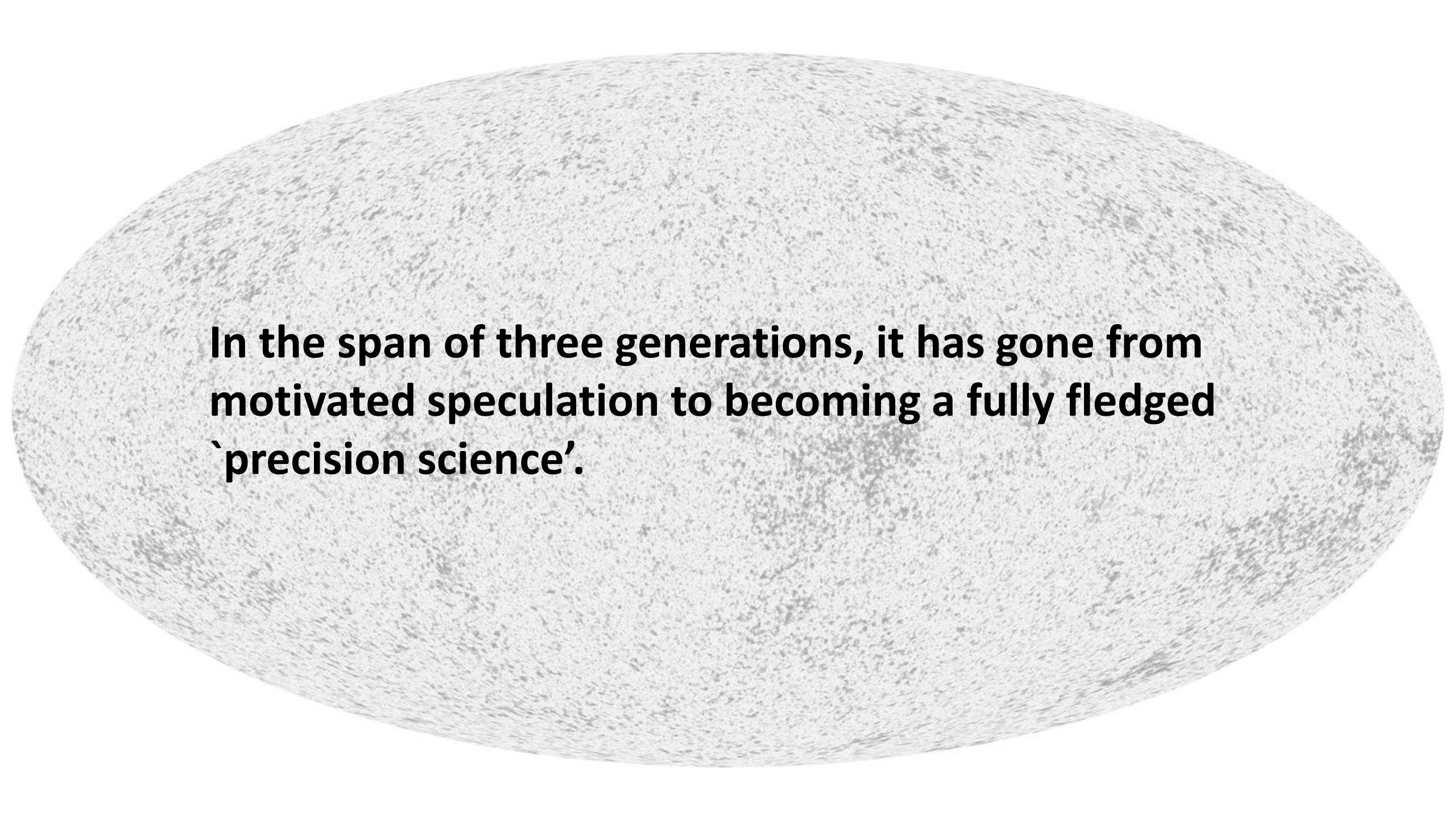


Subodh Patil
NBIA winter school
Jan 3 2019

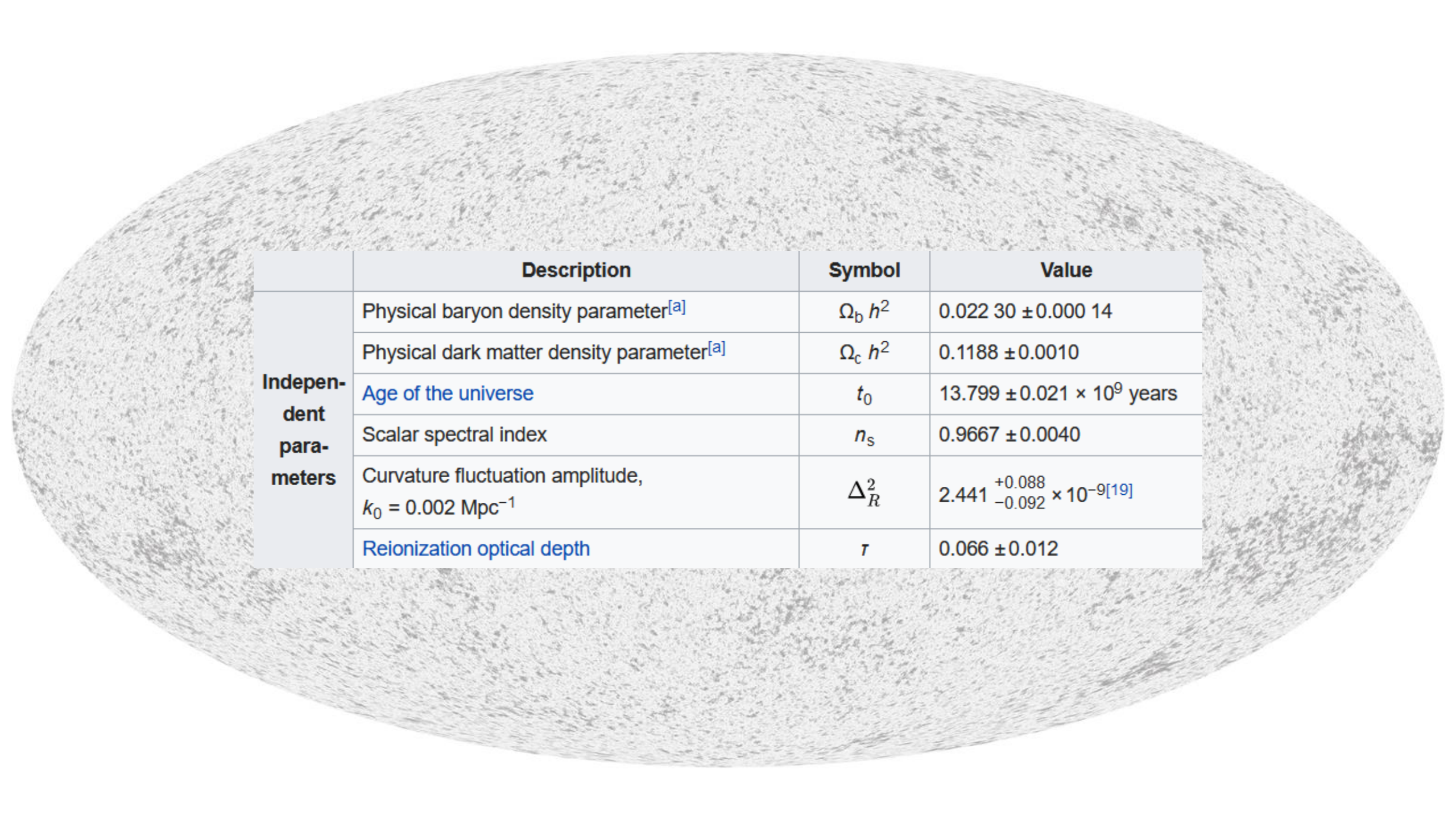
Introduction to Cosmology and Structure Formation



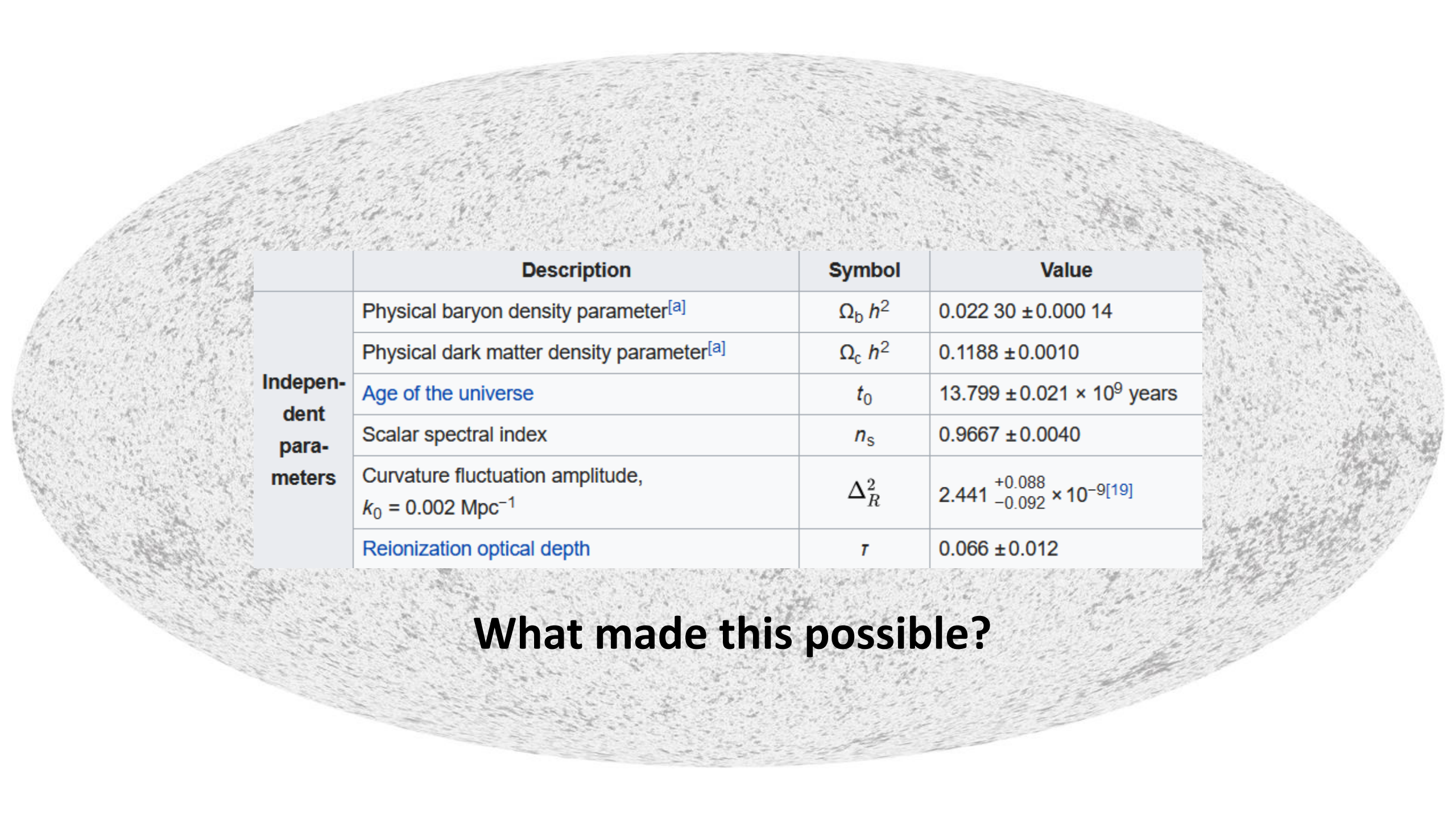
Cosmology is the study of the origin and evolution of the universe.



In the span of three generations, it has gone from motivated speculation to becoming a fully fledged `precision science`.



	Description	Symbol	Value
Independent parameters	Physical baryon density parameter ^[a]	$\Omega_b h^2$	$0.022\,30 \pm 0.000\,14$
	Physical dark matter density parameter ^[a]	$\Omega_c h^2$	0.1188 ± 0.0010
	Age of the universe	t_0	$13.799 \pm 0.021 \times 10^9$ years
	Scalar spectral index	n_s	0.9667 ± 0.0040
	Curvature fluctuation amplitude, $k_0 = 0.002 \text{ Mpc}^{-1}$	Δ_R^2	$2.441^{+0.088}_{-0.092} \times 10^{-9}$ ^[19]
	Reionization optical depth	τ	0.066 ± 0.012



	Description	Symbol	Value
Independent parameters	Physical baryon density parameter ^[a]	$\Omega_b h^2$	$0.022\,30 \pm 0.000\,14$
	Physical dark matter density parameter ^[a]	$\Omega_c h^2$	0.1188 ± 0.0010
	Age of the universe	t_0	$13.799 \pm 0.021 \times 10^9$ years
	Scalar spectral index	n_s	0.9667 ± 0.0040
	Curvature fluctuation amplitude, $k_0 = 0.002 \text{ Mpc}^{-1}$	Δ_R^2	$2.441^{+0.088}_{-0.092} \times 10^{-9}$ ^[19]
	Reionization optical depth	τ	0.066 ± 0.012

What made this possible?

Observations!

- To provide these a consistent theoretical framework, we required—
- A theory of spacetime and its interactions with matter (GR, deviations?)
- A theory of matter (Fields, particles, defects, etc.)

Observations!

- Symmetries: homogeneity, isotropy, thermal equilibrium (a statement of symmetry in state space: equipartition.)
- Luck. e.g. current model suggest at $t \sim 10^{12}$ y, $\lambda_{\text{CMB}} > H_0^{-1}$ i.e. no electromagnetic evidence of the big bang (!)

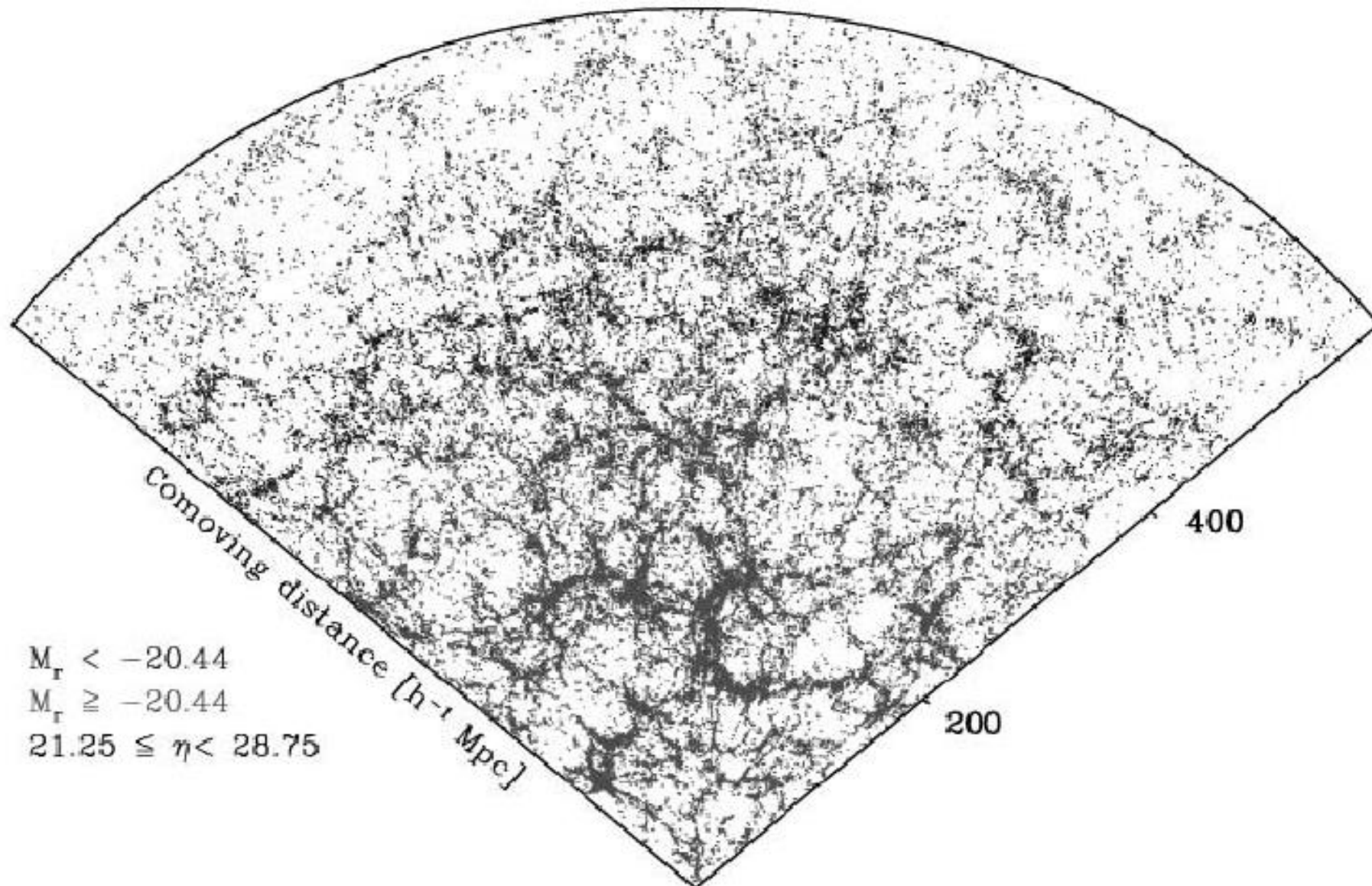
A large, textured, light gray oval shape, possibly representing a planet or a large rock, with a question overlaid. The texture is grainy and uneven, suggesting a rocky or metallic surface. The oval is centered horizontally and vertically on a white background.

