Detecting Gravitational waves on the public O1/ O2 data

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Bottom line

- We have a full fledged pipeline for detecting gravitational wave mergers.
- LVC had announced 10 BBH mergers in the first two observing runs.
- We have detected 9 more events.
 - 7 published, 2 more will join soon.
 - More surprises to come!
- We have a method for fast parameter estimation, that allows fast and cheap parameter estimation.

Similarity to LVC pipelines

- Matched filtering based (akin to pyCBC, GSTLAL)
- Coherent combination of the detectors (akin to pyCBC, GSTLAL)
- Empirical determination of final statistic (akin to GSTLAL)
- Significance based on time slides (akin to pyCBC)

Differences from LIGO data analysis

- Novel template bank construction.
- Automatic routines to detect bad data segments (`glitches") and insulate good data from them.
- Proper accounting for the non-stationary nature of the detector noise
- New signal-quality vetoes at the single-detector level, with rigorously computed False negative.
- New ways to combine results from multiple detectors.
- New way to "Fish" single detector candidates accompanied by a very weak counter signal in the other detector.
- Fast way to compute likelihood values for parameter estimation.

LIGO-VIRGO Detectors



Credit: Carruthers and Reitze (2015)





Looking for GWs



Need a noise model, simplest case is stationary Gaussian random noise

$$\langle s(t+\tau)s(t)\rangle = C(\tau)$$

FT of $C(\tau)$ is the PSD $S(f) \sim \sigma_f^2 \equiv \langle |s(f)|^2 \rangle$

No one tells you what S(f) is! Have to measure, and compare to your guess



Matched Filtering

Suppose you measure a $\sigma^2(f)$ on data d and want to look for a waveform h

Gaussian noise is uncorrelated between frequencies, so compute the inversevariance-weighted overlap of the data and the waveform in the frequency domain

$$Z(h) = \sum_{f} \frac{d(f)h^{\star}(f)}{\sigma^{2}(f)}$$

$$\rho \equiv \text{SNR} = \frac{Z(h)}{\langle Z(h)^{2} \rangle^{1/2}} = \frac{\frac{\sum_{f} d(f)h^{\star}(f)}{\sigma^{2}(f)}}{\left[\frac{\sum_{f} |h(f)|^{2}}{\sigma^{2}(f)}\right]^{1/2}}$$

If everything is OK, the SNR² should be distributed according to a chi-squared distribution

The PSD drifts



PSD drift correction

- PSD estimation requires ~1000s of seconds
- But PSD changes on time-scales of 10s of seconds.
- What happens when adding variables with incorrect coefficients?
- SNR is reduced by $O(\epsilon^2)$
- But standard deviation changes by

 σ_Z $\frac{|h(f)|^2}{S_n(f)}$ $S_{\rm opt} = X1 + X2$ $S_{\text{practical}} = X1 + (1 + \epsilon)X2$ ϵ)

Trigger distributions



What do we do with holes?

- Must not "remove samples" Lines would leak out
- Rephrasing of the problem:
 - Insertion of an infinitely loud white noise process added to the bad segment.
 - Solve the least squares linear algebra problem of measuring amplitude.
 - Identify the relevant data equivalent
 - Solution: Inpaint the samples in the bad segment to the value expected by the rest of the data

Hole correction in practice



GW170817 original data



Applied to GW170817

Vetoing Candidates

- Check for consistency between frequency bands
- Promise:
 - False negative rate is smaller than 1% on Gaussian noise
 - Robust to PSD drift and to template-bank inefficiency



