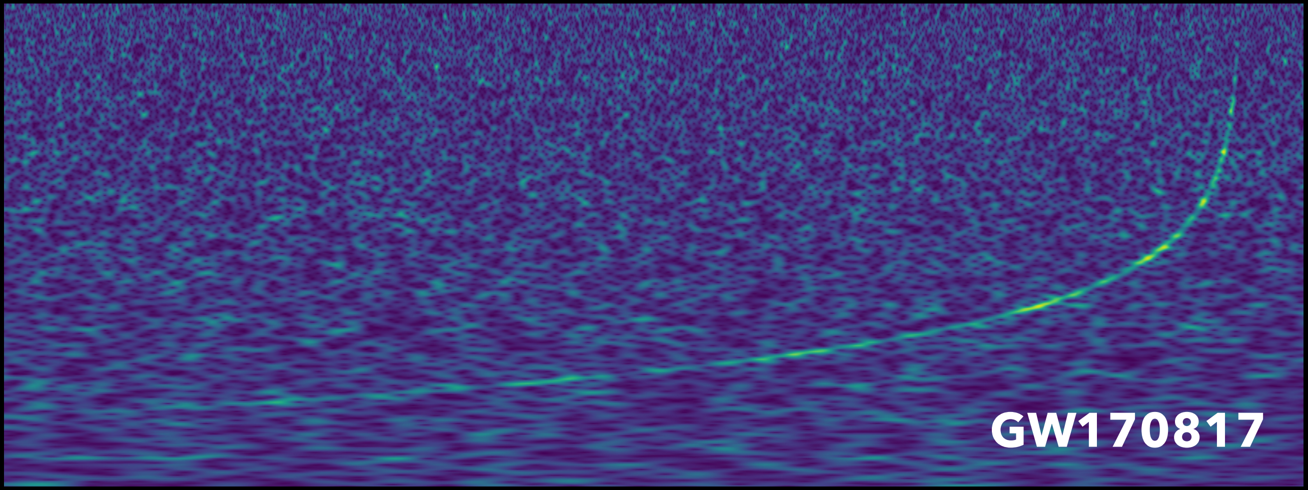
The seismology of Love

the impact of tides on neutron star binary signals



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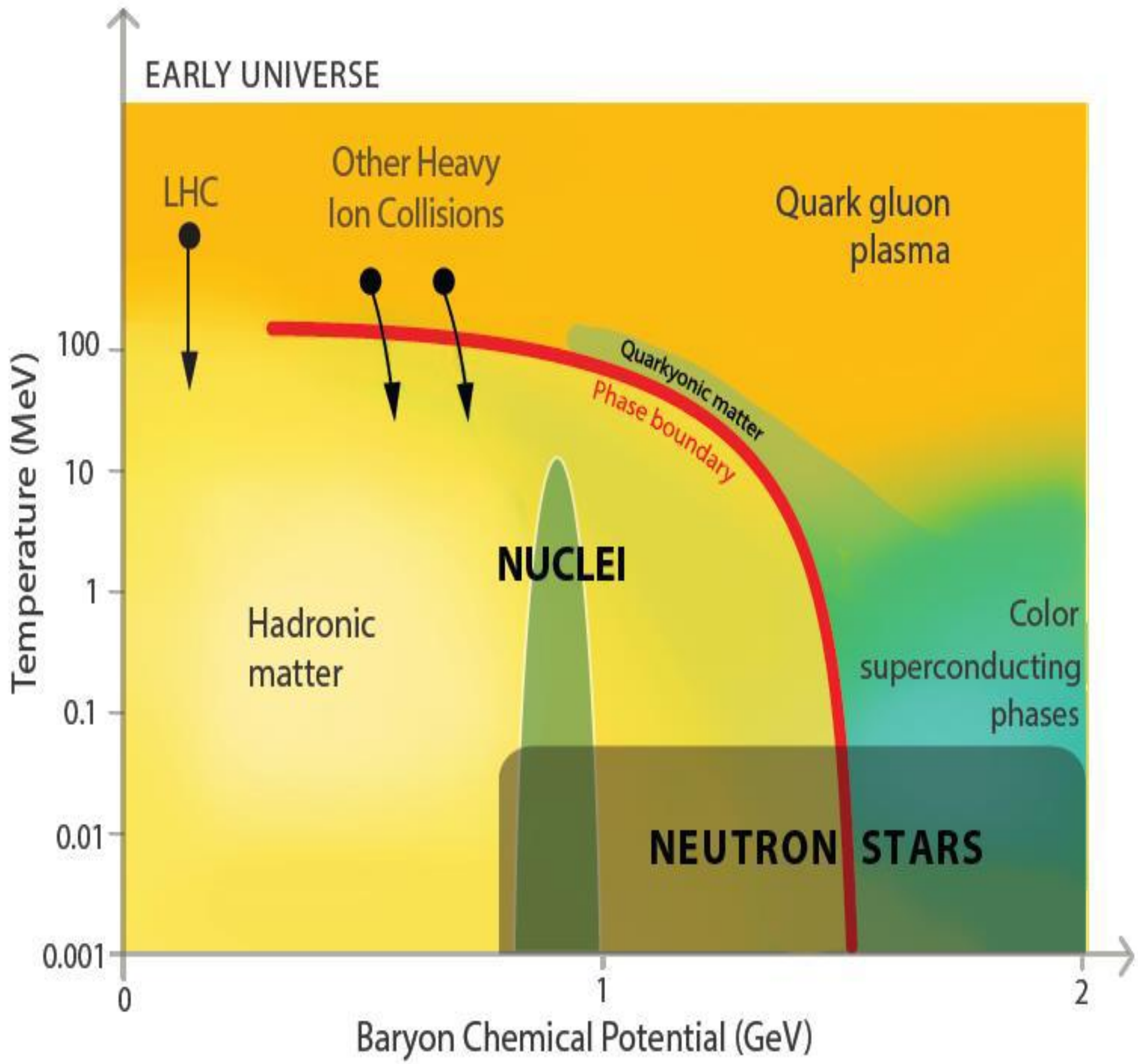
Neutron stars are:

i) “hands-off” laboratories, that

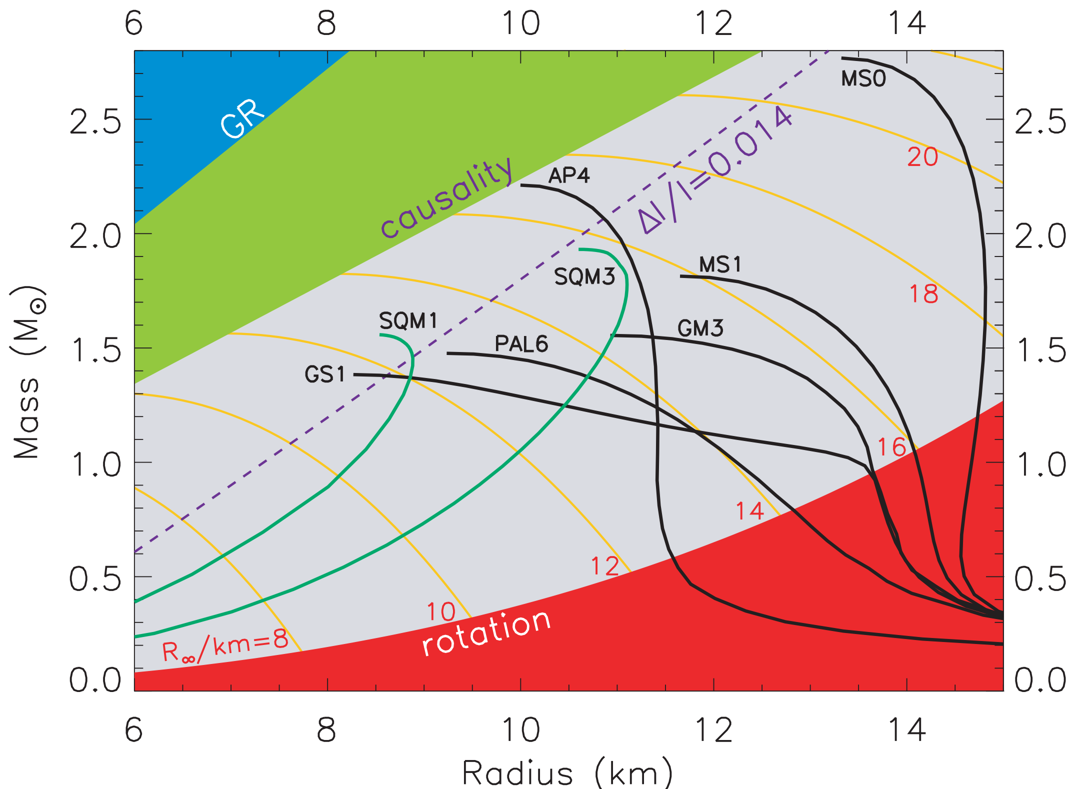
ii) involve a lot of physics…


EARLY UNIVERSE
LHC
Other Heavy Ion Collisions
Quark gluon plasma
Quarkyonic matter
Phase boundary
NUCLEI
Hadronic matter
Color superconducting phases
NEUTRON STARS
Temperature (MeV)
100
10
1
0.1
0.01
0.001
0
1
2
Baryon Chemical Potential (GeV)

[Watts et al]

The macroscopic diagnostic of microscopic many-body interactions is a pressure-density-temperature relation: **the equation of state**.

First principle calculations for many-body QCD systems are problematic at high densities (sign problem).

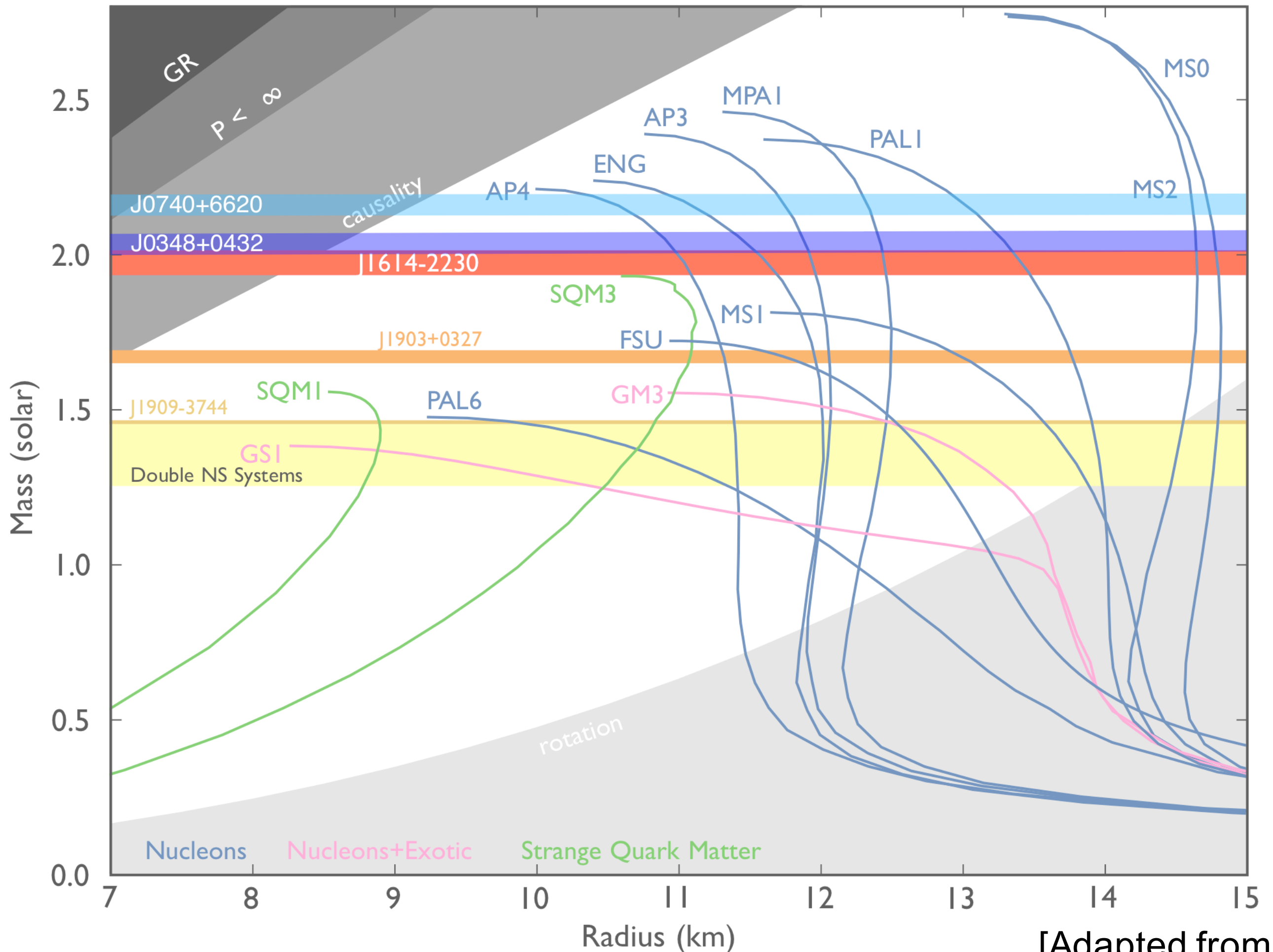


[Lattimer & Prakash]

Basically, three approaches:

- non-relativistic quantum calculations (e.g. APR)
- "phenomenology" (e.g. Skyrme interaction matched to measured nuclear masses)
- relativistic mean-field theory (typically used for hyperons/quarks)