What are these observations?

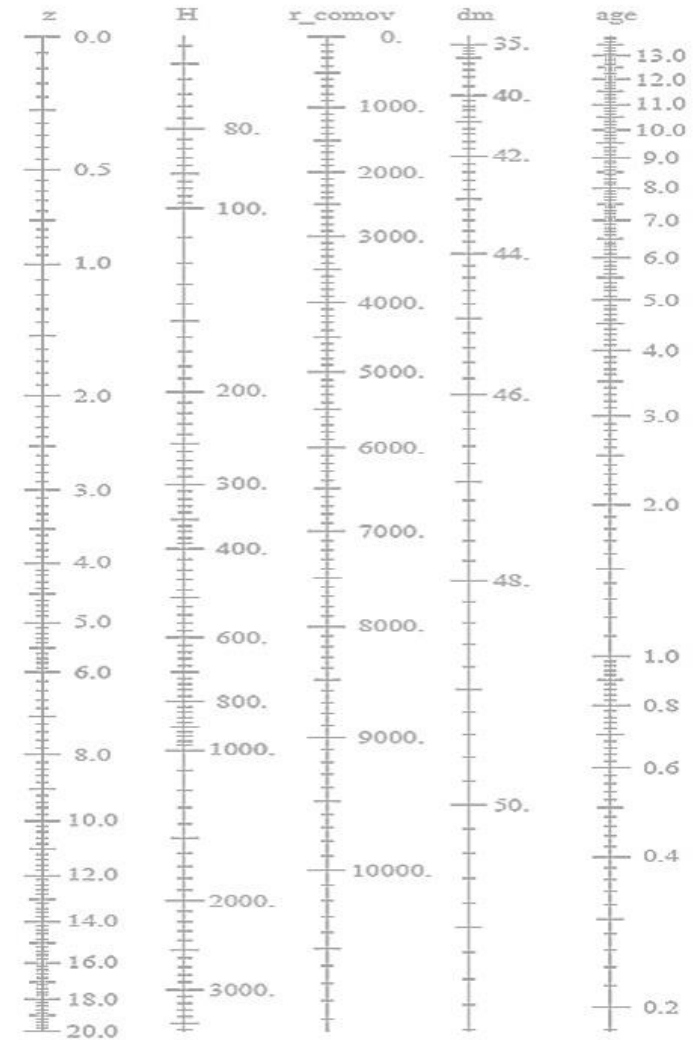
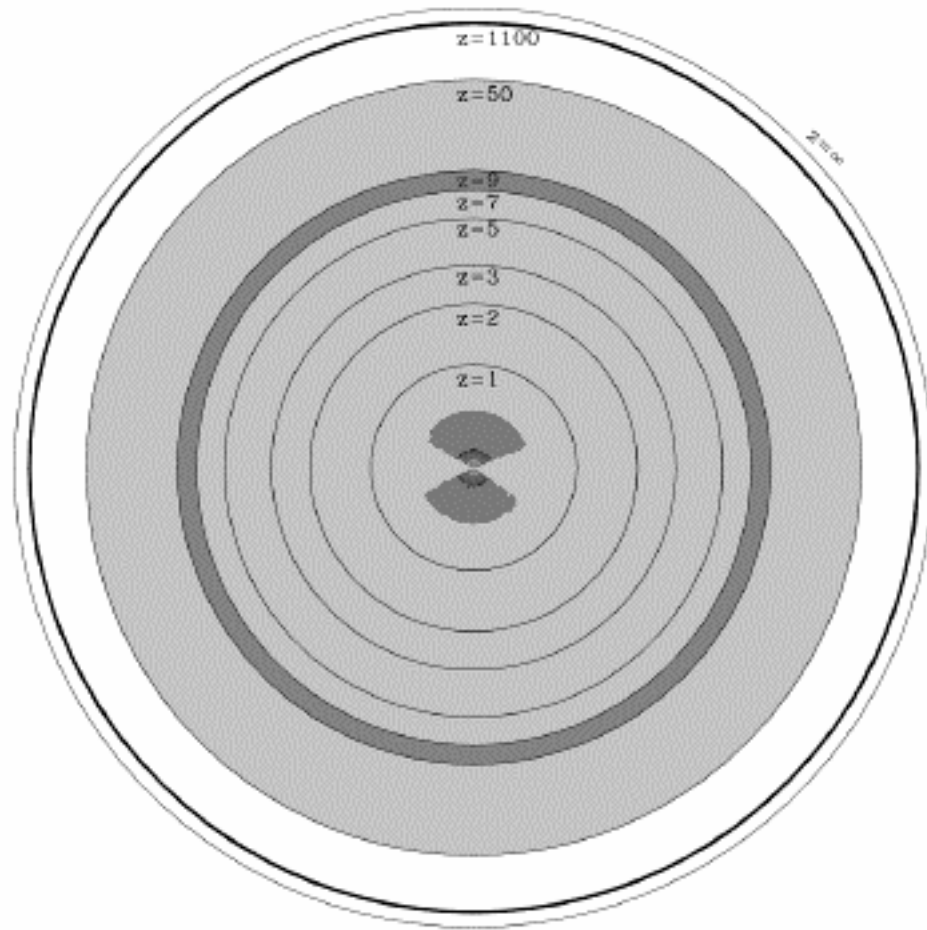
The image displays a full-sky map of the Cosmic Microwave Background (CMB) temperature fluctuations. The map is presented in a Mollweide projection, which is a common way to represent the entire sky on a flat surface. The color scale represents temperature variations, with blue indicating slightly cooler regions and orange/red indicating slightly warmer regions. The fluctuations are most prominent at the poles of the map, where the temperature is lower, and become less pronounced towards the equator. The overall pattern is a complex, grainy texture of small-scale variations, characteristic of the CMB's anisotropy.

The Cosmic Microwave Background

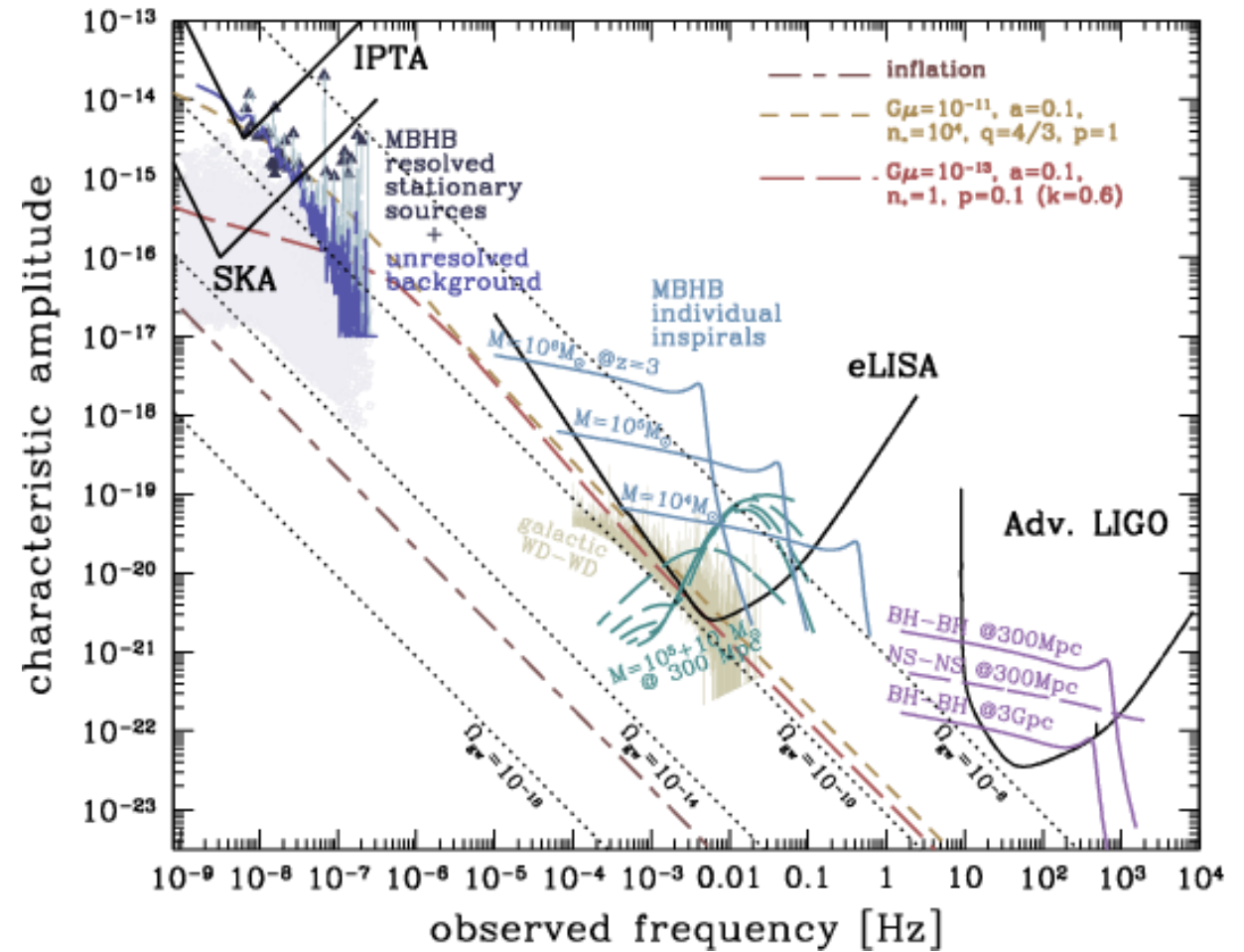
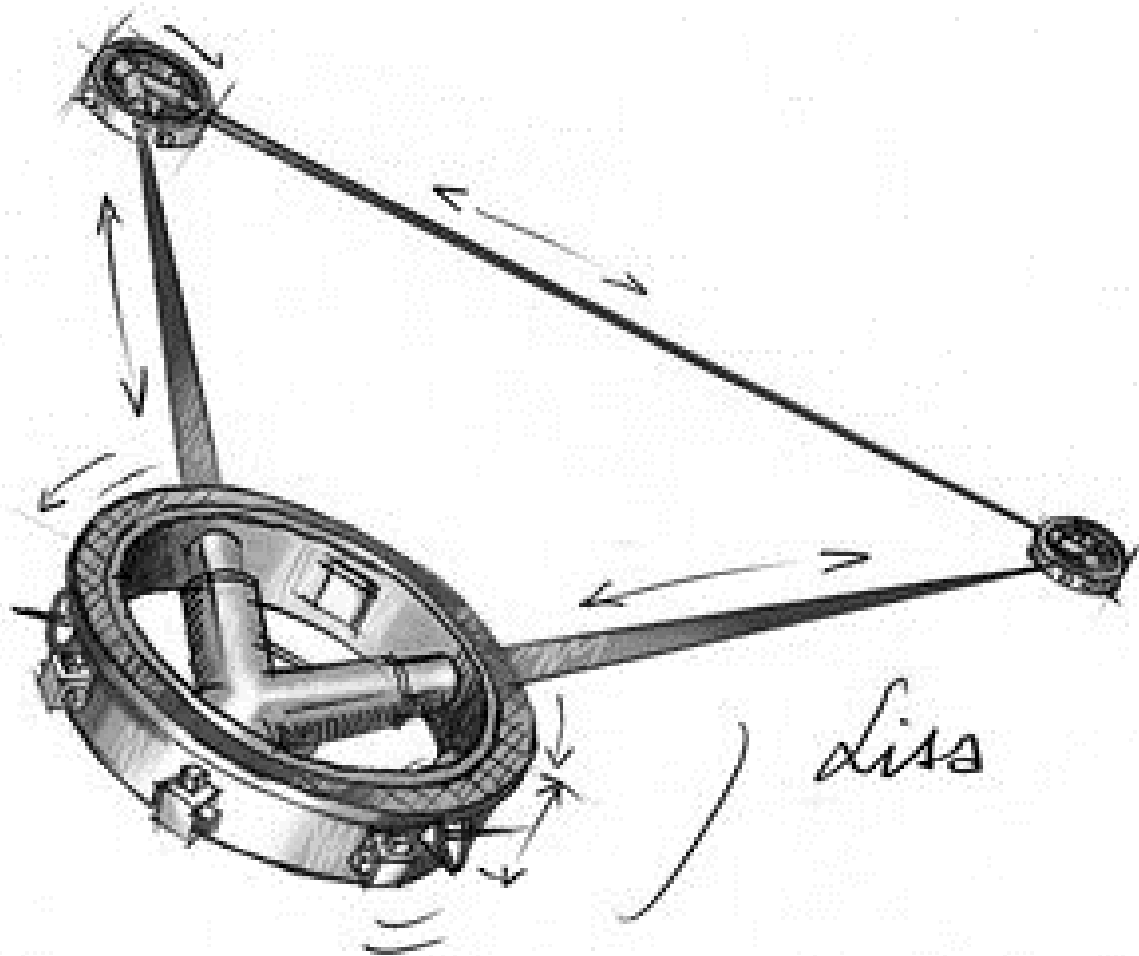
Large Scale Structure Surveys