Ranking Score



Coherent Score

Rank background + candidates according to the ratio

$$p(\rho_1^2, \rho_2^2, \Delta t, \Delta \phi | H1)$$

$$p(\rho_1^2,\rho_2^2,\Delta t,\Delta\phi\,|\,H\!0)$$

Account for the different sensitivities of the detectors, etc We use Monte-Carlo estimates for the numerator

$$h_{+} = A(1 + \cos^{2} i) \cos \phi_{GW}$$

$$h_{\times} = -2A \cos i \sin \phi_{GW}$$

$$h = h_{+}F_{+} + h_{\times}F_{\times}$$

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Coherent Score



Splitting results to banks

| Bank | $ m_1({ m M}_\odot) $ | $m_2({ m M}_\odot)$ | $\mathcal{M}\left(M_{\odot}\right)$ | q_{\min} | $\left \chi_{1,2}\right _{\max}$ | $ \zeta$ | Δc_{α} | $ N_{\rm subbanks} $ | $d_{ m subbanks}$ | $L_{\rm max, subbanks}$ | $N_{\mathrm{templates}}$ |
|--------|-----------------------|---------------------|-------------------------------------|------------|----------------------------------|----------|---------------------|----------------------|-------------------|-------------------------|--------------------------|
| BNS O | | | < 1.1 | | | | | 1 | 2 | 777.0 | 48806 |
| BNS 1 | (1,3) | (1,3) | (1.1, 1.3) | | 0.99 | 0.05 | 0.55 | 1 | 2 | 434.3 | 23856 |
| BNS 2 | | | > 1.3 | | | | | 1 | 2 | 824.6 | 43781 |
| NSBH 0 | | | < 3 | | | | | 1 | 4 | 753.4 | 84641 |
| NSBH 1 | (3, 100) | (1,3) | (3,6) | 1/50 | 0.99 | 0.05 | 0.5 | 2 | 6, 6 | 259.5, 166.8 | 85149 |
| NSBH 2 | | | > 6 | | | | | 3 | 5, 4, 4 | 87.5, 61.2, 9.4 | 15628 |
| BBH 0 | | | < 5 | | | | 0.55 | 1 | 3 | 270.6 | 8246 |
| BBH 1 | | | (5, 10) | | | | 0.55 | 2 | 4, 4 | 113.7, 50.0 | 4277 |
| BBH 2 | (3, 100) | (3, 100) | (10, 20) | 1/18 | 0.99 | 0.05 | 0.5 | 3 | 3,4,3 | 41.5, 33.5, 10.3 | 1607 |
| BBH 3 | | | (20, 40) | | | | 0.45 | 3 | 2, 2, 2 | 11.7, 10.8, 4.9 | 225 |
| BBH 4 | | | > 40 | | | | 0.35 | 5 | 2, 2, 2, 1, 1 | 2.9, 2.0, 1.1, 0.7, 0.1 | 46 |
| Total | | | | | | | | | | | 316262 |
| | | | | | | | | | | | |

- BBH3 has 40 times fewer templates than BBH0 while having most events.
- Prevalence of glitches is very different
- Data cleaning thresholds (and approaches) very different



Volume improvement



New Events!

New event in O1!



| | Flat χ_{eff} prior | Isotropic spin prior |
|---|-----------------------------------|---------------------------------|
| Chirp mass \mathcal{M}^{det} | $31^{+2}_{-3} M_{\odot}$ | $29^{+2}_{-2}M_{\odot}$ |
| Primary mass m_1 | $31^{+13}_{-6} M_{\odot}$ | $38^{+11}_{-11}M_{\odot}$ |
| Secondary mass m_2 | $21^{+5}_{-6} M_{\odot}$ | $16^{+6}_{-3}M_{\odot}$ |
| Mass ratio m_1/m_2 | $1.5^{+1.4}_{-0.4}$ | $2.4^{+1.4}_{-1.1}$ |
| Total mass M | $52^{+9}_{-6} M_{\odot}$ | $54^{+10}_{-8} M_{\odot}$ |
| Primary aligned spin χ_{1z} | $0.86\substack{+0.12 \\ -0.27}$ | $0.73\substack{+0.18 \\ -0.28}$ |
| Secondary aligned spin χ_{2z} | $0.79\substack{+0.19 \\ -0.65}$ | $0.30\substack{+0.51 \\ -0.46}$ |
| Effective aligned spin $\chi_{\rm eff}$ | $0.81\substack{+0.15 \\ -0.21}$ | $0.60\substack{+0.16 \\ -0.18}$ |
| Cosine of inclination $ \cos \iota $ | $0.81\substack{+0.18 \\ -0.52}$ | $0.81\substack{+0.18 \\ -0.51}$ |
| Luminosity distance D_L | $2.4^{+1.2}_{-1.1}\mathrm{Gpc}$ | $2.1^{+1.0}_{-0.9}\mathrm{Gpc}$ |
| Source redshift z | $0.43_{-0.17}^{+0.17}$ | $0.38\substack{+0.15\\-0.15}$ |