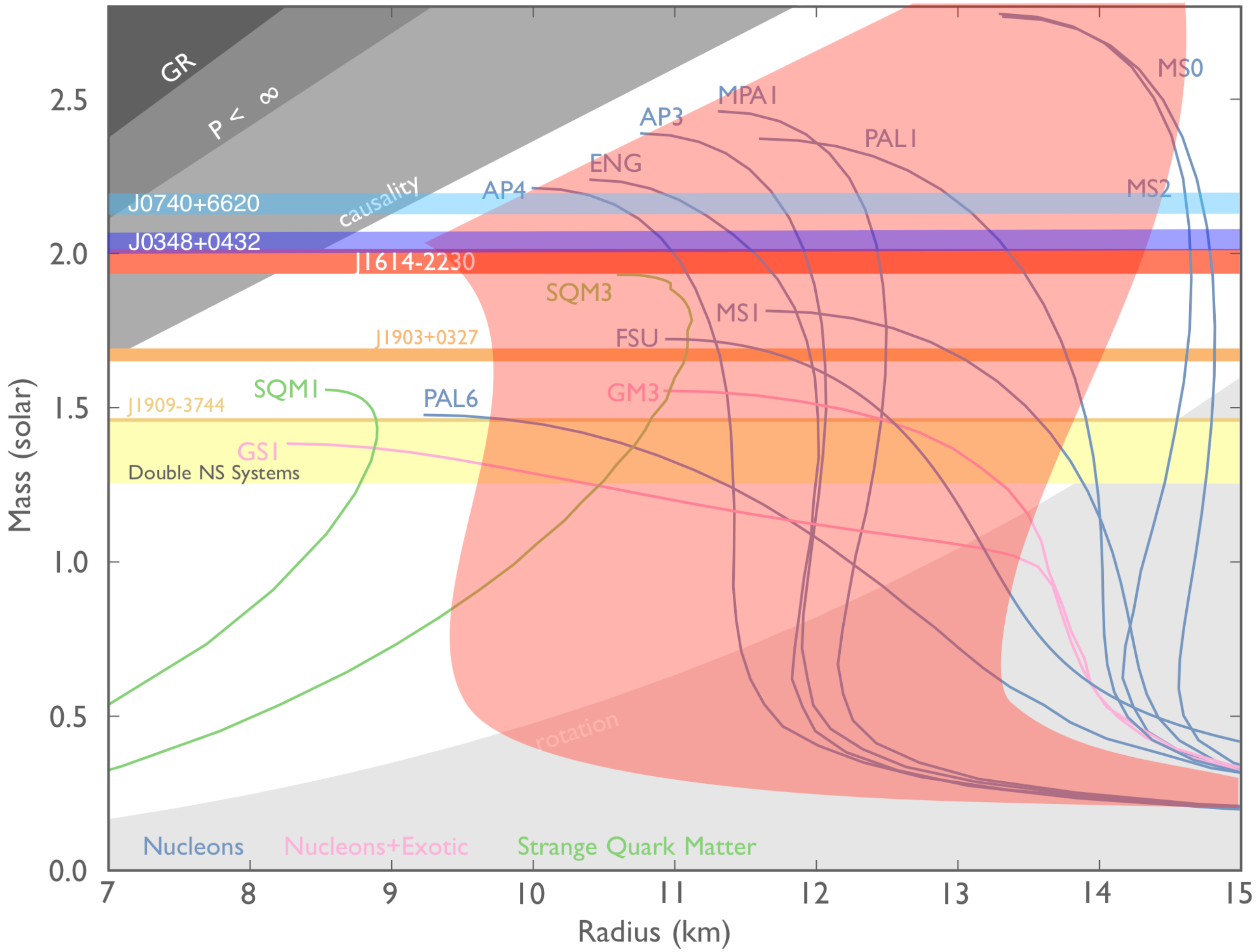
Need experiments and observations to test theory and drive progress.



[Adapted from Antoniadis et al]

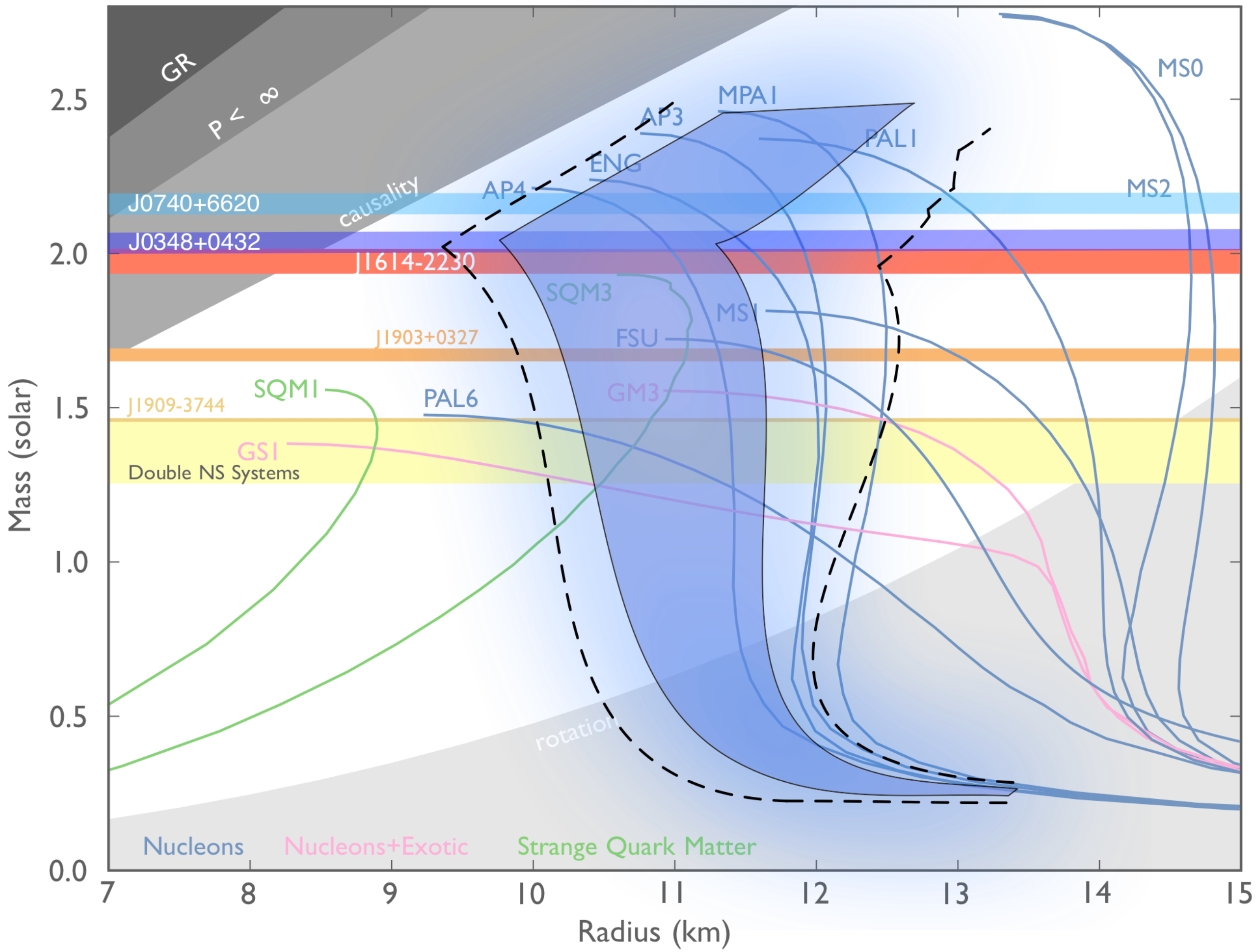
Masses deduced from binary dynamics tend to lie in a relatively narrow range, about 1.1-1.6$M_\odot$. These systems do not constrain nuclear physics (much).

The current record holder is J0348-0432 with (a WD companion) and a mass just over 2$M_\odot$. (Note also the recent evidence for J0740+6620 being 2.17$M_\odot$.)



State-of-the-art chiral effective field theory calculations (Schwenk, Tews and others) provide “reliable” low-density constraints, which can be extrapolated to higher densities (=more massive stars).

Suggests a 1.4$M_\odot$ neutron star should have radius in the range 10-14 km.



The radius is "difficult" to infer from radio data (although… the moment of inertia for the Double Pulsar), but may use accreting systems emitting in x-rays.

Strategy: Construct "empirical" equation of state (from a Bayesian analysis) based on combining data for a set of systems (work by Steiner et al).

Again, constrains the radius to (conservatively) the range 10-14 km.