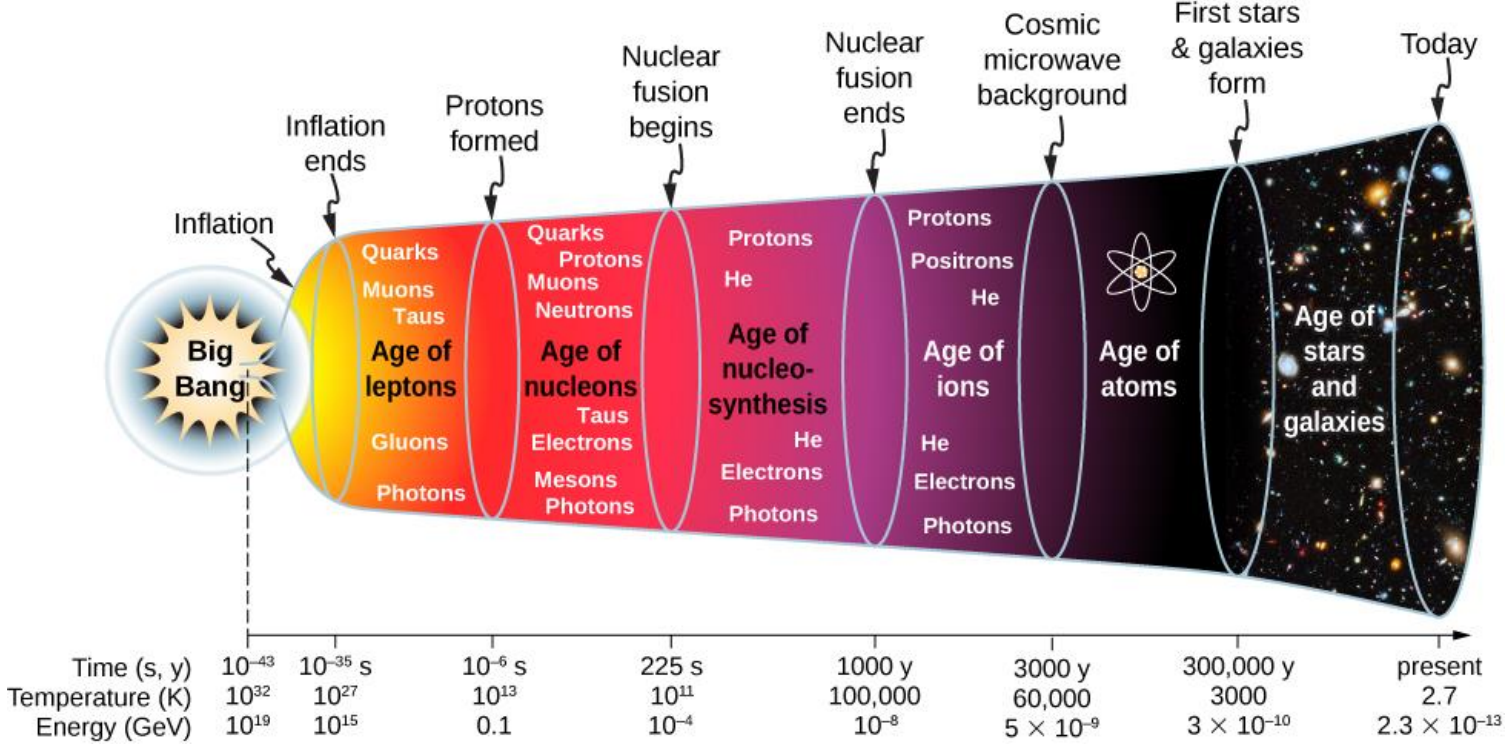
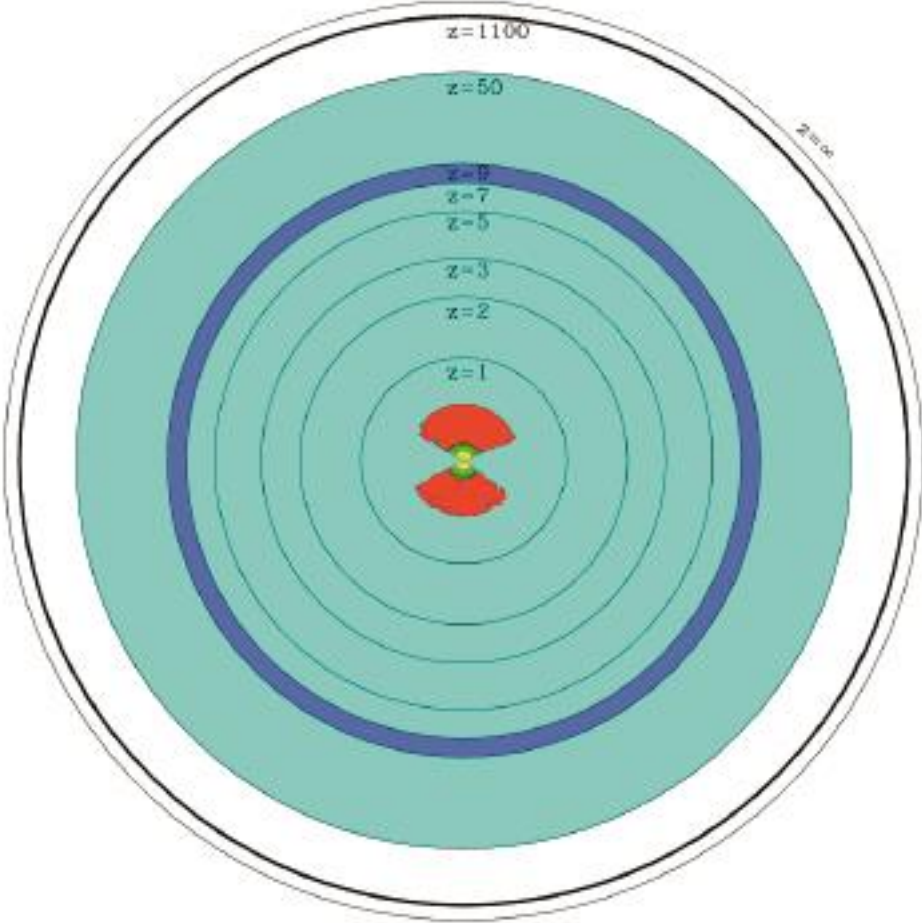
(The future: for things that don't shine at us – 21 cm Tomography?)



(The future cont: Gravitational Waves... cf. Tanja Hinderer's lectures)

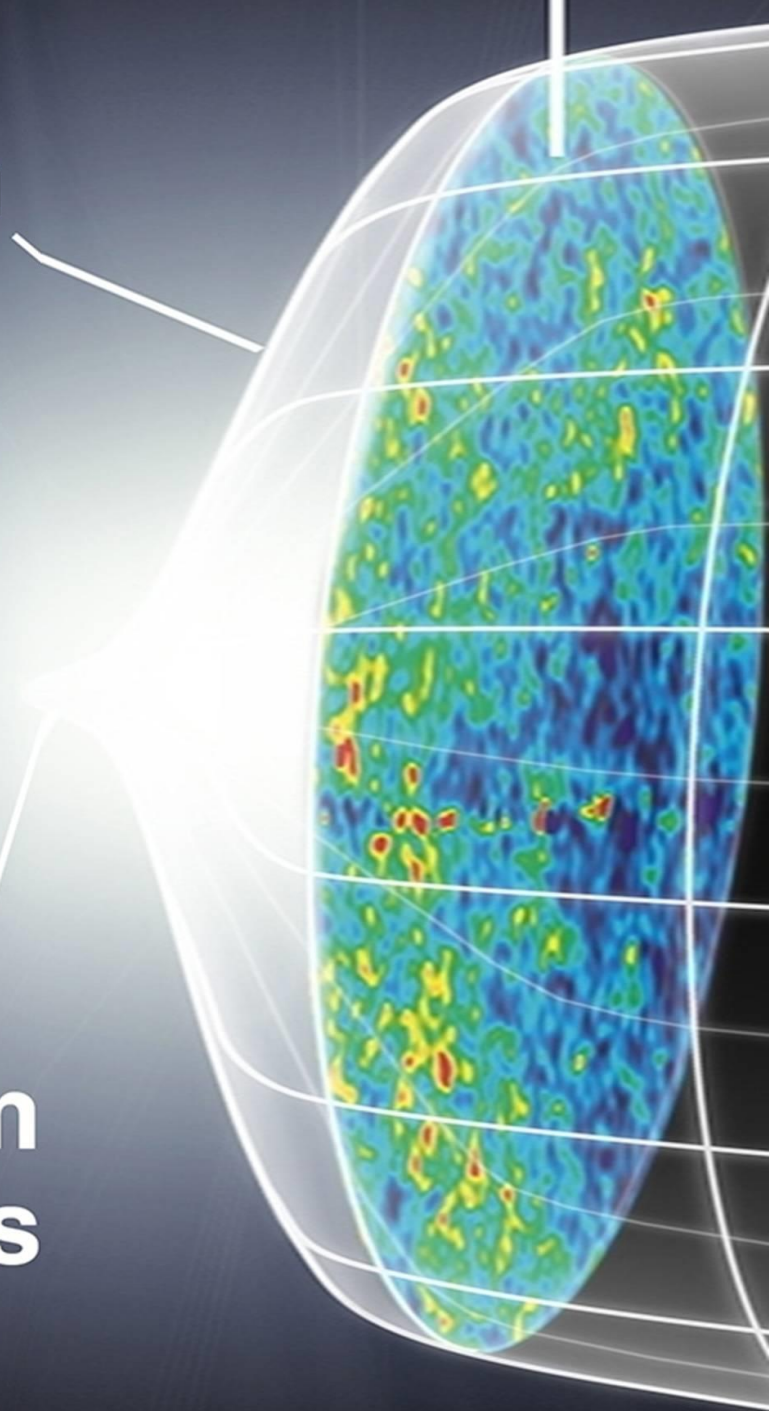


All observed structure is the result of gravitational collapse of initial 'seed' perturbations...

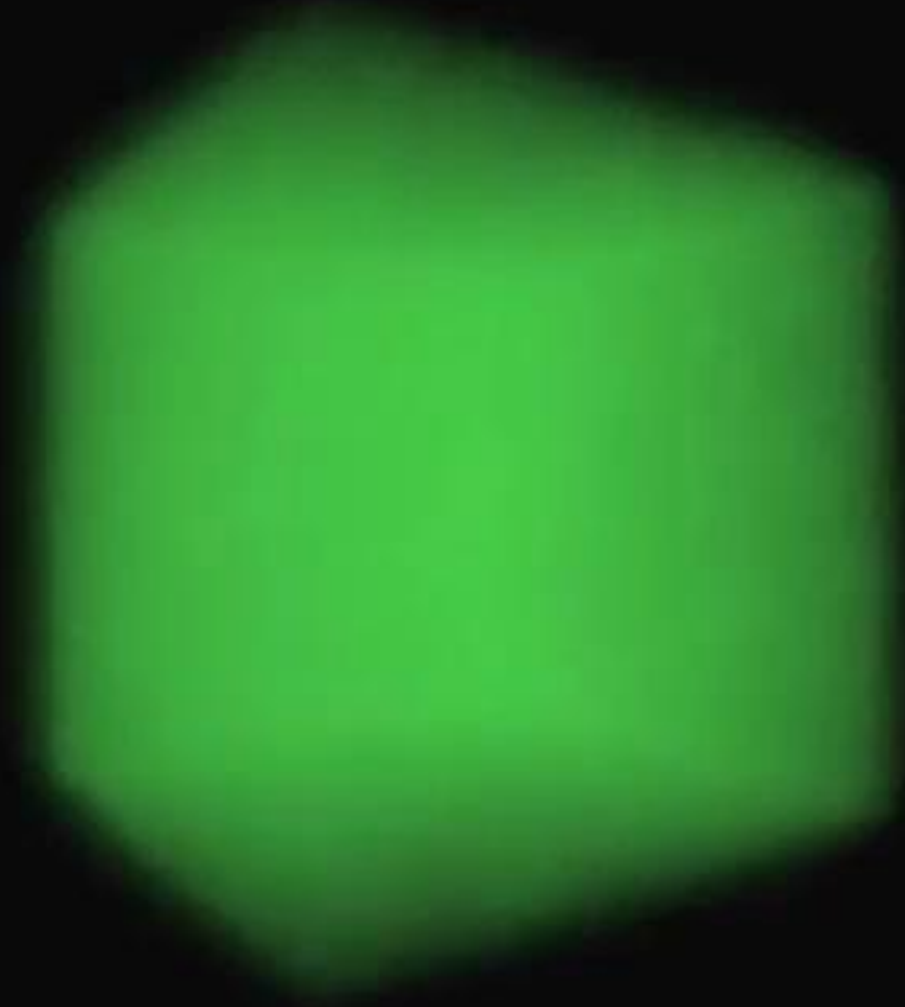


Inflation

**Quantum
Fluctuations**



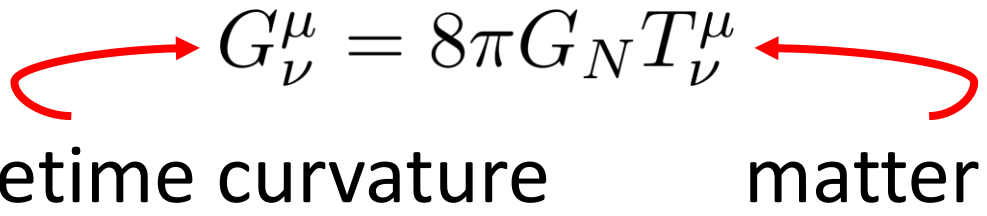
redshift = 24.2



The background – General Relativity + perfect fluids

$$G_{\nu}^{\mu} = 8\pi G_N T_{\nu}^{\mu}$$

spacetime curvature matter



$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \mathcal{L}_M$$

metric tensor $ds^2 = g_{\mu\nu}(x) dx^{\mu} dx^{\nu}$

General Relativity – a conceptual crash course

metric tensor $ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$

(Minkowski space: $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2$)

$$S = -m \int ds = -m \int \sqrt{-g_{\mu\nu}dx^\mu dx^\nu}$$

General Relativity – a conceptual crash course

metric tensor $ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$

(Minkowski space: $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2$)

$$S = -m \int ds = -m \int d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}$$

`affine parameter`

General Relativity – a conceptual crash course

metric tensor $ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$

(Minkowski space: $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2$)

$$S = -m \int ds = -m \int d\tau \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}$$

 `affine parameter`

Minkowski: $p_\mu = \frac{\delta \mathcal{L}}{\delta \dot{x}^\mu} = \frac{m}{\sqrt{1-\dot{\vec{x}}^2}} \frac{dx_\mu}{dt}$ e.o.m: $\frac{dp_\mu}{d\tau} = 0$

General Relativity – a conceptual crash course

metric tensor $ds^2 = g_{\mu\nu}(x)dx^\mu dx^\nu$

(Minkowski space: $ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu = -dt^2 + dx^2 + dy^2 + dz^2$)

$$S = -m \int ds = -m \int d\tau \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}$$

e.o.m on a curved background:

$$\frac{dx^\mu}{d\tau} := u^\mu; \quad \frac{du^\mu}{d\tau} + \Gamma_{\nu\lambda}^\mu u^\nu u^\lambda = 0$$

$$\Gamma_{\nu\lambda}^\mu := \frac{g^{\mu\kappa}}{2} \left(\frac{dg_{\kappa\nu}}{dx^\lambda} + \frac{dg_{\kappa\lambda}}{dx^\nu} - \frac{dg_{\nu\lambda}}{dx^\kappa} \right)$$

General Relativity – a conceptual crash course

e.o.m on a curved background:

$$\frac{dx^\mu}{d\tau} := u^\mu; \quad \frac{du^\mu}{d\tau} + \Gamma_{\nu\lambda}^\mu u^\nu u^\lambda = 0$$

$$\Gamma_{\nu\lambda}^\mu := \frac{g^{\mu\kappa}}{2} \left(\frac{dg_{\kappa\nu}}{dx^\lambda} + \frac{dg_{\kappa\lambda}}{dx^\nu} - \frac{dg_{\nu\lambda}}{dx^\kappa} \right)$$

which is equivalent to $u^\mu \nabla_\mu u^\nu = 0$, where we have defined the *covariant derivative*:

$$\nabla_\mu u^\nu := \partial_\mu u^\nu + \Gamma_{\mu\lambda}^\nu u^\lambda$$

General Relativity – a conceptual crash course

e.o.m on a curved background:

$$\frac{dx^\mu}{d\tau} := u^\mu; \quad \frac{du^\mu}{d\tau} + \Gamma_{\nu\lambda}^\mu u^\nu u^\lambda = 0$$

$$\Gamma_{\nu\lambda}^\mu := \frac{g^{\mu\kappa}}{2} \left(\frac{dg_{\kappa\nu}}{dx^\lambda} + \frac{dg_{\kappa\lambda}}{dx^\nu} - \frac{dg_{\nu\lambda}}{dx^\kappa} \right)$$

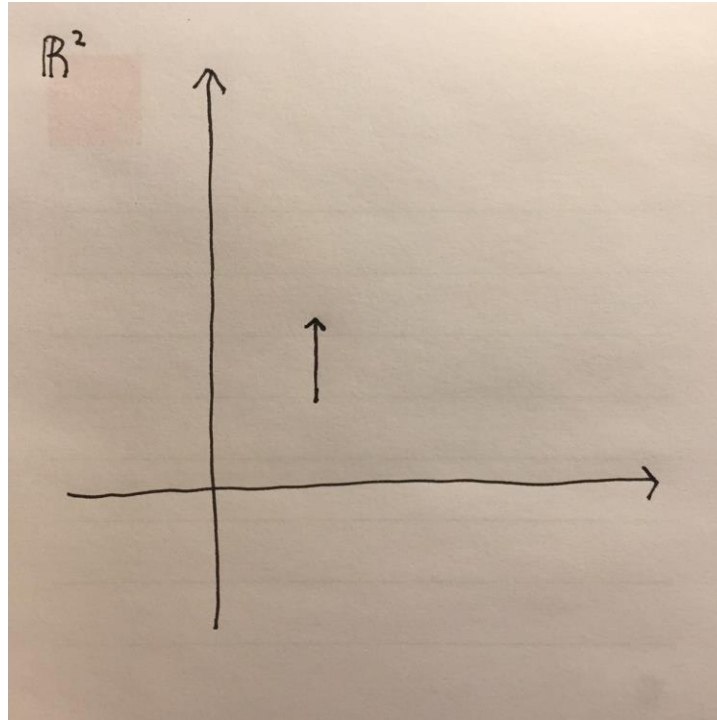
which is equivalent to $u^\mu \nabla_\mu u^\nu = 0$, where we have defined the *covariant derivative*:

$$\nabla_\mu u^\nu := \partial_\mu u^\nu + \Gamma_{\mu\lambda}^\nu u^\lambda$$

‘intrinsic change’

‘coordinate induced’