• Highest spinning system so far !

Six new events in O2!

| Name | Bank | GPS time ^a | $\rho_{\rm H}^2$ | $ ho_{ m L}^2$ | $ FAR^{-1}(O2)^{b} $ | $\frac{W(\text{event})}{\mathcal{R}(\text{event} \mathcal{N})}$ (O2) | $p_{\rm astro}$ |
|----------|-----------|-----------------------|------------------|----------------|----------------------|--|-----------------|
| GW170104 | BBH (3,0) | 1167559936.582 | 85.1 | 104.3 | $> 2 \times 10^4$ | > 100 | > 0.99 |
| GW170809 | BBH (3,0) | 1186302519.740 | 40.5 | 113 | $> 2 \times 10^4$ | > 100 | > 0.99 |
| GW170814 | BBH (3,0) | 1186741861.519 | 90.2 | 170 | $> 2 \times 10^4$ | > 100 | > 0.99 |
| GW170818 | BBH (3,0) | 1187058327.075 | 19.4 | 95.1 | 1.7^{c} | | c |
| GW170729 | BBH (3,1) | 1185389807.311 | 62.1 | 53.6 | $> 2 \times 10^4$ | > 100 | > 0.99 |
| GW170823 | BBH (3,1) | 1187529256.500 | 46.0 | 90.7 | $> 2 \times 10^4$ | > 100 | > 0.99 |

| Name | Bank | $\Big \mathcal{M}^{\rm det}(M_\odot)$ | $\chi_{ m eff}$ | z | $GPS time^{a}$ | $ ho_{ m H}^2$ | $ ho_{ m L}^2$ | $FAR^{-1}(O2)^{b}$ | $\frac{W(\text{event})}{\mathcal{R}(\text{event} \mathcal{N})}$ (O2) | $p_{ m astro}$ |
|----------|-----------|---------------------------------------|----------------------|------------------------|----------------|----------------|----------------|--------------------|--|----------------|
| GW170121 | BBH (3,0) | 29^{+4}_{-3} | $-0.3^{+0.3}_{-0.3}$ | $0.24_{-0.13}^{+0.14}$ | 1169069154.565 | 29.4 | 89.7 | 2.8×10^3 | > 30 | > 0.99 |
| GW170304 | BBH (4,0) | 47^{+8}_{-7} | $0.2^{+0.3}_{-0.3}$ | $0.5^{+0.2}_{-0.2}$ | 1172680691.356 | 24.9 | 55.9 | 377 | 13.6 | 0.985 |
| GW170727 | BBH (4,0) | 42^{+6}_{-6} | $-0.1^{+0.3}_{-0.3}$ | $0.43^{+0.18}_{-0.17}$ | 1185152688.019 | 25.4 | 53.5 | 370 | 11.8 | 0.98 |
| GW170425 | BBH (4,0) | 47^{+26}_{-10} | $0.0^{+0.4}_{-0.5}$ | $0.5^{+0.4}_{-0.3}$ | 1177134832.178 | 28.6 | 37.5 | 15 | 0.65 | 0.77 |
| GW170202 | BBH (3,0) | $21.6^{+4.2}_{-1.4}$ | $-0.2^{+0.4}_{-0.3}$ | $0.27_{-0.12}^{+0.13}$ | 1170079035.715 | 26.5 | 41.7 | 6.3 | 0.25 | 0.68 |
| GW170403 | BBH (4,1) | 48^{+9}_{-7} | $-0.7^{+0.5}_{-0.3}$ | $0.45_{-0.19}^{+0.22}$ | 1175295989.221 | 31.3 | 31.0 | 4.7 | 0.23 | 0.56 |

Fishing two more signals!