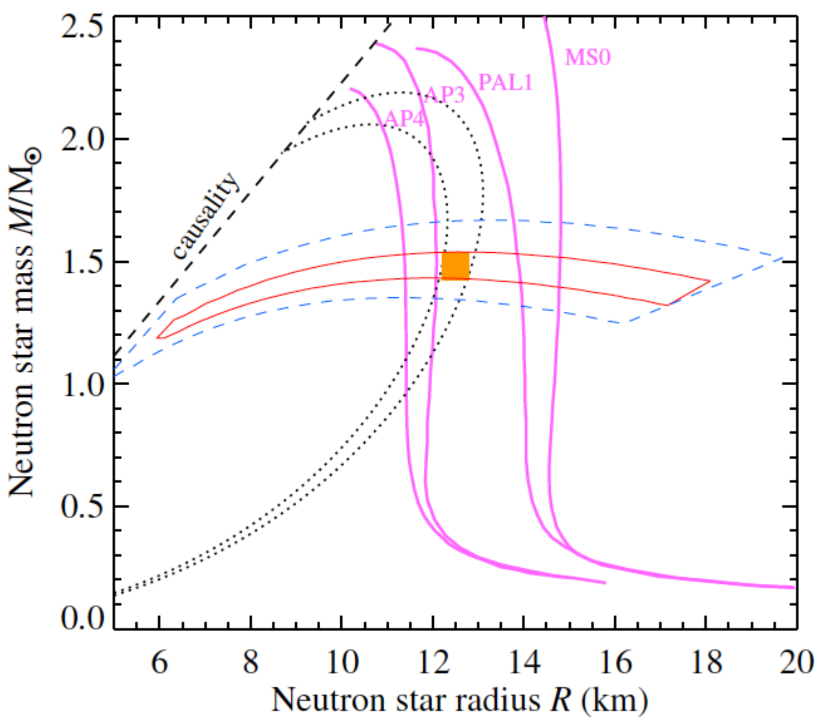
NICER has been taking data since 13 June 2017.

The main aim is to measure pulse profiles associated with non-uniform thermal surface emission of rotation-powered pulsars.

Comparison to theory models leads to estimate of the star's mass and radius.

Preliminary results for PSR J0030+0451 favours two emitting polar caps (=tricky systematics) and a radius in the range 12-15 km.

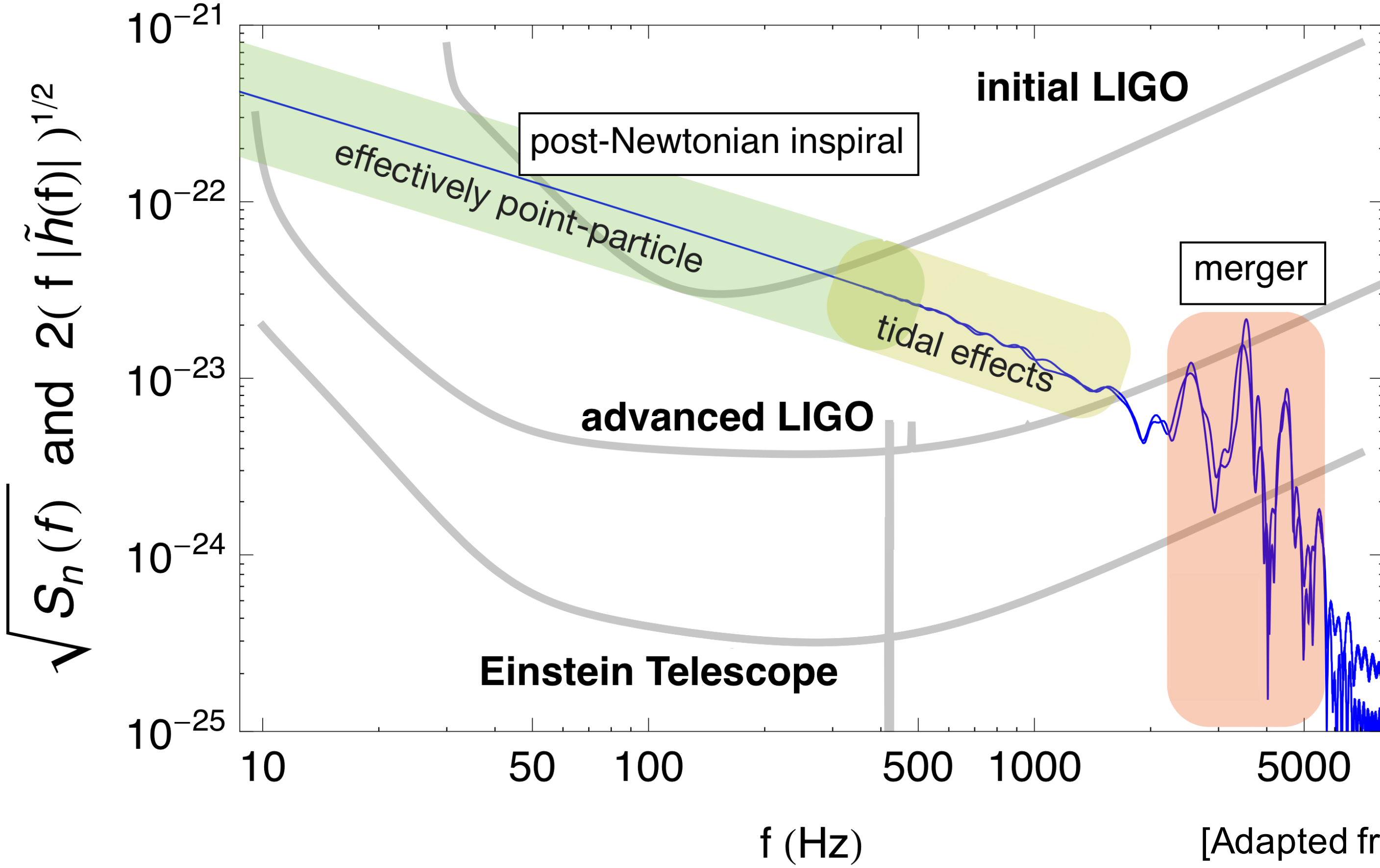
Expect stronger constraints "soon" (e.g. systems with known mass).



Longer term, we need a high-resolution x-ray timing mission (with a large collection area).

The Chinese-European eXTP and the US led STROBE-X missions are designed to explore the state of matter under extreme conditions.

Significant upgrades from previous instruments (e.g. RXTE) and should (finally!) provide mass-radius constraints at the **few % level**.



[Adapted from Read]

Gravitational-wave astronomy provides different opportunities.

Deviations from point-mass dynamics become important during the late stages of binary inspiral .