General Relativity – a conceptual crash course

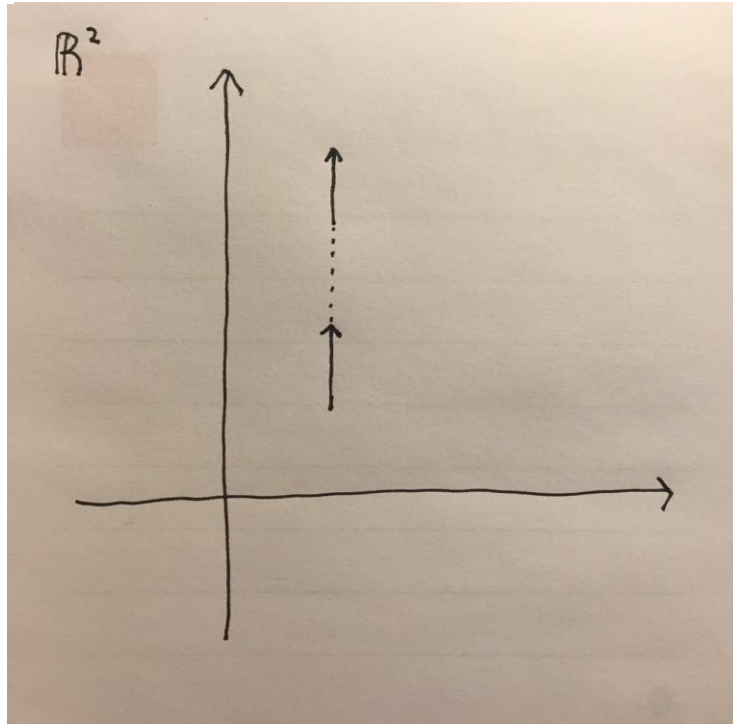


$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma_{\mu\lambda}^{\nu} u^{\lambda}$$

‘intrinsic change’

‘coordinate induced’

General Relativity – a conceptual crash course

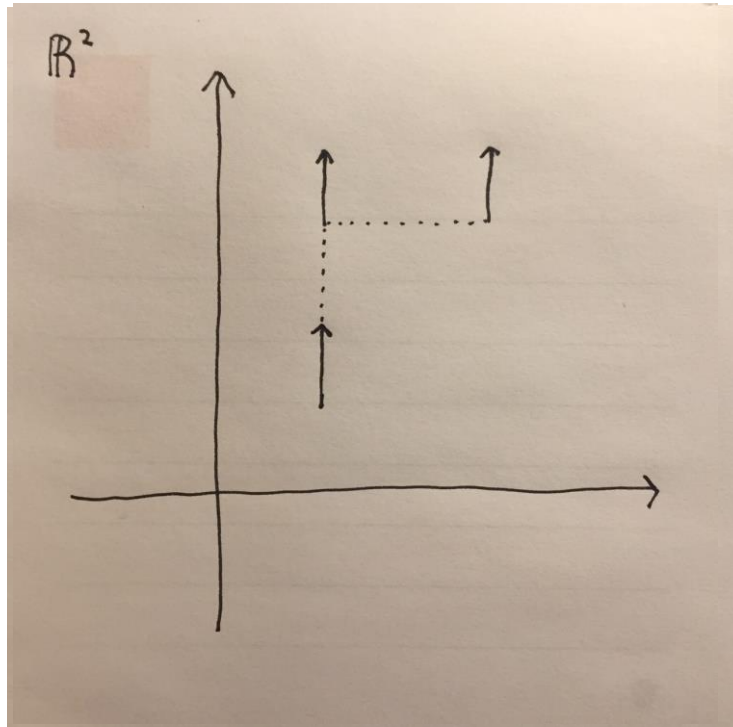


$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma_{\mu\lambda}^{\nu} u^{\lambda}$$

‘intrinsic change’

‘coordinate induced’

General Relativity – a conceptual crash course

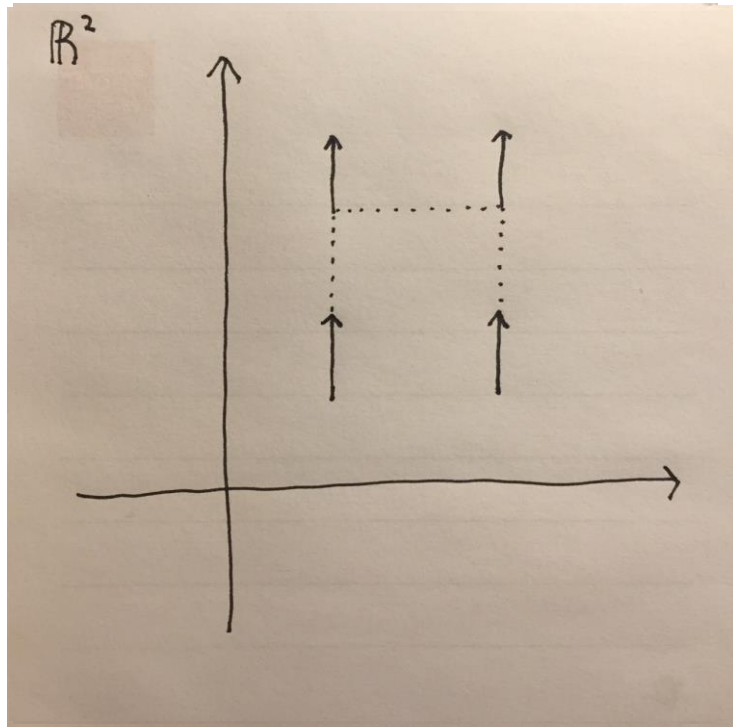


$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma_{\mu\lambda}^{\nu} u^{\lambda}$$

‘intrinsic change’

‘coordinate induced’

General Relativity – a conceptual crash course

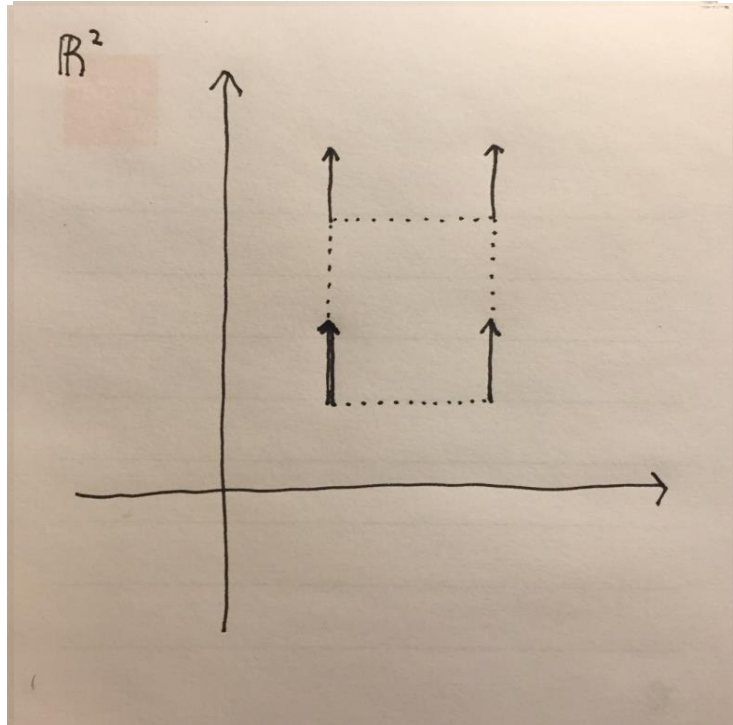


$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma_{\mu\lambda}^{\nu} u^{\lambda}$$

‘intrinsic change’

‘coordinate induced’

General Relativity – a conceptual crash course

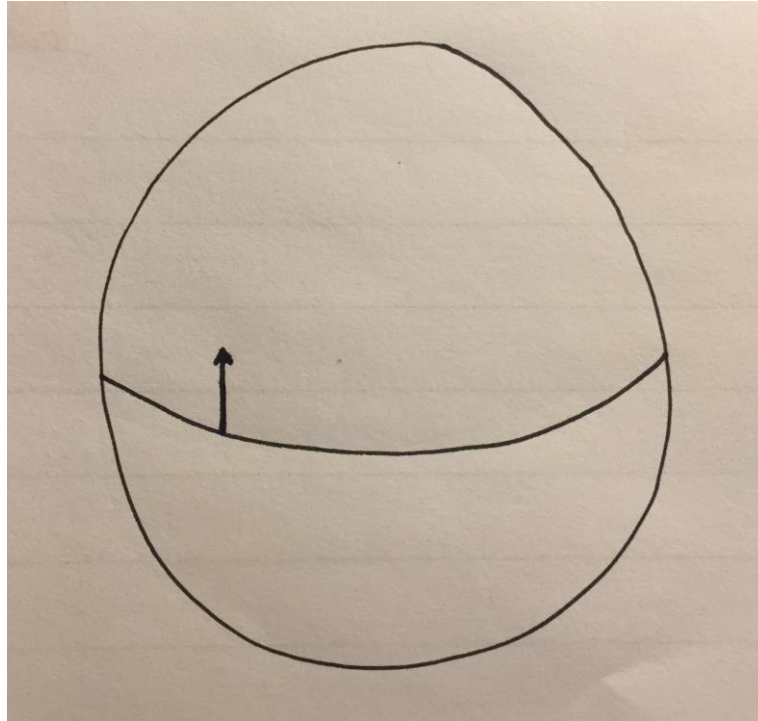


$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma_{\mu\lambda}^{\nu} u^{\lambda}$$

‘intrinsic change’

‘coordinate induced’

General Relativity – a conceptual crash course

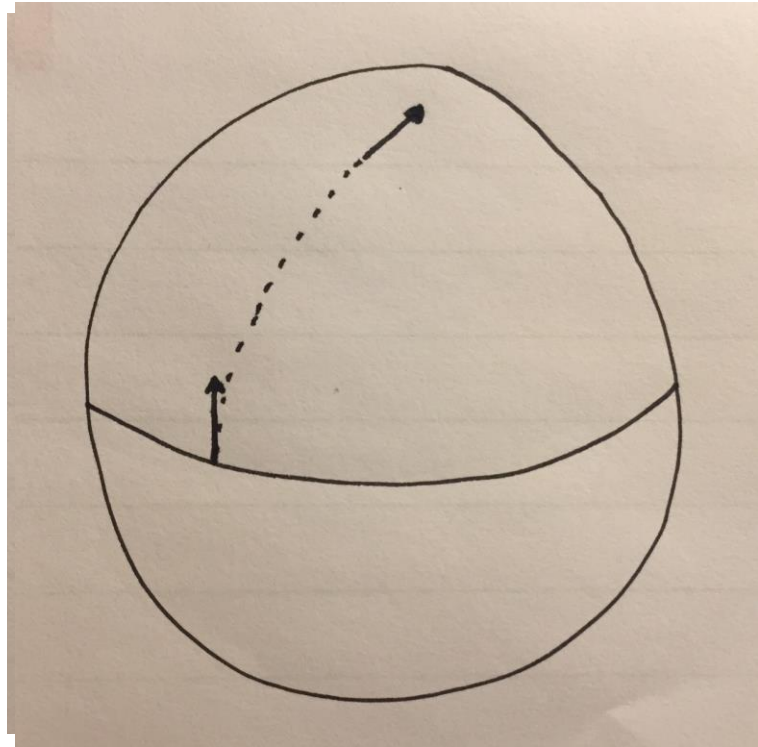


$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma^{\nu}_{\mu\lambda} u^{\lambda}$$

‘intrinsic change’

‘coordinate induced’

General Relativity – a conceptual crash course

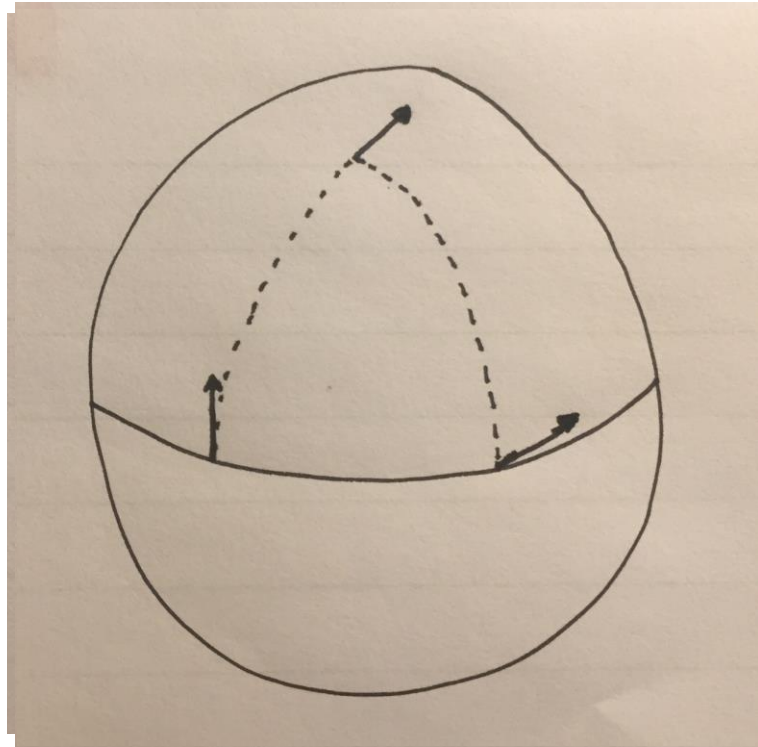


$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma^{\nu}_{\mu\lambda} u^{\lambda}$$

‘intrinsic change’

‘coordinate induced’

General Relativity – a conceptual crash course

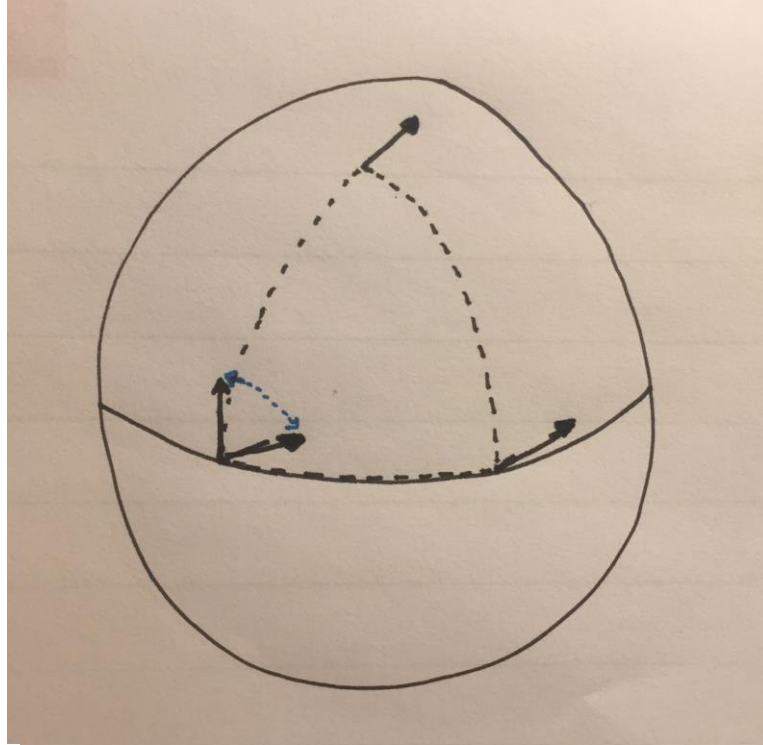


$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma_{\mu\lambda}^{\nu} u^{\lambda}$$

‘intrinsic change’

‘coordinate induced’

General Relativity – a conceptual crash course

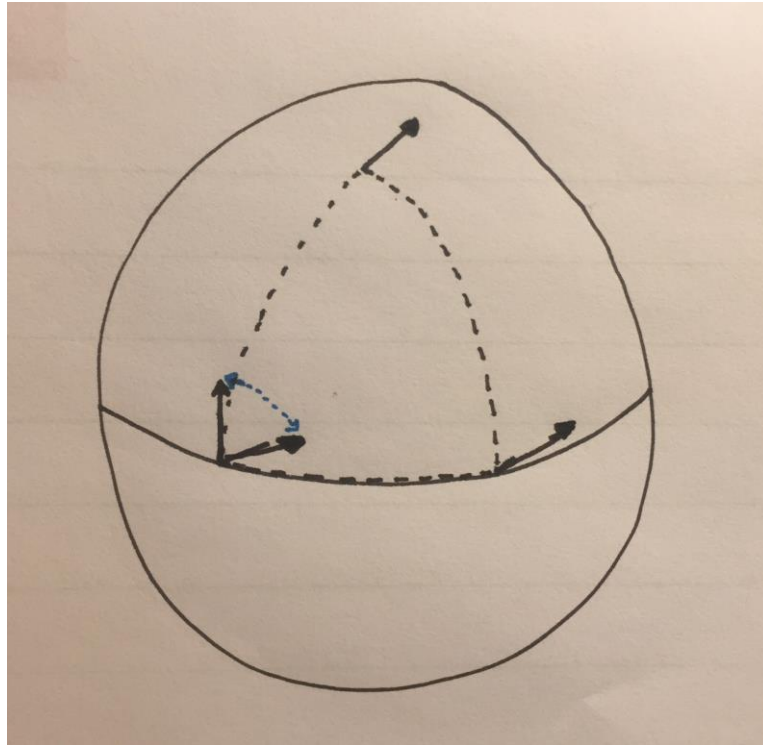


$$\nabla_{\mu} u^{\nu} := \underbrace{\partial_{\mu} u^{\nu}}_{\text{'intrinsic change'}} + \underbrace{\Gamma^{\nu}_{\mu\lambda} u^{\lambda}}_{\text{'coordinate induced'}}$$

'intrinsic change'

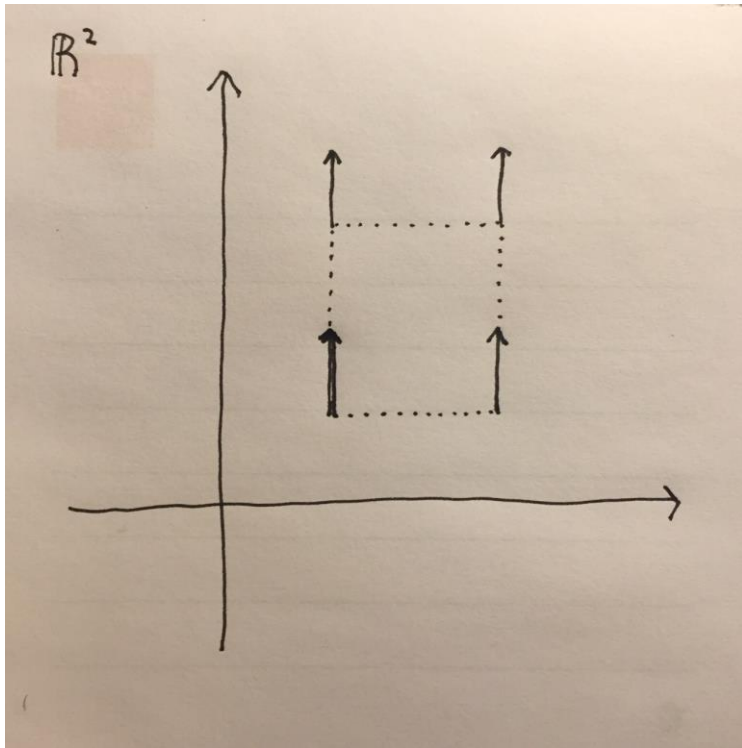
'coordinate induced'

General Relativity – a conceptual crash course



A vector 'parallel transported' around a closed loop will come back to itself on a flat space. Not so on a 'curved' space...

