Field





What is the origin of BBH mergers?

<u>Clusters</u>



<u>Previous studies:</u>

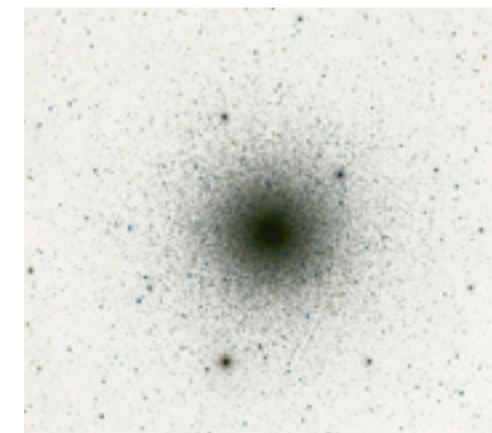
Classical studies (global evolution.):
Piet Hut. John Bahcall. Lyman Spitzer.

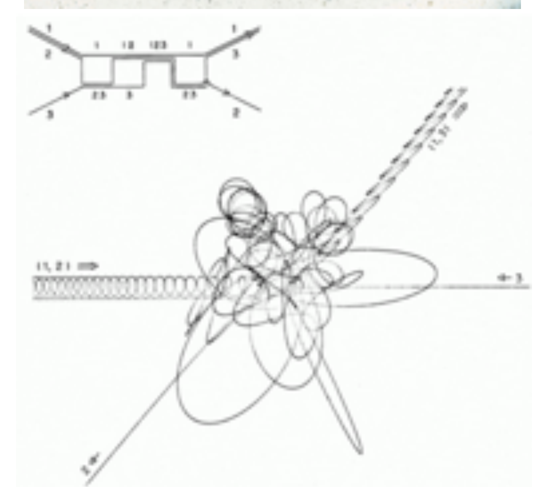
Relativistic Studies (sin-sin GW cap.):
G. Quinlan. S. Shapiro. H.B. Lee.

Black Hole Mergers (analytical):
S. McMillan. S.P. Zwart.

Black Hole Mergers (Numerical):
Fred Rasio. Mirek Giersz.

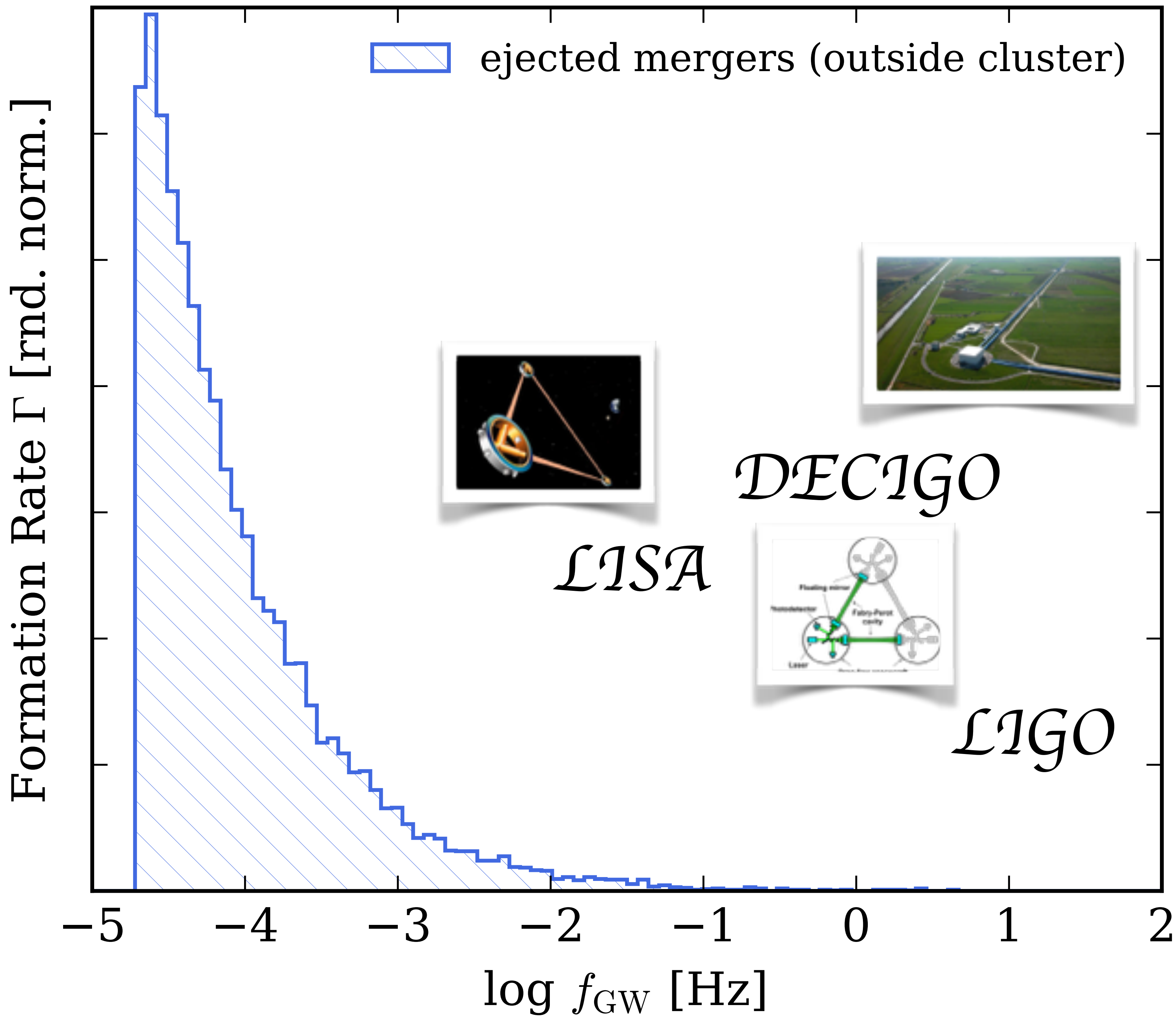
Black Hole Mergers (Relativistic): ←
Ongoing studies…



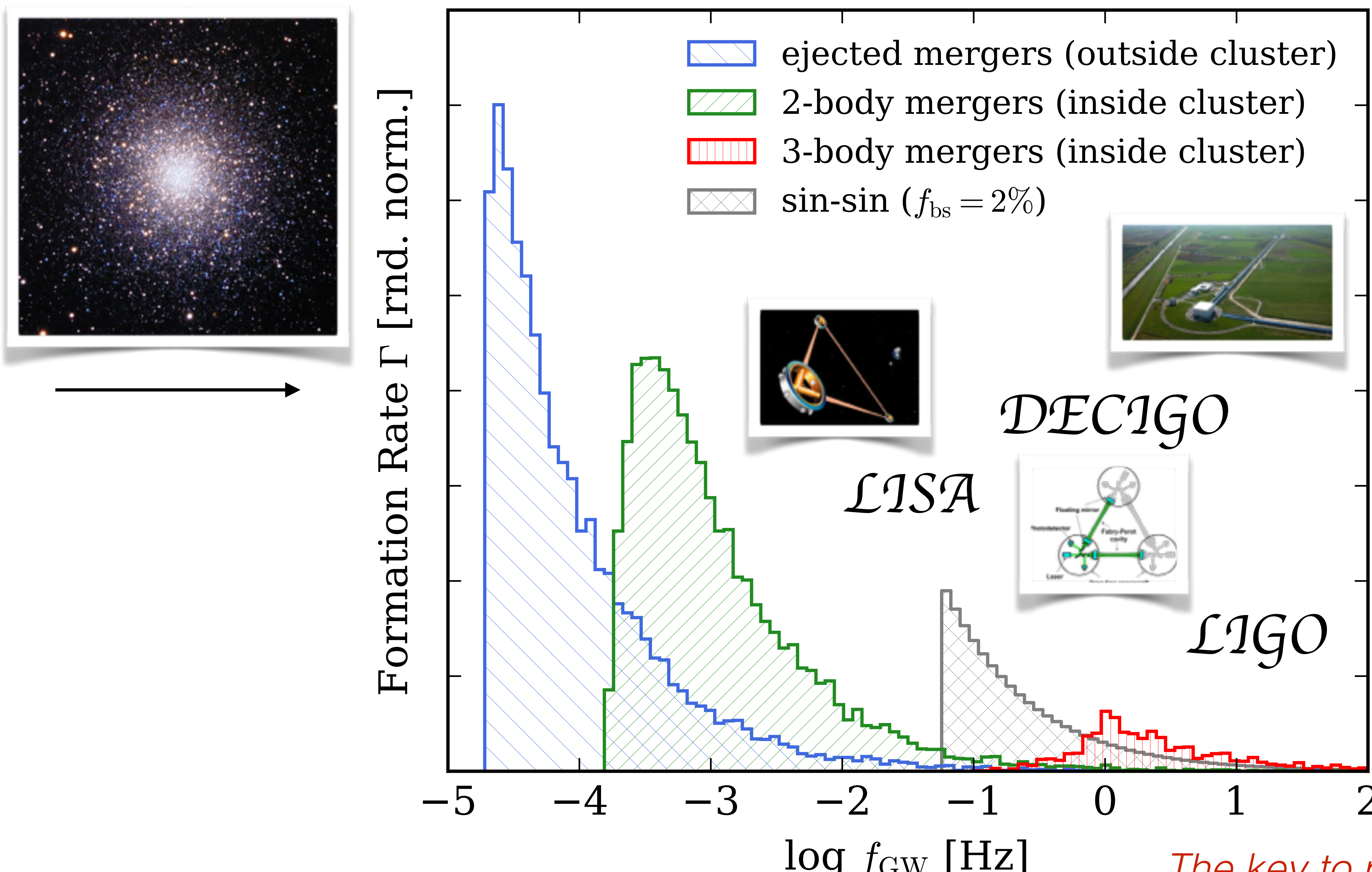


a completely new picture is under development!

Old Newtonian Studies



Our new PN Studies



The key to probe the origin of BBH mergers!

Background

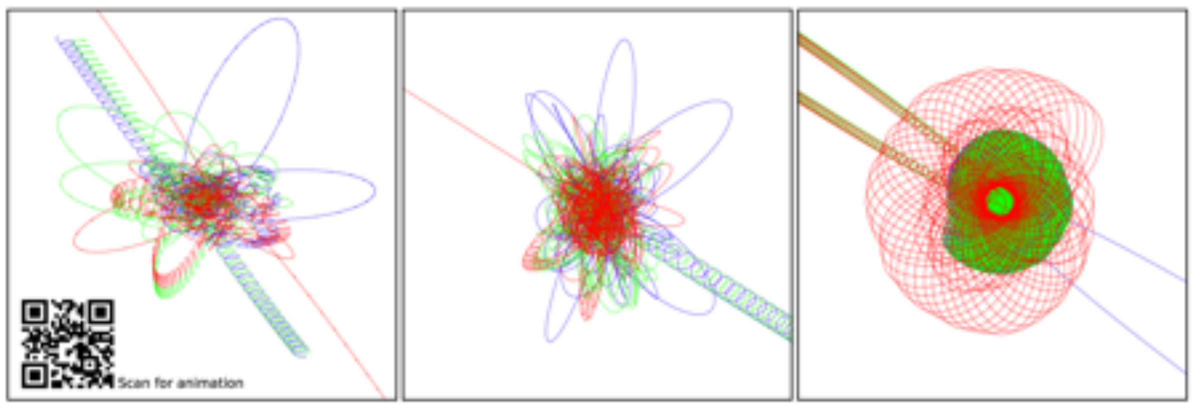
THE FORMATION OF ECCENTRIC COMPACT BINARY INSPIRALS AND THE ROLE OF GRAVITATIONAL WAVE EMISSION IN BINARY-SINGLE STELLAR ENCOUNTERS

JOHAN SAMSING[1], MORGAN MACLEOD[2], ENRICO RAMIREZ-RUIZ[2]

Draft version October 29, 2018

ABSTRACT

The inspiral and merger of eccentric binaries leads to gravitational waveforms distinct from those generated by circularly merging binaries. Dynamical environments can assemble binaries with high eccentricity and peak frequencies within the *LIGO* band. In this paper, we study binary-single stellar scatterings occurring in dense



$$\mathbf{a}_5 = \frac{4}{5}\frac{G^2 m_1 m_2}{r_{12}^3}\left[\left(\frac{2Gm_1}{r_{12}} - \frac{8Gm_2}{r_{12}} - v_{12}^2\right)\mathbf{v}_{12} + (\hat{\mathbf{r}}_{12}\cdot\mathbf{v}_{12})\left(\frac{52Gm_2}{3r_{12}} - \frac{6Gm_1}{r_{12}} + 3v_{12}^2\right)\hat{\mathbf{r}}_{12}\right]$$

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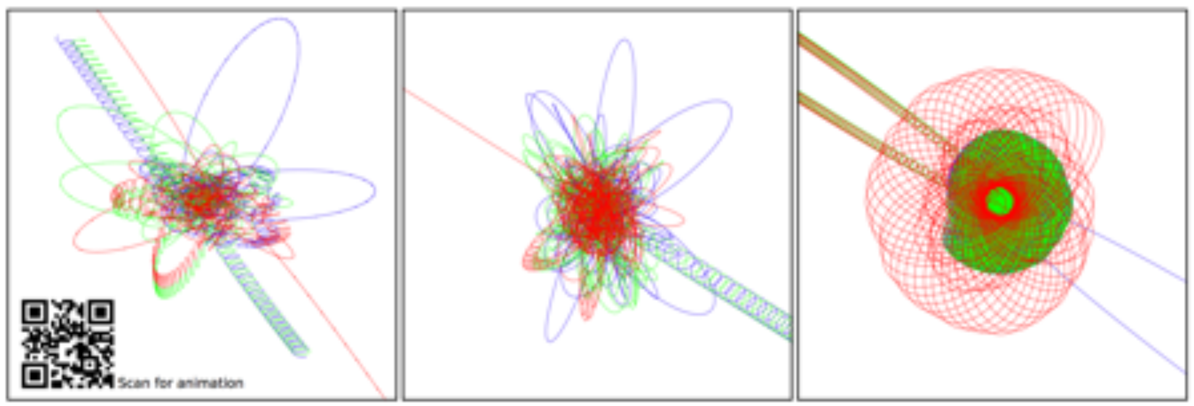
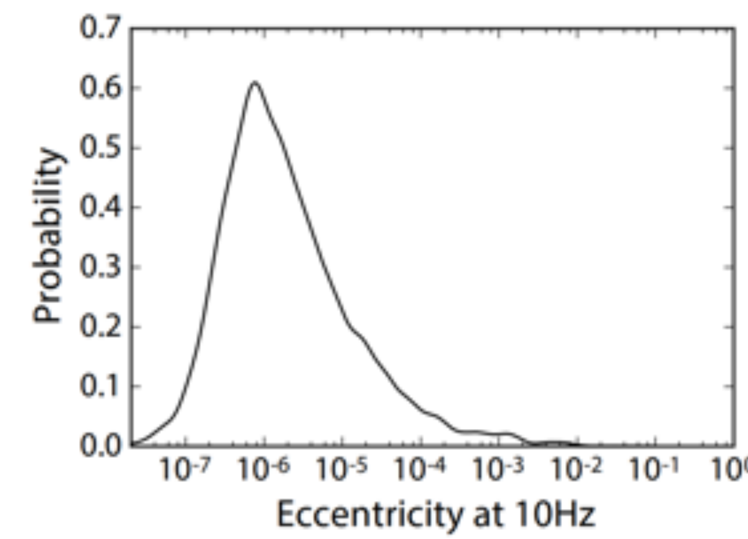
Carl L. Rodriguez,[1] Sourav Chatterjee,[1] and Frederic A. Rasio[1]

[1] *Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA) and Dept. of Physics and Astronomy, Northwestern University, 2145 Sheridan Rd, Evanston, IL 60208, USA*

(Dated: March 25, 2016)

The recent discovery of GW150914, the binary black hole merger detected by Advanced LIGO,





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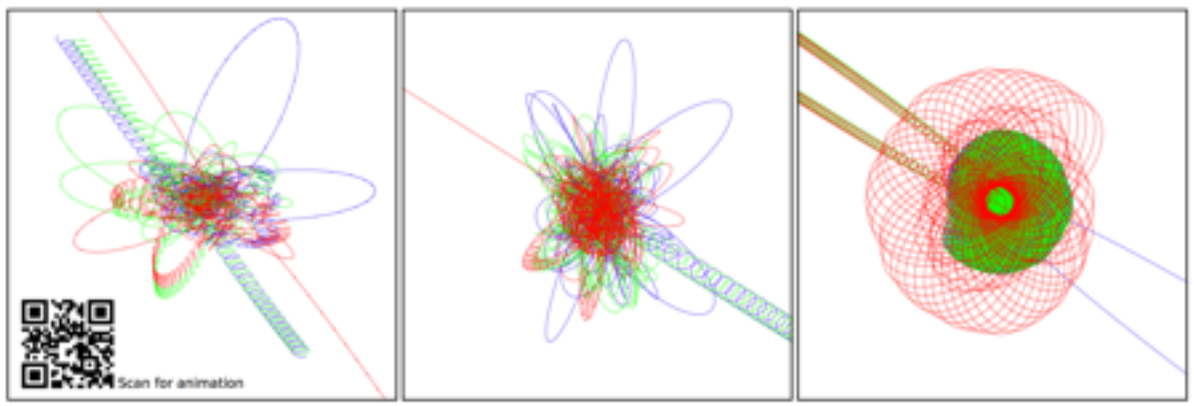

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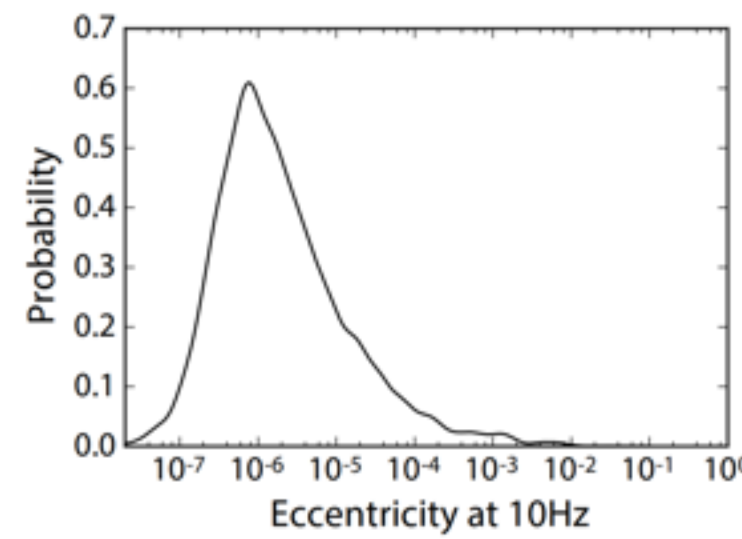
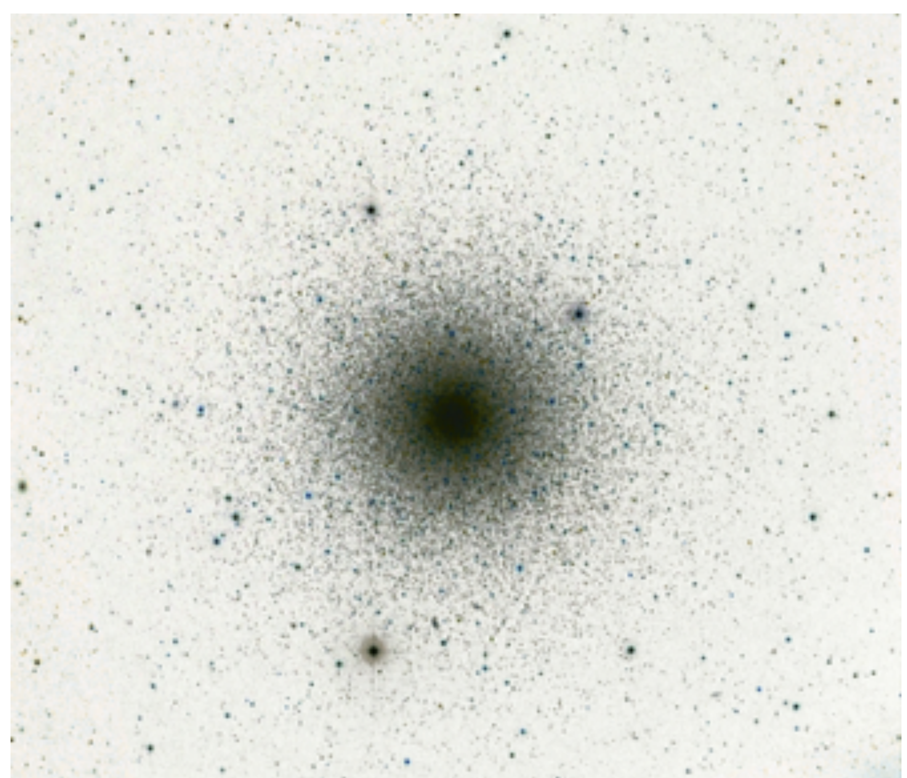


Why is it so difficult?



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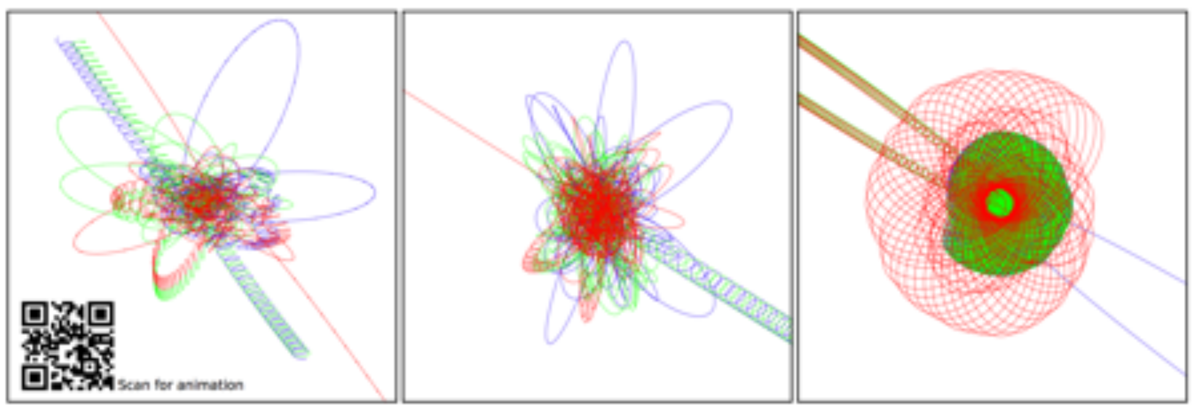

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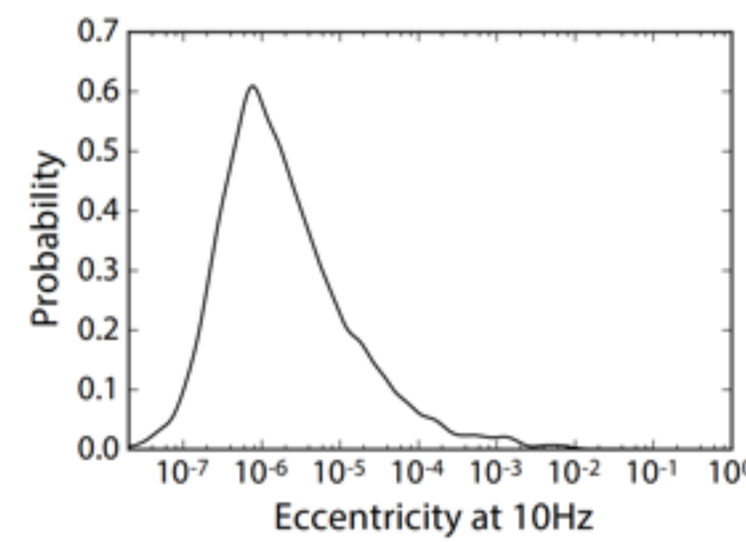


Eccentric Black Hole Mergers Forming in Globular Clusters

Johan Samsing*

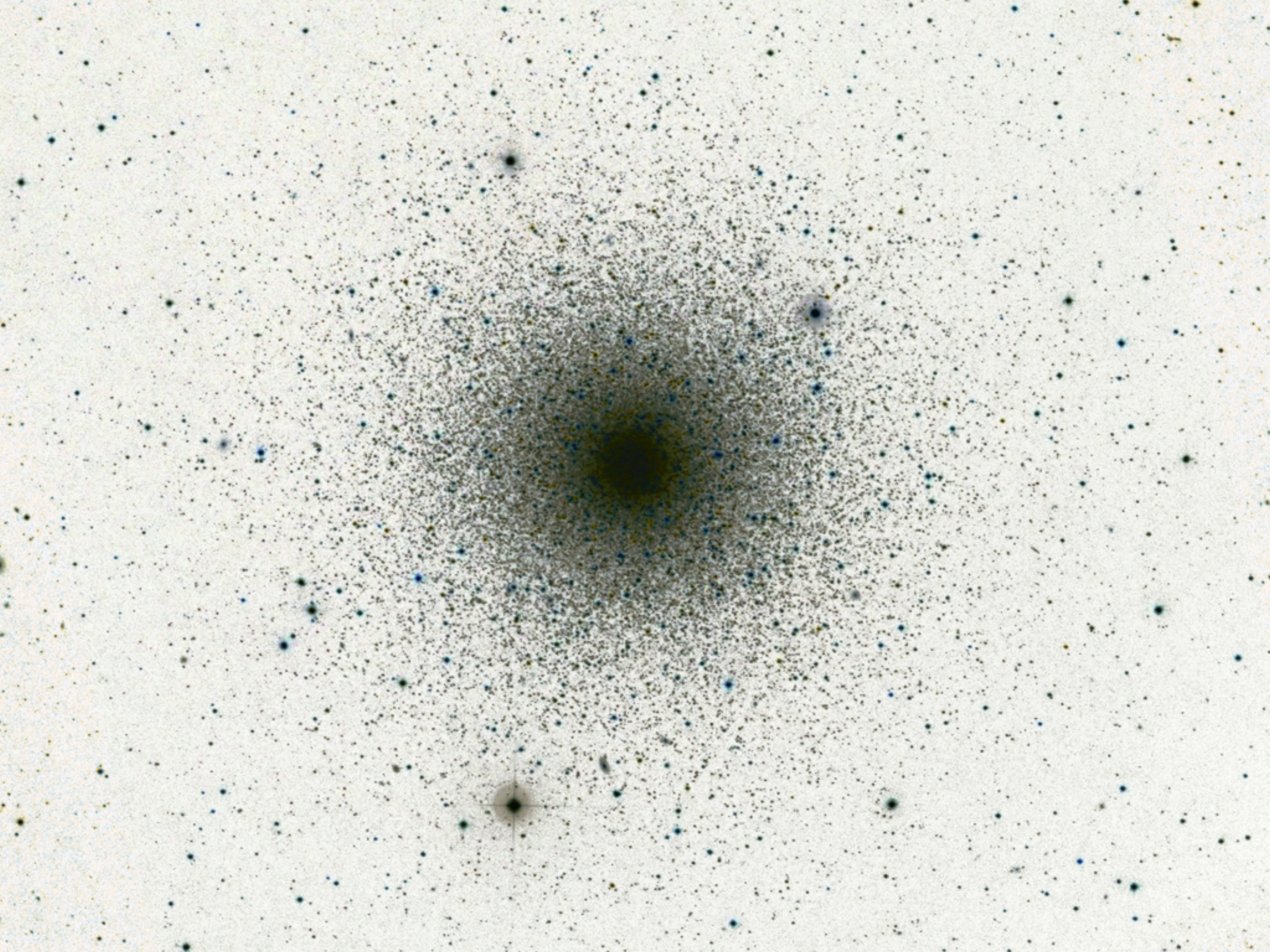
Department of Astrophysical Sciences, Princeton University, Peyton Hall, 4 Ivy Lane, Princeton, NJ 08544, USA.

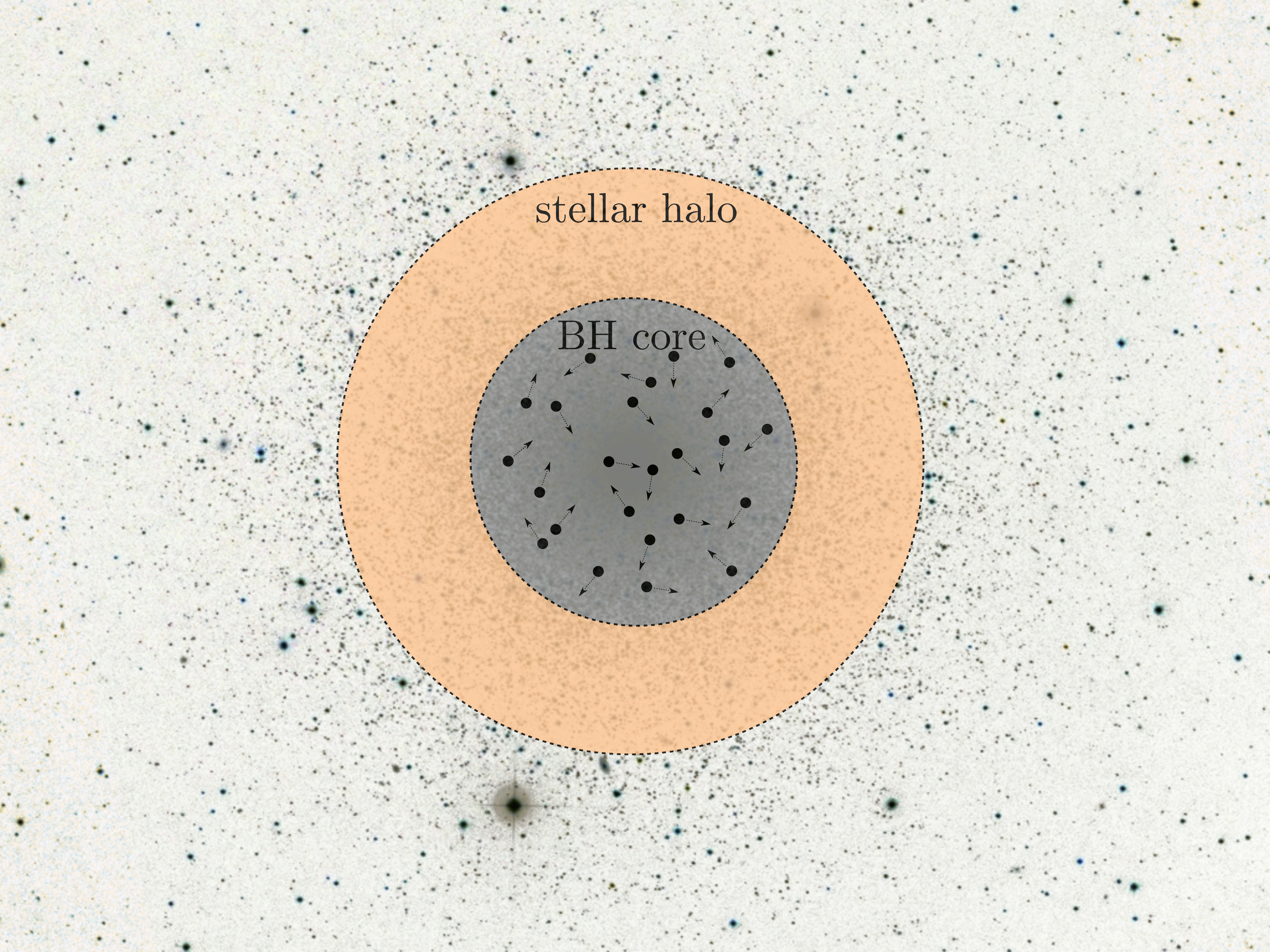
We derive the probability for a newly formed binary black hole (BBH) to undergo an eccentric gravitational wave (GW) merger during binary-single interactions inside a stellar cluster. By integrating over the hardening interactions such a BBH must undergo before ejection, we find that the observable rate of BBH mergers with eccentricity > 0.1 at 10 Hz relative to the rate of circular mergers can be as high as $\sim 5\%$ for a typical globular cluster (GC). This further suggests that BBH mergers forming through GW captures in binary-single interactions, eccentric or not, are likely to constitute $\sim 10\%$ of the total BBH merger rate from GCs. Such GW capture mergers can only

Post-Newtonian Dynamics in Dense Star Clusters: Formation, Masses, and Merger Rates of Highly-Eccentric Black Hole Binaries

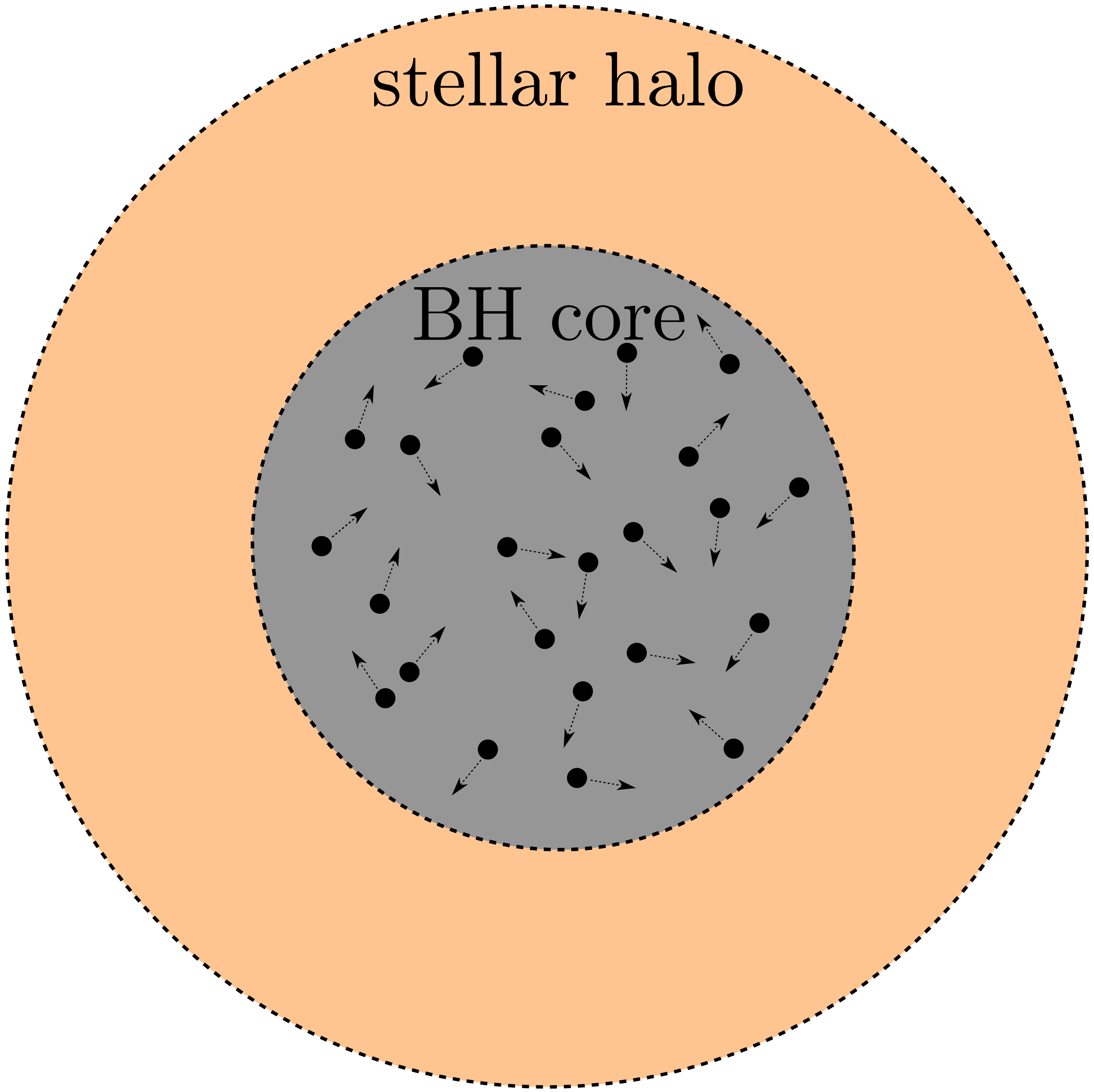
Carl L. Rodriguez,[1] Pau Amaro-Seoane,[2] Sourav Chatterjee,[3] Kyle Kremer,[4] Frederic A. Rasio,[4] Johan Samsing,[5] Claire S. Ye,[4] and Michael Zevin[4]