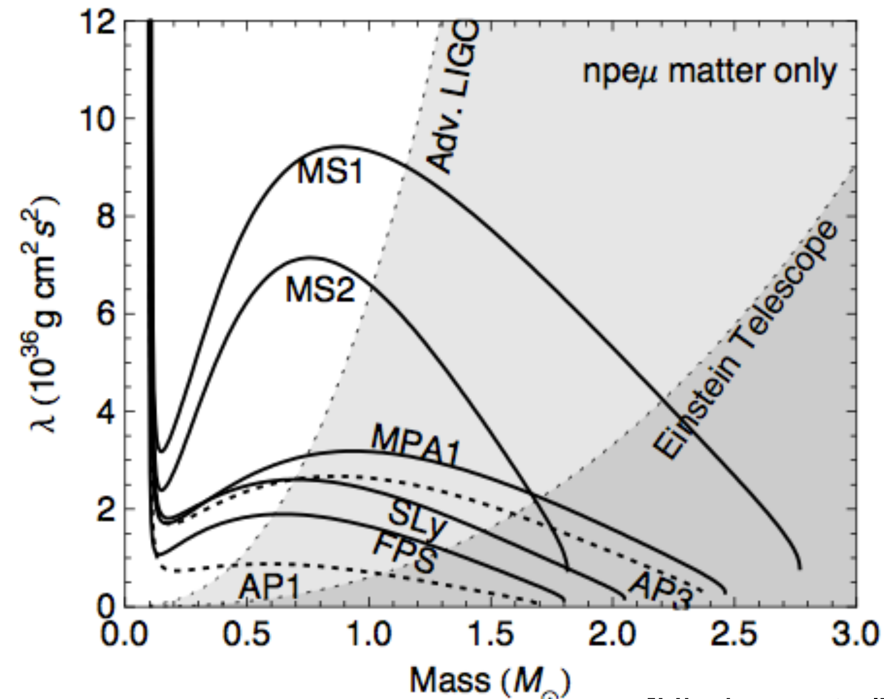
The (main) effect is encoded in the **tidal deformability** (via the Love numbers).

Basically, the tidal interaction affects the number of gravitational-wave cycles

$$\mathcal{N} = \frac{2}{3}\int_{f_a}^{f_b} \frac{E_{\rm orb}}{\dot{E}_{\rm orb}} df \approx \int_{f_a}^{f_b} t_D \left(1 + \frac{E_{\rm r}}{E_{\rm N}} - \frac{\dot{E}_{\rm tide}}{\dot{E}_{\rm gw}}\right) df$$

Template mismatch by (say) half a cycle leads to a significant loss of signal to noise.

Difficult to alter GW phasing (e.g. 10^{46} erg at 100 Hz leads to shift of 10^{-3} radians), but the star's deformability, encoded in the so-called Love number, may lead to a distinguishable secular effect.

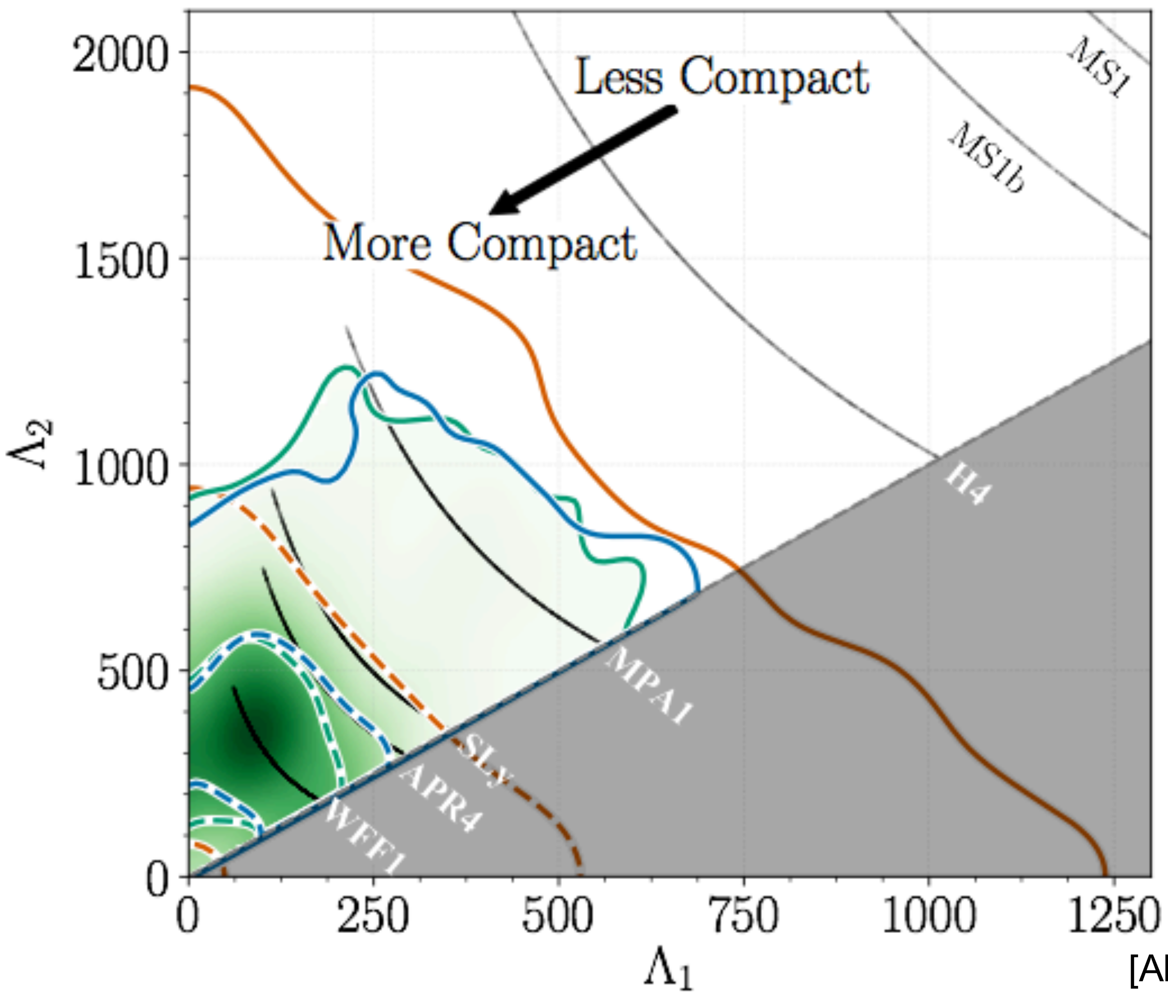


[Hinderer et al]