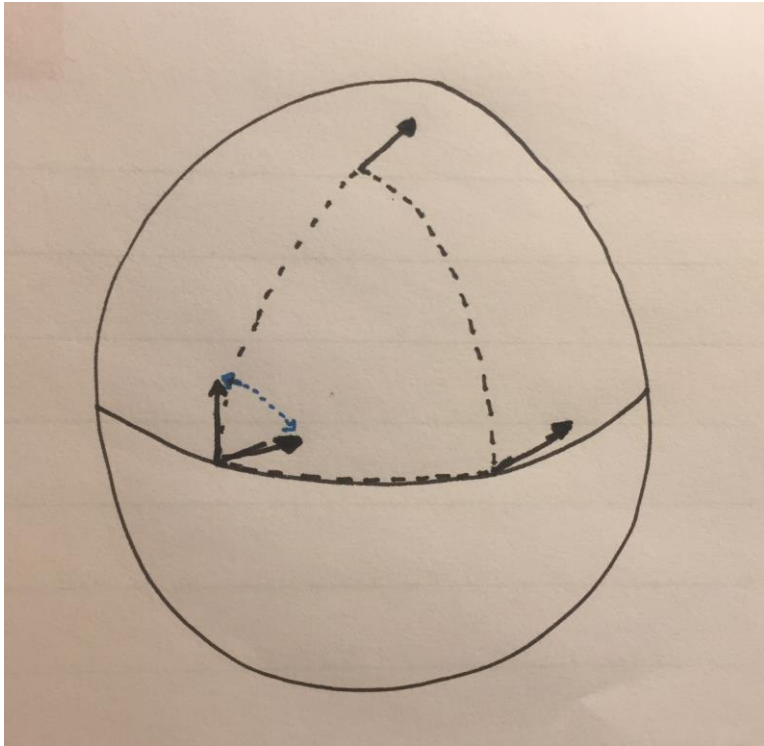
General Relativity – a conceptual crash course



$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma_{\mu\lambda}^{\nu} u^{\lambda}$$

$$(\nabla_{\lambda} \nabla_{\mu} - \nabla_{\mu} \nabla_{\lambda}) u^{\nu} = 0$$

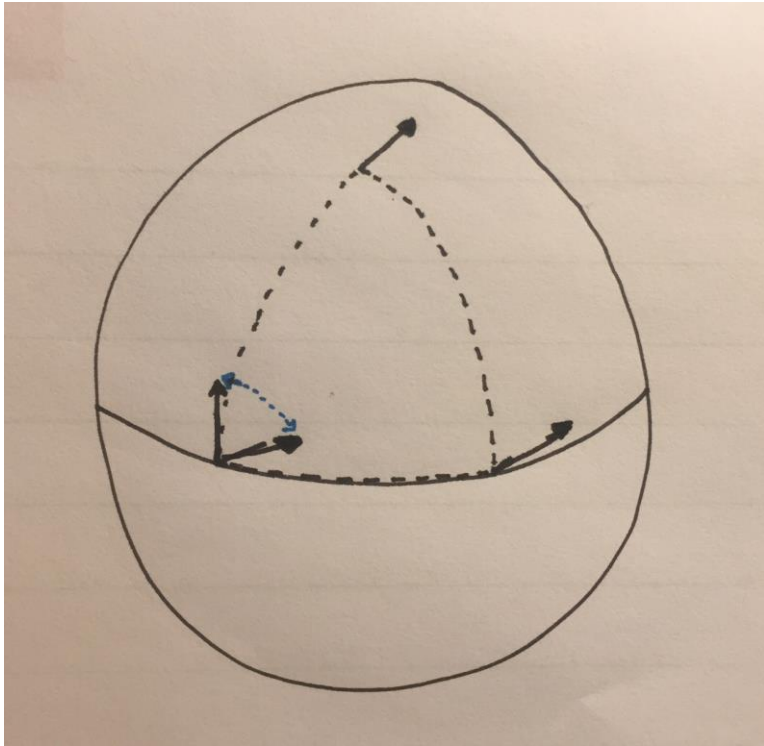
General Relativity – a conceptual crash course



$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma_{\mu\lambda}^{\nu} u^{\lambda}$$

$$(\nabla_{\lambda} \nabla_{\mu} - \nabla_{\mu} \nabla_{\lambda}) u^{\nu} = ?$$

General Relativity – a conceptual crash course

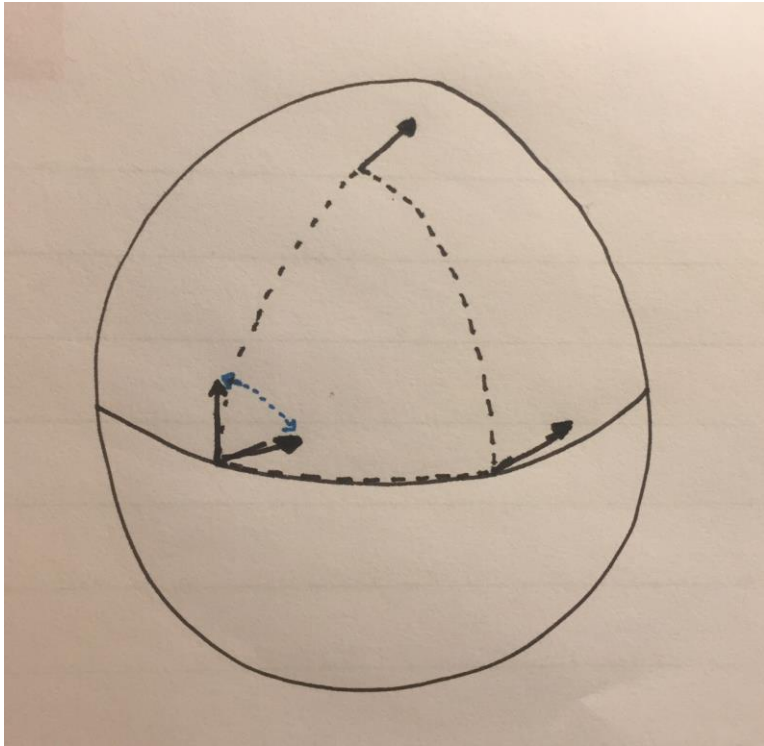


$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma_{\mu\lambda}^{\nu} u^{\lambda}$$

$$(\nabla_{\lambda} \nabla_{\mu} - \nabla_{\mu} \nabla_{\lambda}) u^{\nu} := R_{\rho\lambda\mu}^{\nu} u^{\rho}$$

Riemann curvature tensor

General Relativity – a conceptual crash course



$$\nabla_{\mu} u^{\nu} := \partial_{\mu} u^{\nu} + \Gamma_{\mu\lambda}^{\nu} u^{\lambda}$$

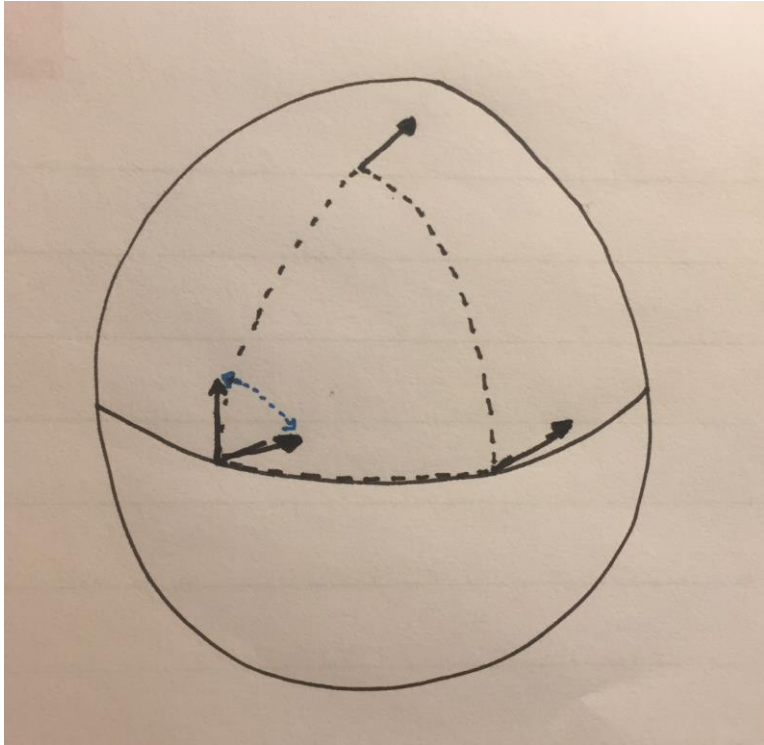
$$(\nabla_{\lambda} \nabla_{\mu} - \nabla_{\mu} \nabla_{\lambda}) u^{\nu} := R_{\rho\lambda\mu}^{\nu} u^{\rho}$$

Riemann curvature tensor

$$R_{\rho\lambda\mu}^{\nu} = \partial_{\lambda} \Gamma_{\rho\mu}^{\nu} - \partial_{\mu} \Gamma_{\rho\lambda}^{\nu} + \Gamma_{\kappa\lambda}^{\nu} \Gamma_{\rho\mu}^{\kappa} - \Gamma_{\kappa\mu}^{\nu} \Gamma_{\rho\lambda}^{\kappa}$$

$$\Gamma_{\nu\lambda}^{\mu} := \frac{g^{\mu\kappa}}{2} \left(\frac{dg_{\kappa\nu}}{dx^{\lambda}} + \frac{dg_{\kappa\lambda}}{dx^{\nu}} - \frac{dg_{\nu\lambda}}{dx^{\kappa}} \right)$$

General Relativity – a conceptual crash course



$$[\nabla_\lambda, \nabla_\mu] u^\nu := R^\nu_{\rho\lambda\mu} u^\rho$$

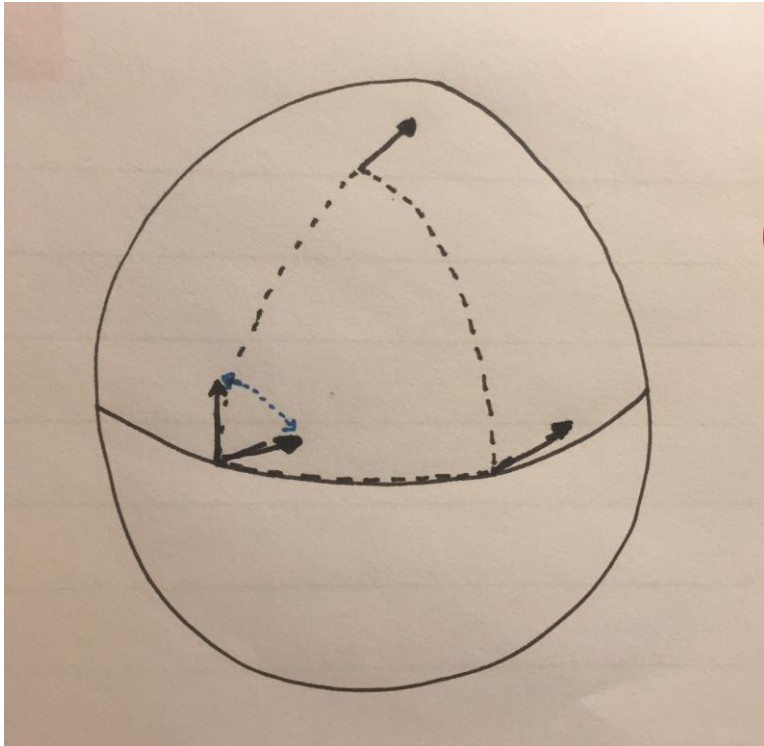
$$R^\nu_{\rho\lambda\mu} = \partial_\lambda \Gamma^\nu_{\rho\mu} - \partial_\mu \Gamma^\nu_{\rho\lambda} + \Gamma^\nu_{\kappa\lambda} \Gamma^\kappa_{\rho\mu} - \Gamma^\nu_{\kappa\mu} \Gamma^\kappa_{\rho\lambda}$$

cf. electromagnetism:

$$D_\mu := \partial_\mu - ieA_\mu$$

$$[D_\mu, D_\nu] = -ie(\partial_\mu A_\nu - \partial_\nu A_\mu) = -ieF_{\mu\nu}$$

General Relativity – a conceptual crash course



$$[\nabla_\lambda, \nabla_\mu] u^\nu := R^\nu_{\rho\lambda\mu} u^\rho$$

$$R^\nu_{\rho\lambda\mu} = \partial_\lambda \Gamma^\nu_{\rho\mu} - \partial_\mu \Gamma^\nu_{\rho\lambda} + \Gamma^\nu_{\kappa\lambda} \Gamma^\kappa_{\rho\mu} - \Gamma^\nu_{\kappa\mu} \Gamma^\kappa_{\rho\lambda}$$

Field strengths!

$$D_\mu := \partial_\mu - ieA_\mu$$

$$[D_\mu, D_\nu] = -ie(\partial_\mu A_\nu - \partial_\nu A_\mu) = -ieF_{\mu\nu}$$

General Relativity is a gauge theory (cf. John Donoghue's lectures)

Electromagnetism: $\mathcal{L}_{\text{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$$\Gamma_{\nu\lambda}^{\mu} := \frac{g^{\mu\kappa}}{2} \left(\frac{dg_{\kappa\nu}}{dx^{\lambda}} + \frac{dg_{\kappa\lambda}}{dx^{\nu}} - \frac{dg_{\nu\lambda}}{dx^{\kappa}} \right)$$

$$R_{\rho\lambda\mu}^{\nu} = \partial_{\lambda}\Gamma_{\rho\mu}^{\nu} - \partial_{\mu}\Gamma_{\rho\lambda}^{\nu} + \Gamma_{\kappa\lambda}^{\nu}\Gamma_{\rho\mu}^{\kappa} - \Gamma_{\kappa\mu}^{\nu}\Gamma_{\rho\lambda}^{\kappa}$$

Gravity: $\mathcal{L}_{\text{GR}} \stackrel{?}{=} [M]^2 g^{\rho\mu} R_{\rho\nu\mu}^{\nu}$

General Relativity is a gauge theory (cf. John Donoghue's lectures)

Electromagnetism: $\mathcal{L}_{\text{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

$$\Gamma_{\nu\lambda}^{\mu} := \frac{g^{\mu\kappa}}{2} \left(\frac{dg_{\kappa\nu}}{dx^{\lambda}} + \frac{dg_{\kappa\lambda}}{dx^{\nu}} - \frac{dg_{\nu\lambda}}{dx^{\kappa}} \right)$$

$$R^{\nu}_{\rho\lambda\mu} = \partial_{\lambda}\Gamma^{\nu}_{\rho\mu} - \partial_{\mu}\Gamma^{\nu}_{\rho\lambda} + \Gamma^{\nu}_{\kappa\lambda}\Gamma^{\kappa}_{\rho\mu} - \Gamma^{\nu}_{\kappa\mu}\Gamma^{\kappa}_{\rho\lambda}$$

Gravity: $\mathcal{L}_{\text{GR}} = \frac{1}{16\pi G_N} g^{\rho\mu} R^{\nu}_{\rho\nu\mu} := \frac{1}{16\pi G_N} R$

 Ricci scalar

Gravity is an effective theory (cf. John and Cliff Burgess' lectures)

Electromagnetism:

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{c_1}{M^4} (F_{\mu\nu}F^{\mu\nu})^2 + \frac{c_2}{M^4} (\epsilon^{\mu\nu\lambda\beta} F_{\mu\nu}F^{\lambda\beta})^2 + \dots$$

Gravity: $\mathcal{L}_{\text{GR}} = \frac{1}{16\pi G_N} R + c_1 R^2 + c_2 R^\mu_{\nu\lambda\beta} R^\nu_{\mu}{}^{\lambda\beta} + \dots$

$$8\pi G_N := \frac{1}{M_{\text{pl}}^2}$$

Gravity is an effective theory (cf. John and Cliff Burgess' lectures)

Electromagnetism:

$$\mathcal{L}_{\text{EM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{c_1}{M^4} (F_{\mu\nu}F^{\mu\nu})^2 + \frac{c_2}{M^4} (\epsilon^{\mu\nu\lambda\beta} F_{\mu\nu}F^{\lambda\beta})^2 + \dots$$

Gravity: $\mathcal{L}_{\text{GR}} = \frac{1}{16\pi G_N} R + c_1 R^2 + c_2 R^\mu_{\nu\lambda\beta} R^\nu_{\mu}{}^{\lambda\beta} + \dots$

$$8\pi G_N := \frac{1}{M_{\text{pl}}^2}$$

(Apologies for the hit and run treatment – an excellent, no-nonsense introduction to GR can be found in *Spacetime and Geometry* by Sean Carroll. A more thorough mathematical intro can be found in *General Relativity* by R. M. Wald)

The background – General Relativity + perfect fluids

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \mathcal{L}_M$$

$$R^\lambda_{\mu\lambda\nu} - \frac{1}{2} g_{\mu\nu} R := G_{\mu\nu} = 8\pi G_N T^\mu_\nu$$

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

want to solve for the geometry given a particular matter content.