Using state-of-the-art dynamical simulations of globular clusters, including radiation reaction during black hole encounters and a cosmological model of star cluster formation, we create a realistic population of dynamically-formed binary black hole mergers across cosmic space and time. We show that in the local universe, 10% of these binaries form as the result of gravitational-wave emission between unbound black holes during chaotic resonant encounters, with roughly half of those events having eccentricities detectable by current ground-based gravitational-wave detectors. The mergers



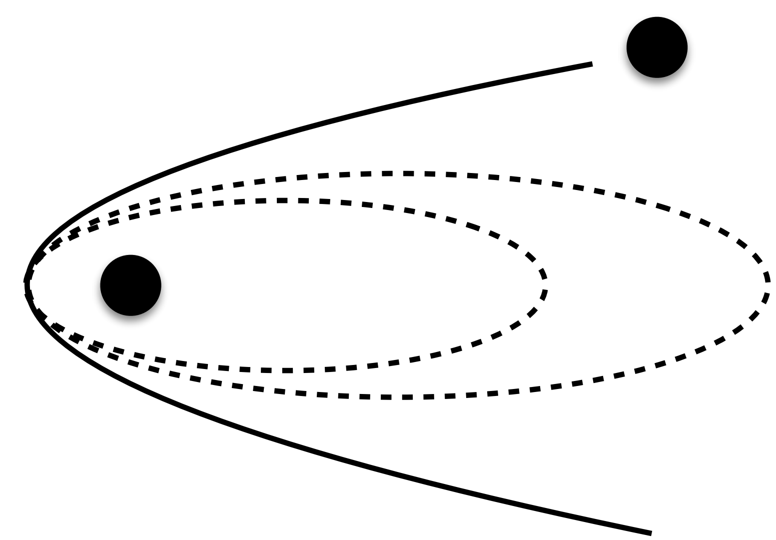

stellar halo
BH core

MODEL:



$$f \approx \frac{1}{\pi}\sqrt{\frac{2Gm}{r_f^3}}$$

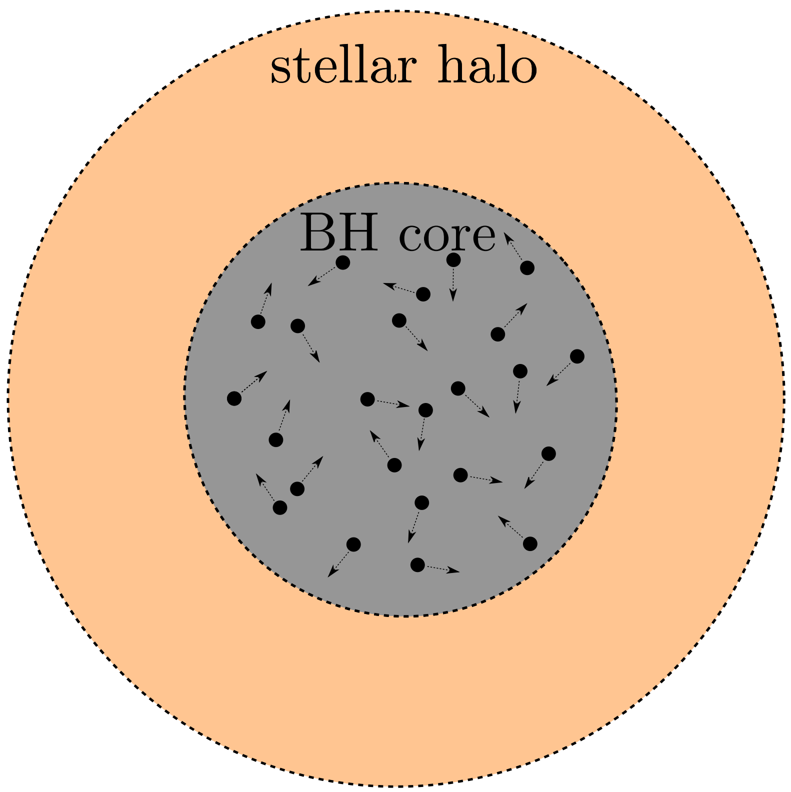
$$r_f \approx \left(\frac{2Gm}{f^2\pi^2}\right)^{1/3}$$

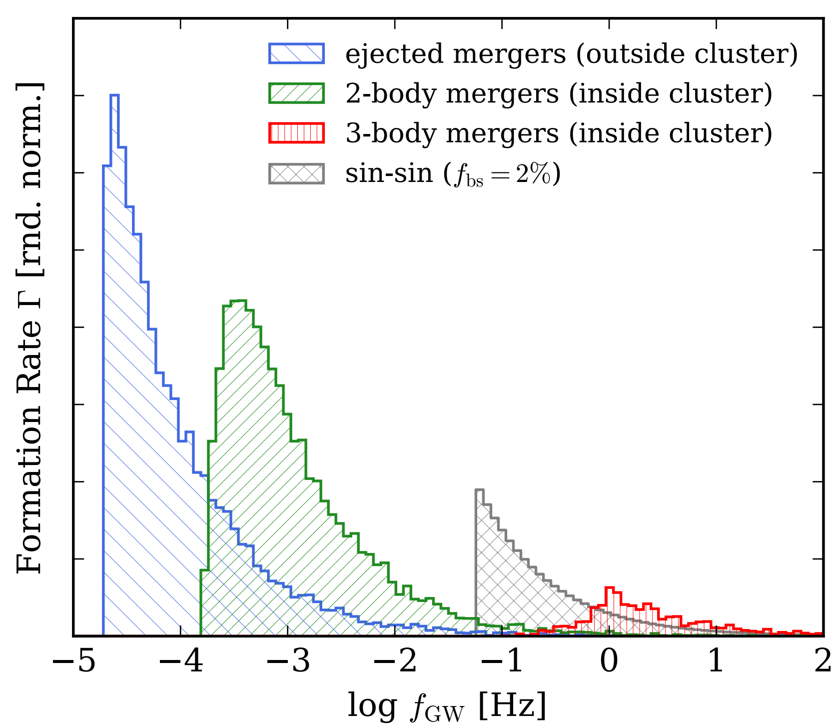


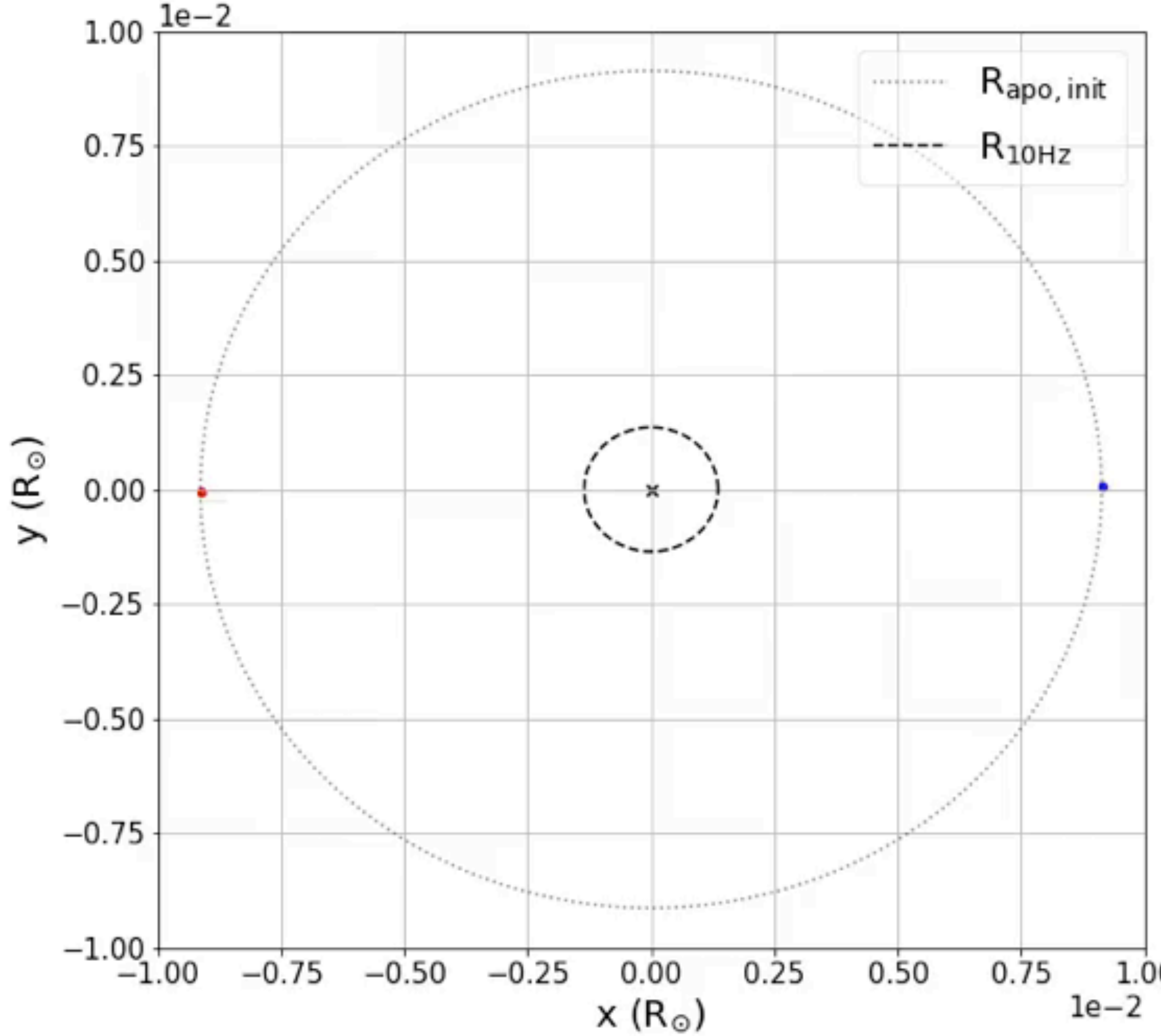
How do BBHs form and merge?

What is the peak freq. dist? Correlate this with e!

MODEL:

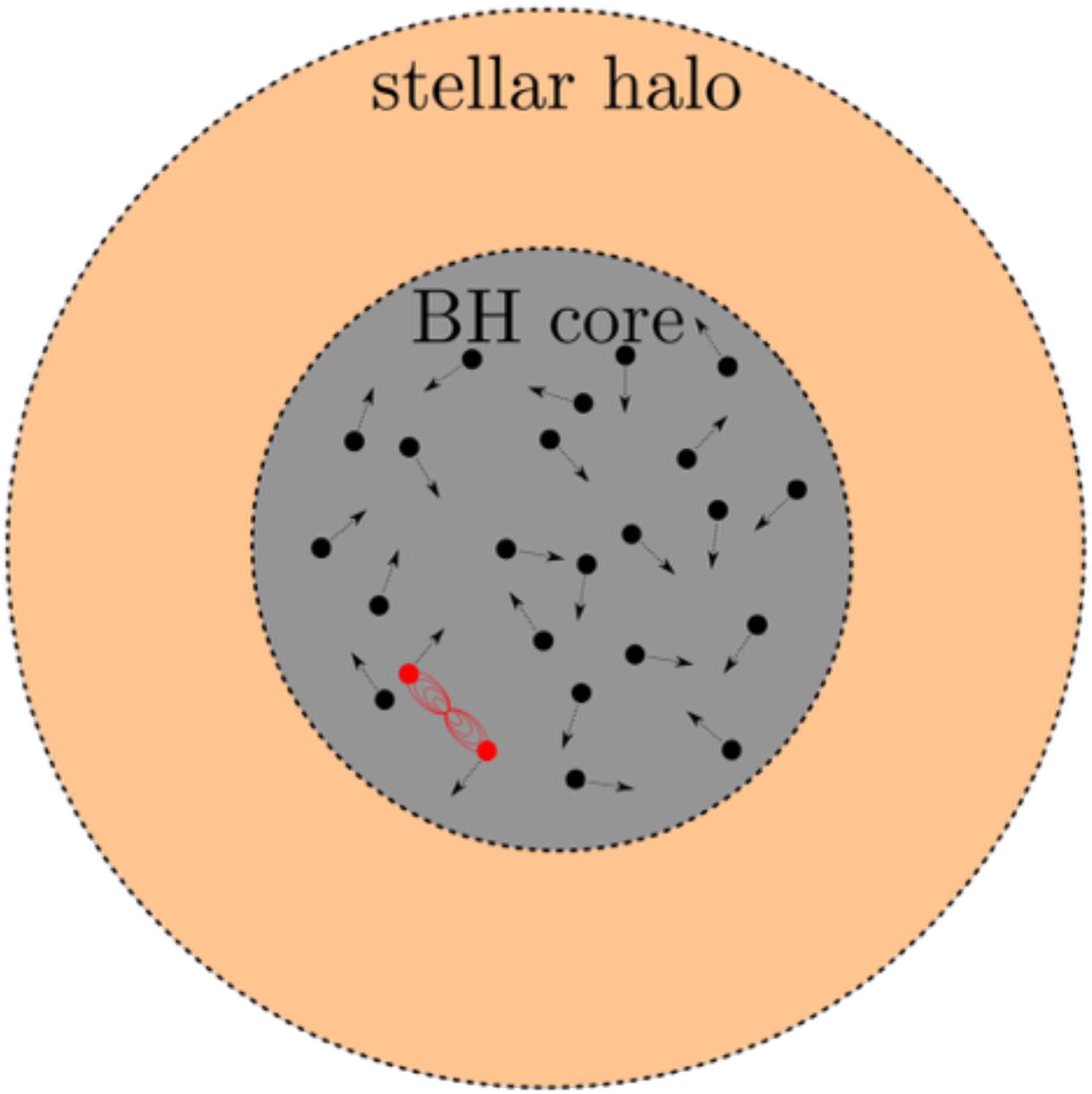




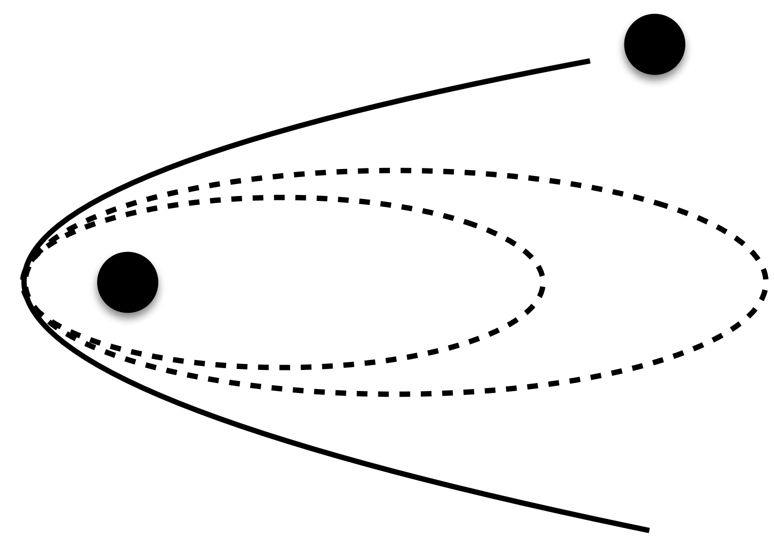


$$f \approx \frac{1}{\pi}\sqrt{\frac{2Gm}{r_f^3}}$$

Merger Type: Single-Single



Capture:

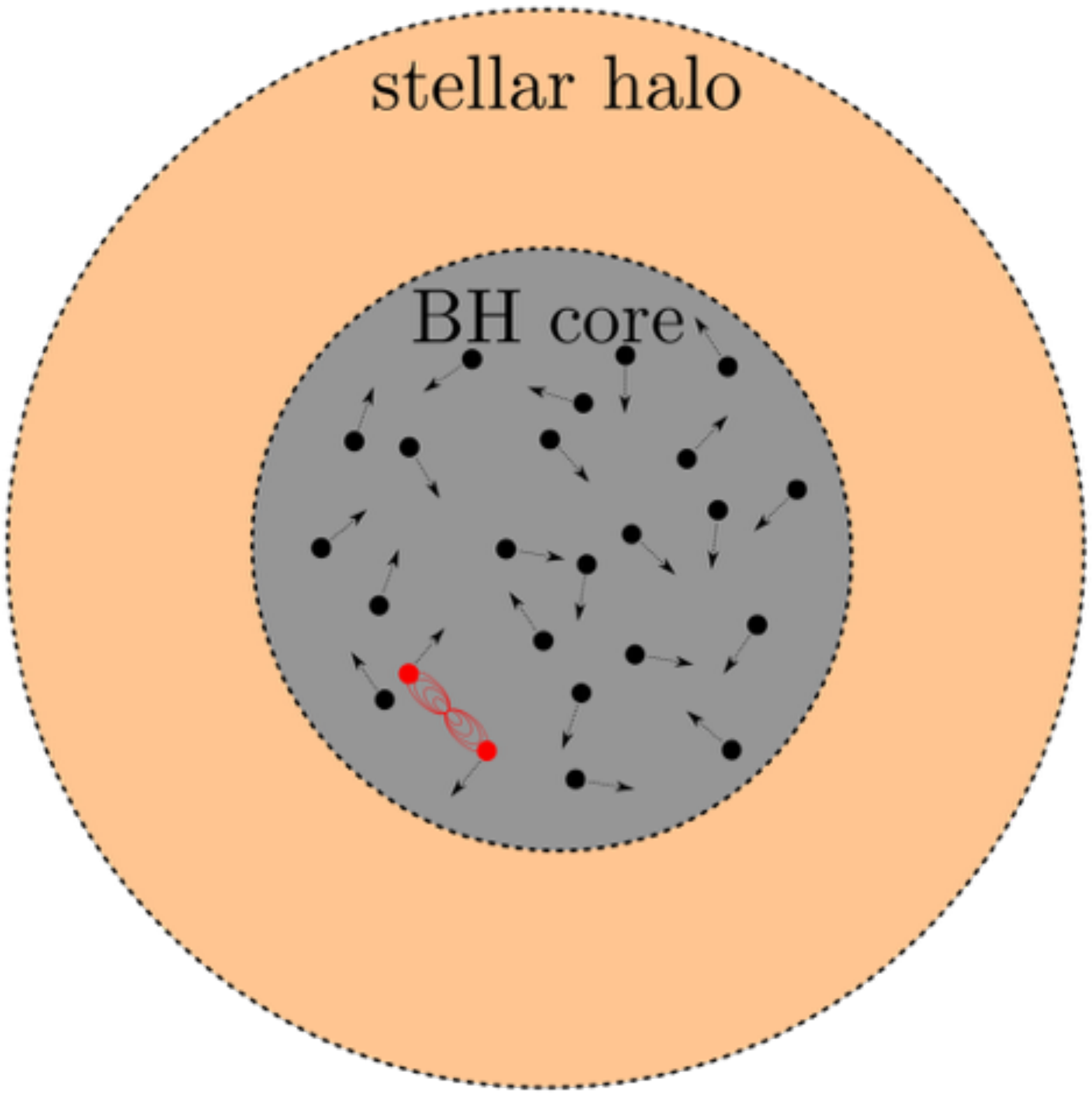


$$\Delta E_{\rm p} \approx (85\pi/12) G^{7/2} c^{-5} m^{9/2} r_{\rm p}^{-7/2}$$

$$E_{\rm ss} \approx \mu v^2/2$$

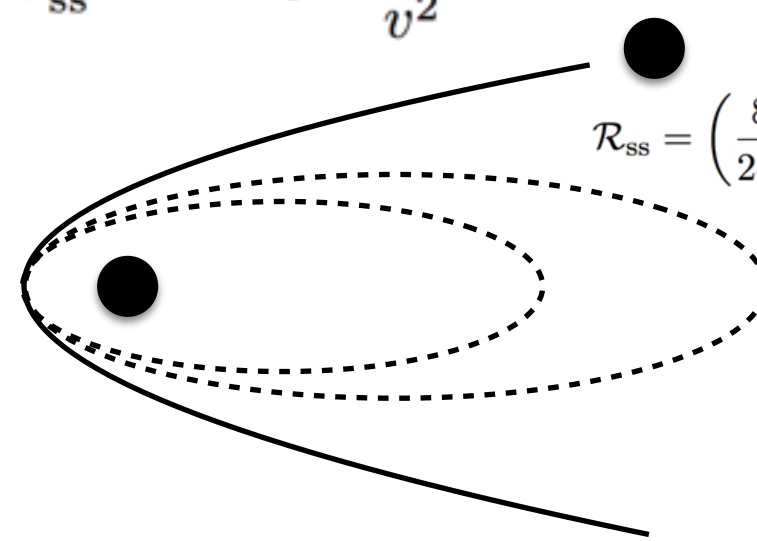
$$\mathcal{R}_{\rm ss} = \left(\frac{85\pi}{24\sqrt{2}}\right)^{2/7} \times \mathscr{R}_{\rm m} \left(\frac{c^2}{v^2}\right)^{2/7}$$

Merger Type: Single-Single



Cross section:

$$\sigma_{\rm ss}^{<R} \approx 2\pi G \frac{2mR_{\rm ss}}{v^2}$$



$$\mathcal{R}_{\rm ss} = \left(\frac{85\pi}{24\sqrt{2}}\right)^{2/7} \times \mathscr{R}_{\rm m} \left(\frac{c^2}{v^2}\right)^{2/7}$$

Rate:

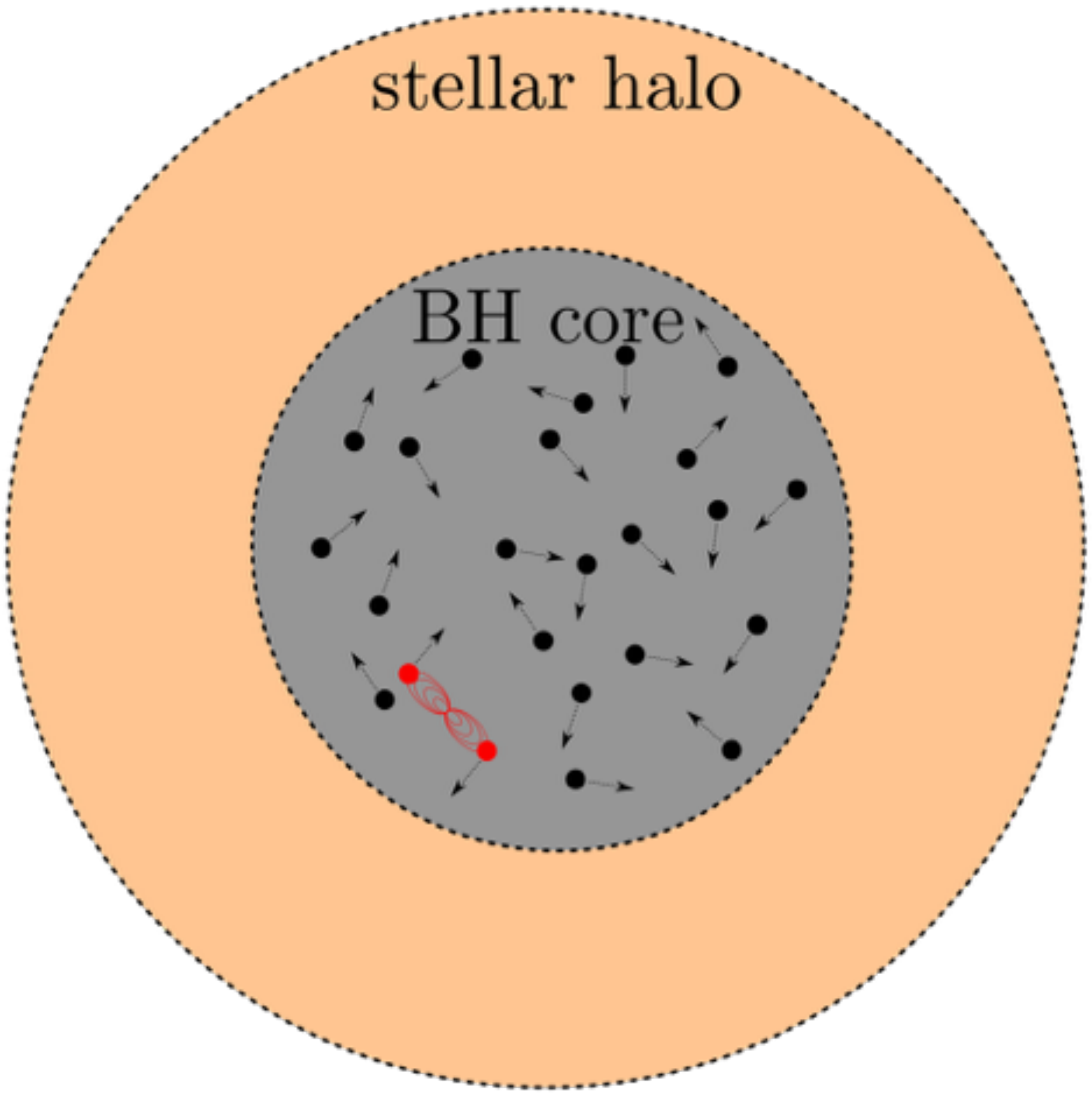
$$d\Gamma_{\rm ss}(r) = n(r)\sigma_{\rm ss}^{<R}(r)v(r) \times n(r)4\pi r^2 dr \times \frac{1}{2}$$

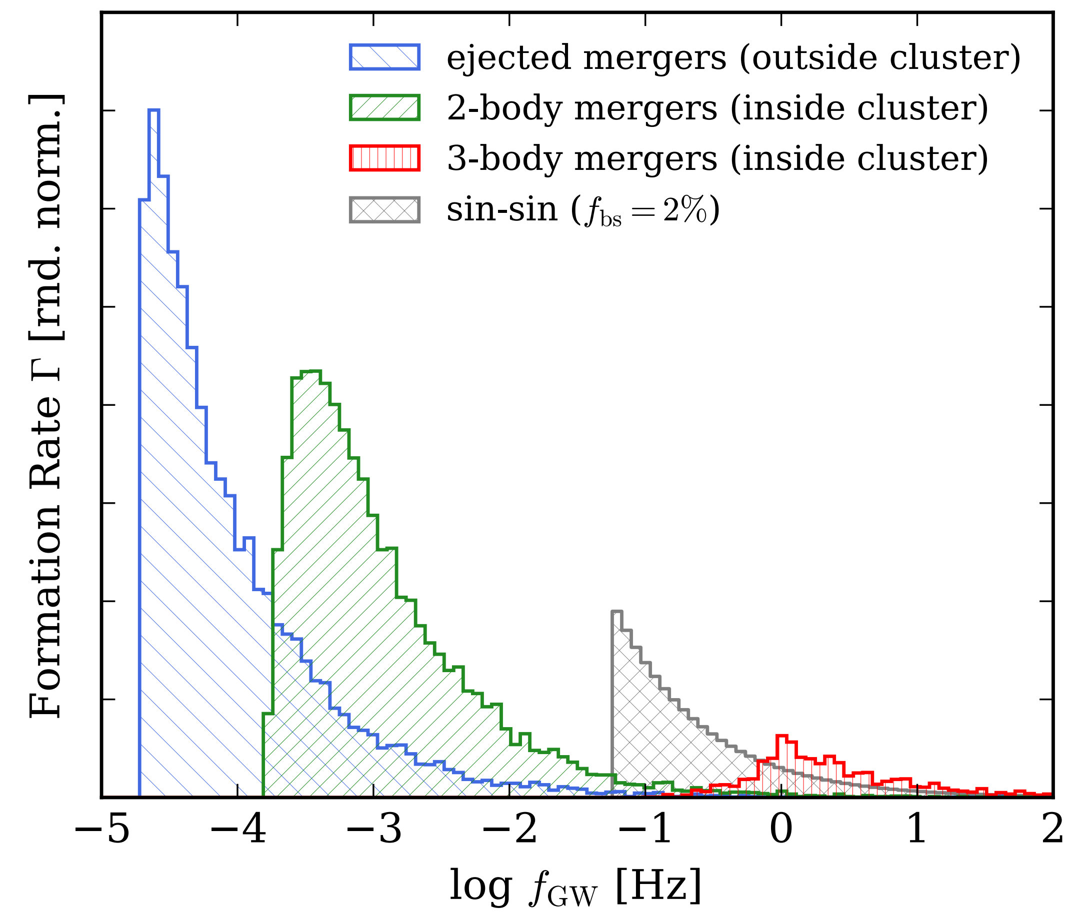
$$\Gamma_{\rm ss} = \frac{8\pi^2 G}{m}\int_0^\infty \frac{R_{\rm ss}\rho^2}{v} r^2 dr$$

$$\Gamma_{\rm ss} = n_0\sigma_{\rm ss,0}^{<R} v_0 \frac{1}{2} N_{\rm s} \times \frac{4}{3}\frac{\pi r_{\rm s}^3 \rho_0}{M}\int_0^\infty \frac{\tilde{R}_{\rm ss}\tilde{\rho}^2}{\tilde{v}} 3\tilde{r}^2 d\tilde{r}$$

$$= n_0\sigma_{\rm ss,0}^{<R} v_0 \frac{1}{2} N_{\rm s} \times \xi_{ss},$$

Merger Type: Single-Single

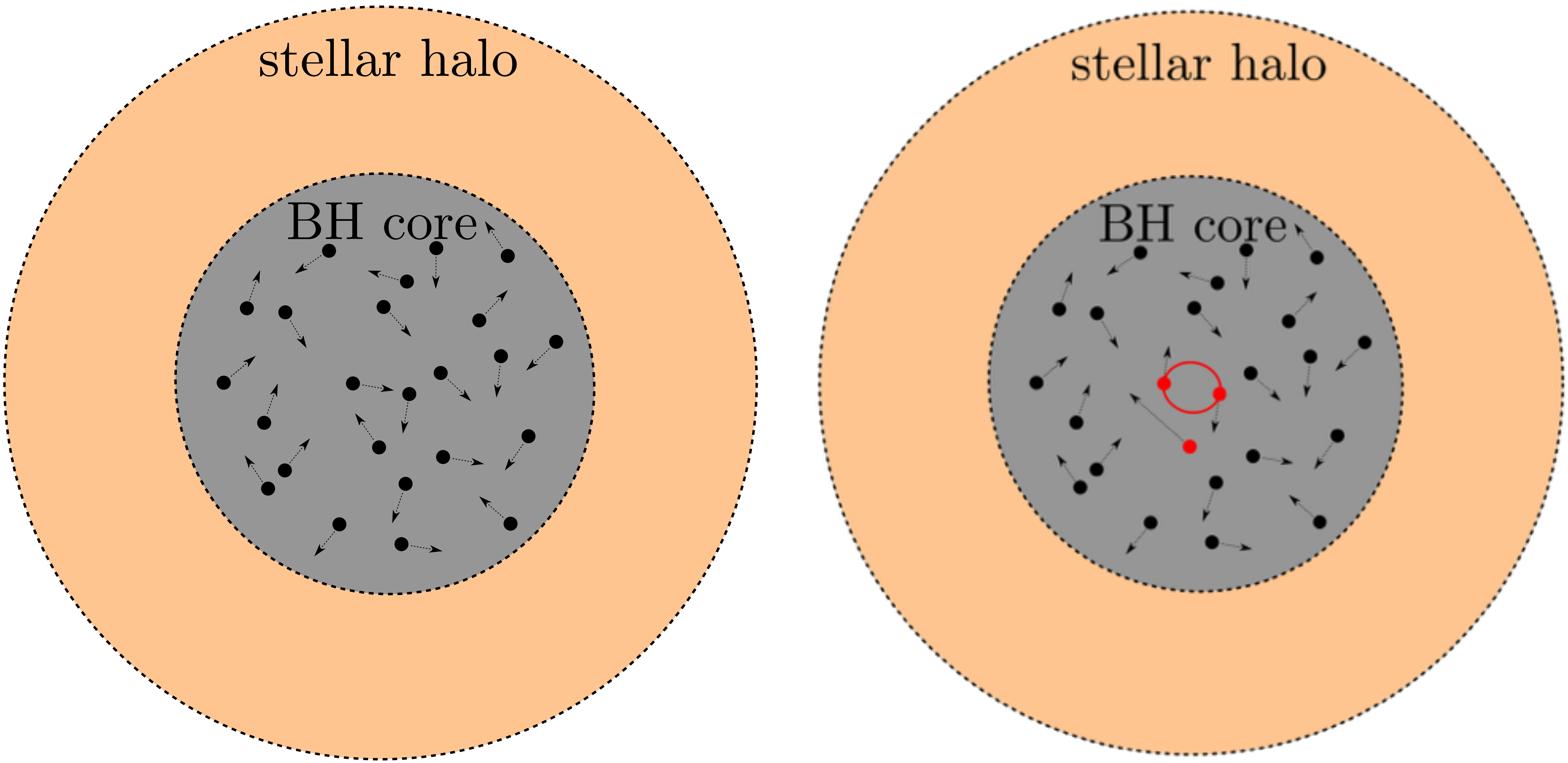




$$\mathcal{R}_{\rm ss} = \left(\frac{85\pi}{24\sqrt{2}}\right)^{2/7} \times \mathscr{R}_{\rm m} \left(\frac{c^2}{v^2}\right)^{2/7}$$