- Framework does not apply to shaded area.
- Criterion for ranking candidates according to L1 alone - number of "similar" glitches observed.
- Exact evidence integral over the sky





More events!

| L1 based rank | GPS time | $ ho_L^2$ | # similar glitches | Bank ID | $C(\mathcal{S} H_0)$ | $C(\mathcal{S} H_1)$ | $\frac{P(\mathcal{S} H_1)}{P(\mathcal{S} H_0)}$ | Comment |
|---------------------------------------|----------------|-----------|--------------------|---------|----------------------|----------------------|---|-------------------|
| 1 | 1187058327.068 | 95.1 | 0 | (3, 0) | $< 10^{-3}$ | 0.16 | 37 | $ m GW170818^{a}$ |
| 2 | 1187529256.504 | 92.4 | 0 | (4, 0) | 0 | - | - | GW170823 |
| 3 | 1169069154.564 | 89.7 | 0 | (3, 0) | 0 | - | - | GW170121 |
| 4 | 1186741861.51 | 172.5 | 1 | (3, 0) | 0 | - | - | GW170814 |
| 5 | 1186302519.731 | 114.7 | 1 | (3, 0) | 0 | - | - | GW170809 |
| 6 | 1167559936.584 | 105.7 | 1 | (3, 0) | 0 | - | - | GW170104 |
| 7 | 1175205128.565 | 74.1 | 1 | (3, 0) | 0.015 | 0.022 | 0.547 | GW170402 |
| 8 | 1186974184.716 | 100.6 | 5 | (4, 2) | 0.028 | 0.055 | 0.98 | GW170817B |
| 9 | 1174043898.842 | 74.3 | 9 | (4, 3) | 0.36 | 0.001 | 0.008 | - |
| 10 | 1181567025.654 | 66.6 | 12 | (3, 1) | 0.34 | 0.009 | 0.015 | - |
| 11 | 1178083239.592 | 75.9 | 24 | (3, 1) | 0.34 | 0.003 | 0.016 | - |
| 12 | 1170885005.109 | 65.3 | 25 | (3, 1) | 0.49 | 0.003 | 0.013 | - |
| $\operatorname{Removed}^{\mathrm{b}}$ | 1173477193.704 | 73.8 | 1 | (3, 1) | 0.38 | 0.014 | 0.011 | Artifacts present |

And they are highly spinning!



Summary

- The availability of the LIGO data gives the community an opportunity to try new ideas and propose new methods. We are very grateful to the LVC.
- We have developed a new, independent pipeline for detecting (and PE) GW mergers, that we estimate it has twice the detection volume compared to previous pipelines.
- We have found 9 new GW events in O2 data.
 - Two are very highly spinning strong indication for non-dynamical formation.
 - Highest mass observed to date
 - An event with substantial negative spin.
- Ongoing:
 - Population study
 - BNS/NSBH searches

Do you see what I see?



Relative Binning Parameter Estimation Template Bank Construction

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Parameter estimation for BNS

Why is parameter estimation for BNS mergers hard?

$$Z(d,h,\Delta t) = 4\sum_{f} \frac{d(f)h^*(f)}{S_n(f)/T} e^{if\Delta t}$$

Frequency resolution set by $T \gtrsim 2 \min$ for O2, and $f_{\max} = 2048 \,\text{Hz}$, say.

If we pick T = 256 s, we have $N \approx 2^{20} \approx 10^6$ frequencies.