The background – General Relativity + perfect fluids

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \mathcal{L}_M$$

$$R^\lambda_{\mu\lambda\nu} - \frac{1}{2} g_{\mu\nu} R := G_{\mu\nu} = 8\pi G_N T^\mu_\nu$$

$$T^\mu_\nu \equiv \langle T^\mu_\nu \rangle$$

What is the state $|\Psi\rangle$ that defines this expectation value?

The background – General Relativity + perfect fluids

On the equivalence between the Boltzmann equation
and classical field theory at large occupation numbers

A.H. Mueller ^a, D.T. Son ^b

^a *Department of Physics, Columbia University, New York, NY 10027, USA*

^b *Institute for Nuclear Theory, University of Washington, Seattle, WA 98195-1550, USA*

Received 17 December 2002; received in revised form 1 December 2003; accepted 10 December 2003

Editor: J.-P. Blaizot

Phys.Lett. B582 (2004) 279-287

The dynamics of any field theory at large enough occupation number (e.g. in $\lambda\phi^4$ theory, with $\lambda N_k \ll 1$) reduces to kinetic theory. For, $k \ll (\rho/m)^{1/3}$ this is hydrodynamics*.

*(Non)-equilibrium hydro/ kinetic theory \longleftrightarrow (non)-equilibrium QFT at large occupation.

The history of the Universe in a line:

$$G_{\mu\nu} = 8\pi G_N \langle T_{\nu}^{\mu} \rangle$$

$$\hat{\rho}_{\Psi} : |\phi\rangle\langle\phi| \rightarrow \sum_i e^{-\beta E_i} |E_i\rangle\langle E_i| \rightarrow \sum_a c_a |n_a\rangle\langle n_a|; \quad (n_i \gg 1)$$

e.g. $c_a \sim e^{-\beta_a E_a}$ at freeze-out

Inflation



(p)re-heating to a thermalized universe



perfect fluid (radiation matter domination)

The background – ISO(3) invariance (homogeneity, Isotropy)

$$G_{\mu\nu} = 8\pi G_N \langle T_{\nu}^{\mu} \rangle$$

$$T_{\nu}^{\mu} = (\rho + P)u^{\mu}u_{\nu} + P g_{\mu\nu}$$

u^{μ} is the four velocity vector of a fluid element, can adapt our coordinate system such that there is no momentum flux across spatial slices (i.e. u^{μ} defines the time direction).

$$T_{\nu}^{\mu} = \text{diag}(-\rho, p, p, p)$$

$$ds^2 = -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2)$$

The background – inflation

$$G_{\mu\nu} = 8\pi G_N \langle T_{\nu}^{\mu} \rangle \quad T_{\nu}^{\mu} = \text{diag}(-\rho, p, p, p)$$

$$ds^2 = -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2)$$

$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p = \frac{\dot{\phi}^2}{2} - V(\phi)$$

$$\rho \approx -p \quad \text{if} \quad \frac{\dot{\phi}^2}{2} \ll V(\phi)$$

$$\frac{3\dot{a}^2}{a^2} = \frac{1}{M_{\text{pl}}^2} \left(\frac{\dot{\phi}^2}{2} + V(\phi) \right) \approx \frac{V(\phi)}{M_{\text{pl}}^2}$$

The background – inflation

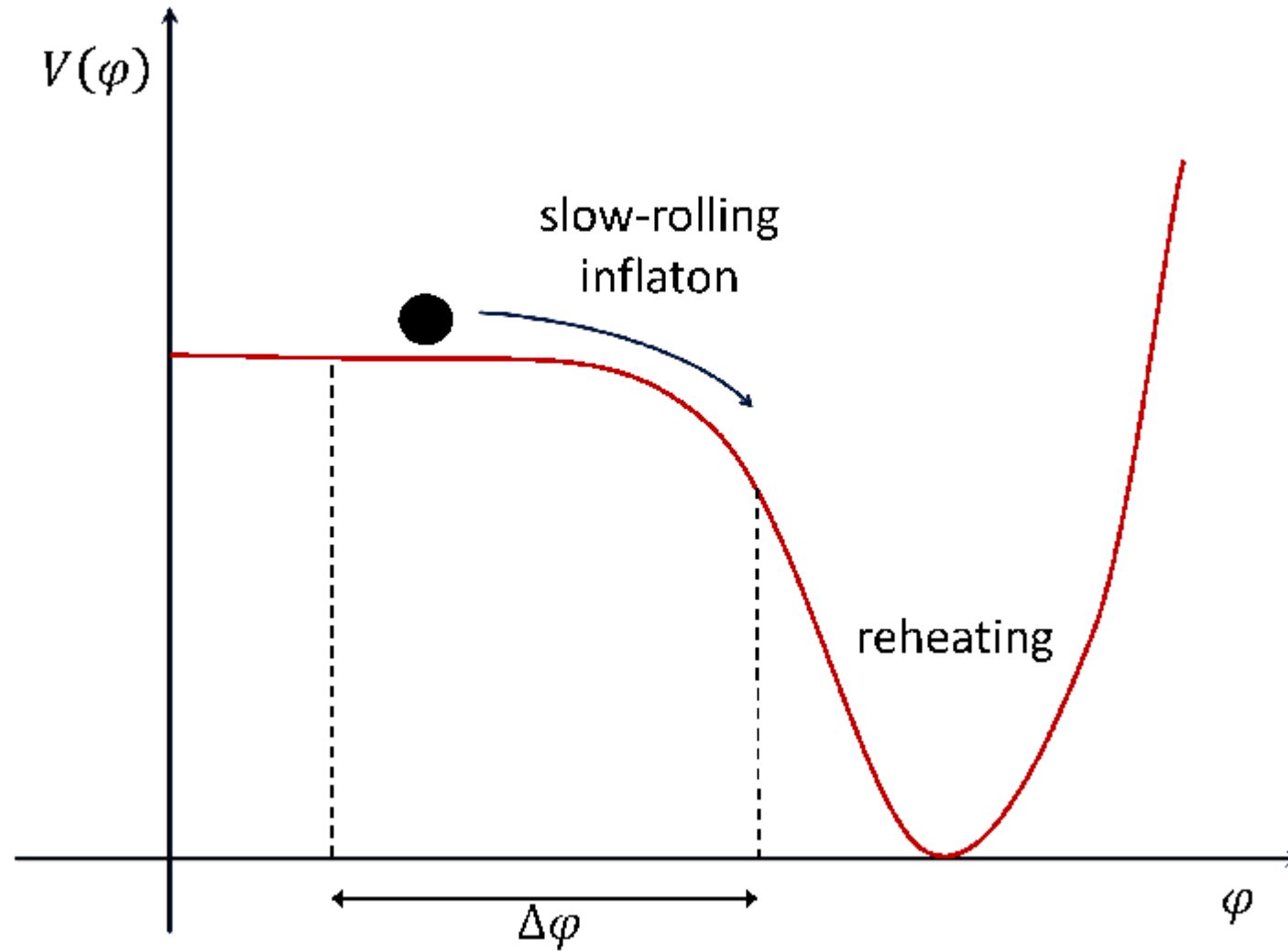
$$\rho = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p = \frac{\dot{\phi}^2}{2} - V(\phi)$$

$$\rho \approx -p \quad \text{if} \quad \frac{\dot{\phi}^2}{2} \ll V(\phi)$$

$$\frac{3\dot{a}^2}{a^2} := 3H^2 = \frac{1}{M_{\text{pl}}^2} \left(\frac{\dot{\phi}^2}{2} + V(\phi) \right) \approx \frac{V(\phi)}{M_{\text{pl}}^2}$$

$$a(t) \propto e^{Ht} \quad \text{if} \quad \epsilon := \frac{\dot{\phi}^2}{2H^2 M_{\text{pl}}^2}, \eta := \frac{\dot{\epsilon}}{H\epsilon} \ll 1$$

The background – inflation



The background – radiation, matter domination

$$p = w\rho, \quad \nabla_\mu T_0^\mu \rightarrow \dot{\rho} + 3H(\rho + p) = 0$$

$$\rho \propto \frac{1}{a^{3(1+w)}}$$

$$a(t) \propto t^{1/2} \quad \text{if } w = 1/3 \text{ (radiation domination)}$$

$$a(t) \propto t^{2/3} \quad \text{if } w = 0 \text{ (matter domination)}$$

The background – General Relativity + perfect fluids

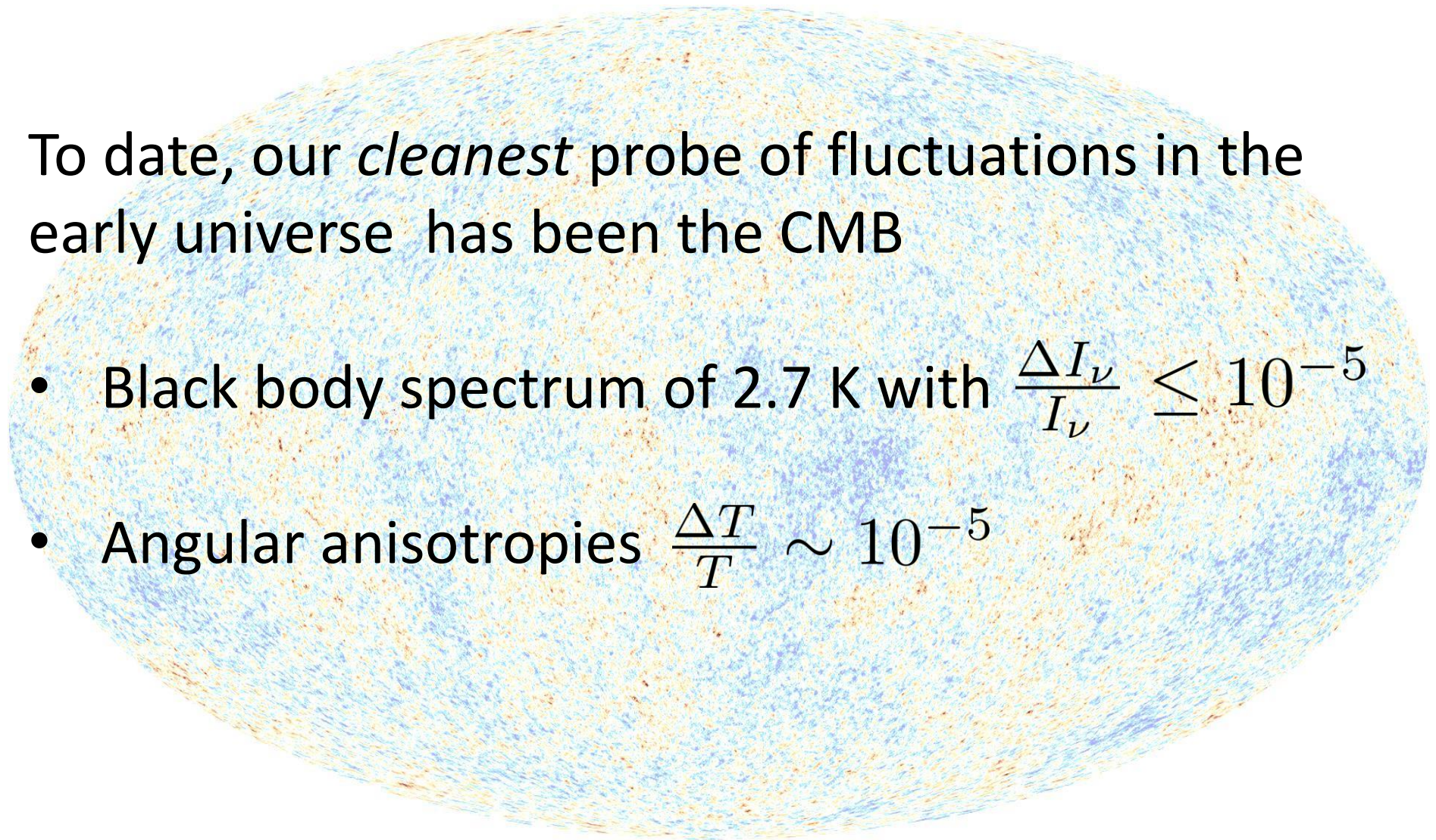
$$G_{\mu\nu} = 8\pi G_N T_{\nu}^{\mu}$$

Inflation, thermalization, radiation domination, matter domination, dark energy domination...

Perturbations – from quantum fields to galaxies

$$\delta G_{\mu\nu} = 8\pi G_N \delta T_{\nu}^{\mu}$$

perturbations: $g_{\mu\nu} = g_{\mu\nu}^{(0)} + \delta g_{\mu\nu}$



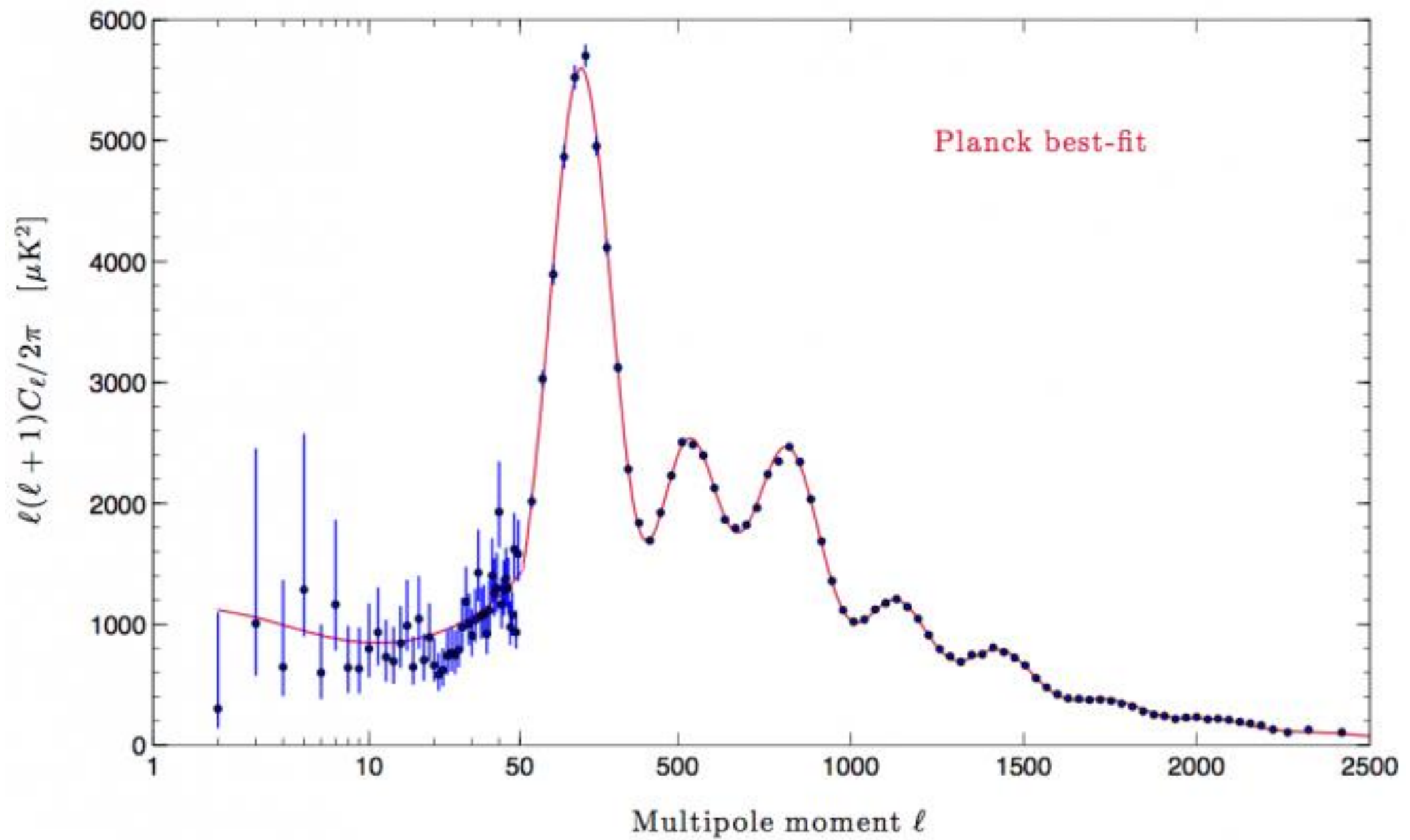
To date, our *cleanest* probe of fluctuations in the early universe has been the CMB

- Black body spectrum of 2.7 K with $\frac{\Delta I_\nu}{I_\nu} \leq 10^{-5}$
- Angular anisotropies $\frac{\Delta T}{T} \sim 10^{-5}$



What is the information content of the CMB?