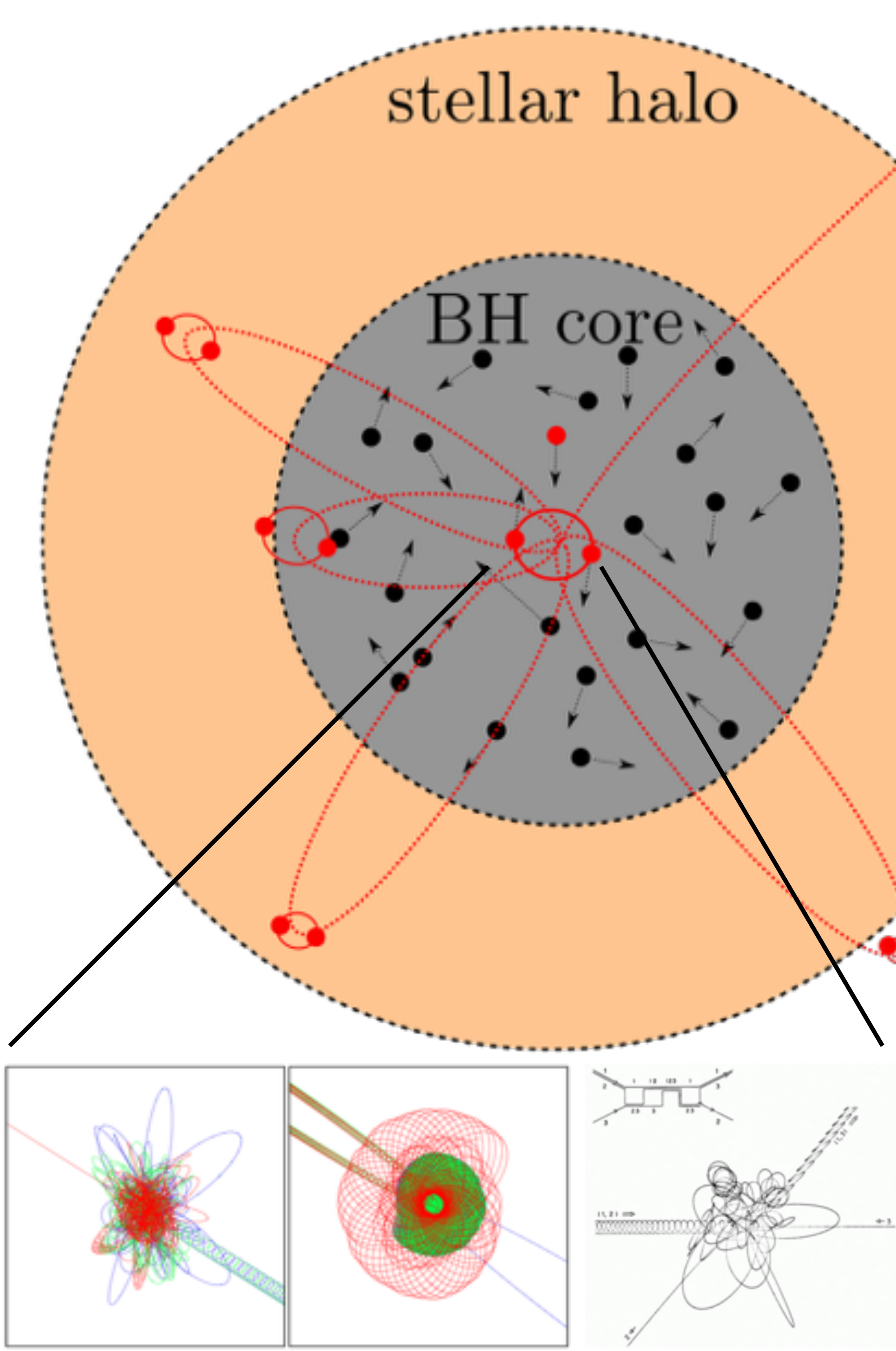
$$f \approx \frac{1}{\pi}\sqrt{\frac{2Gm}{r_f^3}}$$

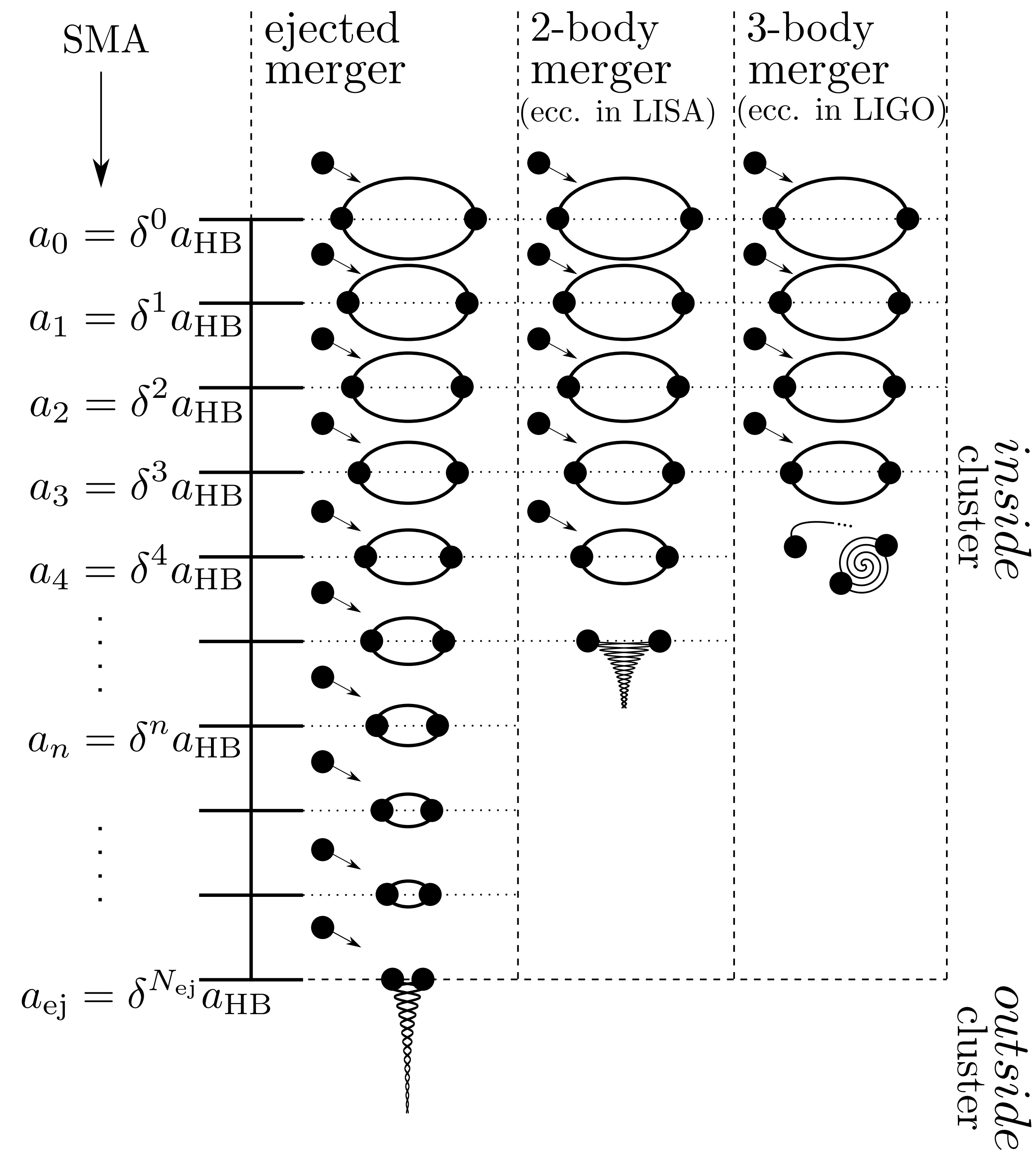
Few-body BBH mergers: Formation of a BBH



$$a_{\rm HB} = \frac{3}{2}\frac{Gm_{\rm BH}}{v_{\rm dis}^2}$$

Merger Type: Ejected Merger





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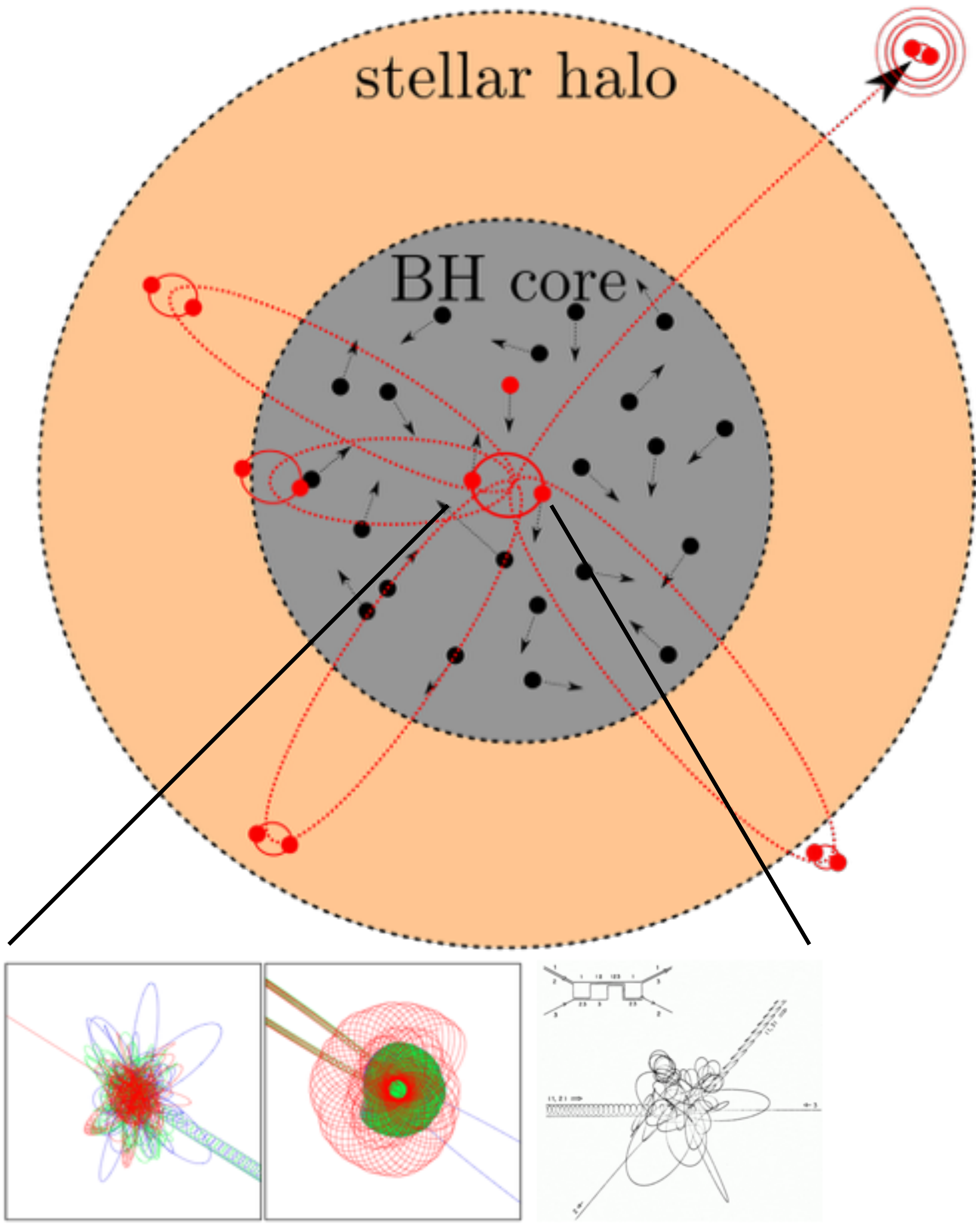
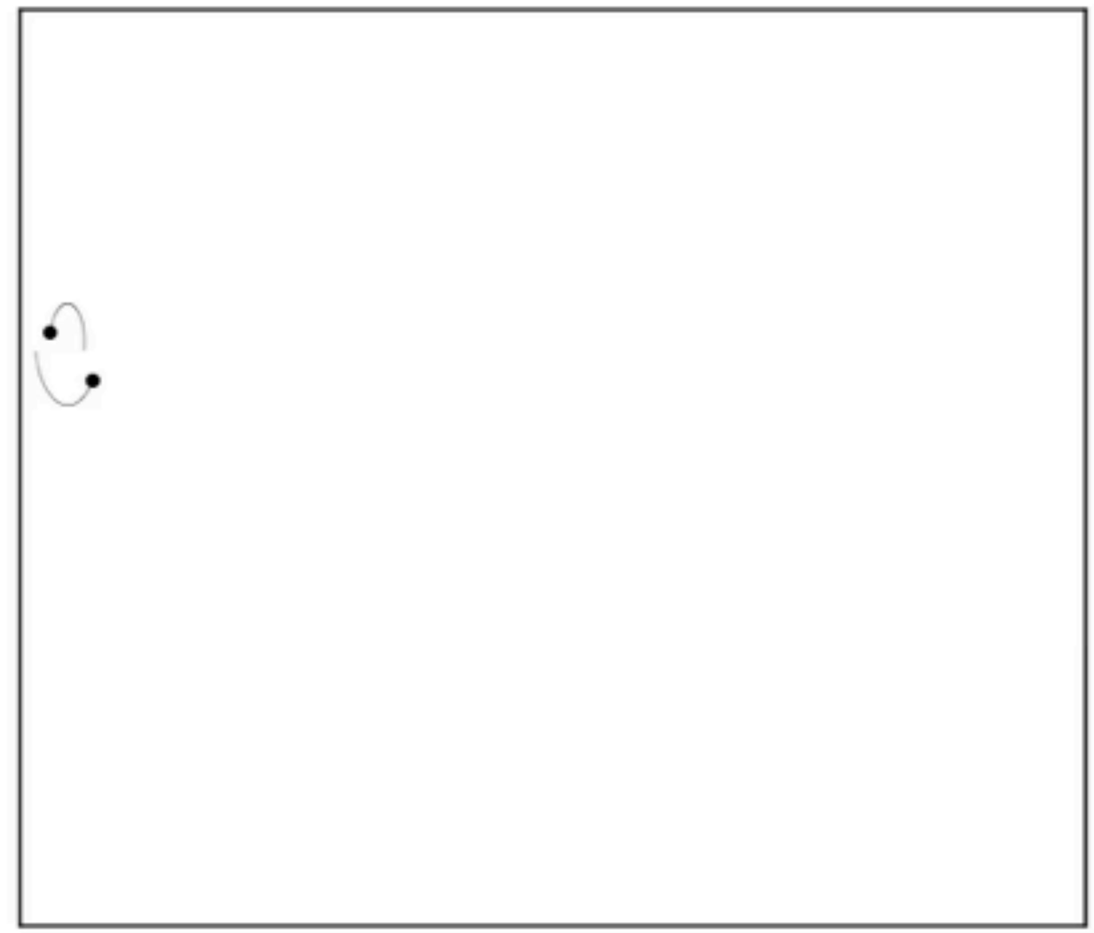
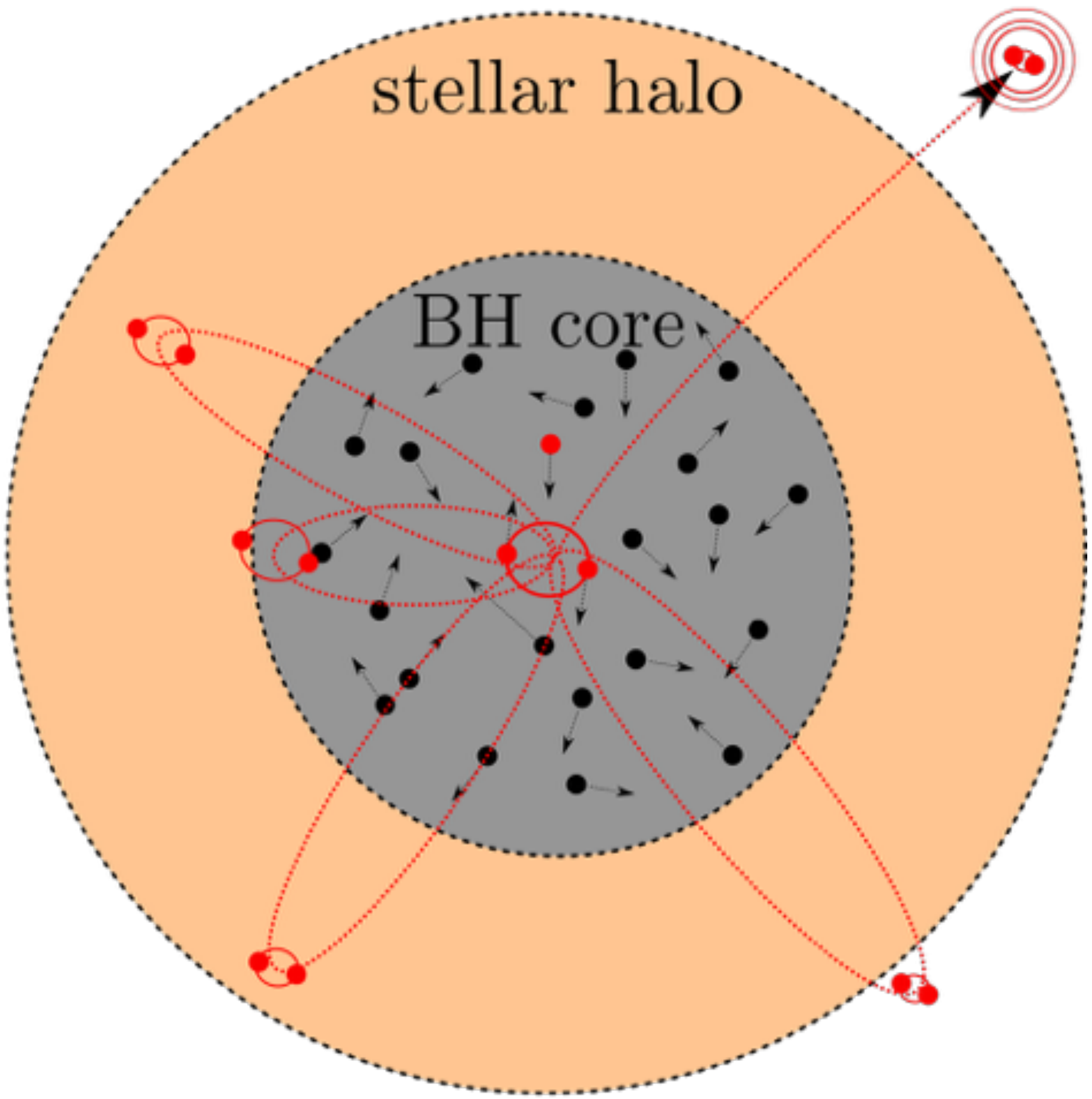




Merger Type: Ejected Merger



$$N_{\rm bs}(a_{\rm in}, a_{\rm ej}) = \int_{a_{\rm ej}}^{a_{\rm in}} \frac{1}{1-\delta}\frac{1}{a}da = \frac{1}{1-\delta}\ln\left(\frac{a_{\rm in}}{a_{\rm ej}}\right)$$

Classical way of forming BBH mergers
Suggested by: P. Zwart, S. McMillan

$$\Delta E_{\rm bs} = (1/\delta - 1) \times E_{\rm B}(a)$$

$$a_{\rm ej} \approx \frac{1}{6}\left(\frac{1}{\delta} - 1\right)\frac{Gm}{v_{\rm esc}^2}$$

$$a_{\rm HB} = \frac{3}{2}\frac{Gm_{\rm BH}}{v_{\rm dis}^2}$$

$$a(k) = a_{\rm HB}\delta^k$$

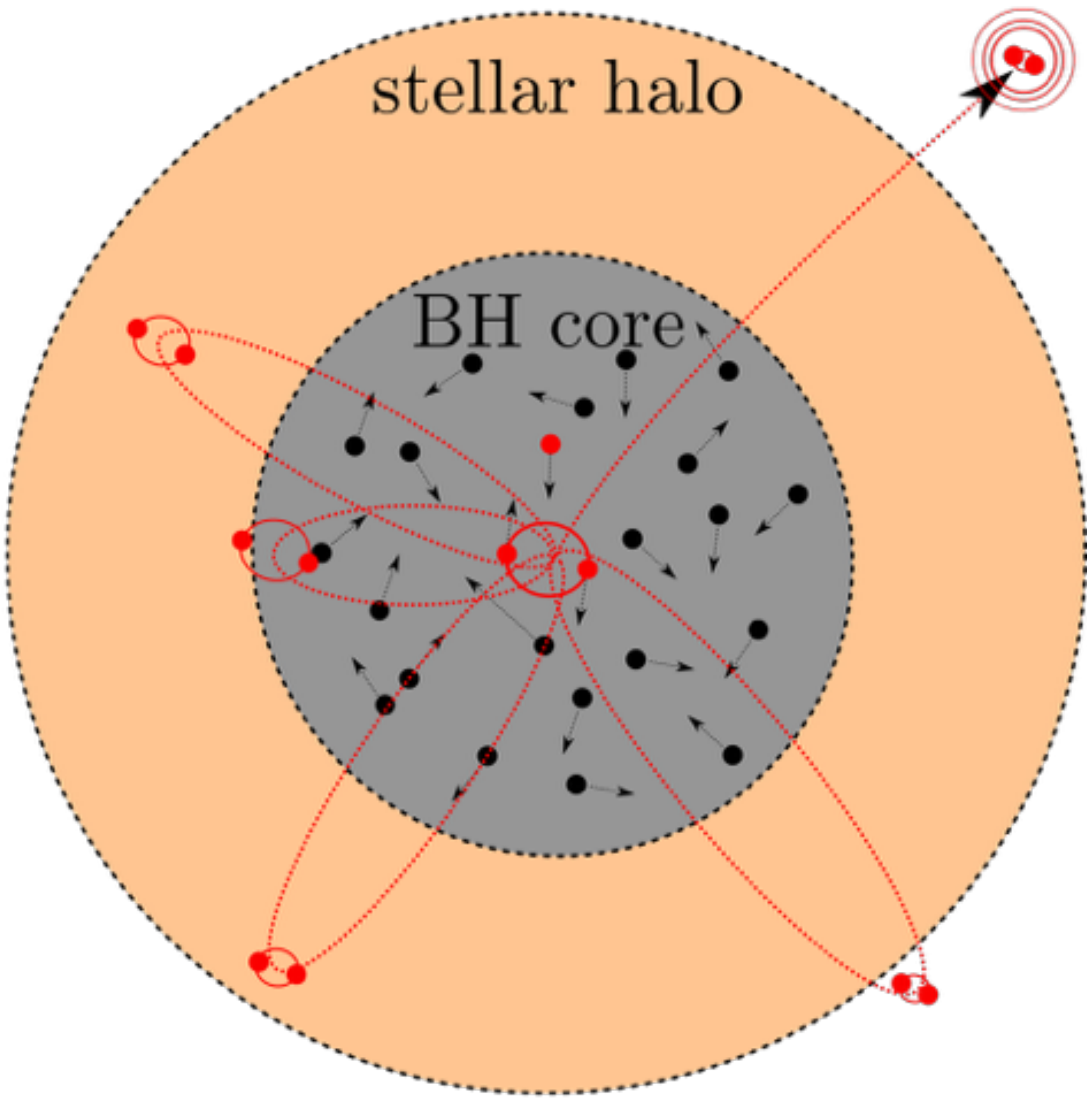
$$da = -a(1-\delta)dk$$

$$T_{\rm ej} = \int_{a_{\rm ej}}^{a_{\rm HB}} \frac{1}{n_0 \sigma_{\rm bs} v_0}\frac{da}{a(1-\delta)}, \approx \frac{(6\pi G)^{-1}}{(1-\delta)}\frac{v_0}{n_0}\frac{m^{-1}}{a_{\rm ej}},$$

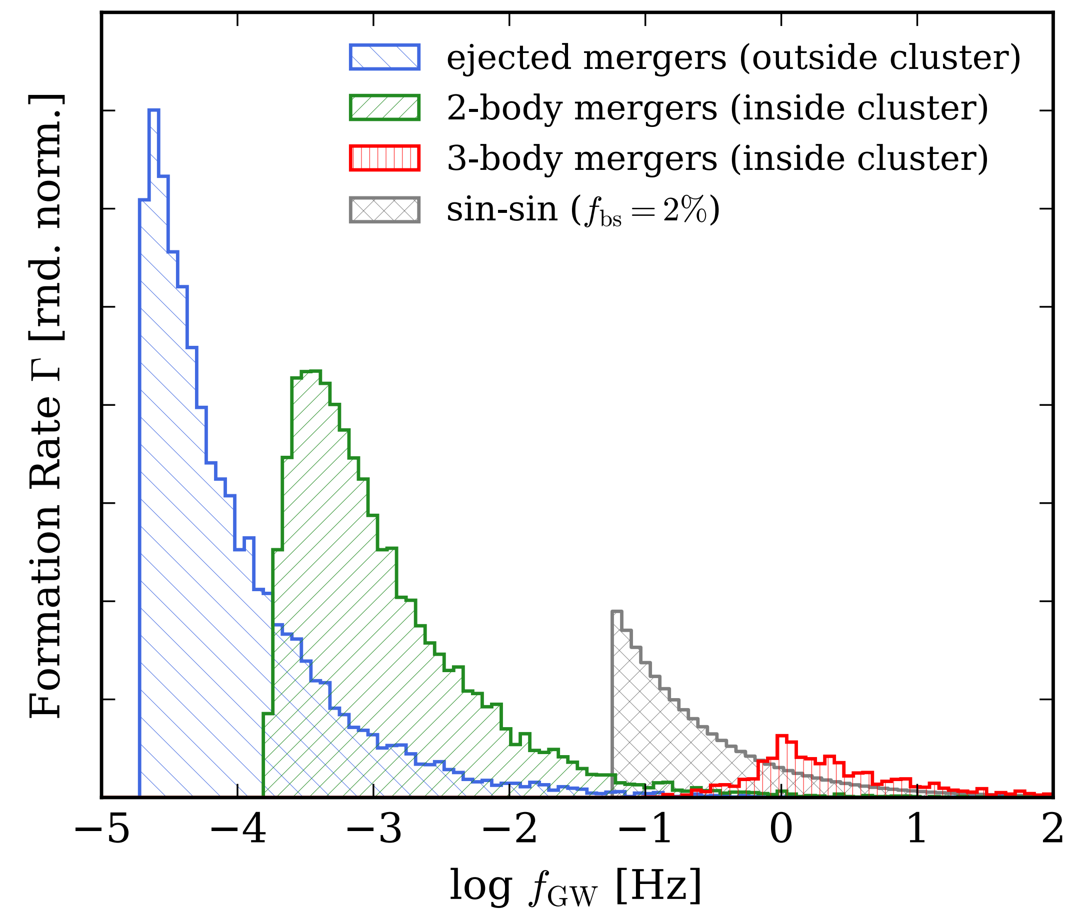
Newtonian outcome.

Leads to interesting rate,
but standard circular mergers.

Merger Type: Ejected Merger

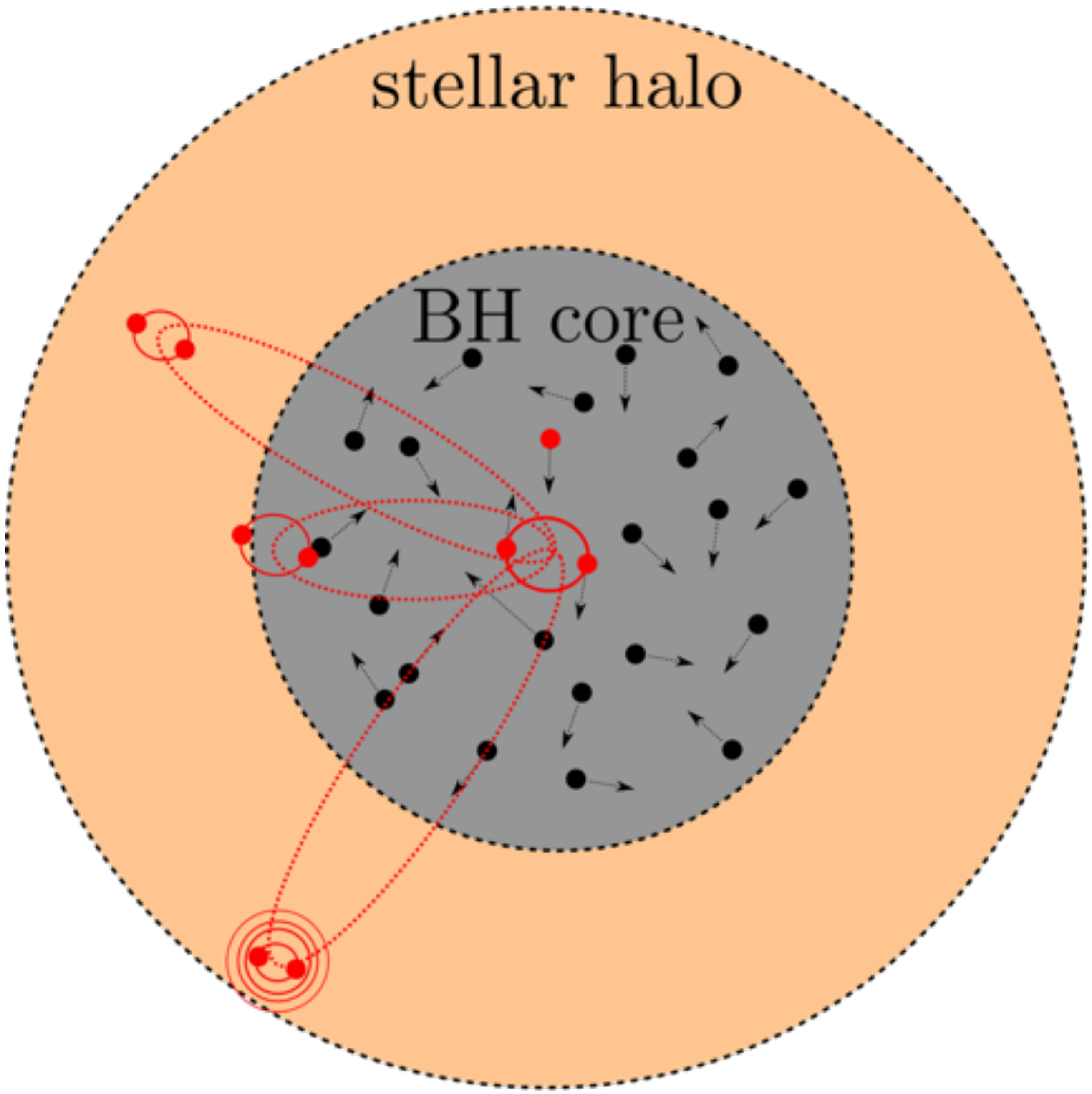


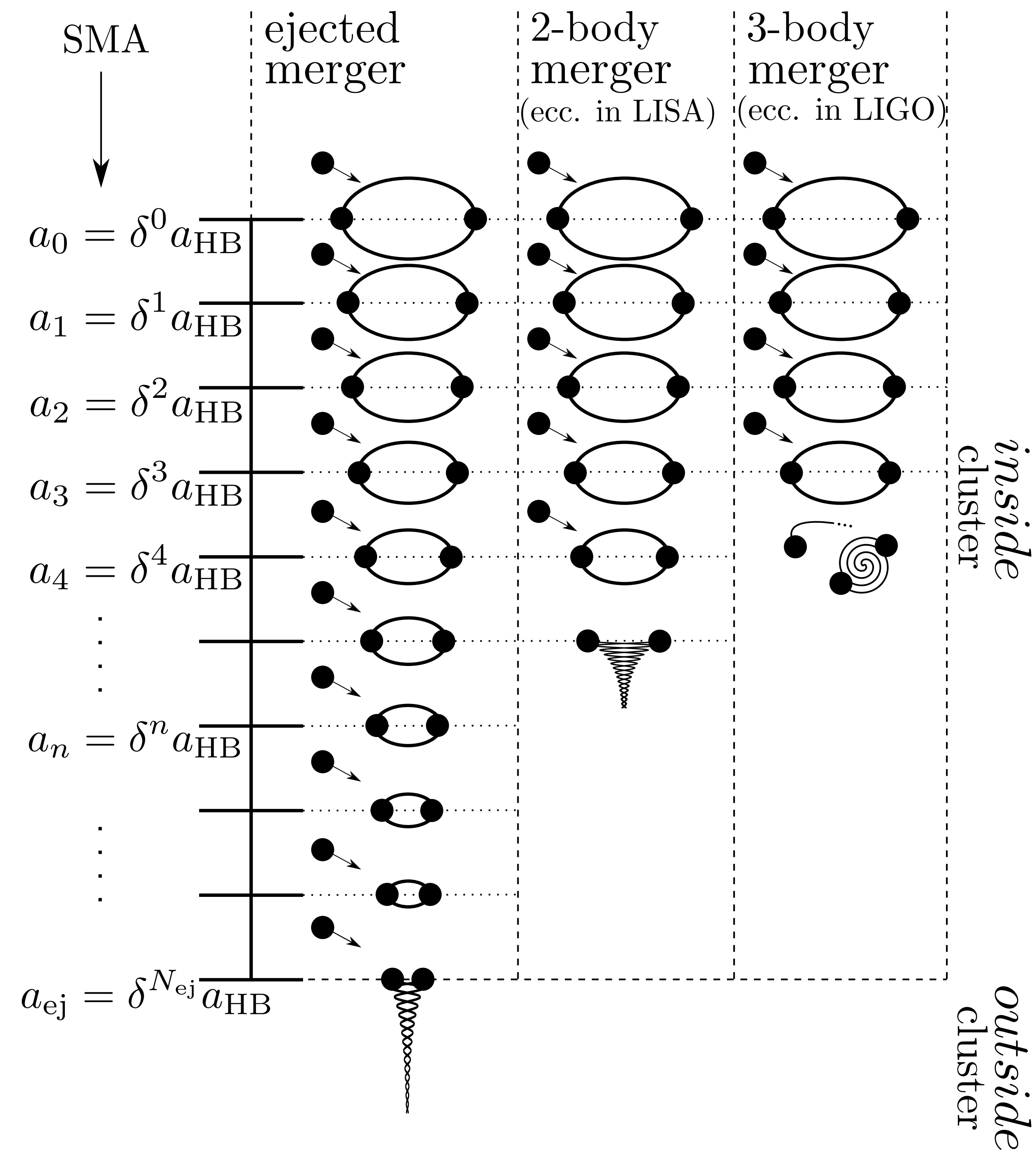
Circular GW sources



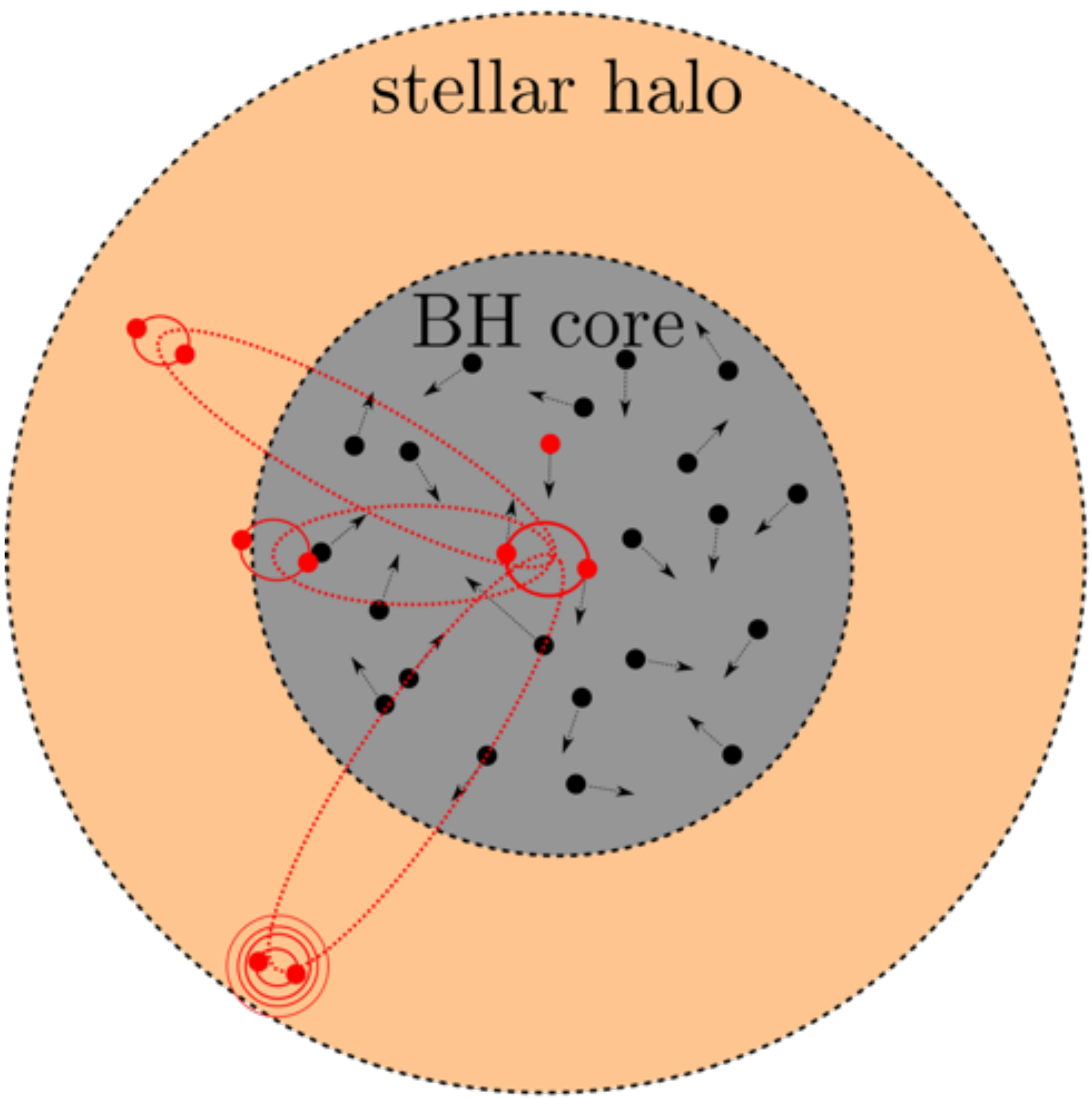
$$f_{r,0}^{\rm peak}(\mathcal{T}) \approx 2{\cdot}10^{-5}\ {\rm Hz} \left(\frac{\mathcal{T}}{10^{10}{\rm yrs}}\right)^{-3/7} \left(\frac{a}{0.5{\rm au}}\right)^{3/14} \left(\frac{m}{30M_{\odot}}\right)^{-11/14}$$