The FFT requires $O(N \log N) \approx 10^7$ flops.

Parameter estimation requires us to evaluate the FFT above for many ($\sim 10^8$) parameter choices

Speeding up parameter estimation

- Speeding up template generation
 - Analytical frequency domain waveforms
 - Reduced order modeling (e.g., Pürrer 2014).
 - Multi-band interpolation (Vinciguerra, S., et. al., (2018))
- Speeding up matched filtering
 - Reduced order quadrature (Smith, R., et. al. (2016))
- Speeding up the sampling method
 - Separating variables as much as possible.
 - Using sophisticated sampling methods.

Are CBC templates smooth?

- Not in time domain
- Not in frequency domain

 $A(t)e^{i(\alpha_1t^{\gamma_1}+\alpha_2t^{\gamma_2}+\dots)}$ $A(f)e^{i(\alpha_1f^{\gamma_1}+\alpha_2f^{\gamma_2}+\dots)}$

• But when you **phase unwrap**:

$$\phi(f) = (\alpha_1 f^{\gamma_1} + \alpha_2 f^{\gamma_2} + ...)$$

Waveform with high overlap are "similar"

- Not in physical parameters that's why we need parameter estimation
- Rule of thumb: templates with high overlap do not lose phase (inside the sensitive band)

$$|\phi(f) - \phi_0(f)| < 1$$

$$\frac{A(f)}{A_0(f)}e^{i(\phi(f)-\phi_0(f))} = \frac{A(f)}{A_0(f)}e^{i((\alpha-\alpha_0)f^{\gamma}+\dots}$$

Template ratios are smooth and low dimensional



Relative binning

Compute
$$r(f) = \frac{h(f)}{h_0(f)} = r_0(h, b) + r_1(h, b) (f - f_m(b)) + \cdots$$

on a coarse set of frequency bins, for a fiducial template

$$Z(d,h) = 4 \sum_{f} \frac{d(f)h^{*}(f)}{S_{n}(f)/T} \approx \sum_{b} \left[A_{0}(b) r_{0}^{*}(h,b) + A_{1}(b) r_{1}^{*}(h,b) \right]$$

where
$$A_{0}(b) = 4 \sum_{f \in b} \frac{d(f) h_{0}^{*}(f)}{S_{n}(f)/T}$$
$$A_{1}(b) = 4 \sum_{f \in b} \frac{d(f) h_{0}^{*}(f)}{S_{n}(f)/T} (f - f_{m}(b))$$

are computed once and for all on a fine frequency grid.

Choosing bin-boundaries to control approximation accuracy

- No reason to chose equal spacing
- Spacing chosen such that total deviation of each component in the PN expansion to < 0.5 rad.

$$\phi(f) = (\alpha_1 f^{\gamma_1} + \alpha_2 f^{\gamma_2} + ...)$$

 Assume that a waveform is sampled by parameter estimation if

$$|\phi(f) - \phi_0(f)| < 2\pi$$

Relative binning





Building a template bank using this understanding

- Can apply "reduced order modeling" on phase unwrapped waveforms
 - Trivially low dimension.
 - Very few examples required.
- Can build optimally gridded template banks by applying SVD on unwrapped waveform phases.
 - Similar to Tanaka & Tagoshi (2000).
 - Does not require analytical understanding of $\phi(f)$

Outlook

- Parameter estimation speed up at little cost for any reasonable accuracy.
- Will become essential for longer waveforms. O3 BNS, A+, ET, LISA parameter estimation.
 - Without relative binning $N_{\rm bins} \sim N_{\rm cycles} \sim 10^6$
 - With relative binning -

 $N_{\rm bins} \sim 60$

Refs:

Zackay, B. Dai, L., TV, arXiv:1806.08792 Dai, L., TV, Zackay B., arXiv:1806.08793 Code at <u>https://bitbucket.org/dailiang8/gwbinning</u>