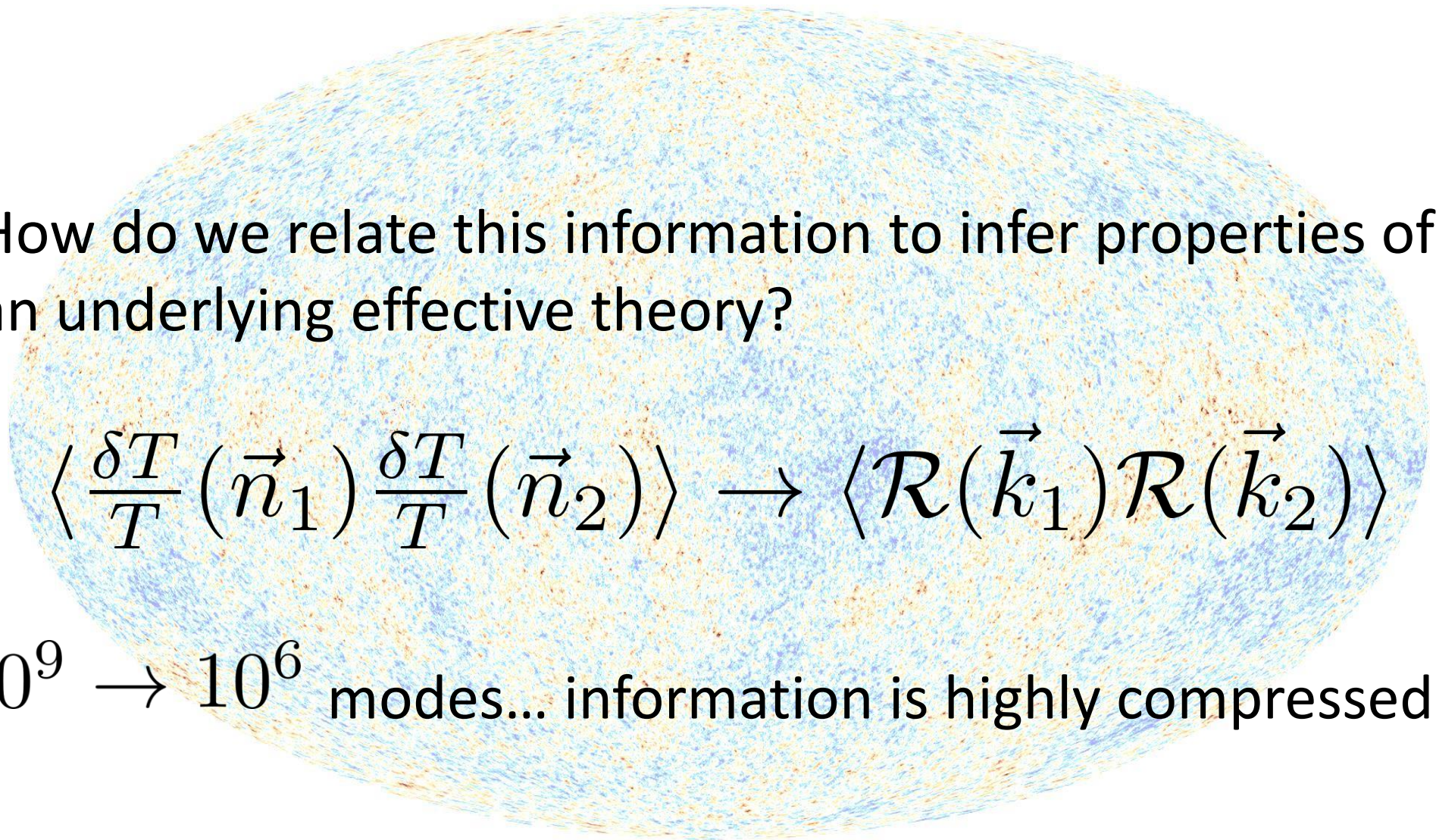
- Temperature (T) and polarization (E,B) in each direction $\leftrightarrow 10^6$ 'pixels' in the sky.
- Spectrum of incident photons in a given direction (new information only w/ deviations from blackbody).



What is the information content of the CMB?

- For T, radio sources and (SZ) clusters start to dominate at $\ell \sim 2500$
- For E, foregrounds subdominant until $\ell \sim 5000$
- Damping tail to be measured more precisely (SPTPol, ACTPol, CMB S4...)
- Forecast $\sum_\nu m_\nu \sim 0.05$ eV
- Cosmological measurement of a BSM parameter?

$$\mathcal{L} \supset \frac{H^\dagger H}{\Lambda} \bar{\psi} \psi; \quad \Lambda \sim 10^{16} \text{ GeV}$$



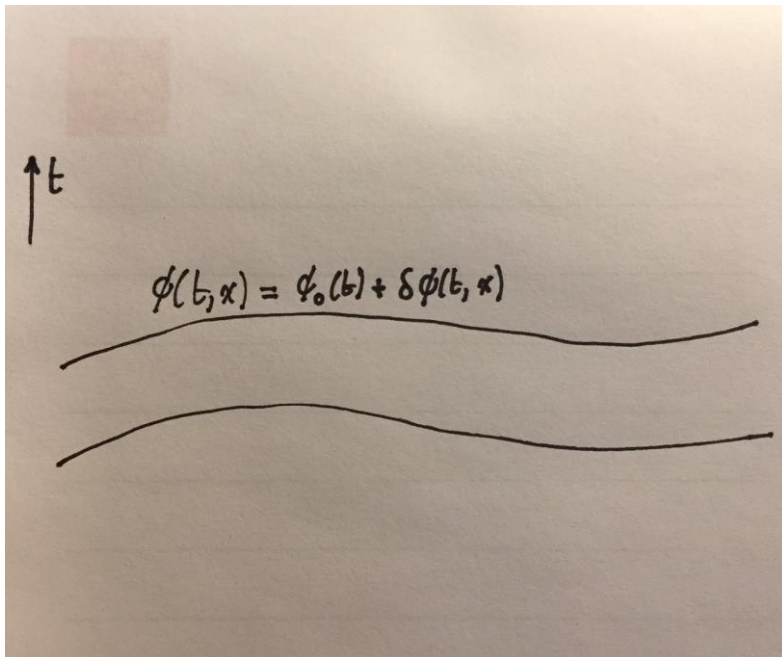
How do we relate this information to infer properties of an underlying effective theory?

$$\left\langle \frac{\delta T}{T}(\vec{n}_1) \frac{\delta T}{T}(\vec{n}_2) \right\rangle \rightarrow \left\langle \mathcal{R}(\vec{k}_1) \mathcal{R}(\vec{k}_2) \right\rangle$$

$10^9 \rightarrow 10^6$ modes... information is highly compressed!

How do we relate this information to infer properties of an underlying effective theory?

$$\mathcal{L}_{\text{tot}} = \frac{M_{\text{pl}}^2}{2} R + \dots + \mathcal{L}(\phi, \nabla\phi, \nabla^2\phi, \dots)$$



$$\phi(x, t) = \phi_0(t) + \delta\phi(t, x)$$

$$t \rightarrow t + \pi$$

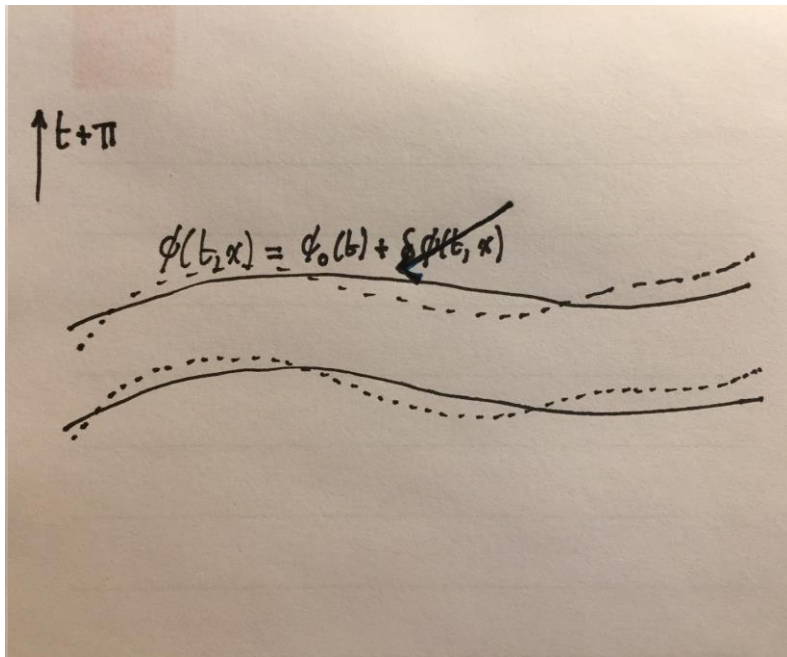
$$\phi_0 + \delta\phi(t, x) \rightarrow \phi_0 + \delta\phi(t, x) - \dot{\phi}_0\pi \equiv \phi_0(t)$$

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$h_{ij} = a^2(t)e^{2\mathcal{R}}\delta_{ij}$$

How do we relate this information to infer properties of an underlying effective theory?

$$\mathcal{L}_{\text{tot}} = \frac{M_{\text{pl}}^2}{2} R + \dots + \mathcal{L}(\phi, \nabla\phi, \nabla^2\phi, \dots)$$



$$\phi(x, t) = \phi_0(t) + \delta\phi(t, x)$$

$$t \rightarrow t + \pi$$

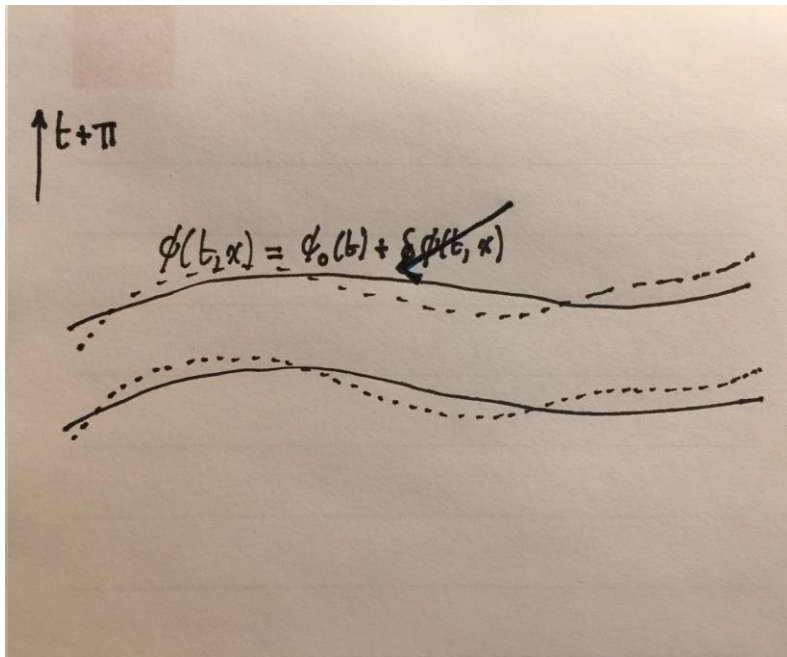
$$\phi_0 + \delta\phi(t, x) \rightarrow \phi_0 + \delta\phi(t, x) - \dot{\phi}_0\pi \equiv \phi_0(t)$$

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$h_{ij} = a^2(t)e^{2\mathcal{R}}\delta_{ij}$$

How do we relate this information to infer properties of an underlying effective theory?

Q) Where did the scalar perturbation go?



$$\phi(x, t) = \phi_0(t) + \delta\phi(t, x)$$

$$t \rightarrow t + \pi$$

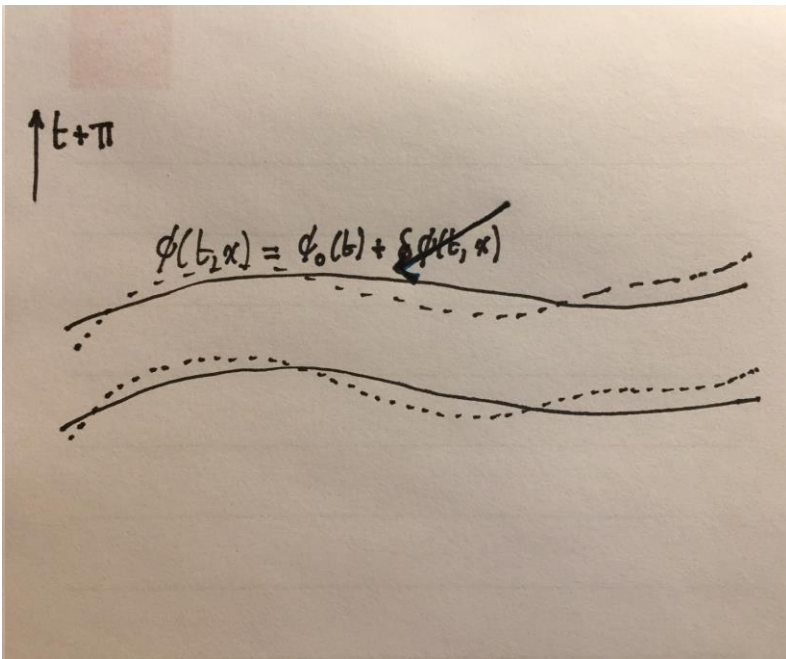
$$\phi_0 + \delta\phi(t, x) \rightarrow \phi_0 + \delta\phi(t, x) - \dot{\phi}_0\pi \equiv \phi_0(t)$$

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$h_{ij} = a^2(t)e^{2\mathcal{R}}\delta_{ij}$$

How do we relate this information to infer properties of an underlying effective theory?