Merger Type: 2-body Merger





Merger Type: 2-body Merger



Time scales:

$$t_{\rm life}(a,e) \approx t_{\rm life}(a)(1-e^2)^{7/2}$$

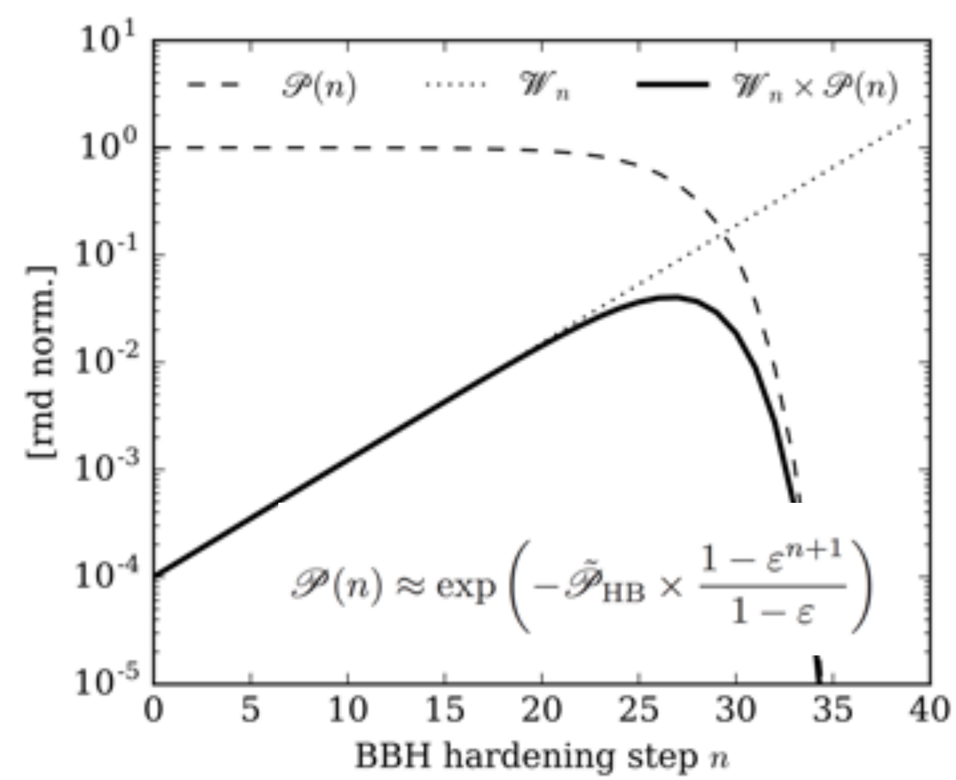
$$t_{\rm bs}(a) \;\approx\; 1/\Gamma_{\rm bs} \;\approx\; (n_{\rm s}\sigma_{\rm bs}v_{\rm disp})^{-1}$$

Eccentricity:

$$P(e) = 2e$$

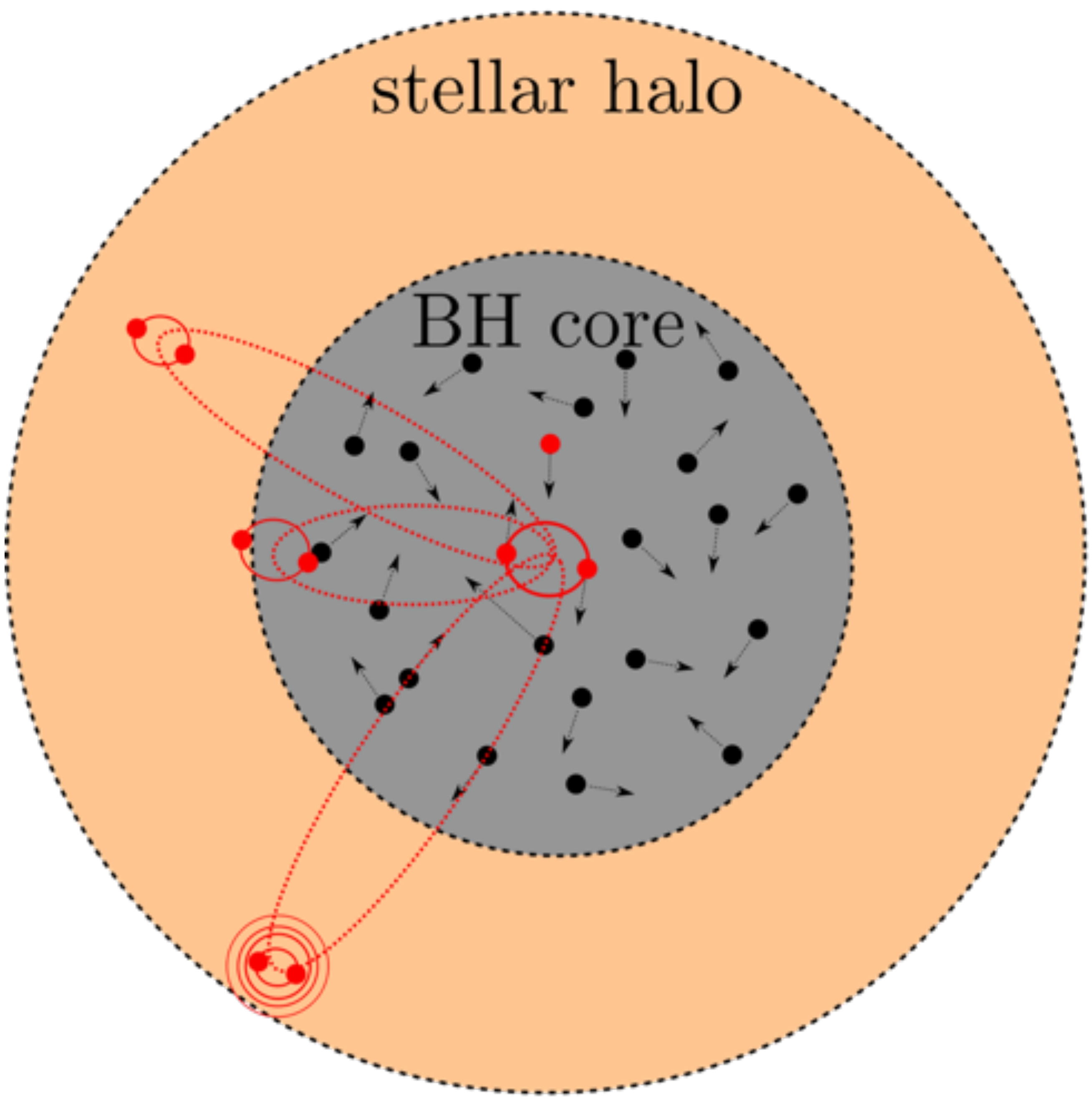
Probability:

$$P_{\rm IM}(a) \approx (t_{\rm bs}(a)/t_{\rm life}(a))^{2/7}$$



$$P_{\rm IM}(a_{\rm in}, a_{\rm ej}) \approx \frac{1}{1-\delta}\int_{a_{\rm ej}}^{a_{\rm in}} \frac{P_{\rm IM}(a)}{a}da \approx \frac{7}{10}\frac{P_{\rm IM}(a_{\rm ej})}{1-\delta}$$

Merger Type: 2-body Merger



Time scales:

$$t_{\rm life}(a,e) \approx t_{\rm life}(a)(1-e^2)^{7/2}$$

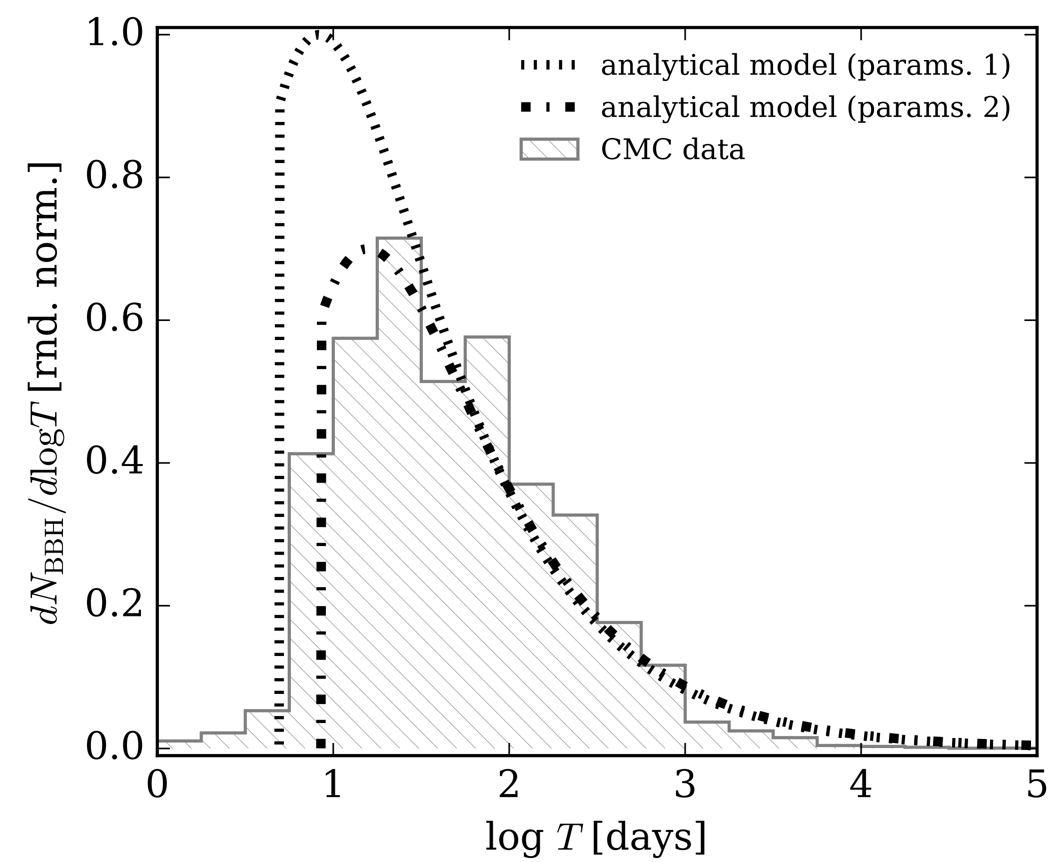
$$t_{\rm bs}(a) \approx 1/\Gamma_{\rm bs} \approx (n_{\rm s}\sigma_{\rm bs}v_{\rm disp})^{-1}$$

Eccentricity:

$$P(e) = 2e$$

Probability:

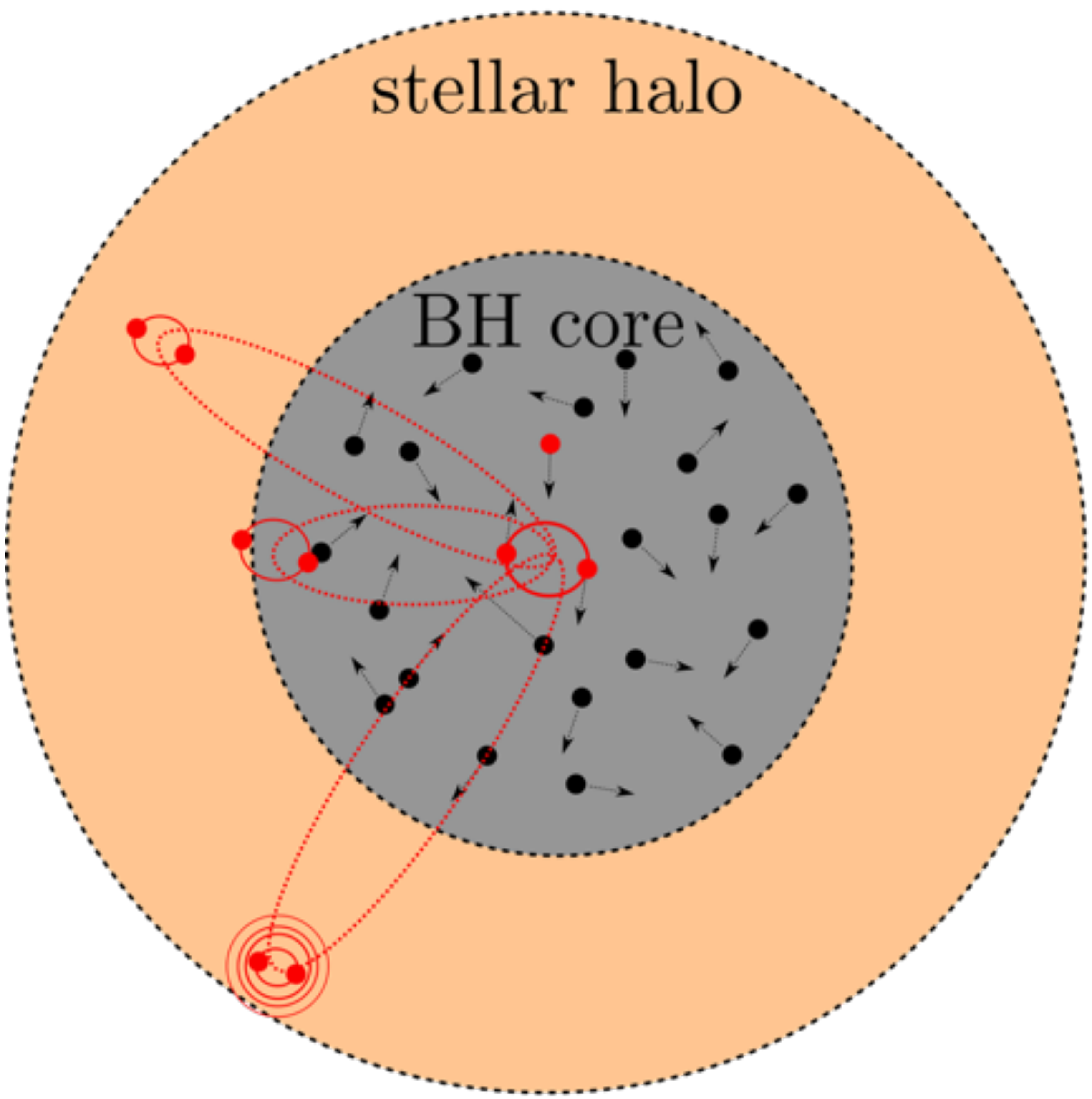
$$P_{\rm IM}(a) \approx (t_{\rm bs}(a)/t_{\rm life}(a))^{2/7}$$



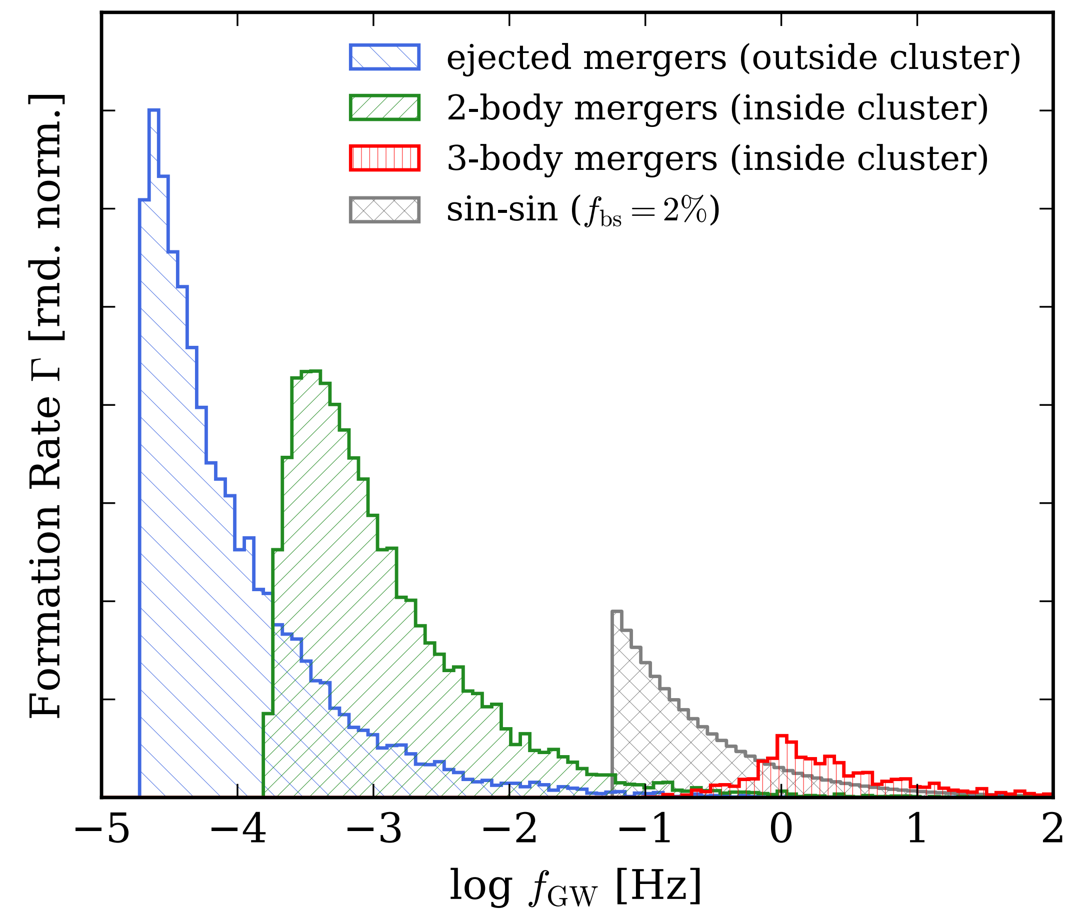
$$\frac{d\Gamma_{\rm TD}}{d\log T} \propto \frac{dN_{\rm BBH}}{dn} \propto \frac{dN_{\rm BBH}}{d\log T}$$

$$\propto T^{-2/3}\exp\left(-\tilde{\mathscr{P}}_{\rm HB}\times\frac{1-\varepsilon(T/T_{\rm HB})^{-20/21}}{1-\varepsilon}\right)$$

Merger Type: 2-body Merger

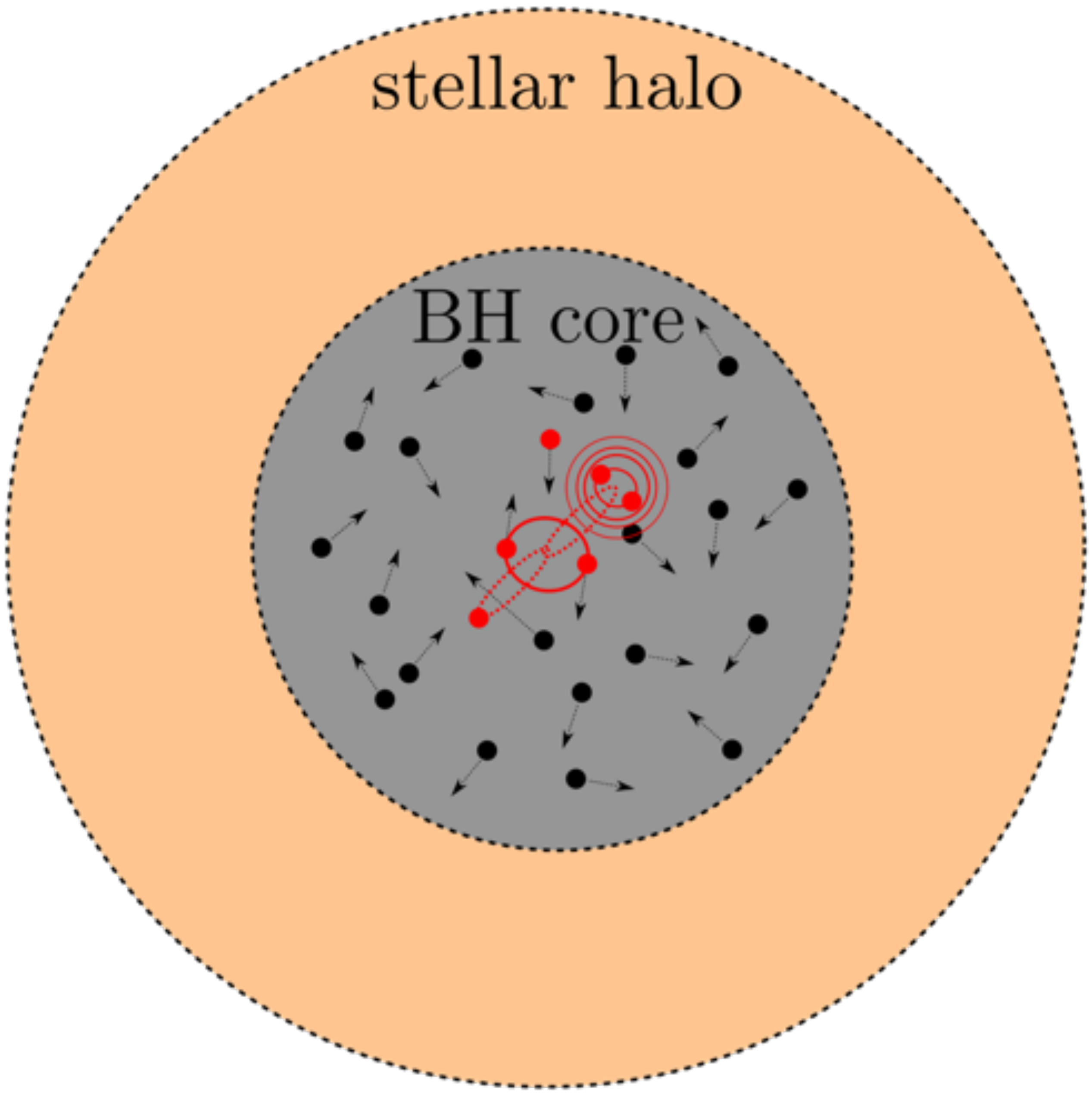


Eccentric LISA sources



$$f_{r,0}^{\rm peak}(\mathcal{T}) \approx 2\cdot 10^{-5}\,{\rm Hz}\left(\frac{\mathcal{T}}{10^{10}{\rm yrs}}\right)^{-3/7}\left(\frac{a}{0.5{\rm au}}\right)^{3/14}\left(\frac{m}{30M_\odot}\right)^{-11/14}$$