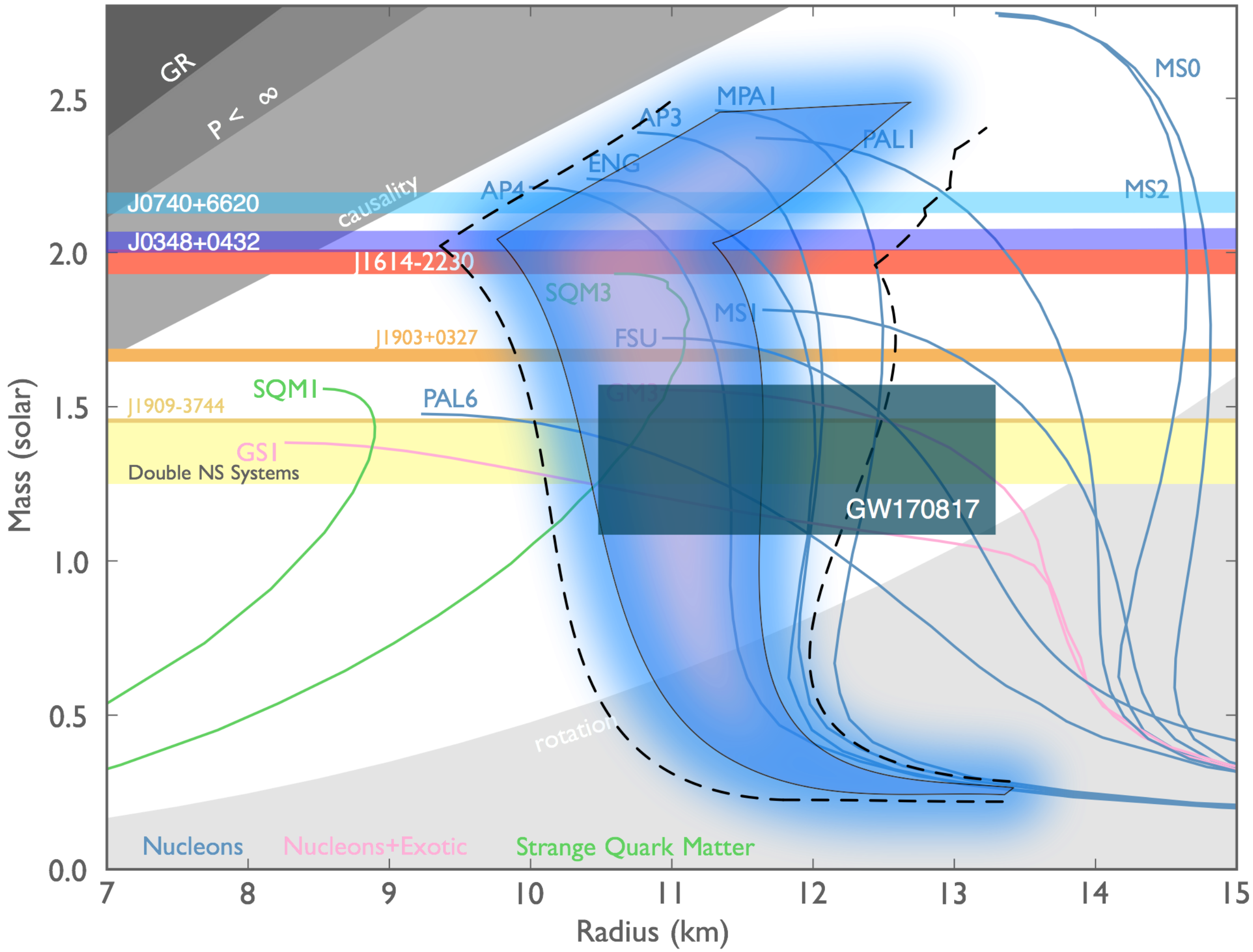
$$\Phi_l = -\frac{4\pi G}{2l+1}\frac{I_l}{r^{l+1}} \qquad \chi_l = \frac{4\pi}{2l+1} d_l r^l \qquad G I_l = -2k_l R^{2l+1} d_l$$

Demonstrated by the GW170817 signal.

$$\lambda = \frac{2}{3} k_2 R^5 = \frac{Q}{E} = \frac{\text{quadrupole deformation}}{\text{strength of tidal field}}$$



[Abbott et al]



"Best" constraints on the tidal deformability (assuming the same equation of state, slow spins and maximum mass indicated by pulsar data) suggests a radius in the range

R=10.5-13.3 km

(similar to the x-ray results…).

Question:

At what level does the "gory" neutron-star physics enter?

Answer:

Need to go beyond the "static" tide.

— individual oscillation modes may become resonant as the binary spirals through the sensitivity band of advanced interferometers (g-modes depend on interior composition),

— the f-mode resonance is the strongest, but probably does not become resonant before merger (dynamical tide),

— nonlinear coupling between p- and g-modes may lead to an instability,

— the elastic crust will leave a faint imprint on the tidal signal (<1% effect?),

— the superfluid interior affects the mode spectrum and may play a role,

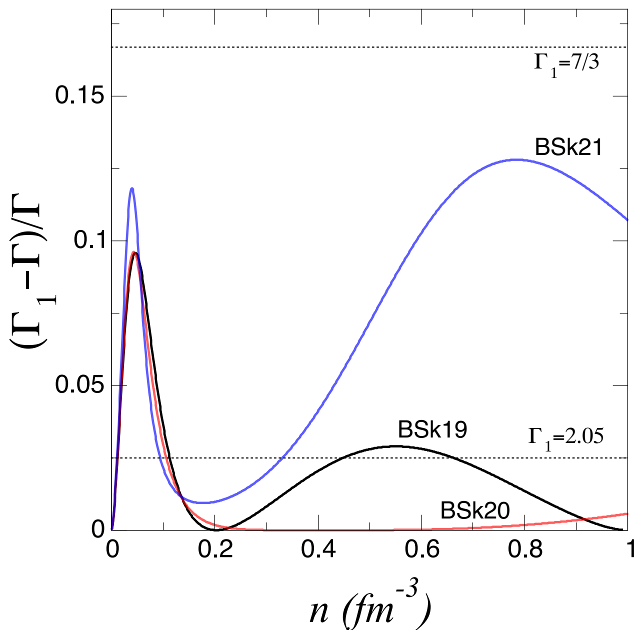
— oscillations of the post-merger remnant rely on the hot equation of state.

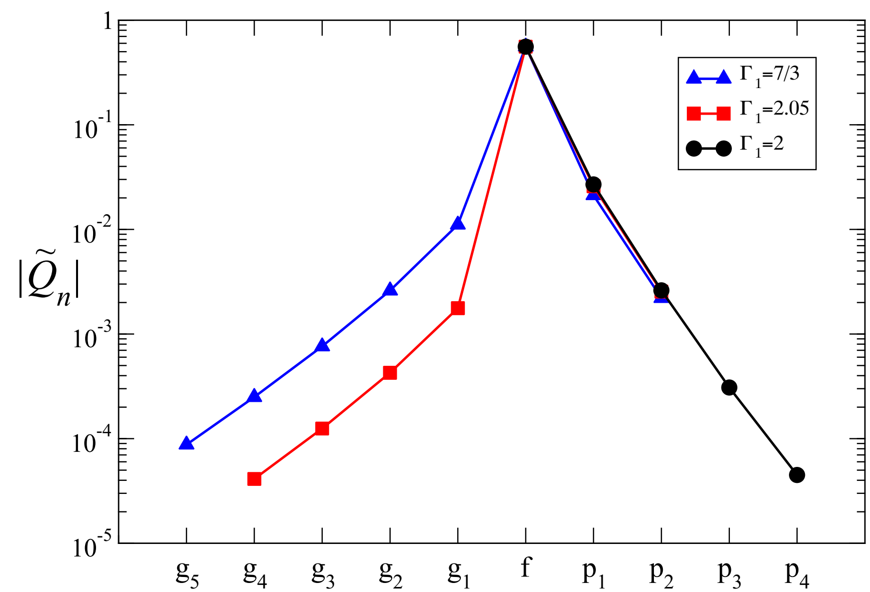
Dynamical tides are usually discussed in terms of the overlap integral between each mode (amplitude a_n) and the tidal potential.