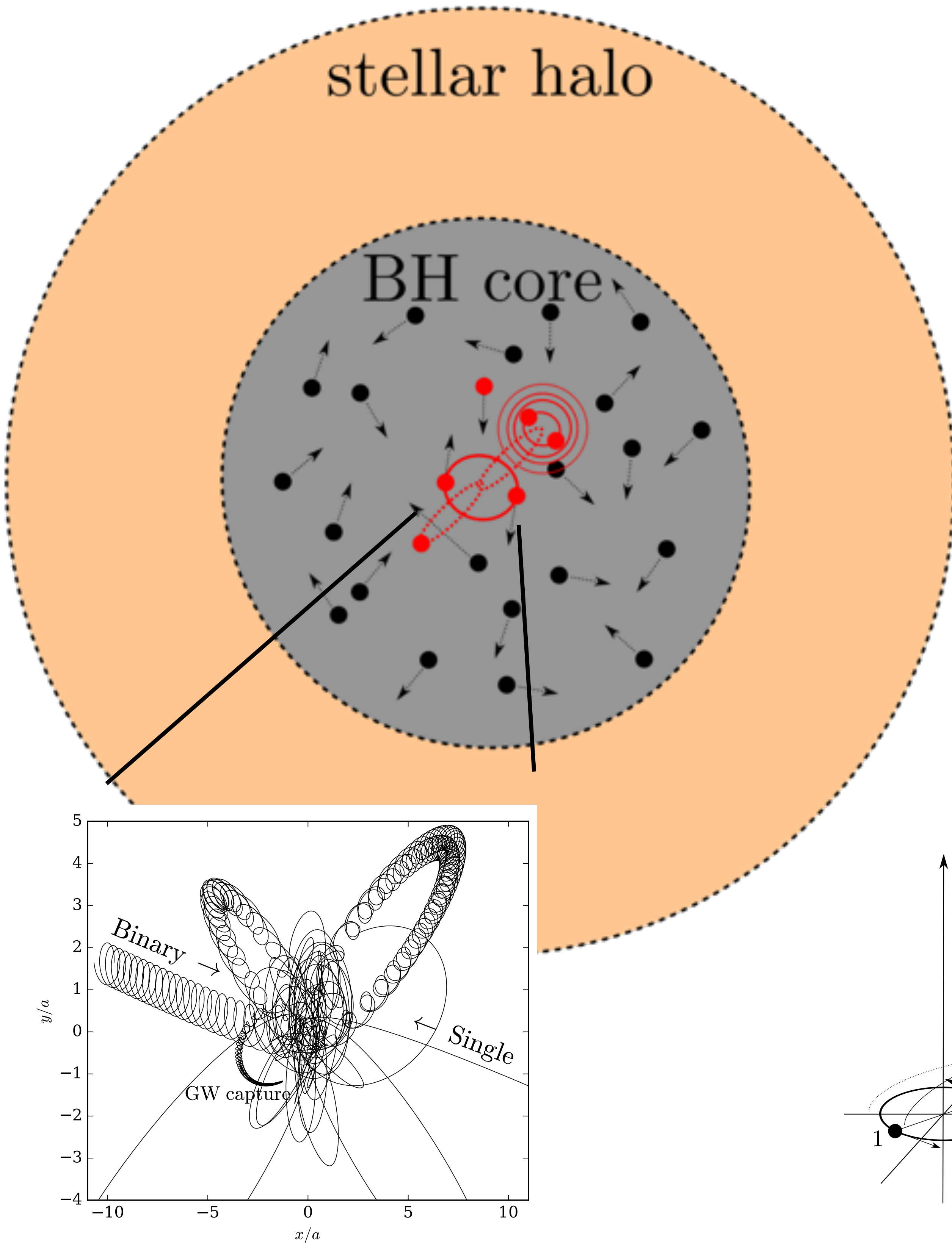
Merger Type: 3-body Merger

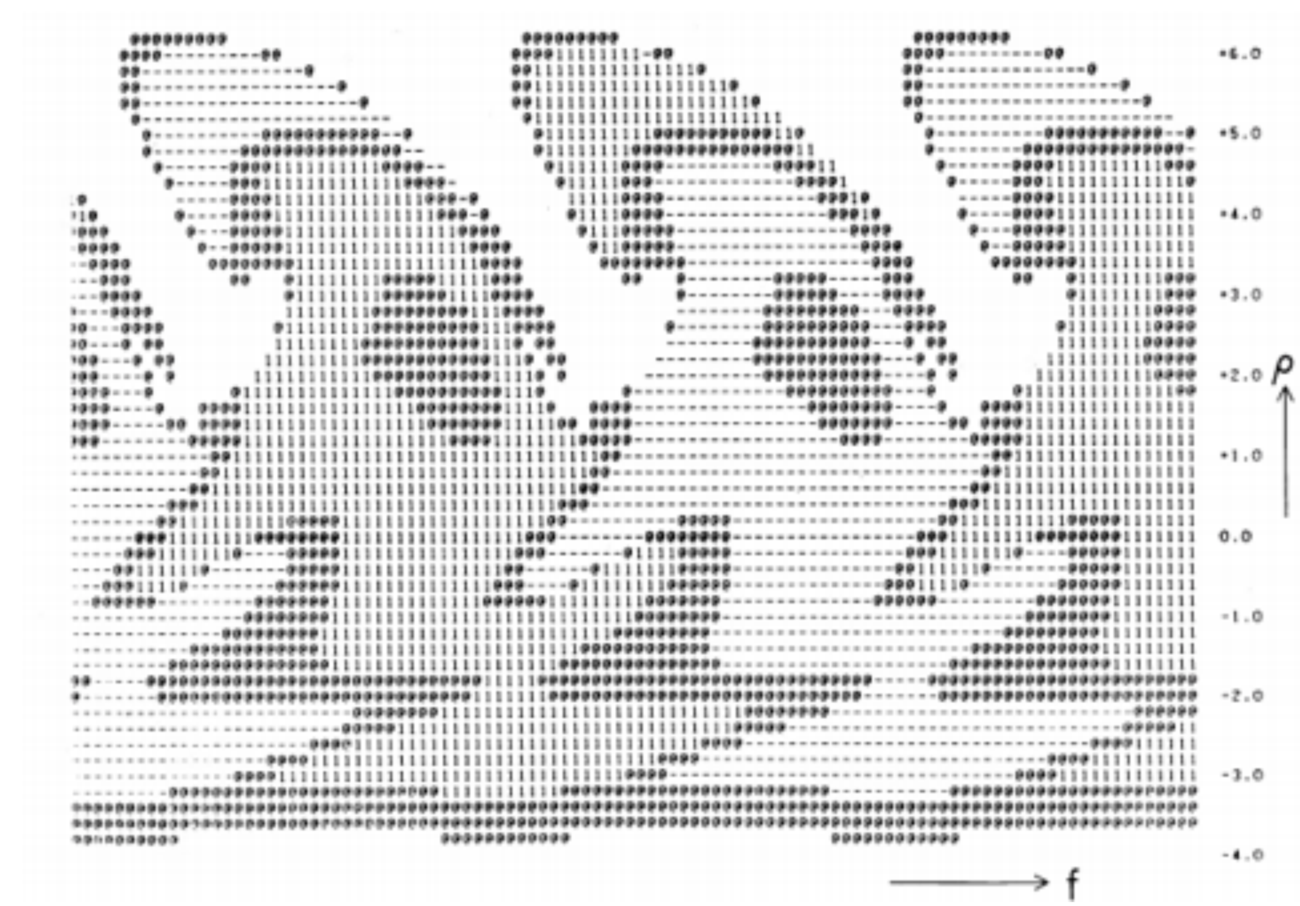


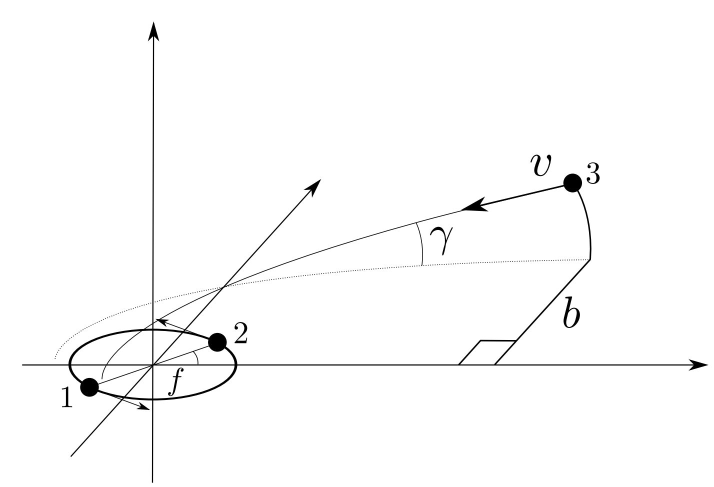
Post-Newtonian

Merger Type: 3-body Merger

Topology

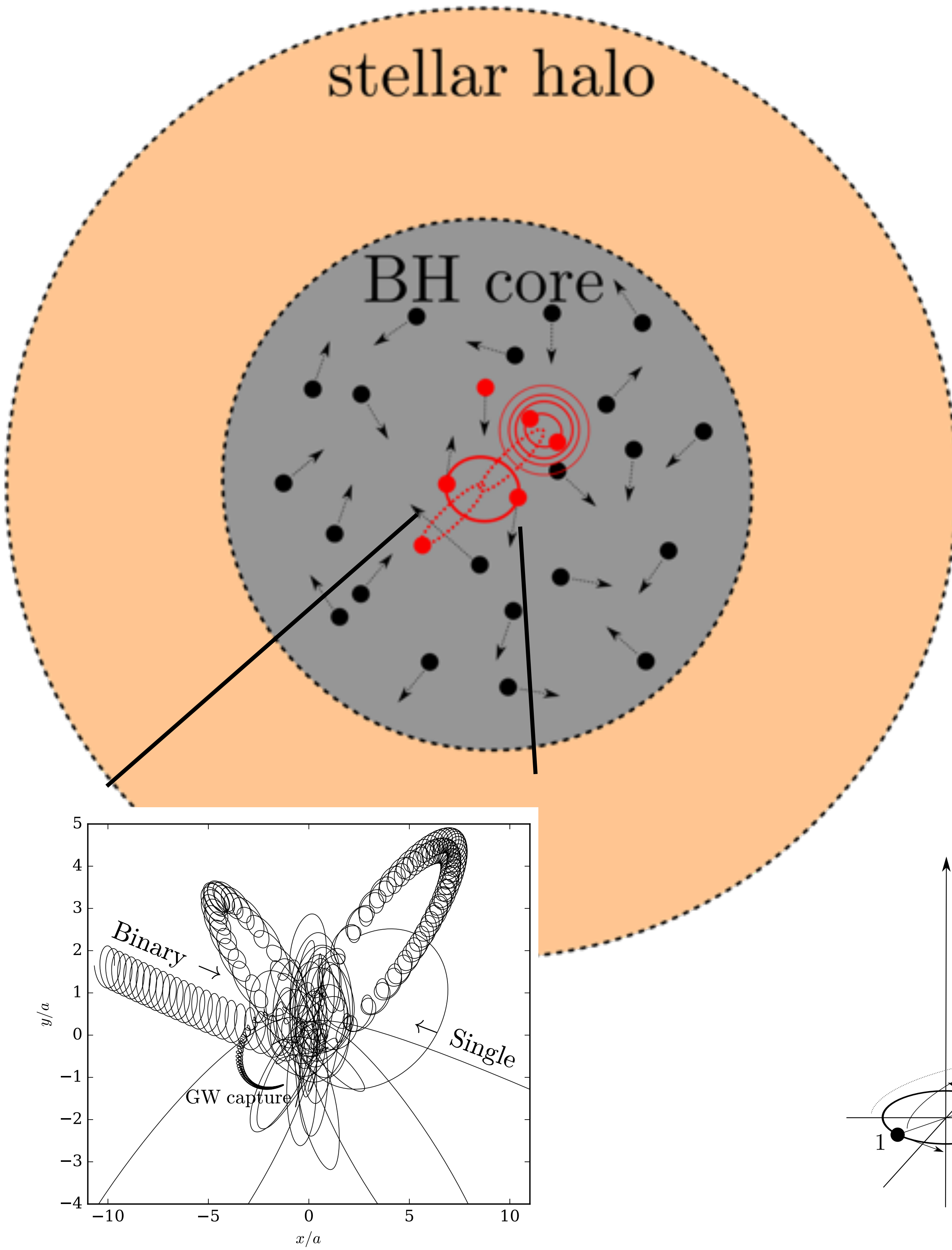


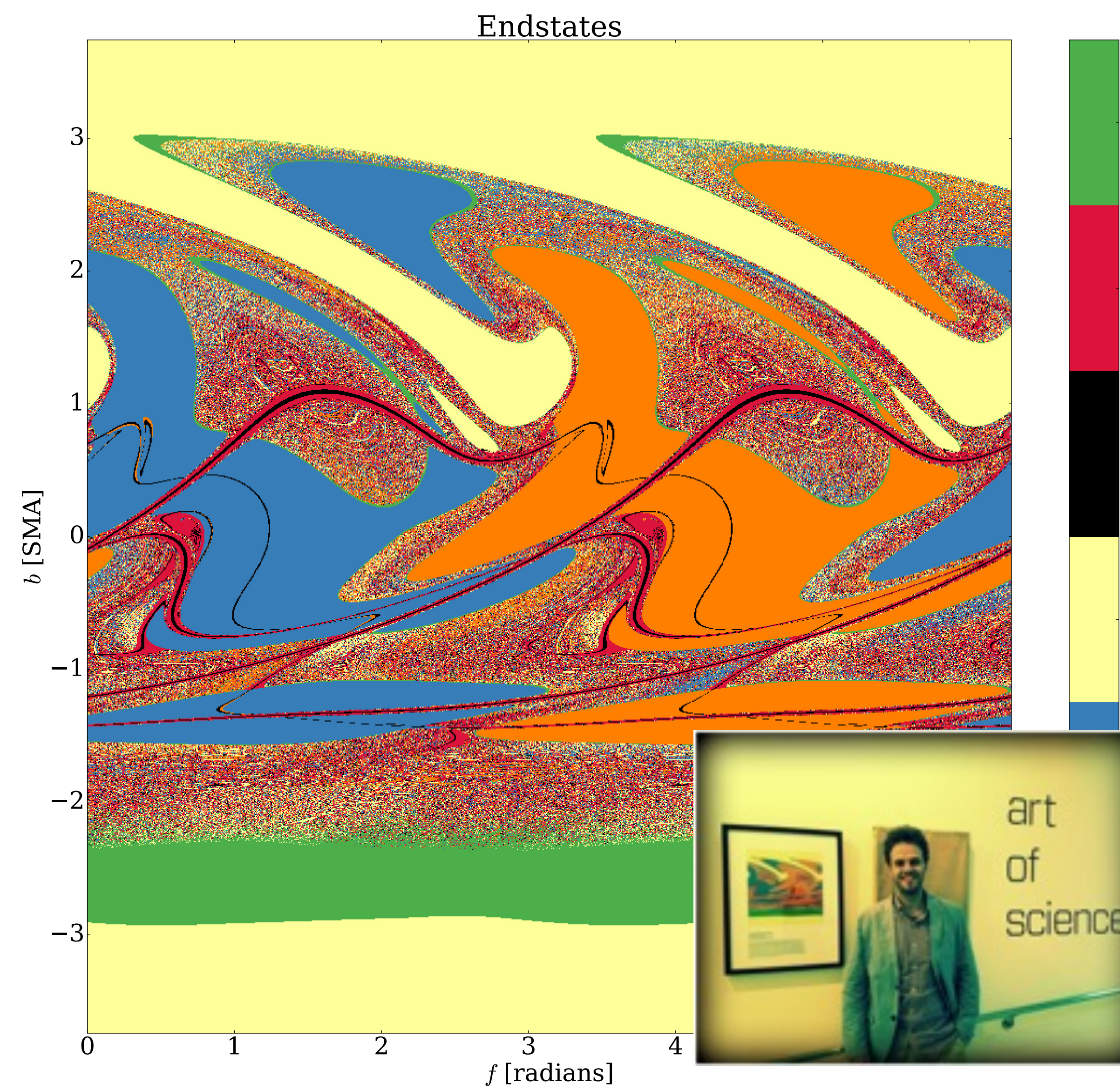


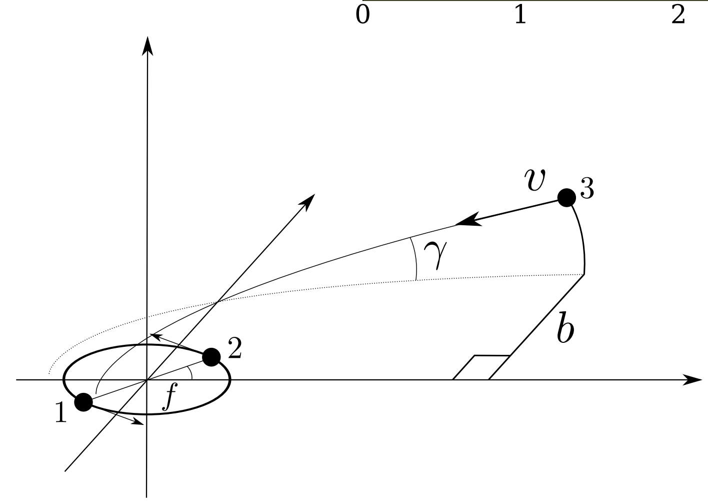


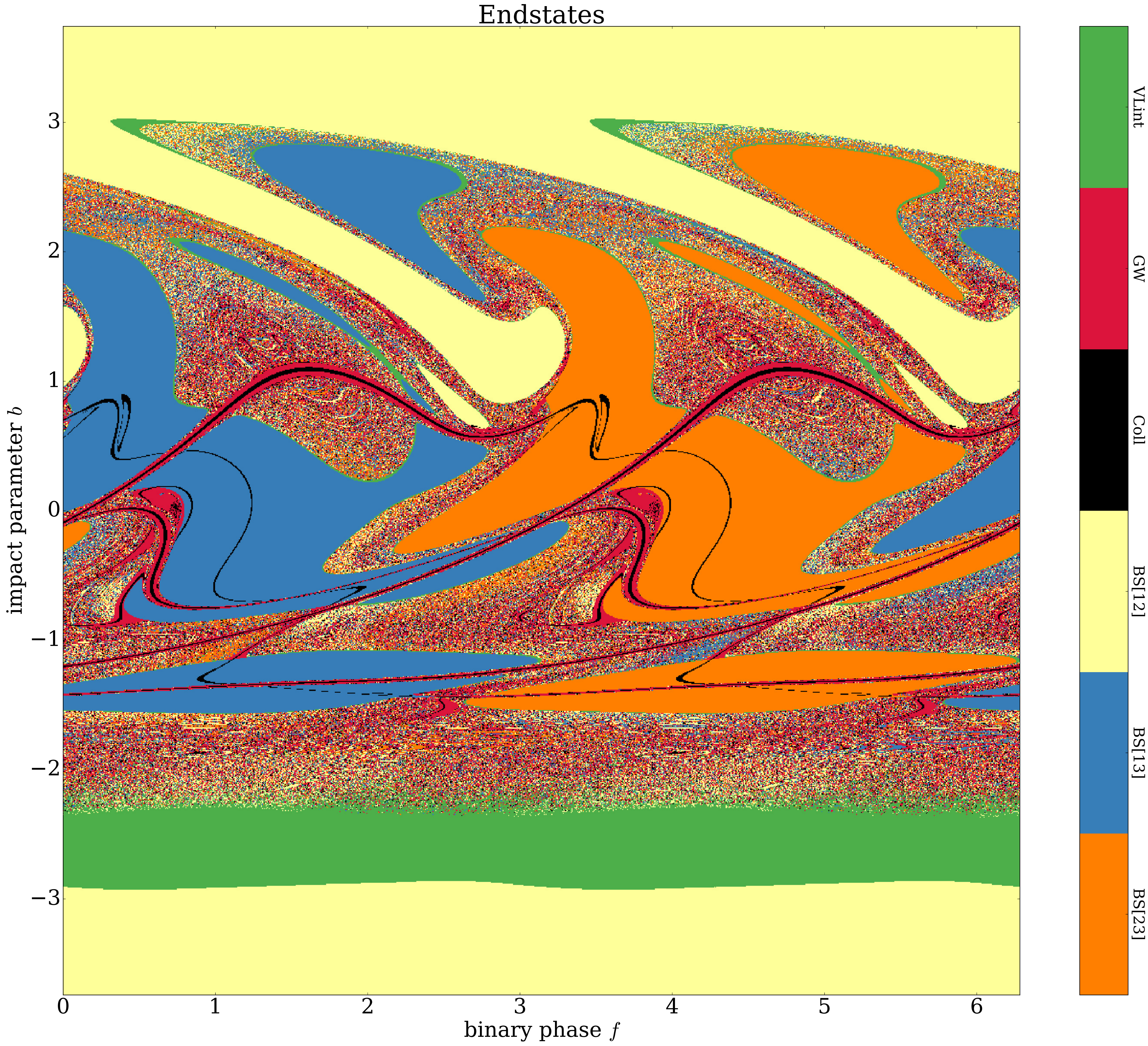
Merger Type: 3-body Merger

Topology



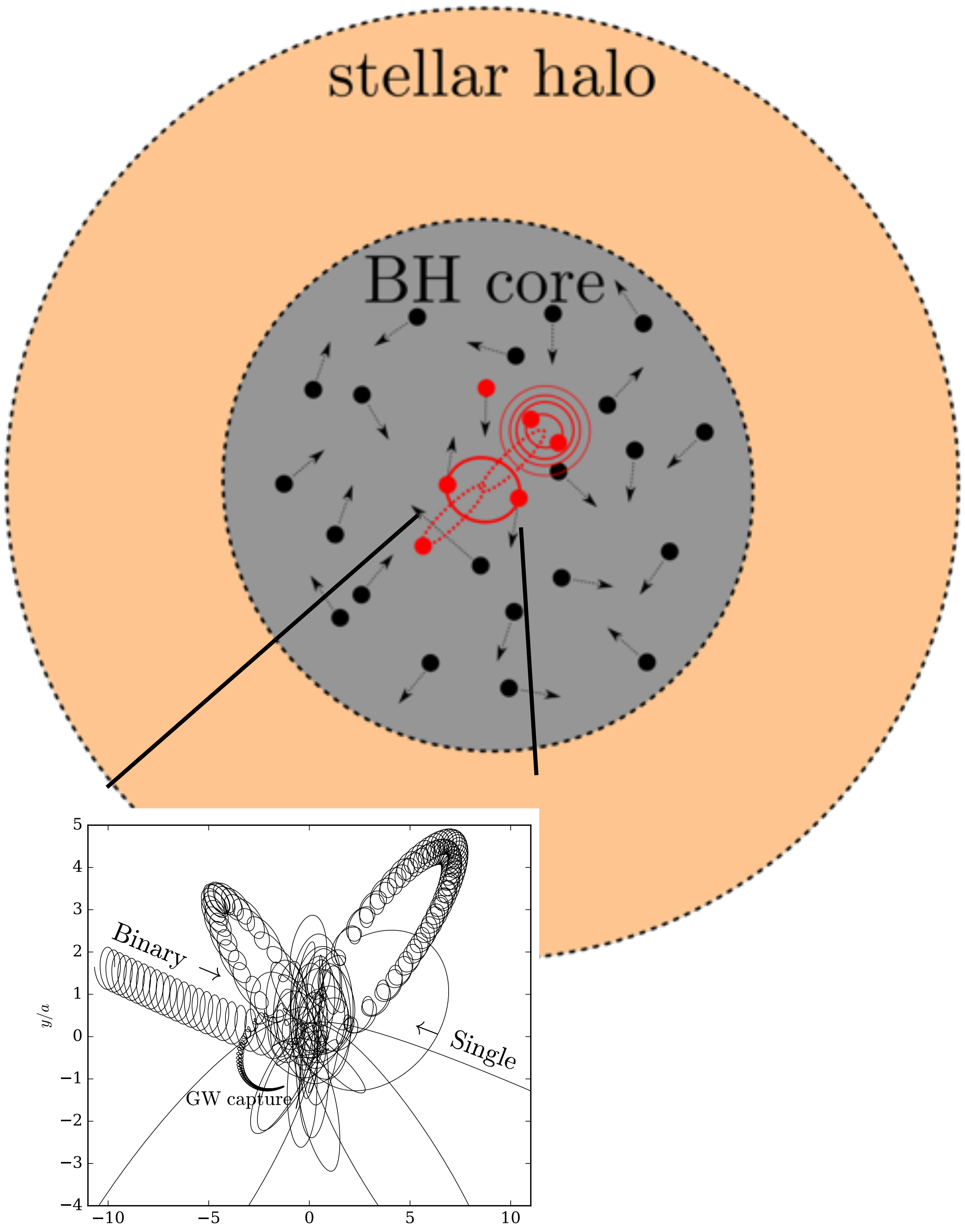




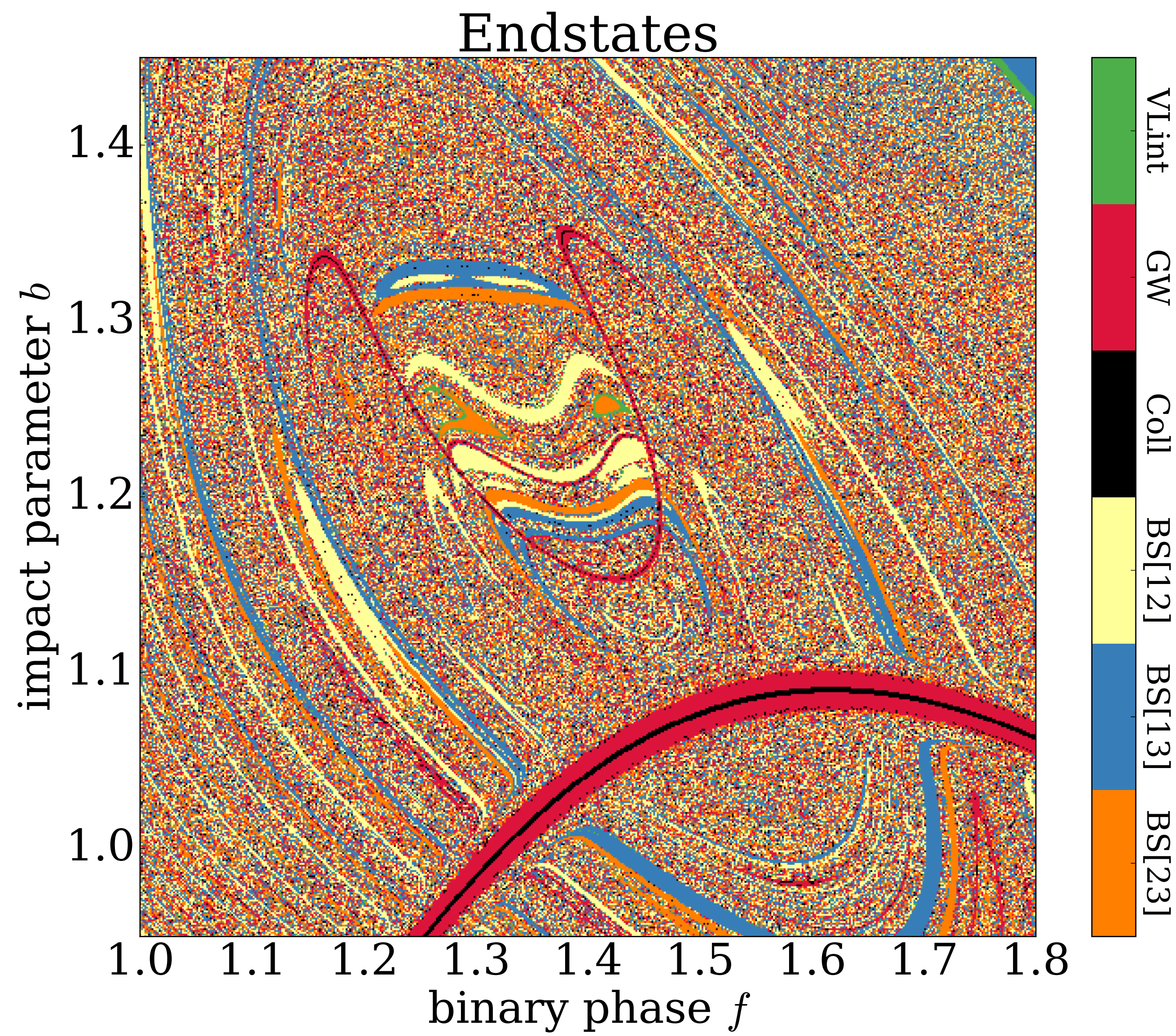


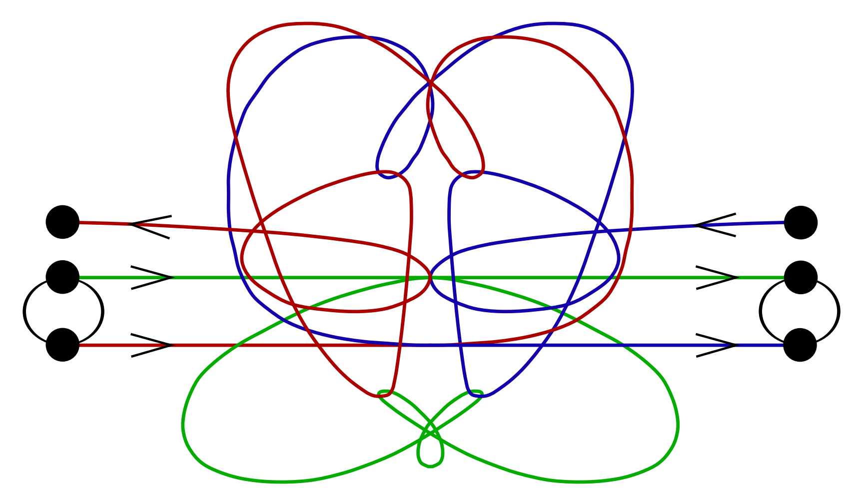
Endstates
impact parameter b
binary phase f
3
2
1
0
−1
−2
−3
0
1
2
3
4
5
6
VLint
GW
Coll
BS[12]
BS[13]
BS[23]

Merger Type: 3-body Merger

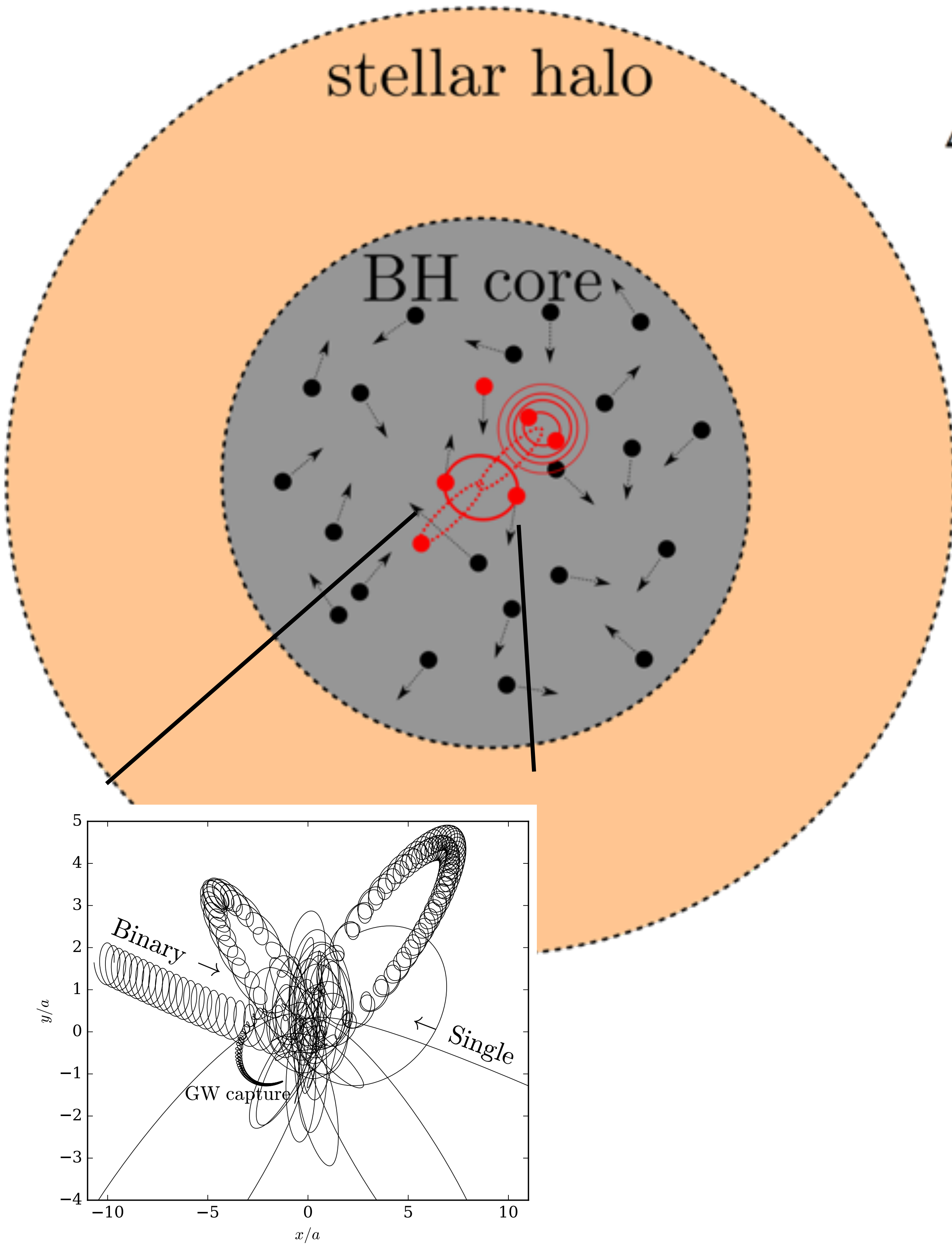


Topology





Merger Type: 3-body Merger

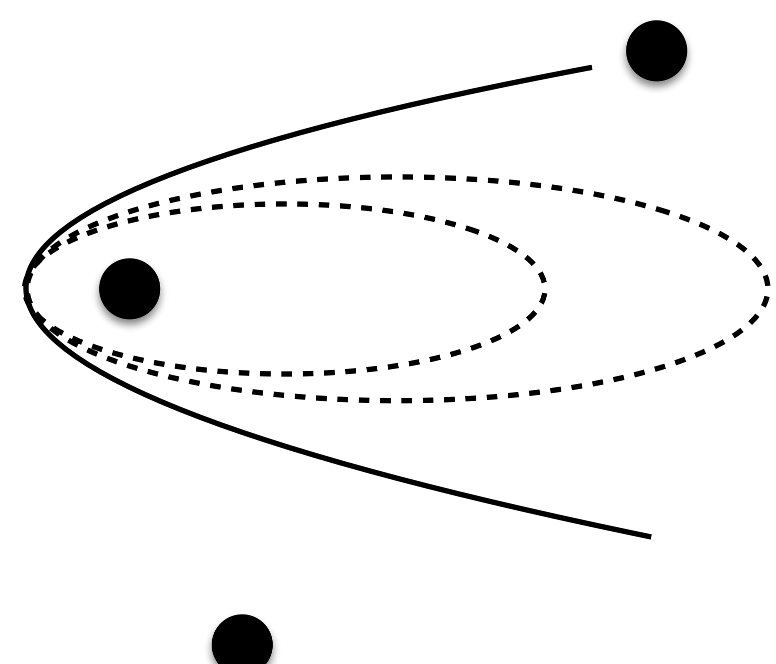


3-body GW capture:

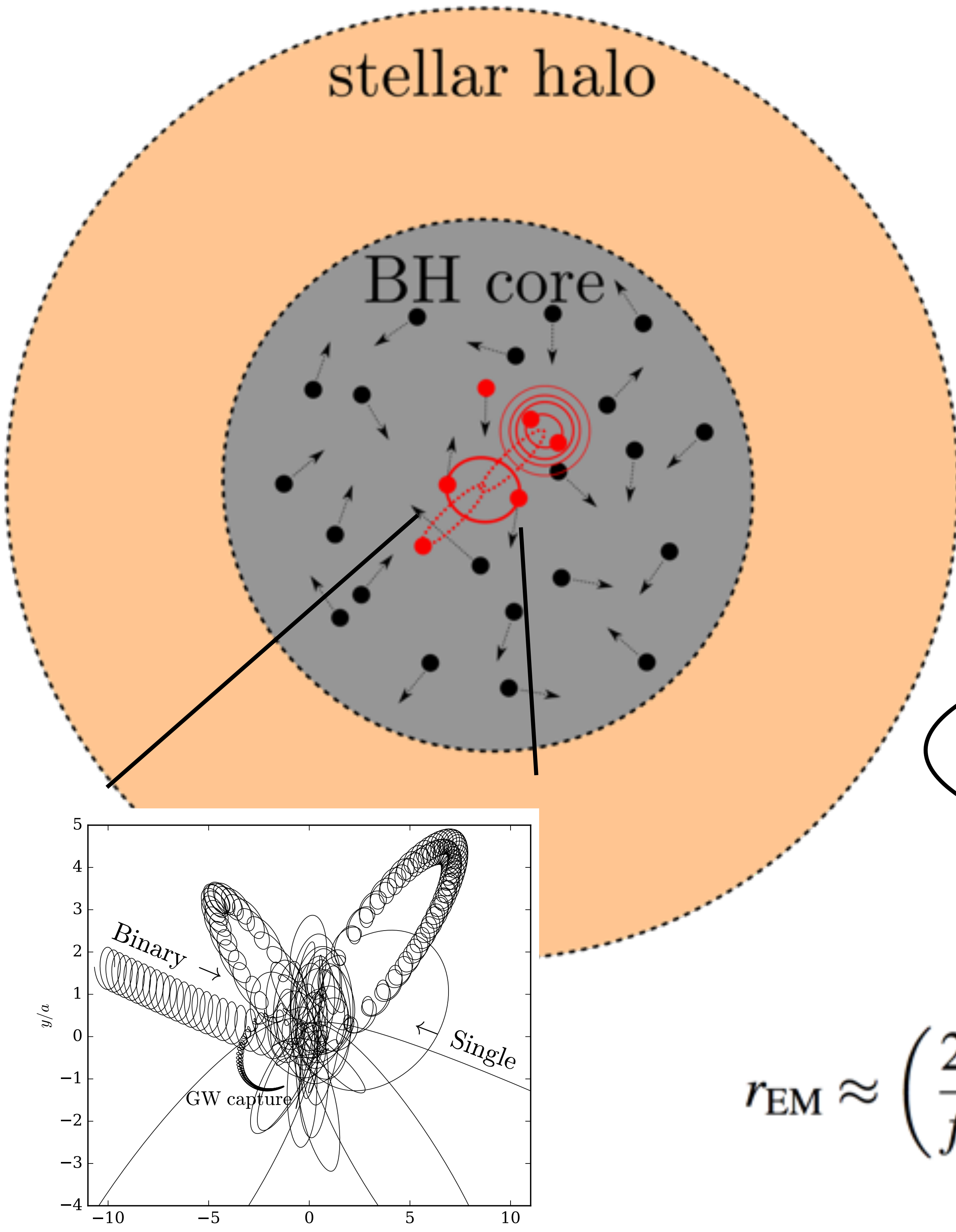
$$E_{\mathrm{B}} \approx Gm^2/(2a)$$

$$\Delta E_{\mathrm{p}} \approx (85\pi/12) G^{7/2} c^{-5} m^{9/2} r_{\mathrm{p}}^{-7/2}$$

$$\mathcal{R}_{\mathrm{bs}} \approx \left(\frac{85\pi}{24\sqrt{2}}\right)^{2/7} \times \mathscr{R}_{\mathrm{m}} \left(\frac{a}{\mathscr{R}_{\mathrm{m}}}\right)^{2/7}$$



Merger Type: 3-body Merger

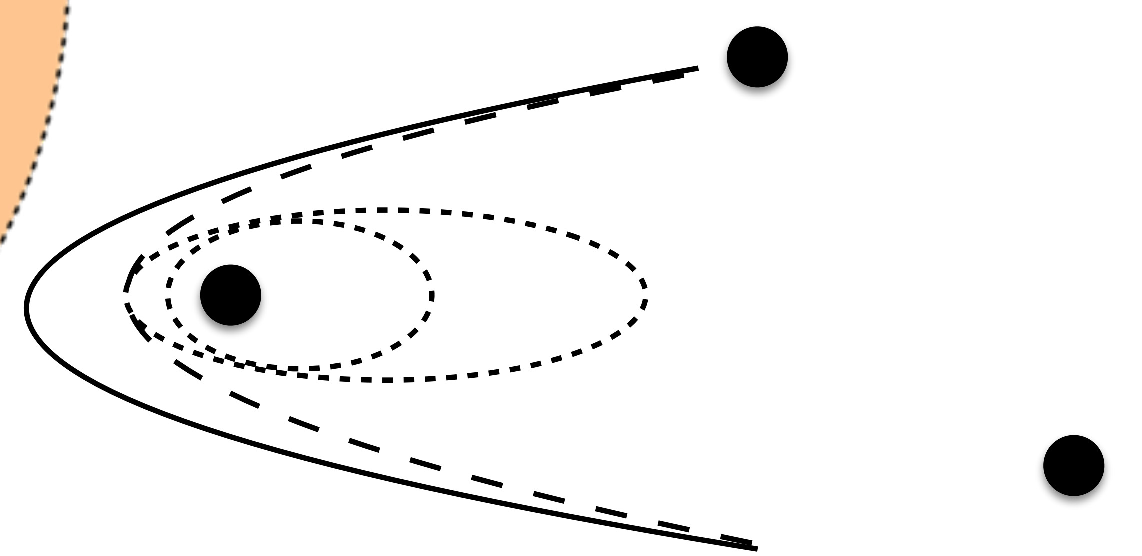


3-body Ecc. encounter:

$$r_f \approx \left(\frac{2Gm}{f^2\pi^2}\right)^{1/3}$$

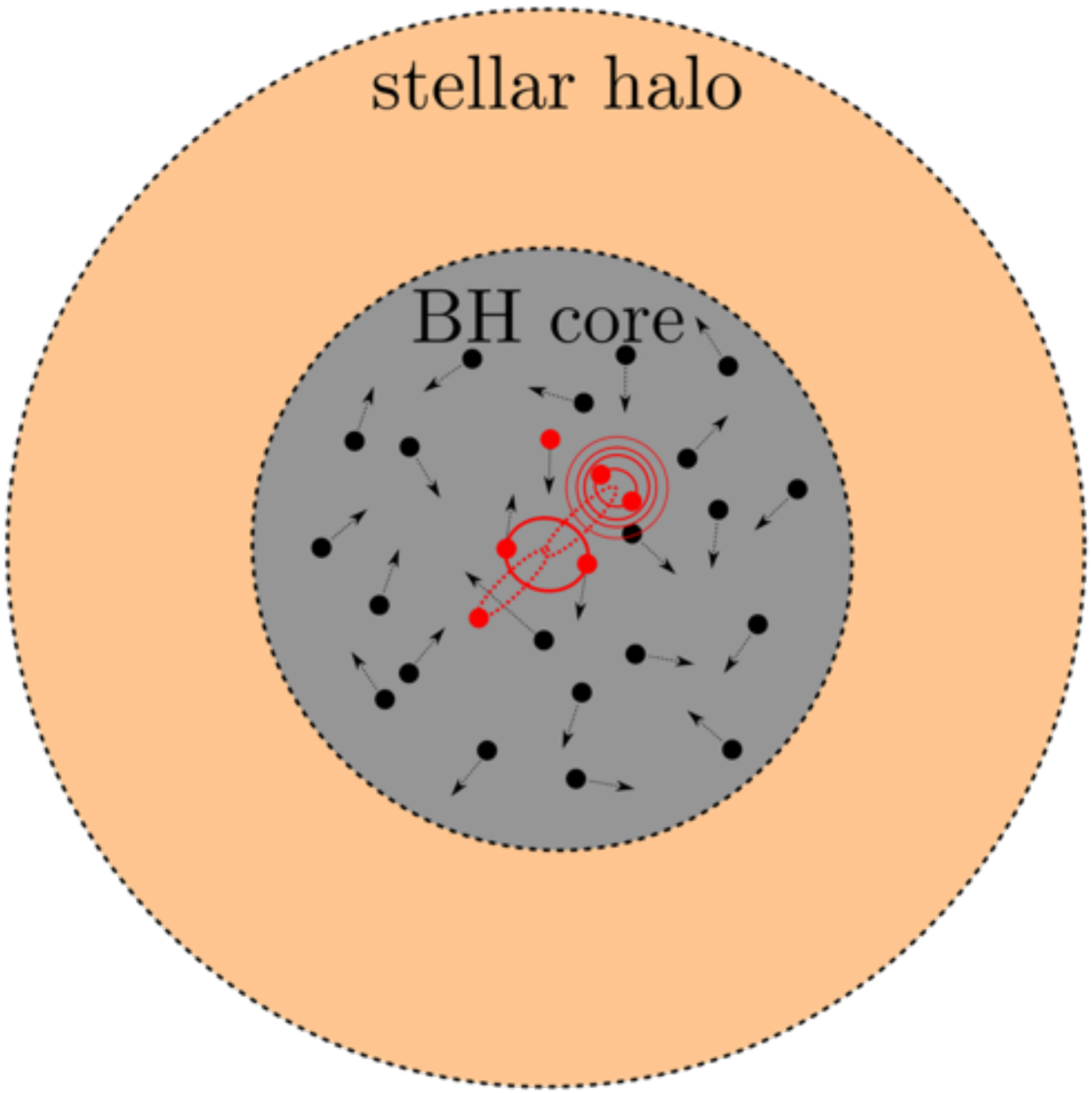
$$r_{\rm EM} \approx r_f \times \frac{1}{2F(e_f)} \left(\frac{425}{304}\right)^{870/2299}$$

$$F(e) = \frac{e^{12/19}}{1+e}\left(1+\frac{121}{304}e^2\right)^{870/2299}$$

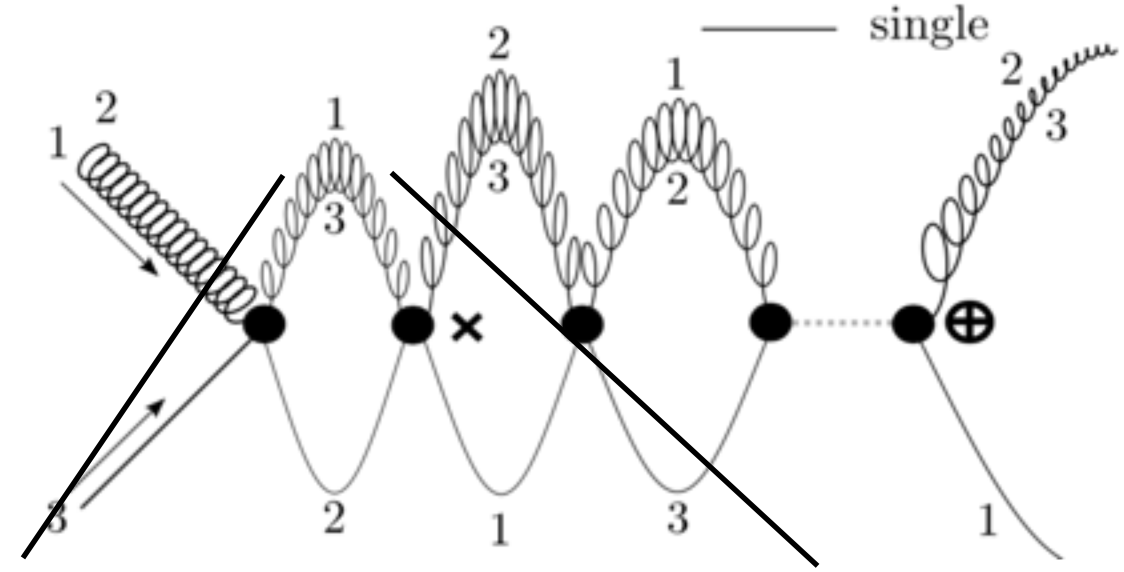


$$r_{\rm EM} \approx \left(\frac{2Gm}{f^2\pi^2}\right)^{1/3} \frac{1}{2}\,\frac{1+e_f}{e_f^{12/19}}\left[\frac{425}{304}\left(1+\frac{121}{304}e_f^2\right)^{-1}\right]^{870/2299}$$

Merger Type: 3-body Merger



3-body Ecc. Probability:



$$r_{\rm p} = a(1-e) \quad P(e) = 2e$$

$$P_{\rm EM}(a) \approx \frac{2r_{\rm EM}}{a} \times N_{\rm IMS}$$

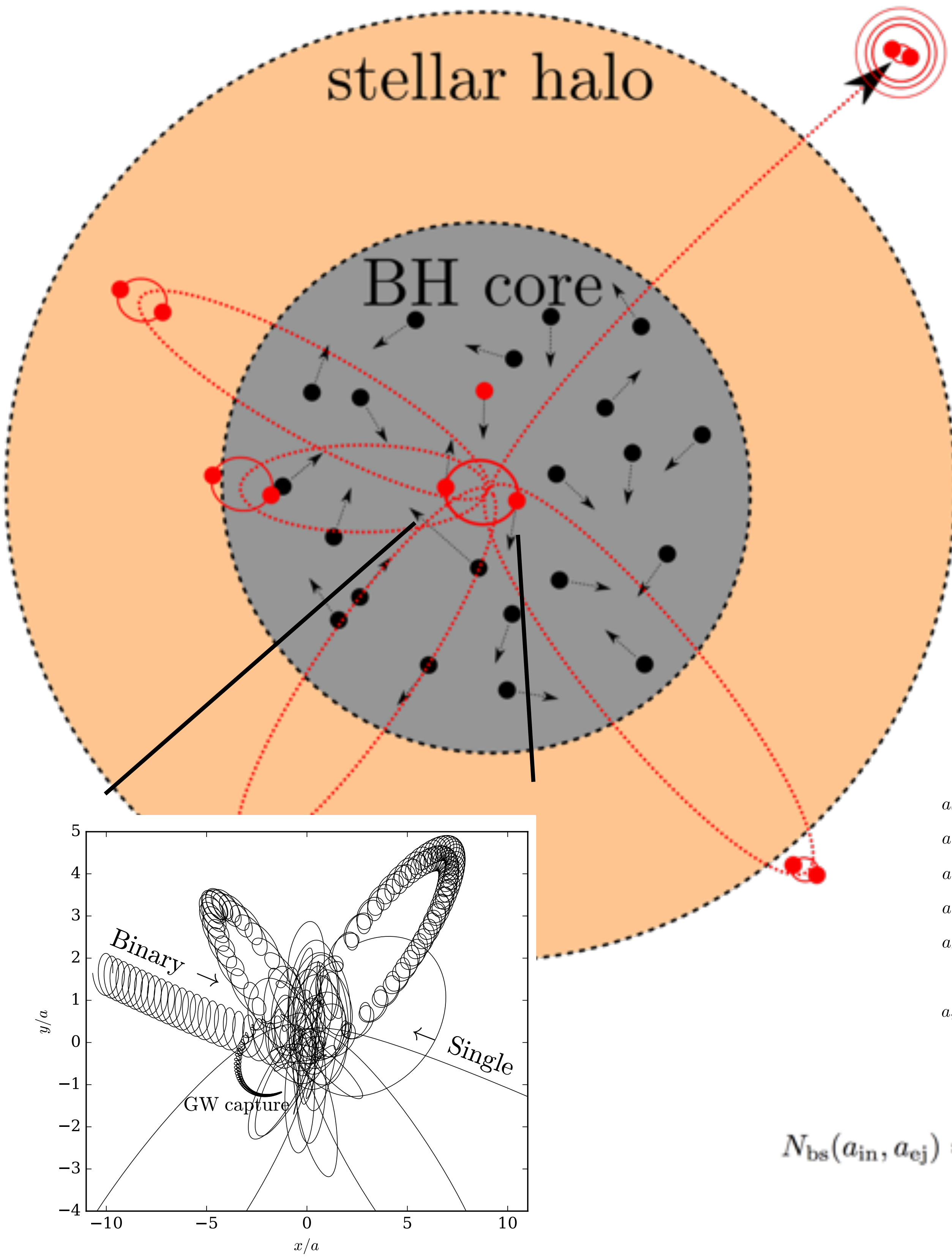
$$r_{\rm EM} \approx \left(\frac{2Gm}{f^2\pi^2}\right)^{1/3} \frac{1}{2}\frac{1+e_f}{e_f^{12/19}}\left[\frac{425}{304}\left(1+\frac{121}{304}e_f^2\right)^{-1}\right]^{870/2299}$$

$$P_{\rm EM}(a_{\rm in}, a_{\rm ej}) = \frac{1}{1-\delta}\int_{a_{\rm ej}}^{a_{\rm in}} \frac{P_{\rm EM}(a)}{a} da \approx \frac{P_{\rm EM}(a_{\rm ej})}{1-\delta}$$

Merger Type: 3-body Merger

Eccentric Black Hole Mergers Forming in Globular Clusters
Authors: Johan Samsing

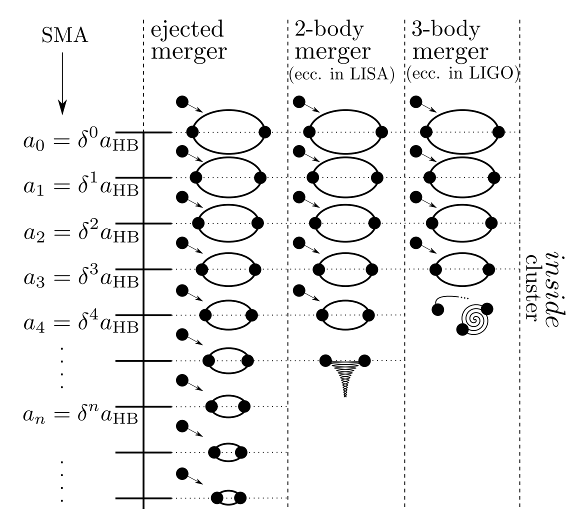
Newtonian codes underestimate the rate by more than a factor of 100!



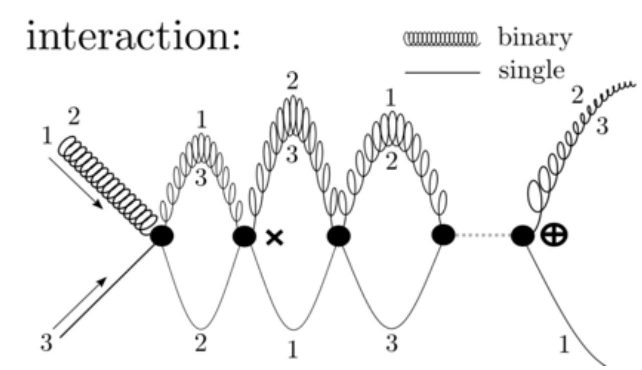
number of tries

- GR: **1x1 = 1**

+GR: **20x20 = 400**



X

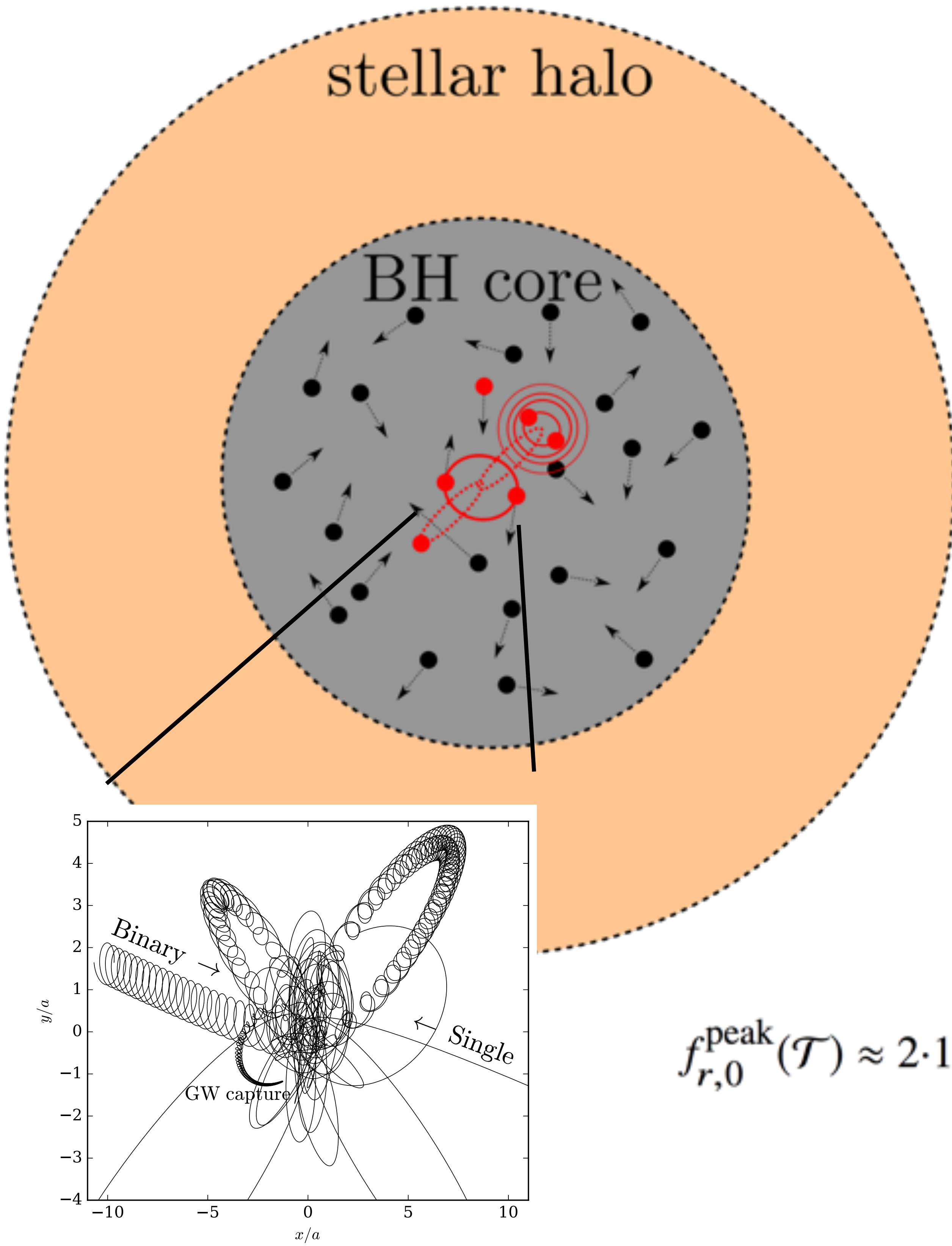


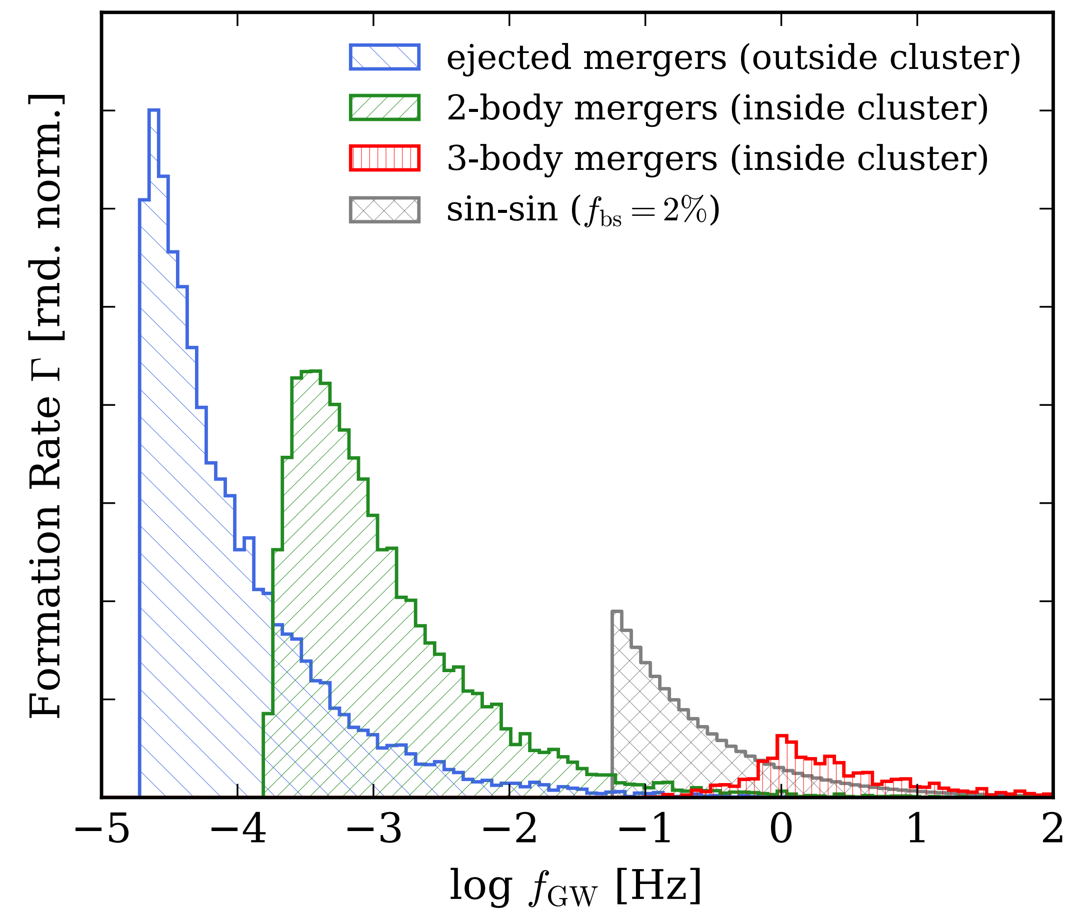
$$N_{\rm bs}(a_{\rm in}, a_{\rm ej}) = \int_{a_{\rm ej}}^{a_{\rm in}} \frac{1}{1-\delta}\frac{1}{a}da = \frac{1}{1-\delta}\ln\left(\frac{a_{\rm in}}{a_{\rm ej}}\right)$$

$$N_{\rm IMS} \approx (\max(a_{\rm IMS})/a)^{7/2}$$

Merger Type: 3-body Merger

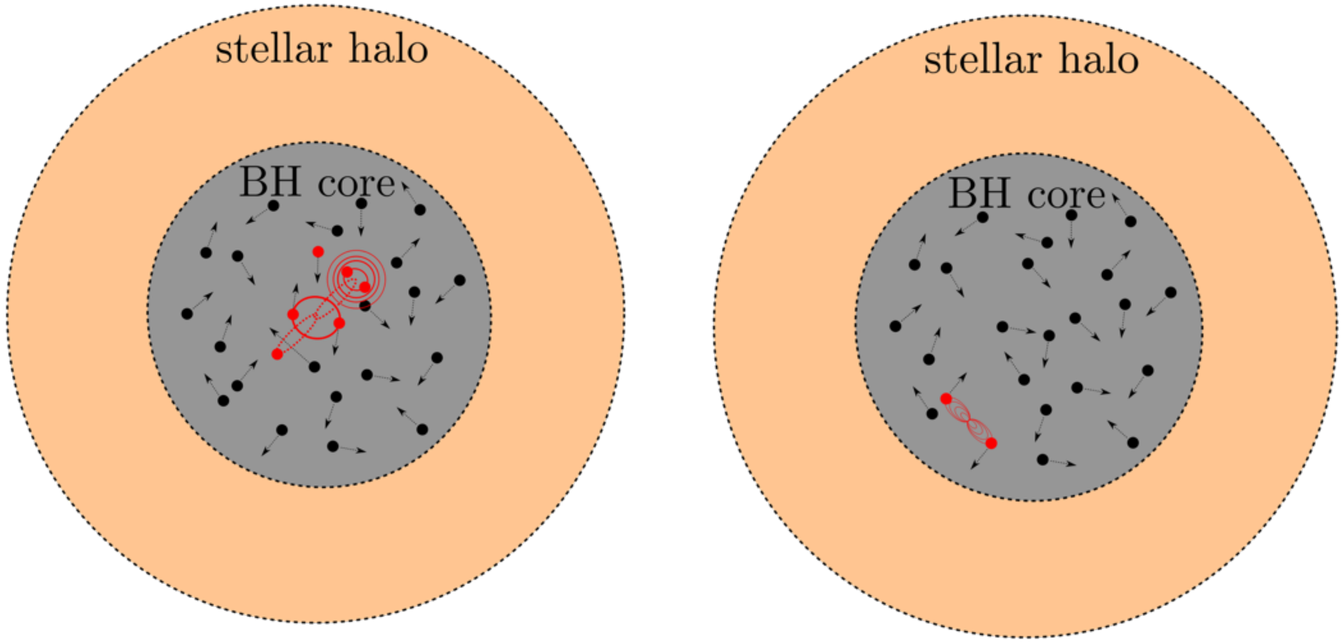
Eccentric LIGO sources



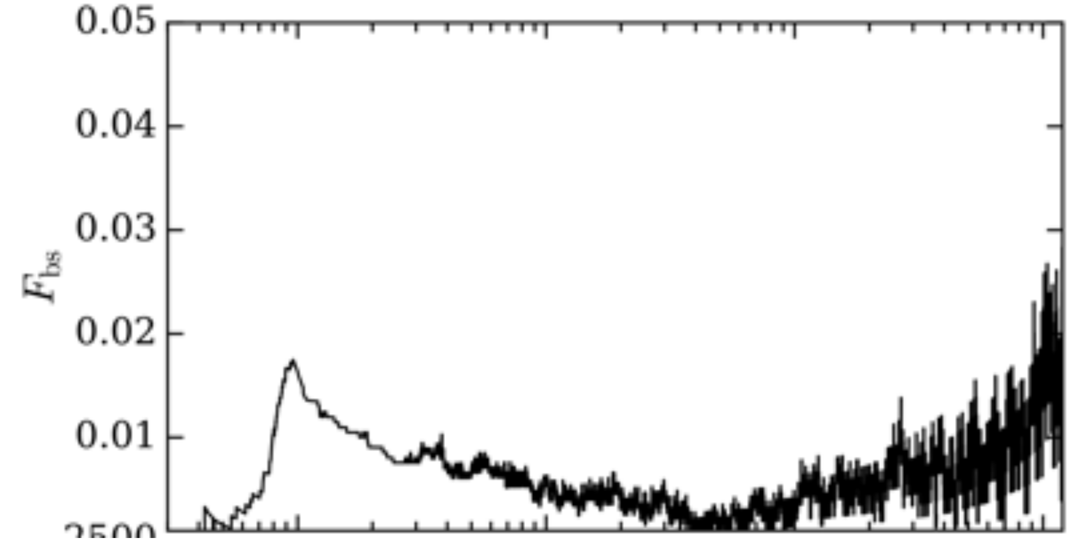


$$f_{r,0}^{\mathrm{peak}}(\mathcal{T}) \approx 2\cdot 10^{-5}\ \mathrm{Hz} \left(\frac{\mathcal{T}}{10^{10}\mathrm{yrs}}\right)^{-3/7} \left(\frac{a}{0.5\mathrm{au}}\right)^{3/14} \left(\frac{m}{30 M_\odot}\right)^{-11/14}$$

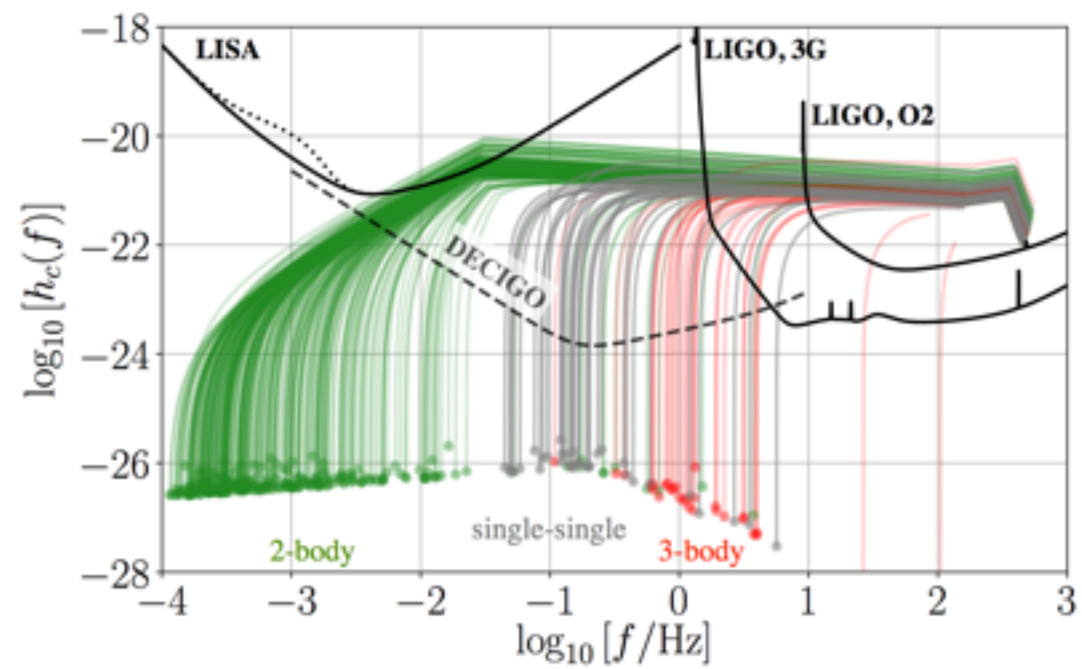
Comparing binary-single and single-single:



binary fraction



f distribution

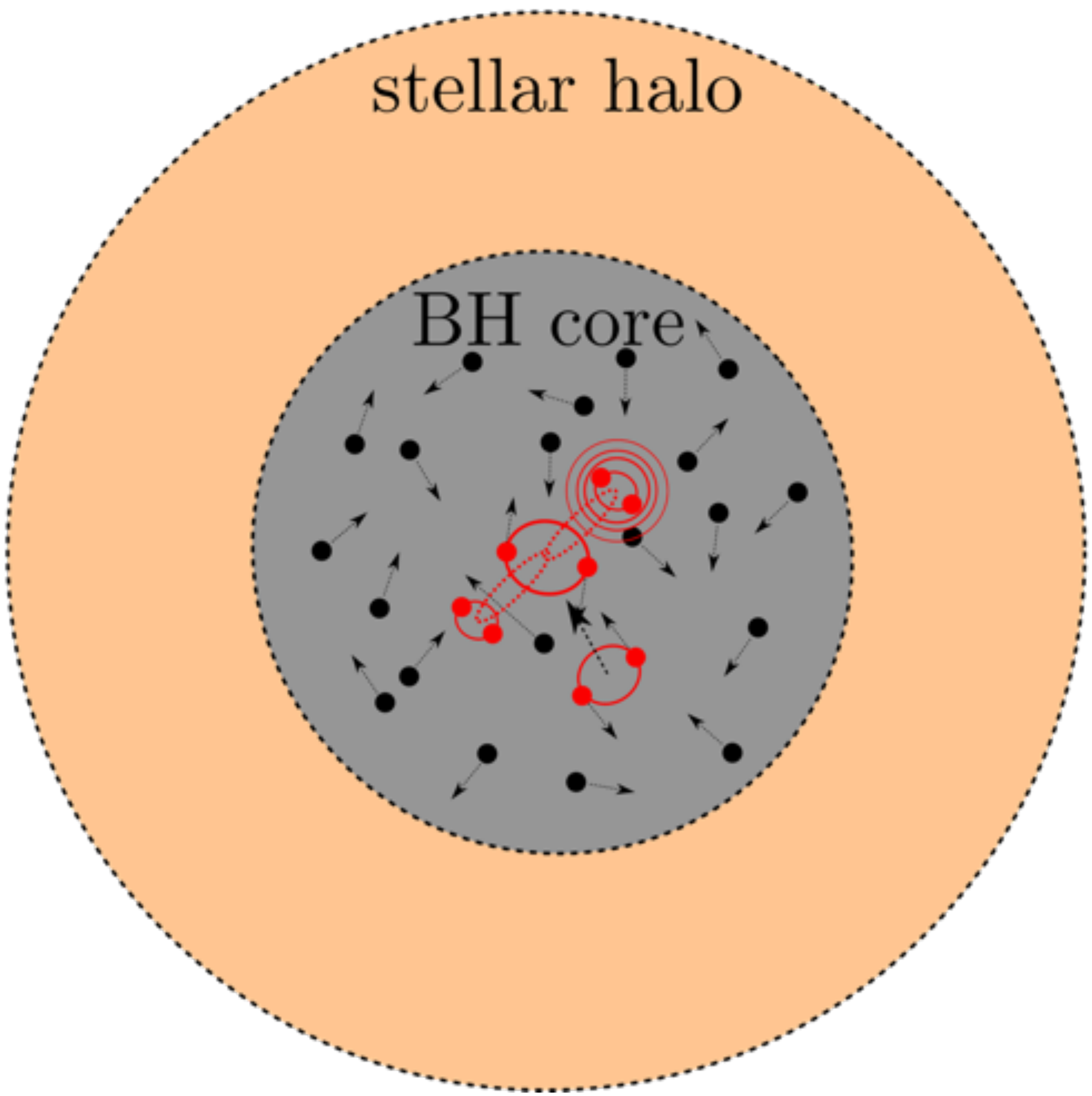


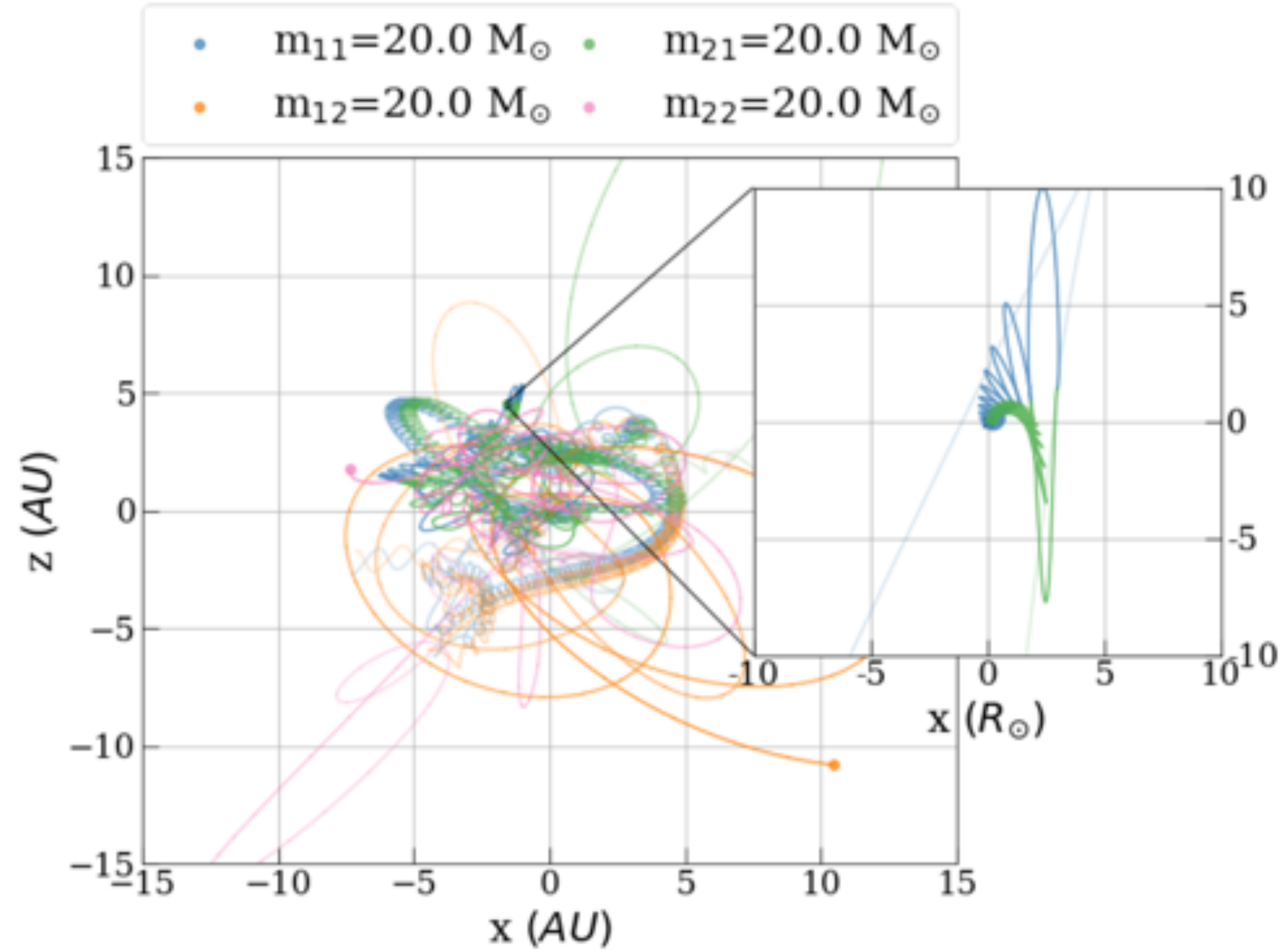
uniform sphere:

$$\frac{\Gamma_{\rm bs}}{\Gamma_{\rm ss}} \approx 6F_{\rm bs}\mathcal{N} \times \left(\frac{\phi - 1}{12 f_{\rm ed}^2}\right)^{2/7} \frac{7}{5}$$

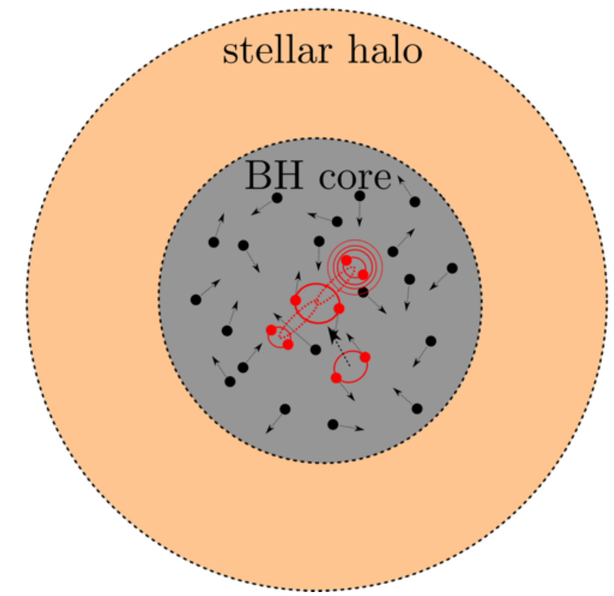
Can we here probe the BH core properties?