$$-\omega^2 a_n + \omega_n^2 a_n = v_l Q_n$$

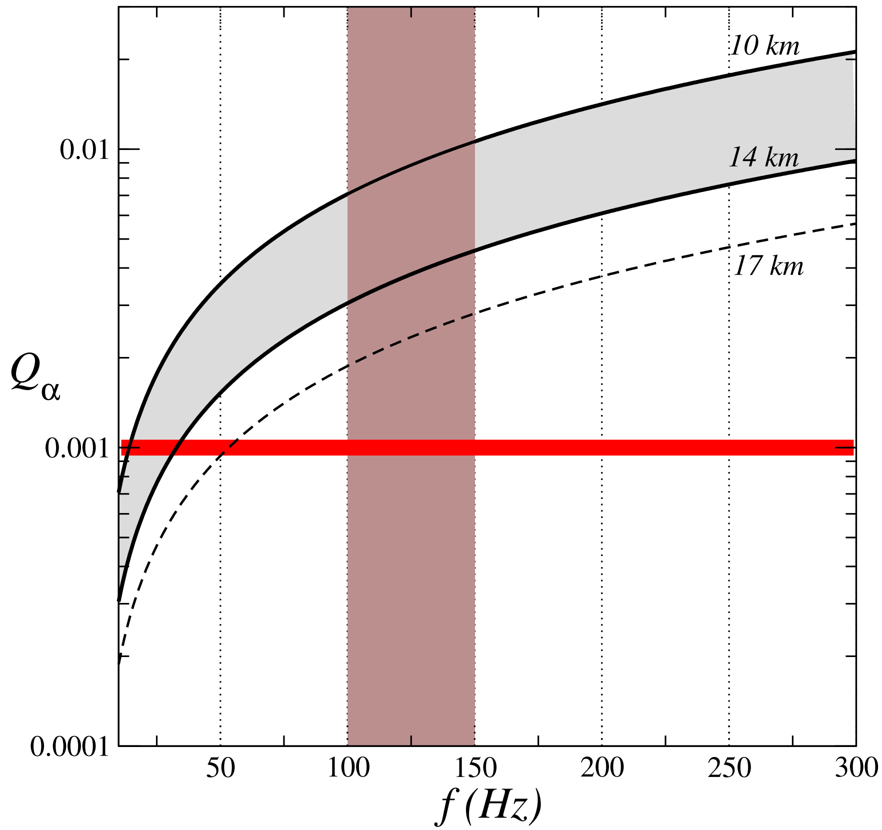
$$Q_n = -\int \delta\rho_n^* r^{l+2} dr = \frac{2l+1}{4\pi G} R^{l+1} \delta\Phi_n(R) = I_n$$

The g-modes enter the problem for "frozen" composition (nuclear reactions slower than inspiral).





[Andersson & Pnigouras]



[Andersson & Ho]

Example: For typical equation of state, we have

$$Q_n \leq 10^{-2} \left(\frac{f}{100 \text{ Hz}} \right) |\Delta \mathcal{N}|^{1/2}$$

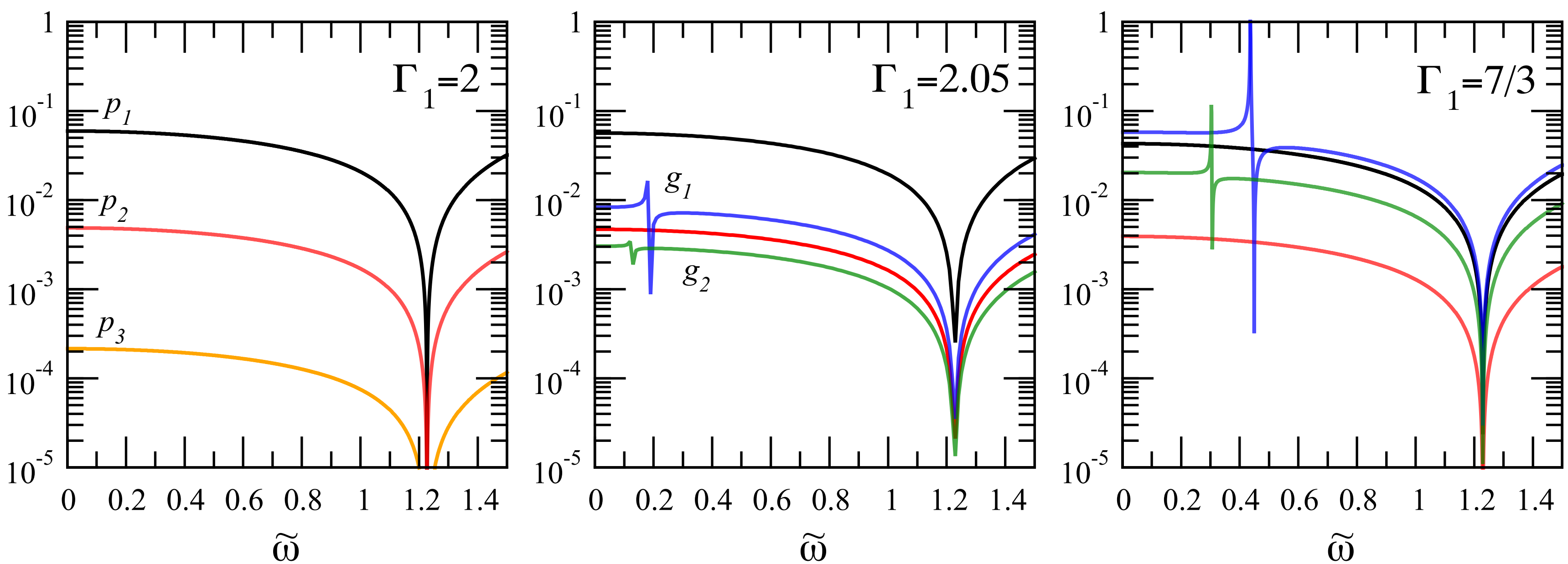
May use data to "constrain" the overlap integral. Absence of resonant feature in a given frequency interval provides an upper limit.

Alternatively, use the "fact" that the modes are complete to expand the tidal response as a mode-sum:

$$k_l = -\frac{1}{2} + \frac{1}{2R^l} \sum_n \frac{Q_n}{A_n^2} \frac{1}{\omega_n^2 - \omega^2} \left[\omega^2 V_n(R) - \frac{GM}{R^3} W_n(R) \right]$$

This gives us (first time!) an idea of the role of internal composition variations.

The effect may be at the (few) % level and could (just about…) be within reach of 3G detectors, like the Einstein Telescope or Cosmic Explorer.



[Andersson & Pnigouras]

Since the f-mode contribution dominates, we have a simple expression for the dynamical tide:

$$k_l^{\rm eff} = -\frac{1}{2} + \frac{A_f}{\tilde\omega_f^2 - \tilde\omega^2}\left[1 - \tilde\omega^2 B_f\right]\left[1 - \tilde\omega_f^2 B_f\right]^{-1}$$

This relation would be “exact” for incompressible (Newtonian) stars.

Using “universal relation” between (the relativistic) f-mode and the Love number we have a simple 2-parameter expression*:

$$k_l^{\rm eff} \approx -\frac{1}{2} + \frac{\bar\omega_f^2}{\bar\omega_f^2 - \delta(2\bar\Omega)^2}\left(k_l + \frac{1}{2}\right)\left[1 - \frac{(2\bar\Omega)^2}{\mathcal{C}^3}\frac{\epsilon}{l}\right]$$

One of the parameters represent the horizontal/radial ratio of the eigenfunction at the surface.