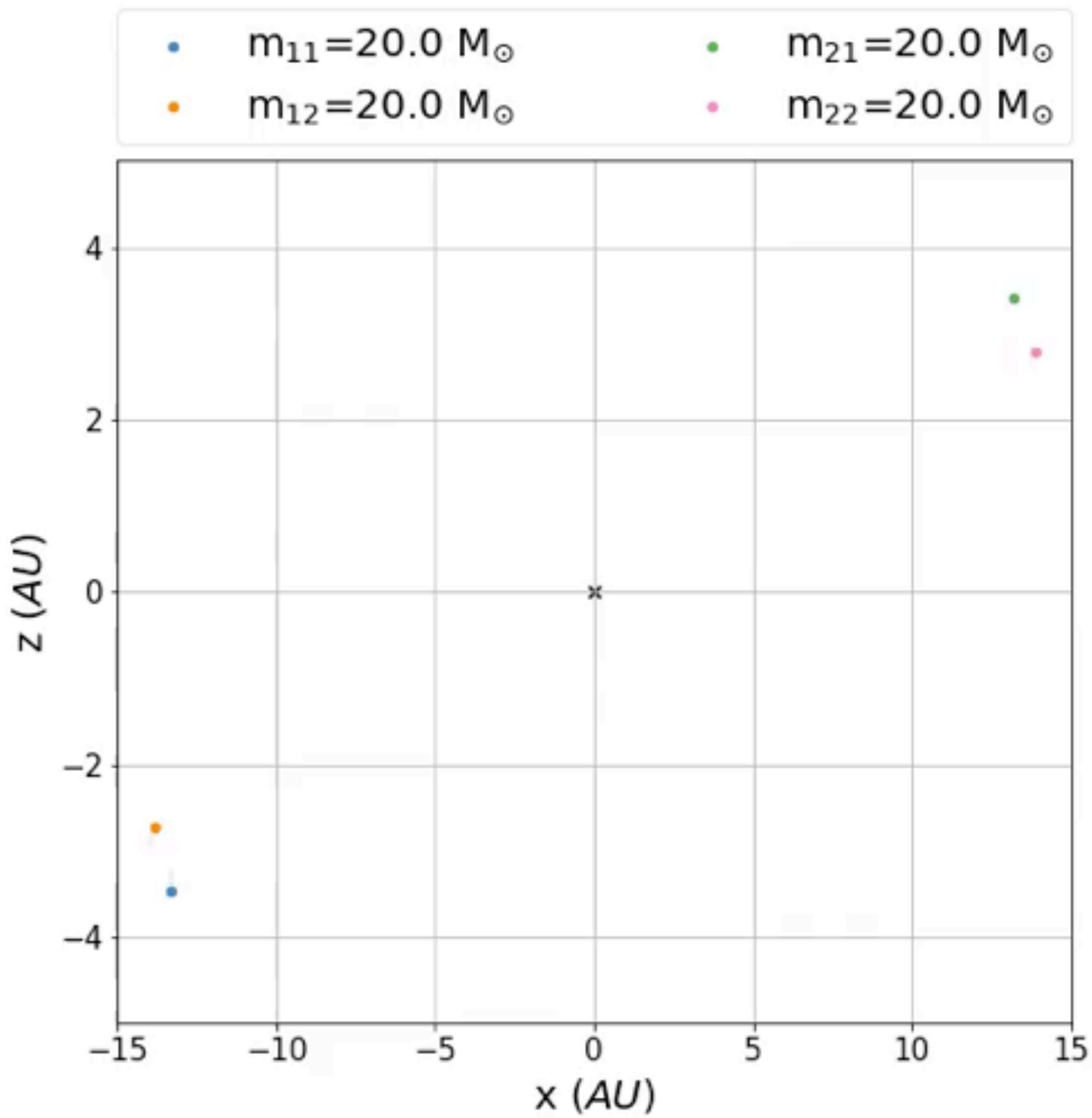
Merger Type: 4-body Merger

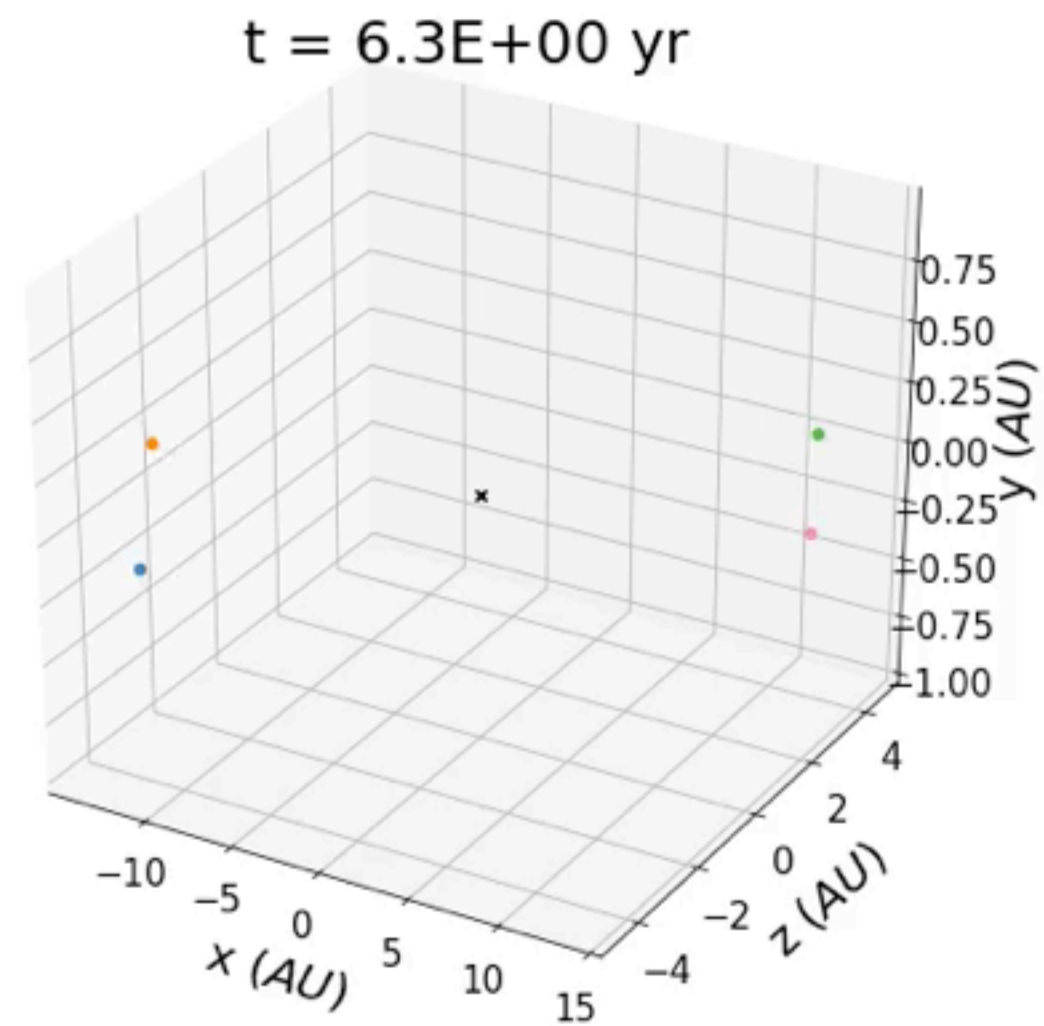




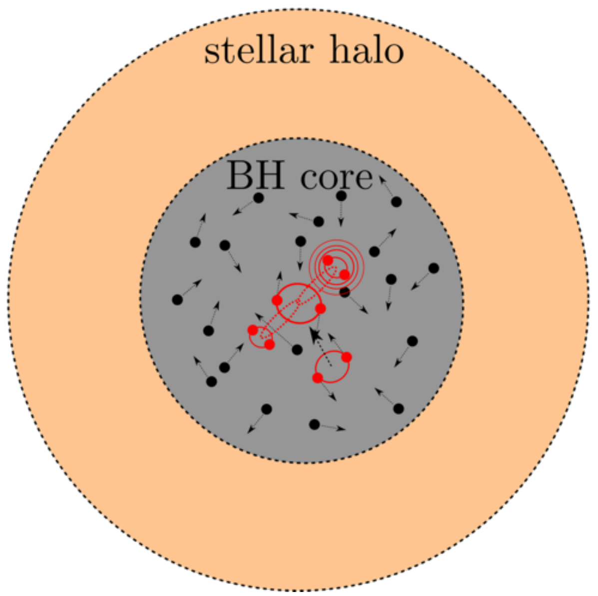
Merger Type: 4-body Merger

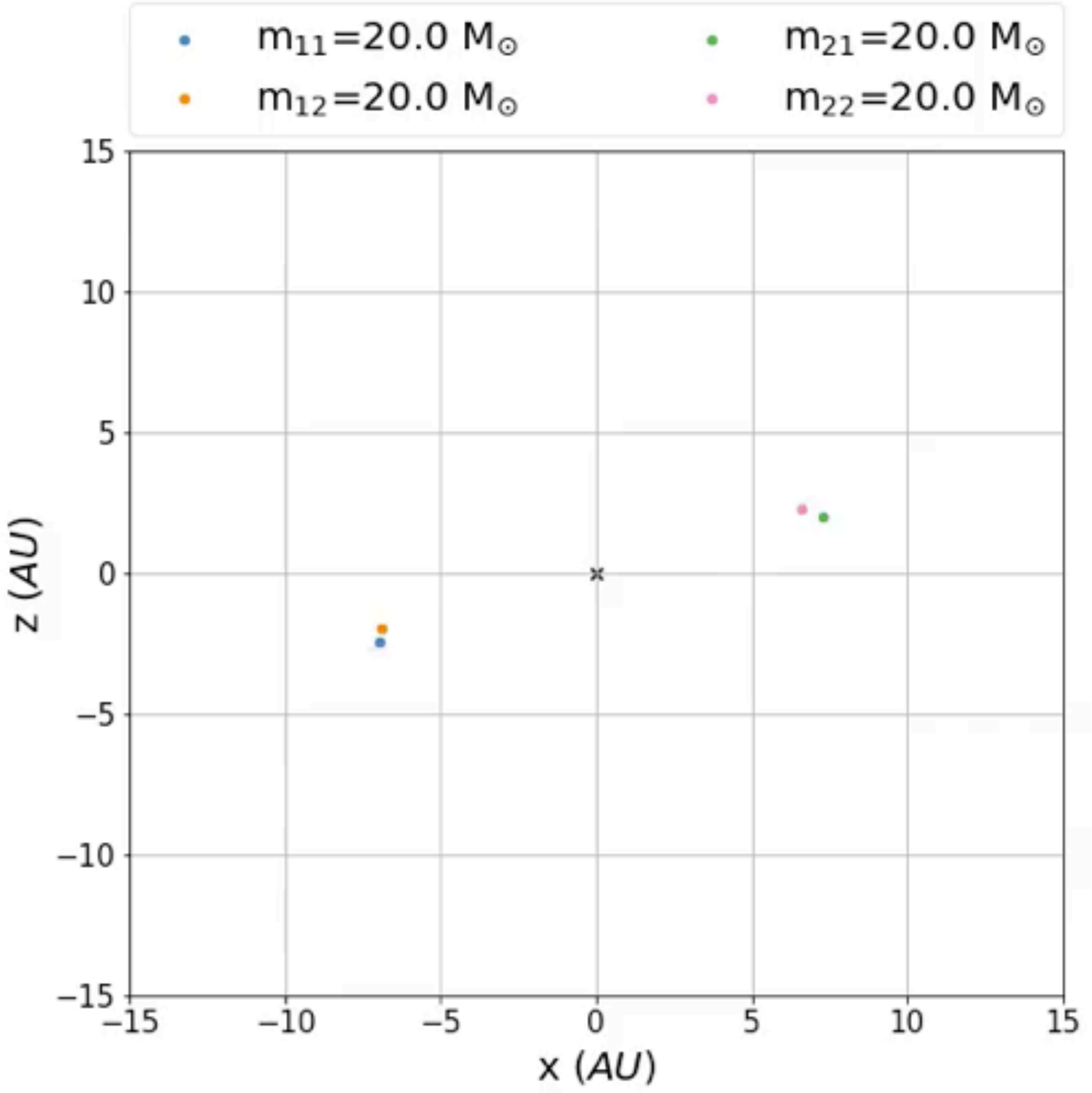


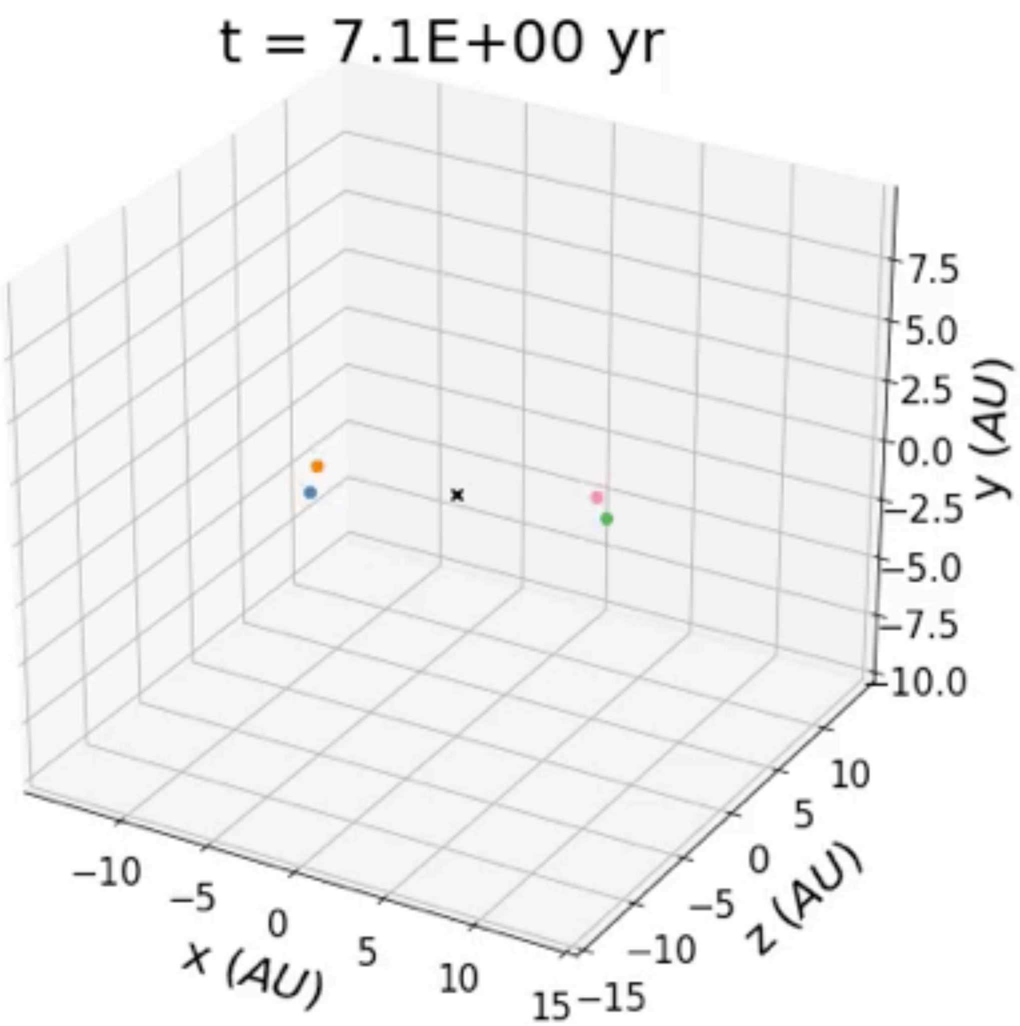




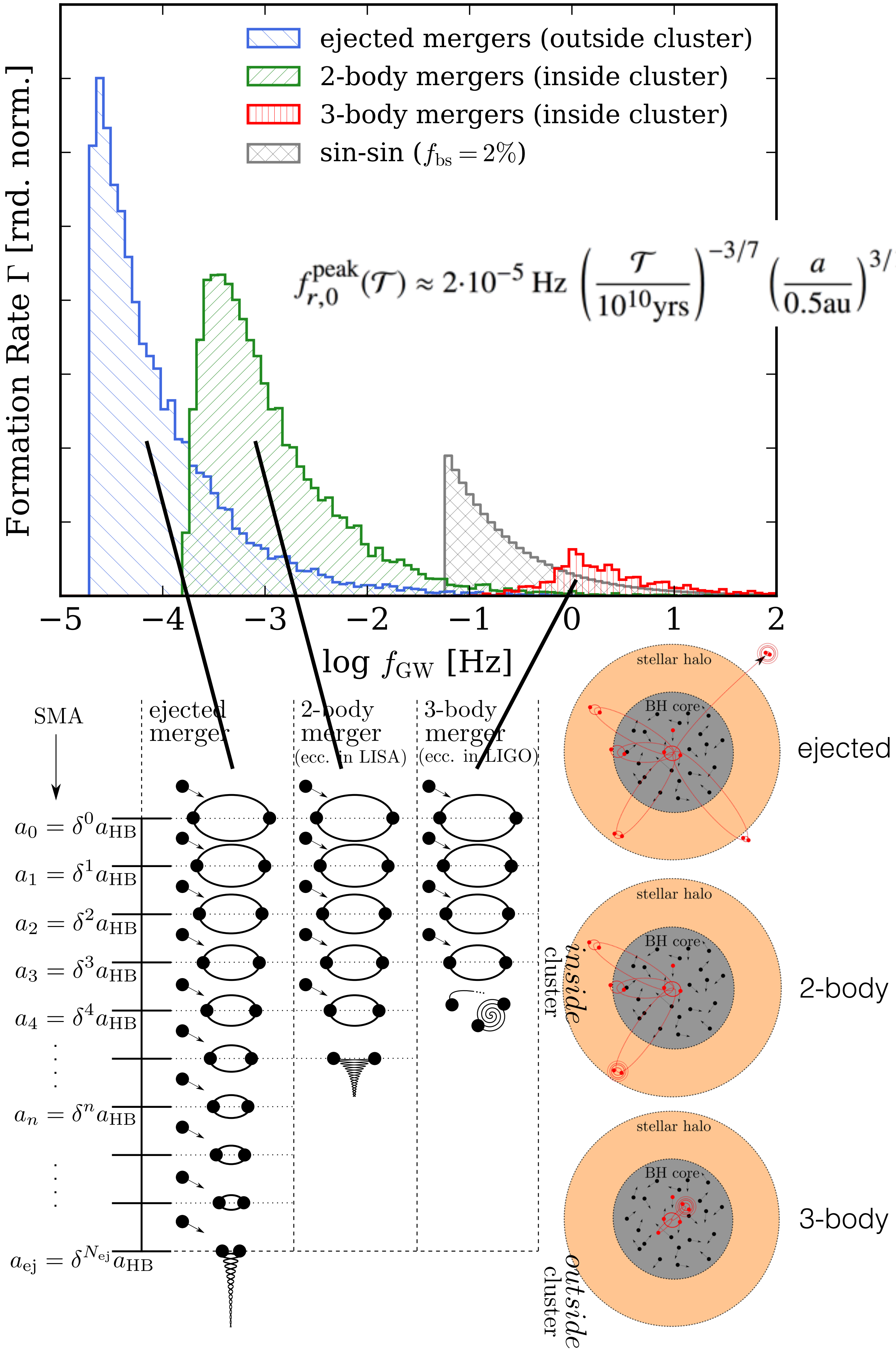
Merger Type: 4-body Merger







Peak Normalizations:



$$f_{r,0}^{\rm peak}(\mathcal{T}) \approx 2\cdot 10^{-5}\,{\rm Hz}\left(\frac{\mathcal{T}}{10^{10}{\rm yrs}}\right)^{-3/7}\left(\frac{a}{0.5{\rm au}}\right)^{3/14}\left(\frac{m}{30M_{\odot}}\right)^{-11/14}$$

$$P_i \approx F_i \times \left(\frac{\tau_i(a_{\rm ej})}{t_{\rm GW}^{e=0}(a_{\rm ej})}\right)^{2/7}$$

$$F_{\rm in} \approx (7/10)/(1-\delta) \approx 3$$

$$F_{\rm GW} \approx (7/5)/(1-\delta) \times N_{\rm MS} \approx 120$$

$$a_{\rm ej} \sim 0.5\ {\rm AU} \qquad M \sim 30 M_{\odot}$$

$$\tau_{\rm in} \sim 10^7\ {\rm years} \qquad \tau_{\rm GW} \sim 0.1\ {\rm year}$$

$$P_{\rm in} \approx 0.15 \quad P_{\rm GW} \approx 0.03$$

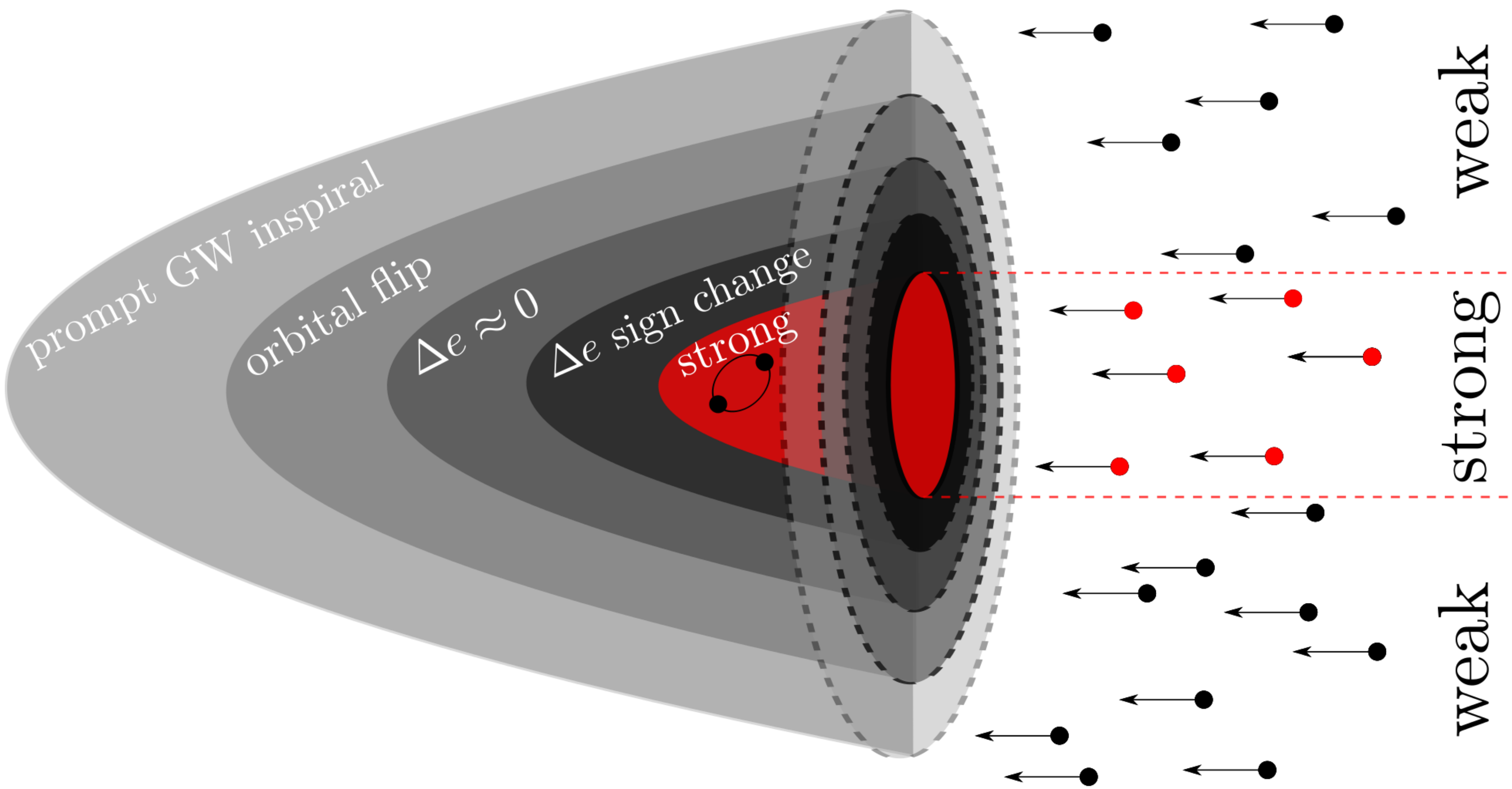
$$P(t_{\rm GW}(a_{\rm ej}) < T_{\rm H}) \approx 0.35$$

$$0.82 \times 0.35 \approx 0.3$$

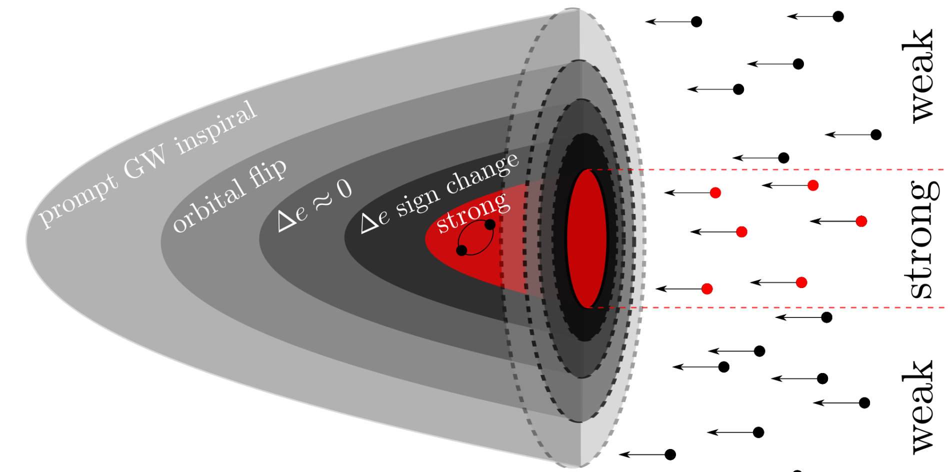
start to reach 10% high ecc. LIGO mergers

Merger Type: Secular-processes

- work done with Adrian Hamers (IAS)

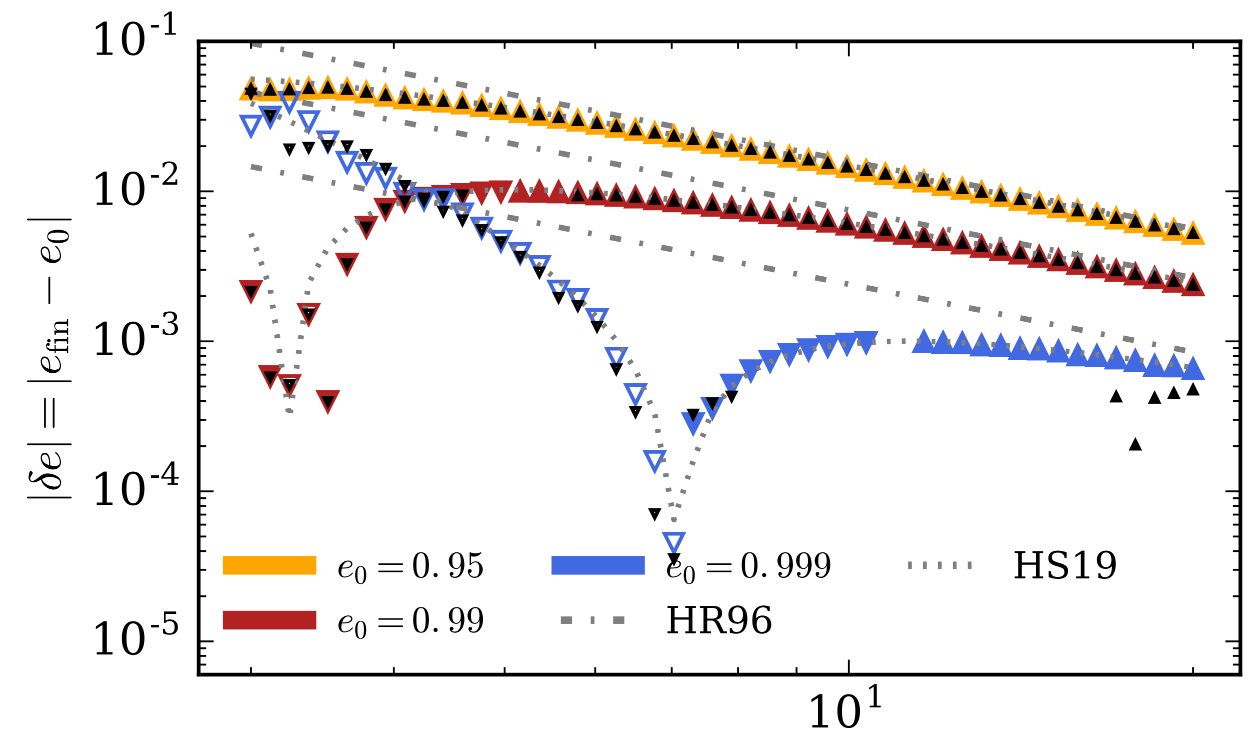


Merger Type: Secular-processes



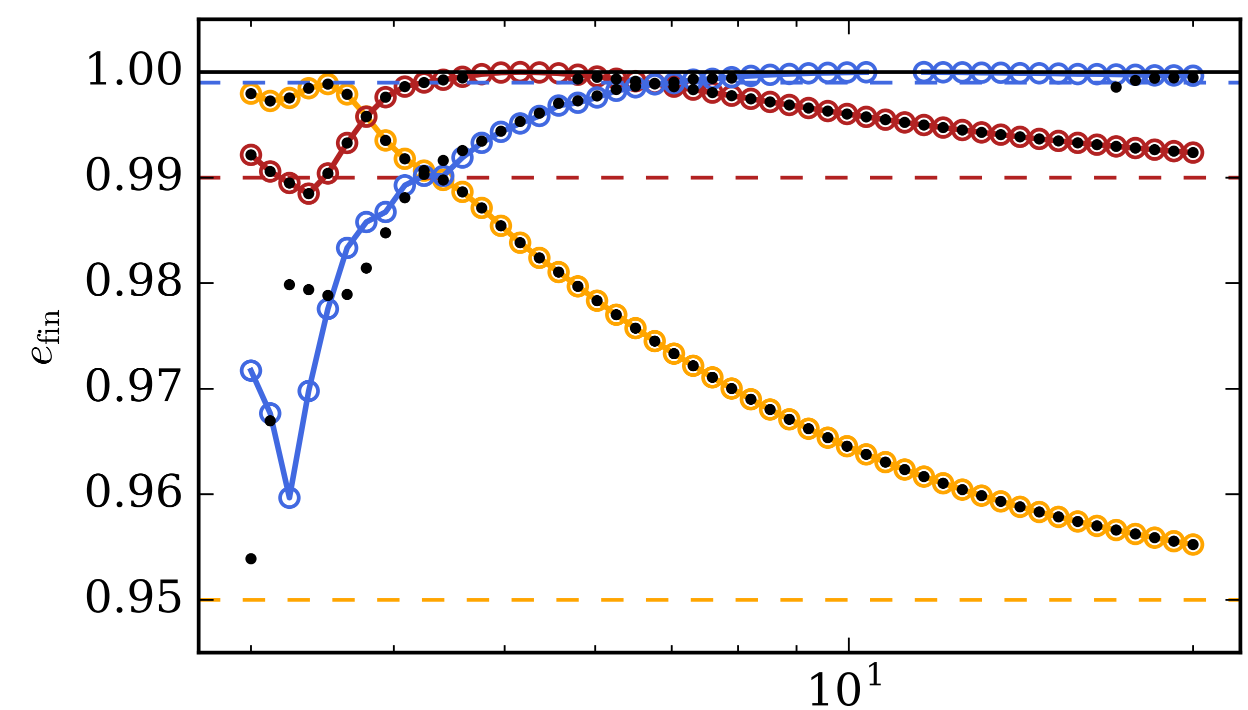
1.order (Heggie, Rasio 96)

$$\delta e = -\frac{15\pi}{16}\left(\frac{2m_3^2 a^3}{M_{123}M_{12}r_{\mathrm{p}}^3}\right)^{1/2} e\sqrt{1-e^2}\sin 2\Omega \sin^2 i$$

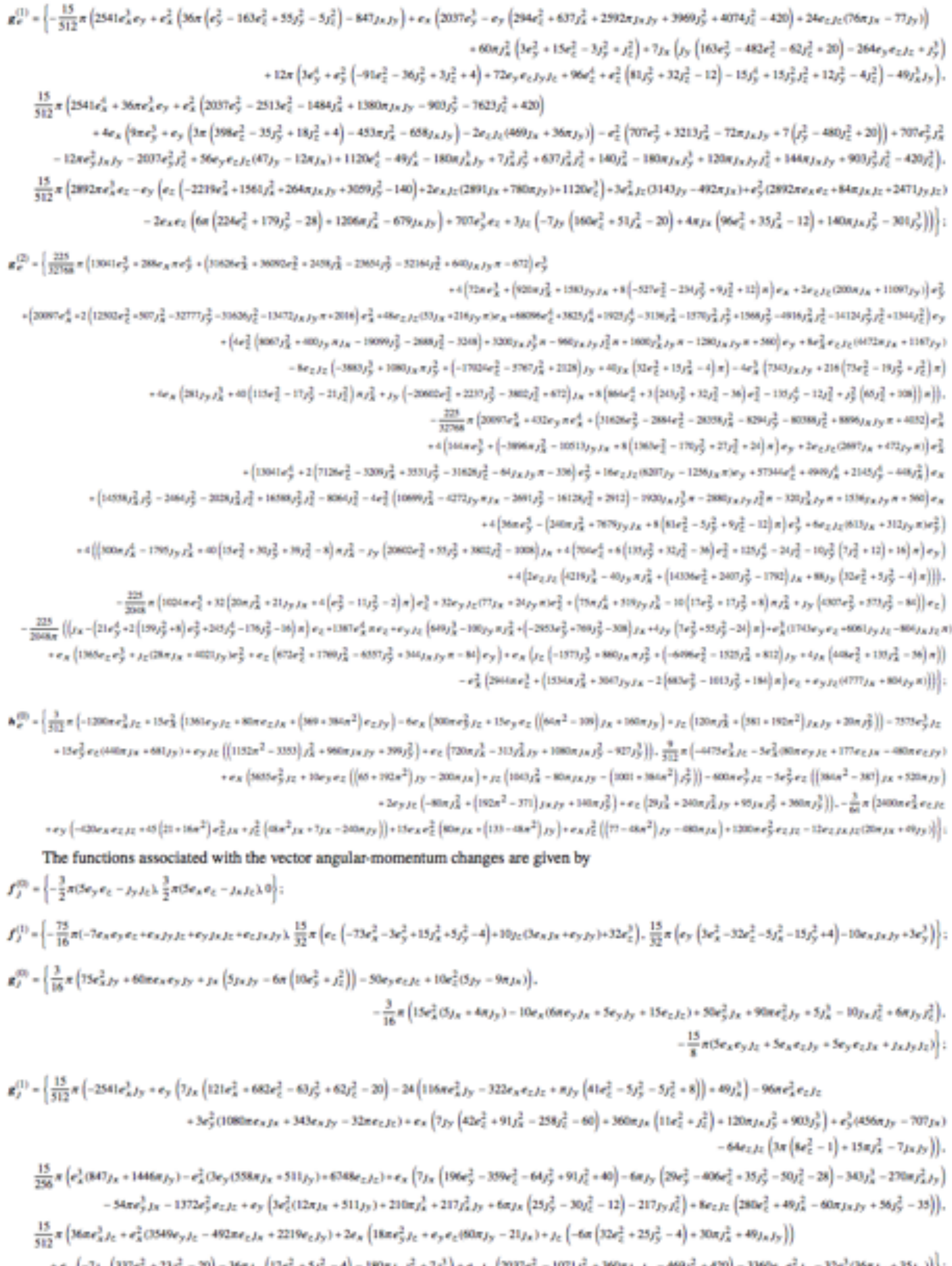


2.order (Hamers, Samsing 19)

$$\begin{aligned}\Delta e_{\mathrm{SO}} =& \Delta e_{\mathrm{FO}} + \epsilon^2 \frac{3}{512}\pi e_0 \Big[-100\left(1-e_0^2\right)\sin 2\Omega \\ & \Big\{(5\cos i + 3\cos 3i)\cos 2\omega + 6\sin i \sin 2i\Big\} \\ & + 4\cos 2i \Big\{3\pi\left(81e_0^2 - 56\right) - 200\left(1-e_0^2\right) \\ & \cos 2\Omega \sin 2\omega\Big\} + 3\pi\Big\{200 e_0^2 \sin^4 i \cos 4\Omega \\ & + 8\left(16e_0^2 + 9\right)\sin^2 2i \cos 2\Omega \\ & + \left(39e_0^2 + 36\right)\cos 4i - 299e_0^2 + 124\Big\}\Big],\end{aligned}$$



non-parabolic limit…


16 *Hamers & Samsing*

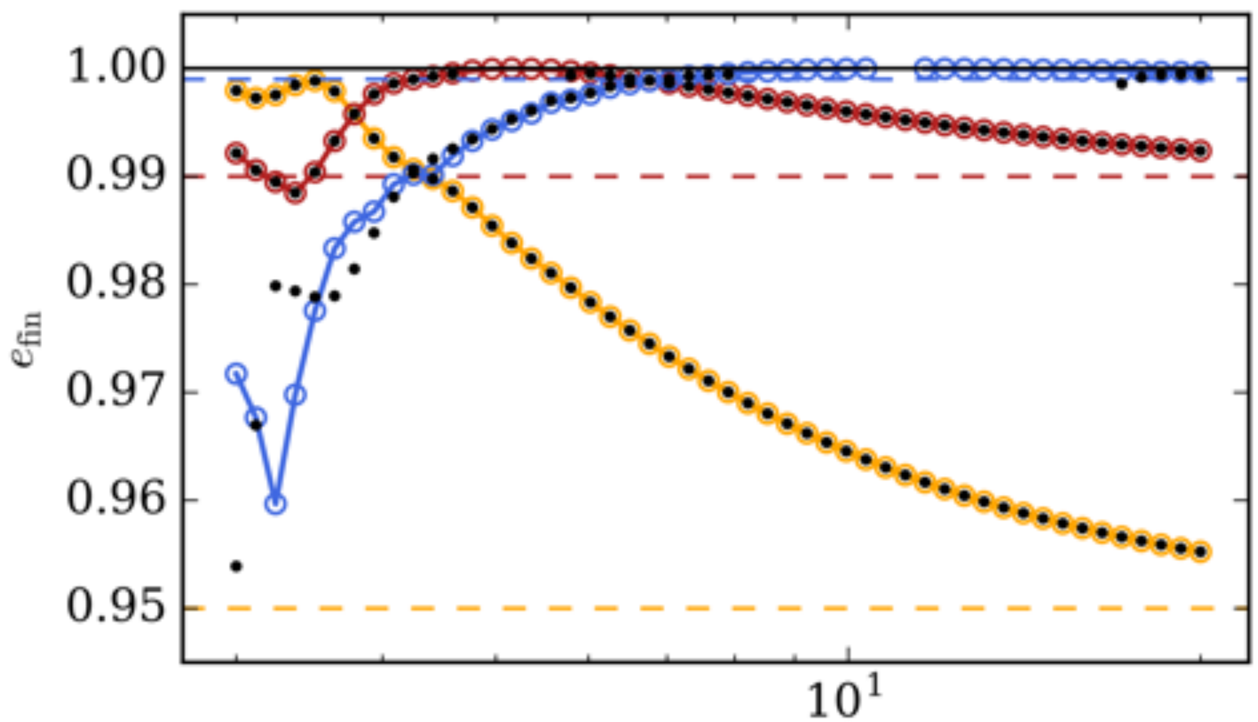
The functions associated with the vector angular-momentum changes are given by

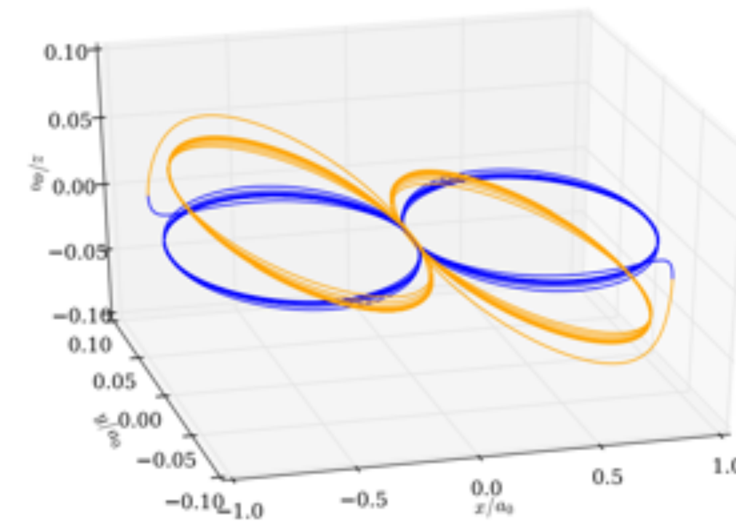
MNRAS 000, 1–17 (2019)

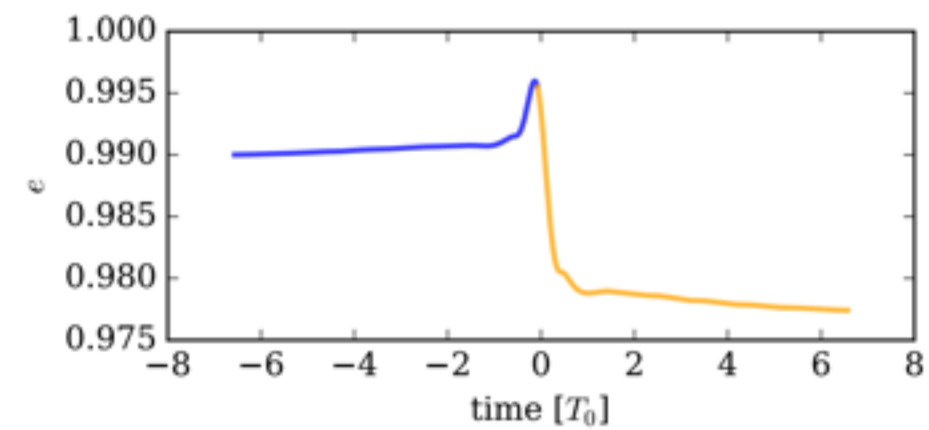
2 *Hamers & Samsing*

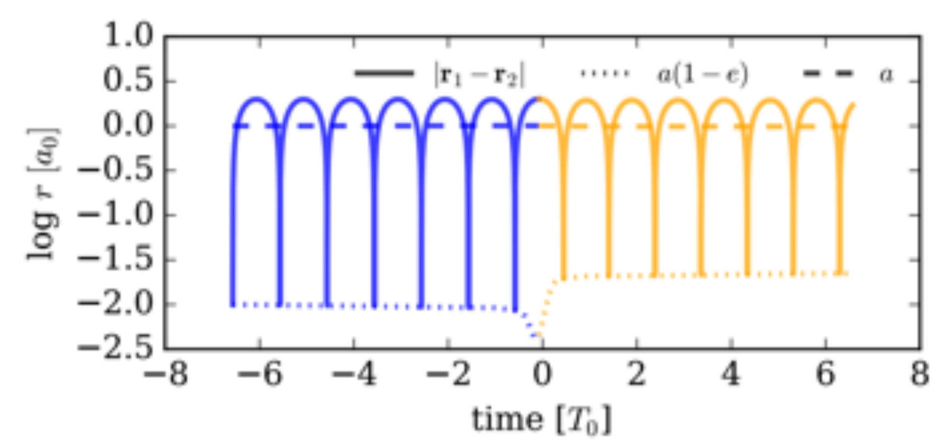
Terms of order	Number of terms in Δe $E = 1$	$E > 1$
ϵ_{SA} (all)	16	60
ϵ_{SA}	2	8
$\epsilon_{\mathrm{SA}}\epsilon_{\mathrm{oct}}$	14	52
ϵ_{SA}^2 (all)	193	55,895
ϵ_{SA}^2	17	1,871
$\epsilon_{\mathrm{SA}}^2\epsilon_{\mathrm{oct}}$	60	16,035
$\epsilon_{\mathrm{SA}}^2\epsilon_{\mathrm{oct}}^2$	116	37,989
ϵ_{SA}^3 (all)	1,146	2,931,541
ϵ_{SA}^3	54	38,366
$\epsilon_{\mathrm{SA}}^3\epsilon_{\mathrm{oct}}$	175	289,496
$\epsilon_{\mathrm{SA}}^3\epsilon_{\mathrm{oct}}^2$	311	856,072
$\epsilon_{\mathrm{SA}}^3\epsilon_{\mathrm{oct}}^3$	606	1,747,607

Merger Type: Secular-processes





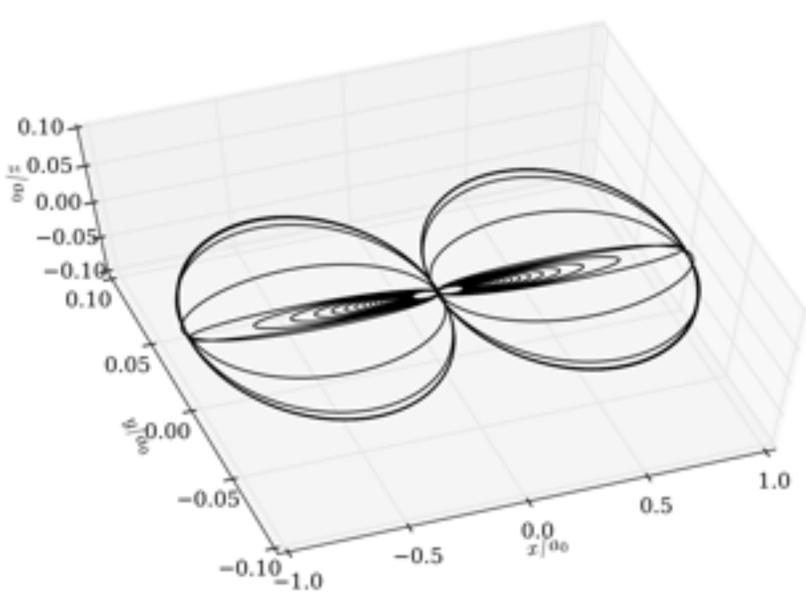




PN effects?

2.5

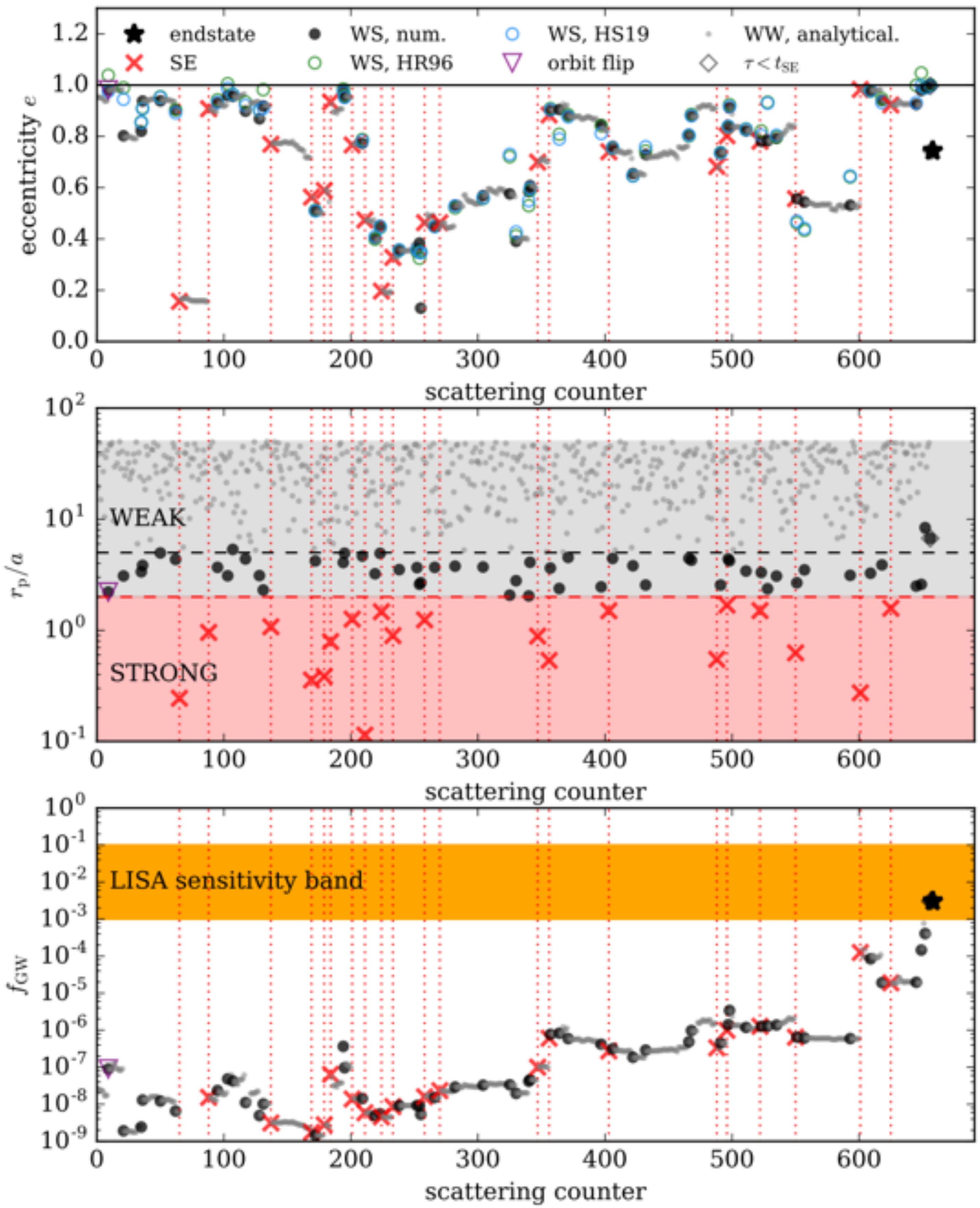
1,2



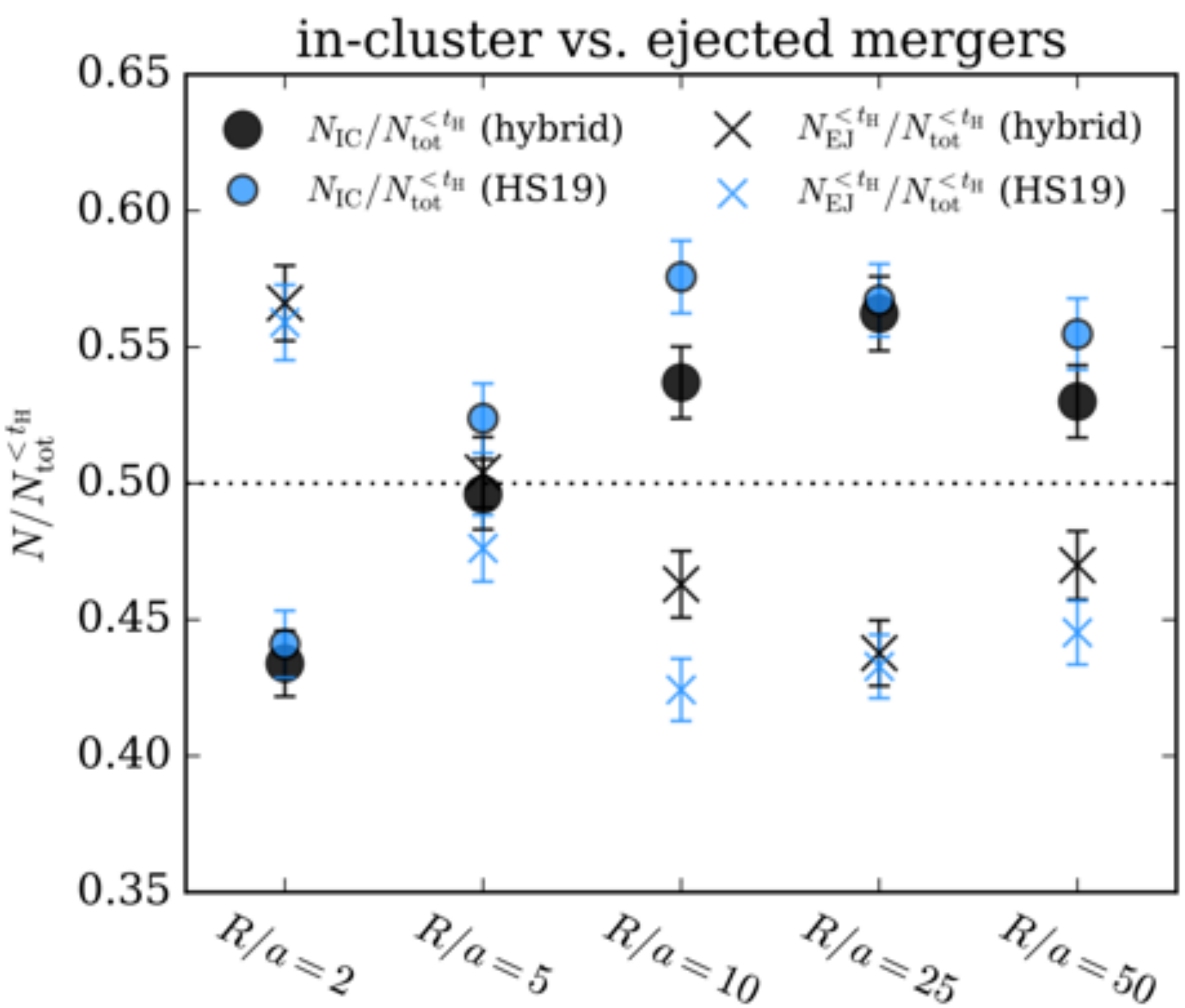
$$\frac{r_{\rm p}}{a_0} \gtrsim \frac{a_0^{2/3}}{\mathscr{R}_{\rm m}^{2/3}}(1-e_0^2)^{2/3}$$

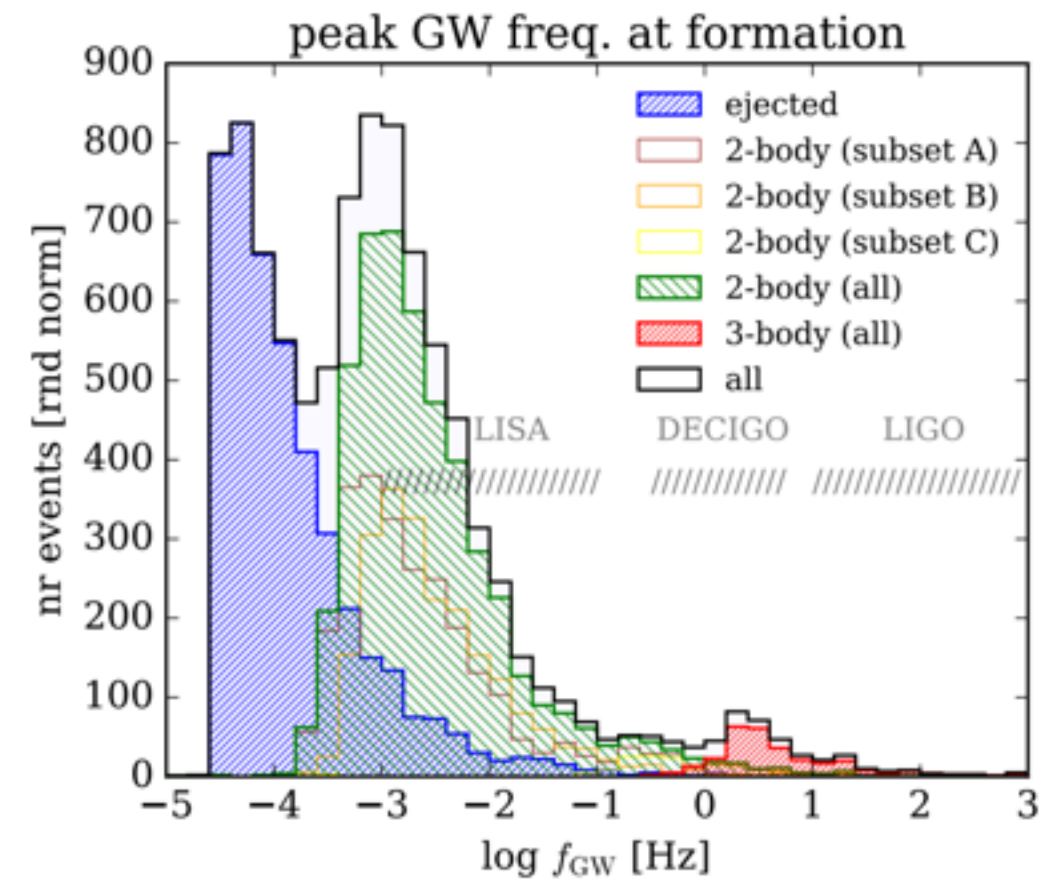
$$\gtrsim 10^3 \times \left(\frac{a_0}{0.5{\rm AU}}\right)^{2/3}\left(\frac{m}{20M_\odot}\right)^{-2/3}\left(1-(e_0/0.99)^2\right)^{2/3}$$

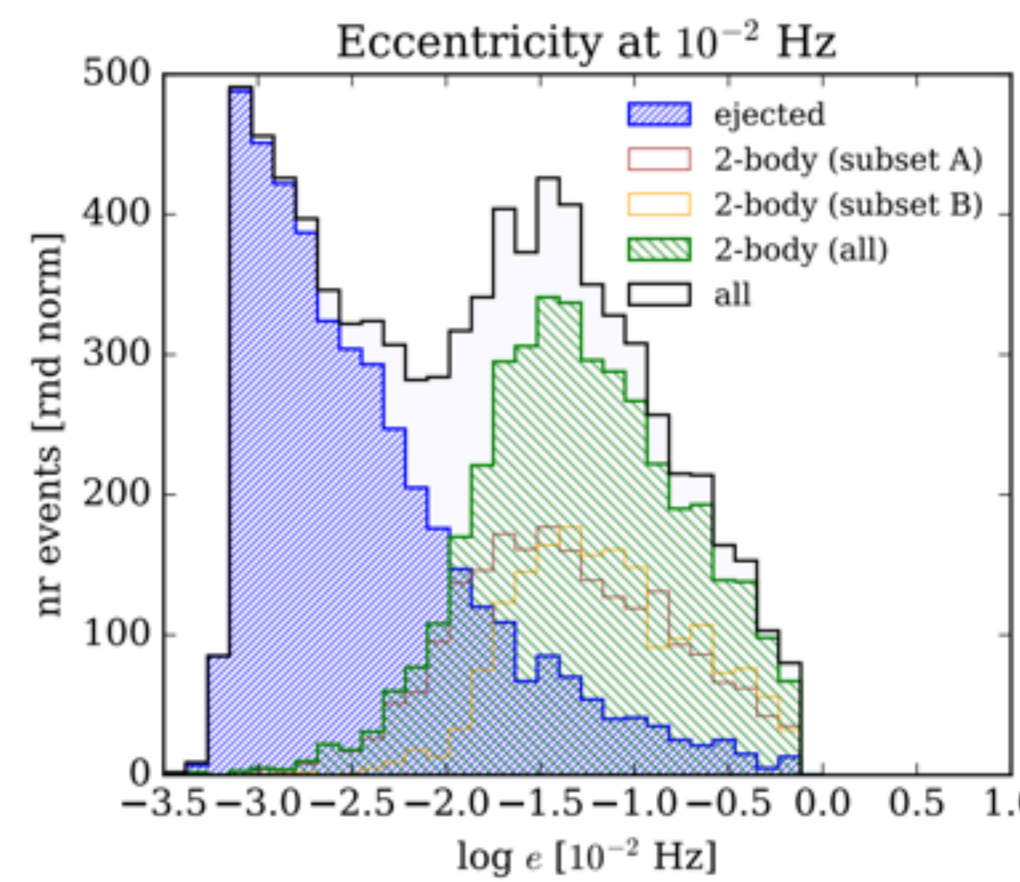
Merger Type: Secular-processes



Results from our MC code

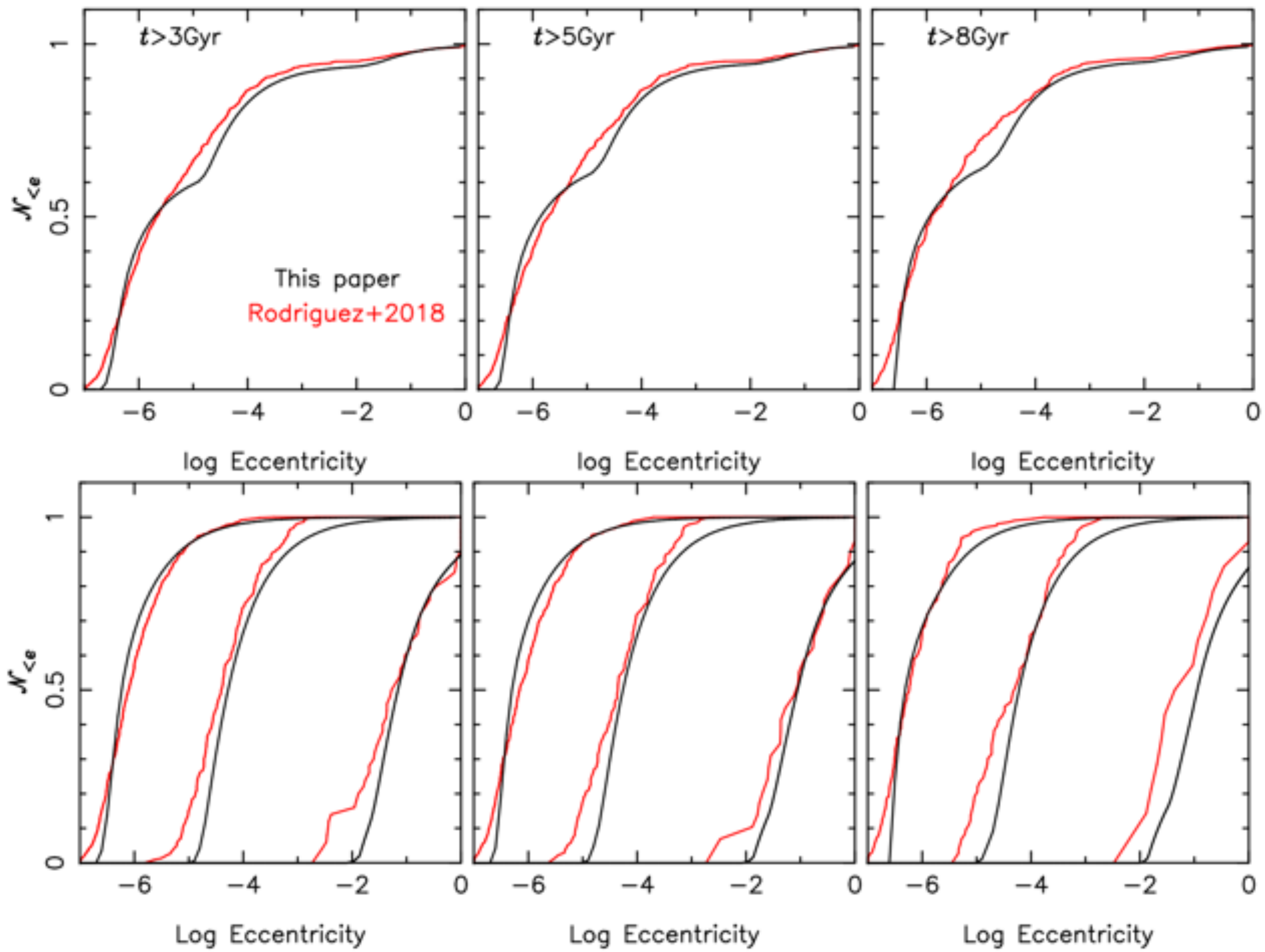




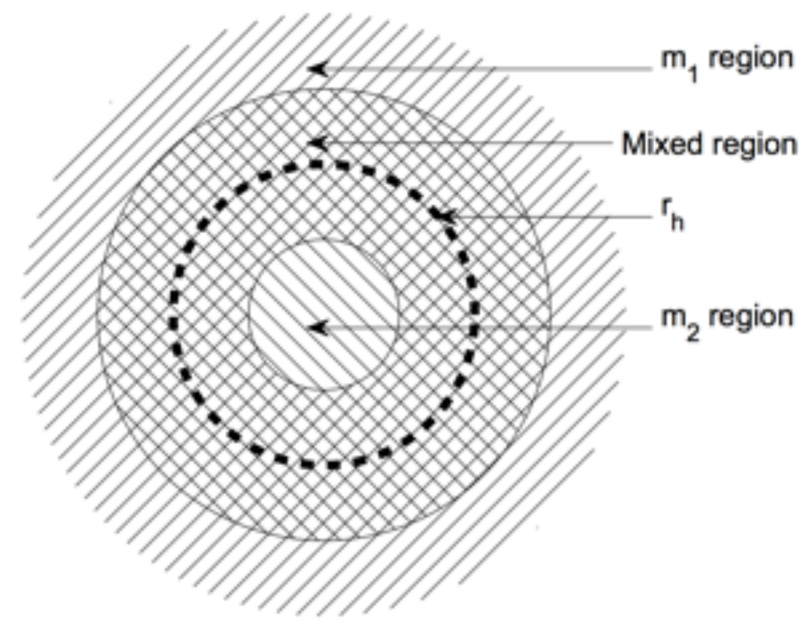


more BBHs are driven to merger!

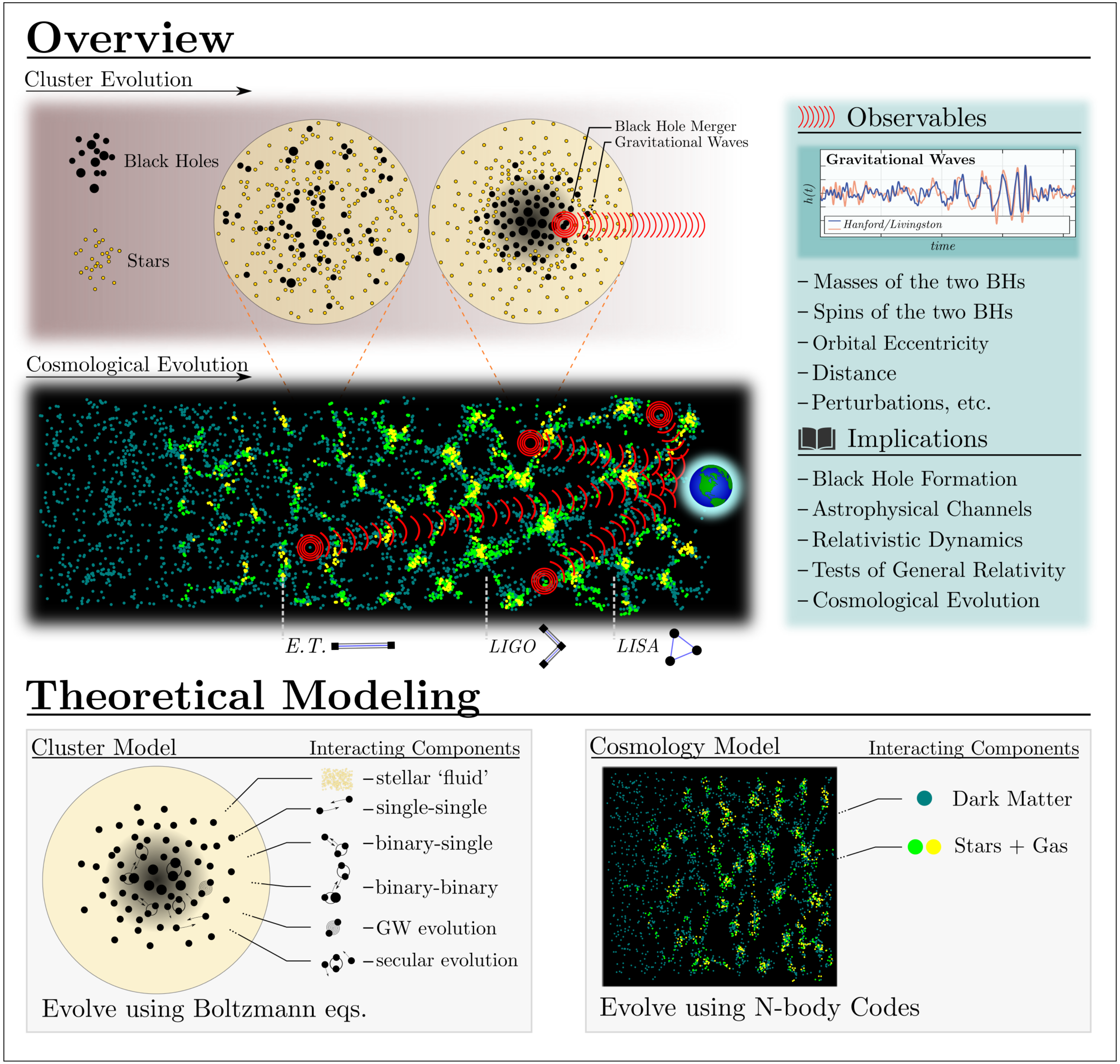
Promising..



Antonini et al. 2019



Future


Overview
Cluster Evolution
Black Holes
Stars
Black Hole Merger
Gravitational Waves
Observables
Gravitational Waves
h(t)
Hanford/Livingston
time
– Masses of the two BHs
– Spins of the two BHs
– Orbital Eccentricity
– Distance
– Perturbations, etc.
Implications
– Black Hole Formation
– Astrophysical Channels
– Relativistic Dynamics
– Tests of General Relativity
– Cosmological Evolution
Cosmological Evolution
E.T.
LIGO
LISA
Theoretical Modeling
Cluster Model
Interacting Components
–stellar 'fluid'
–single-single
–binary-single
–binary-binary
–GW evolution
–secular evolution
Evolve using Boltzmann eqs.
Cosmology Model
Interacting Components
Dark Matter
Stars + Gas
Evolve using N-body Codes


THANK YOU