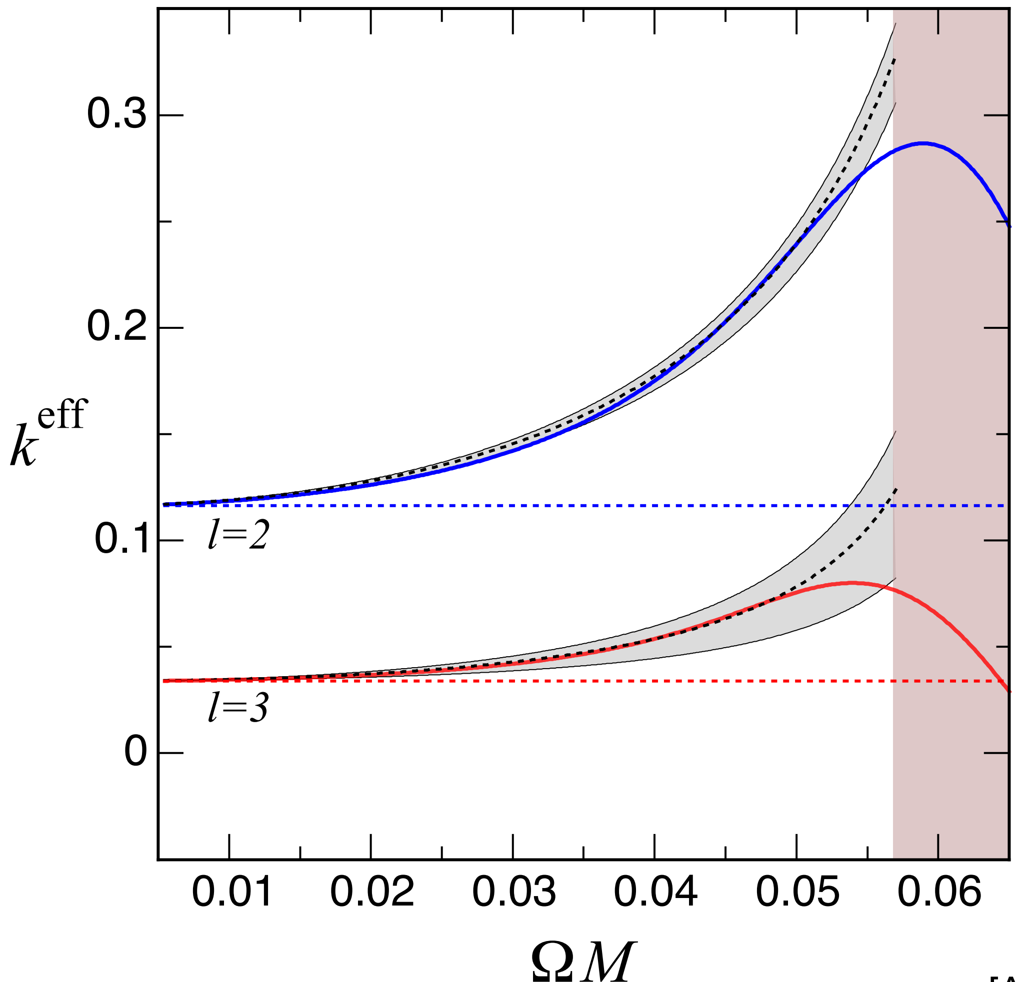
The other parameter can be used to account for redshift (and so on).

The “simplest” option (which turns out to work well…) is to simply “remove” the gravitational redshift from the mode frequency.

*In the perfect world one would like to “derive” this within relativistic perturbation theory, but…

Very simple phenomenology, but agrees well with (only) previous attempt to model dynamical tide.

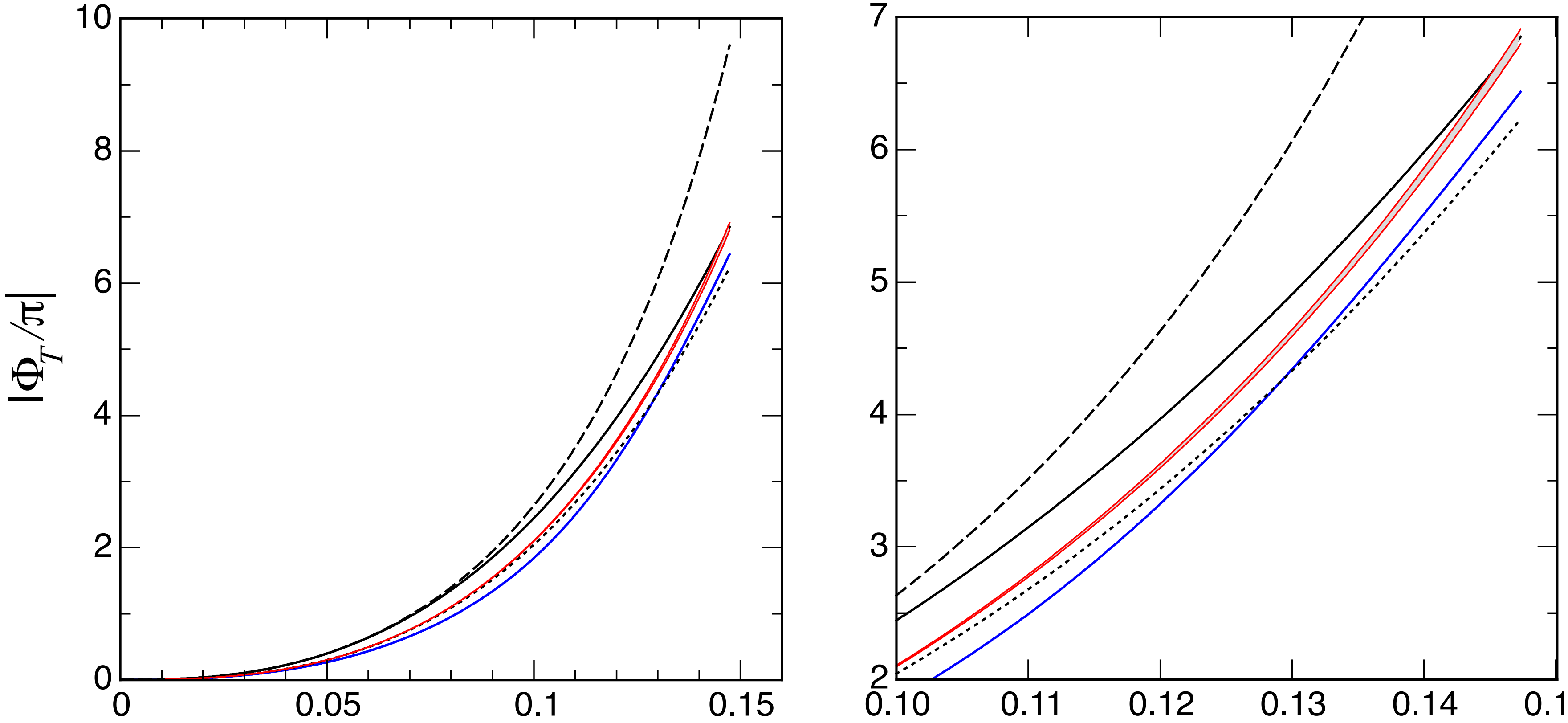
Note: Can remove divergence at resonance by Taylor expansion.



[Andersson & Pnigouras]

Could also lead to an efficient evaluation of the gravitational-wave phasing (although… need a closer look at the interface with pN terms).

$$\frac{d\Phi_T}{dx} = -\frac{65}{2^5}\frac{k_2}{\mathcal{C}^5}x^{3/2}f(x)$$



$$x = (\Omega M)^{2/3}$$

[Andersson & Pnigouras]

take home message

With increasingly sensitive instruments, observations are beginning to constrain neutron star theory…

In order to match the precision of the next generation of instruments we need "better models" (e.g. state and composition of matter).

The dynamical tide provides an instructive example.

Additional effects may not be "leading order" (e.g. internal composition) but could nevertheless introduce systematics that need to be accounted for.

Need to understand the "error budget": observations+physics.

Natural to consider the tidal response from the "seismology" point of view (not naturally expressed in terms of the pN expansion parameter).

The next steps are "hard":

- develop relativistic Lagrangian perturbation theory for tides to use realistic matter models,
- keep track of frozen composition (reactions) in numerical simulations (and get a better handle* on the "accuracy").

*In absence of "convergence", it is not clear how you "compare" simulations.