A) It got 'eaten' by the metric, which now propagates a longitudinal polarization...



$$\phi(x, t) = \phi_0(t) + \delta\phi(t, x)$$

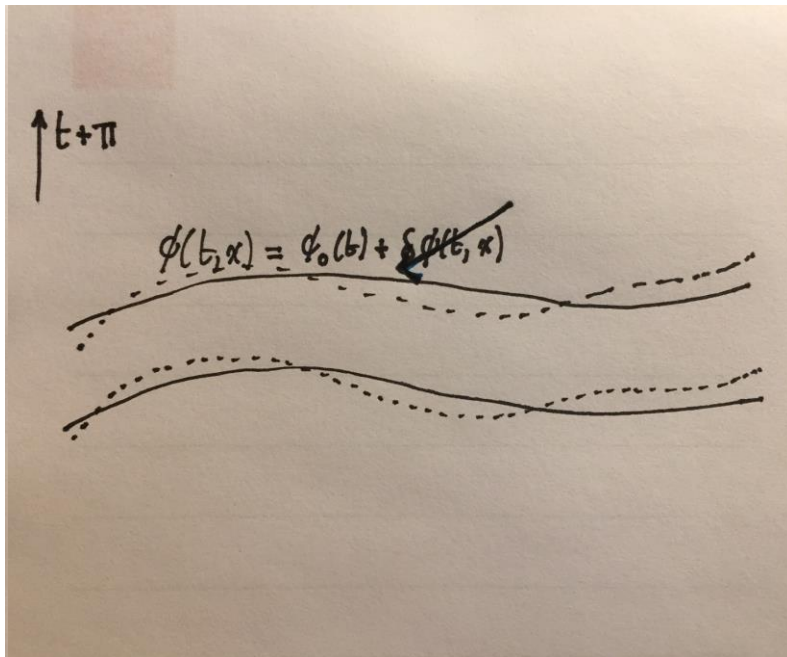
$$t \rightarrow t + \pi$$

$$\phi_0 + \delta\phi(t, x) \rightarrow \phi_0 + \delta\phi(t, x) - \dot{\phi}_0\pi \equiv \phi_0(t)$$

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$h_{ij} = a^2(t)e^{2\mathcal{R}}\delta_{ij}$$

Since \mathcal{R} is a Goldstone, $\mathcal{R} = \text{const.}$ will *always* be a solution for $k \ll 1$ to any order in perturbation theory since only derivative interactions. This is what imprints anisotropies on the CMB...



$$\phi(x, t) = \phi_0(t) + \delta\phi(t, x)$$

$$t \rightarrow t + \pi$$

$$\phi_0 + \delta\phi(t, x) \rightarrow \phi_0 + \delta\phi(t, x) - \dot{\phi}_0 \pi \equiv \phi_0(t)$$

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt)$$

$$h_{ij} = a^2(t) e^{2\mathcal{R}} \delta_{ij}$$

$$2\pi^2 \delta^3(\vec{k}_1 + \vec{k}_2) \mathcal{P}_{\mathcal{R}}(k_1) = k_1^3 \langle \mathcal{R}(\vec{k}_1) \mathcal{R}(\vec{k}_2) \rangle$$

$$\frac{\Delta_{\hat{T}}}{\hat{T}}(\vec{n}) = \sum_{\ell m} a_{\ell m}^T Y_{\ell m}(\vec{n})$$

$$|\Psi\rangle = |0\rangle$$

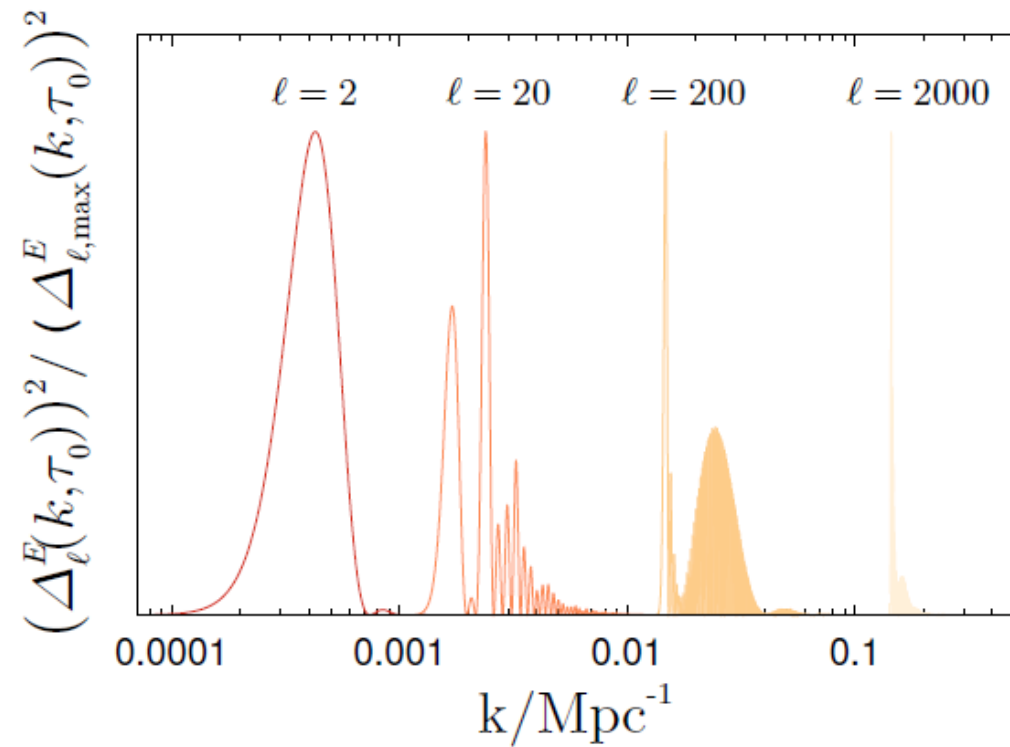
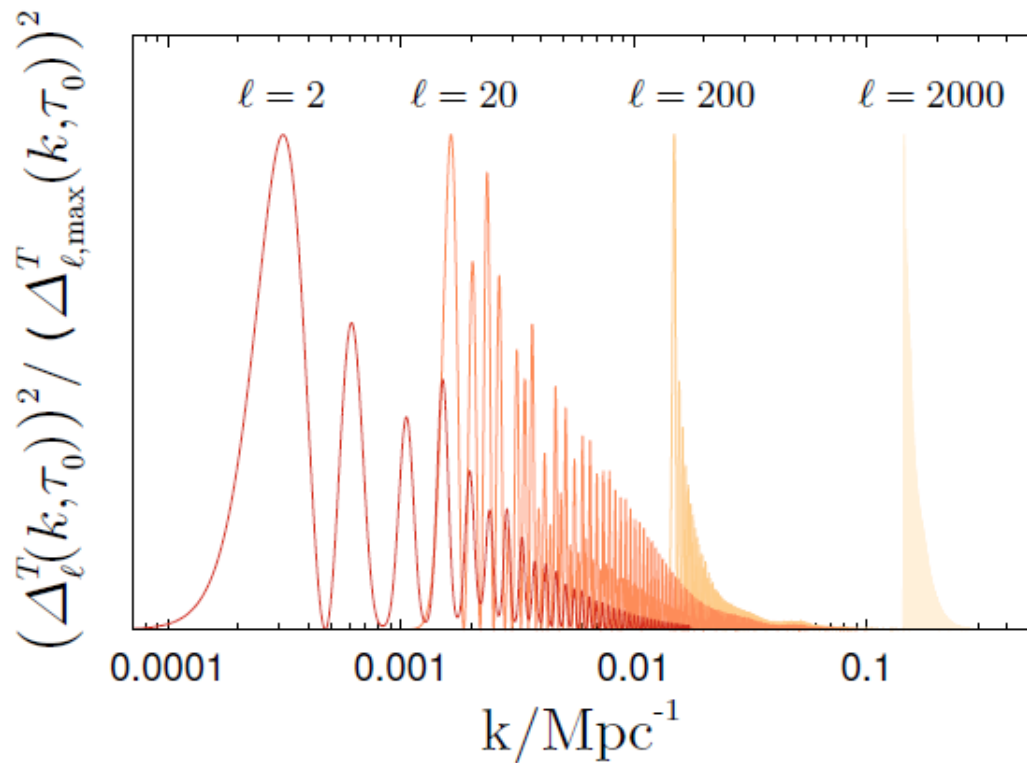
$$\langle a_{\ell m}^X a_{\ell' m'}^{Y*} \rangle = C_{\ell}^{XY} \delta_{\ell\ell'} \delta_{mm'}$$

$$C_{\ell}^{XY} = \frac{1}{2\pi^2} \int d \ln k \Delta_{\ell}^X(k, \tau_0) \Delta_{\ell}^Y(k, \tau_0) \mathcal{P}_{\mathcal{R}}(k)$$

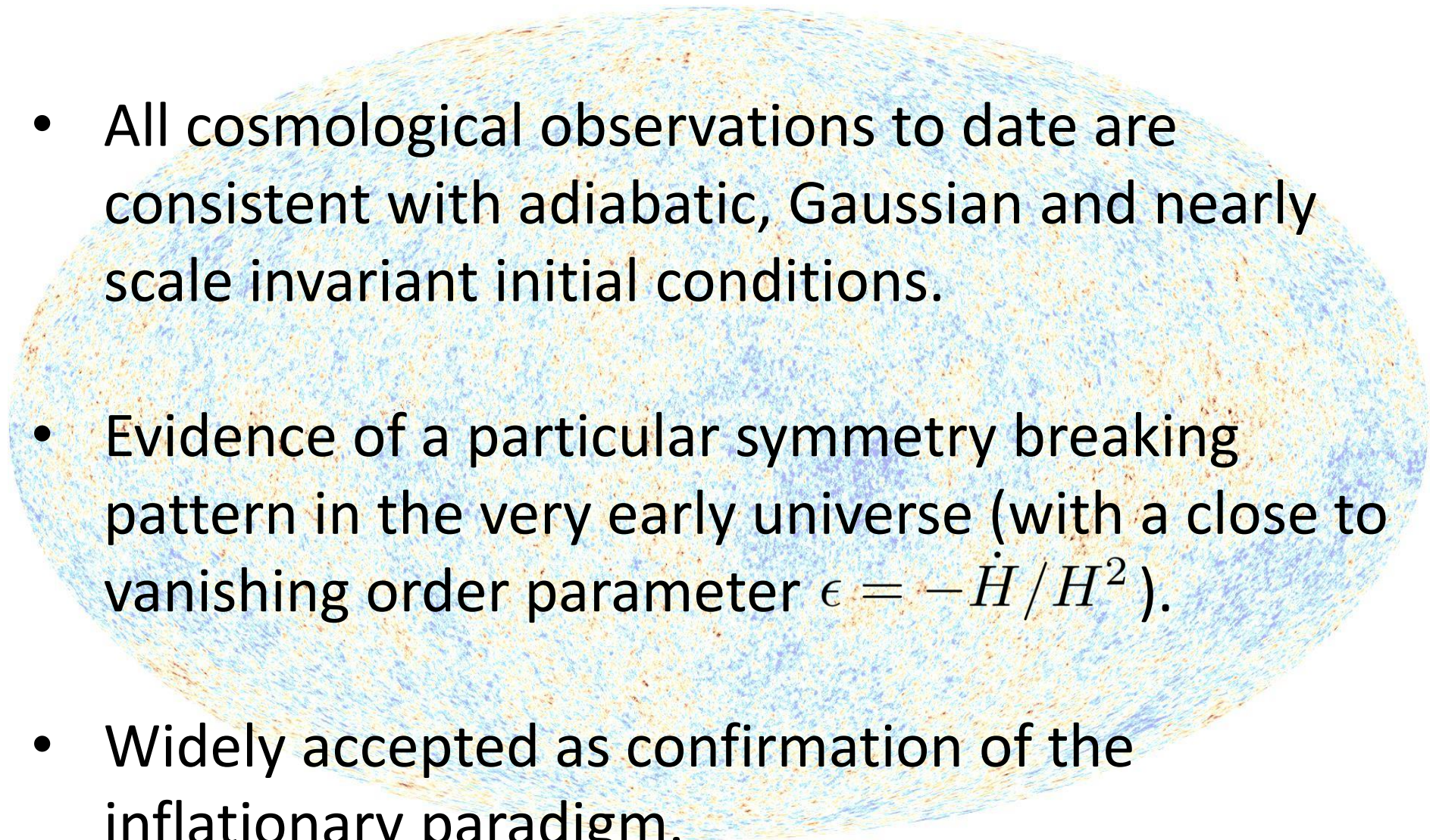
$$\Delta_{\ell}^X(k, \tau_0) = \int_{\tau_{\text{ini}}}^{\tau_0} d\tau S^X(k, \tau) j_{\ell}(k(\tau - \tau_0))$$

non-primordial cosmology

geometry



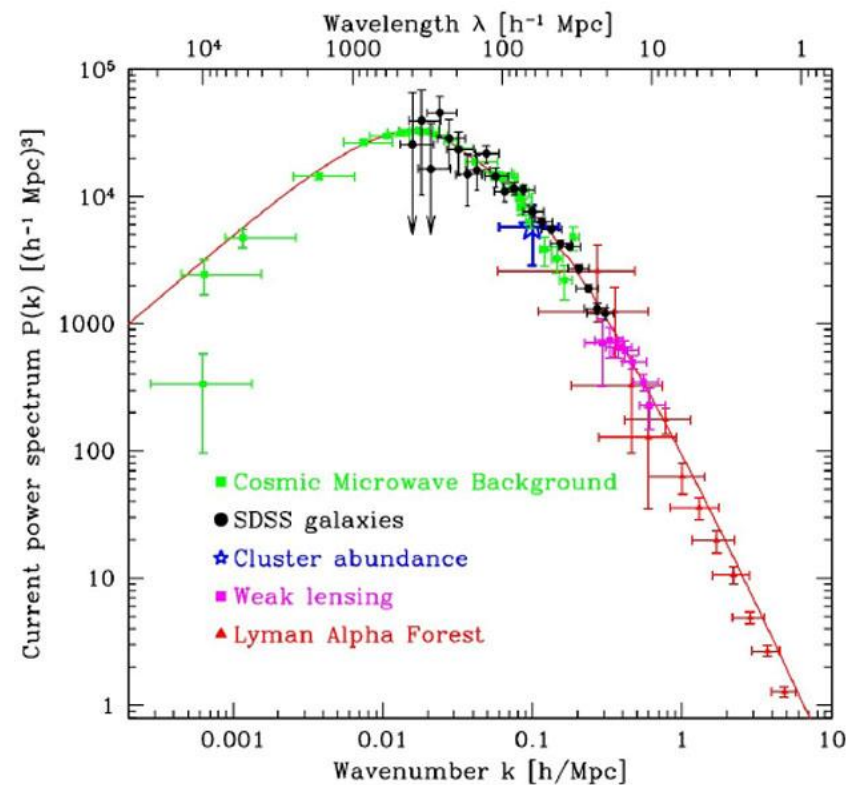
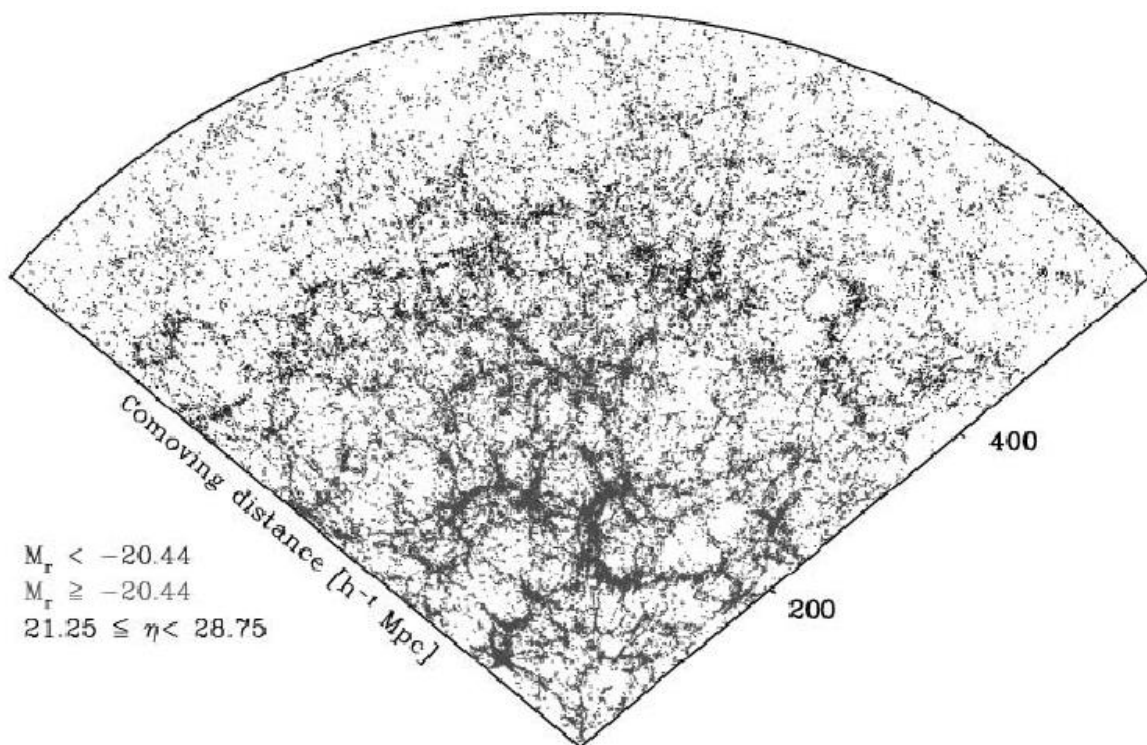
Are interesting things hiding in plain sight?

- 
- All cosmological observations to date are consistent with adiabatic, Gaussian and nearly scale invariant initial conditions.
 - Evidence of a particular symmetry breaking pattern in the very early universe (with a close to vanishing order parameter $\epsilon = -\dot{H}/H^2$).
 - Widely accepted as confirmation of the inflationary paradigm.

Large Scale Structure

$$\delta := \frac{\delta\rho}{\rho} \propto \nabla^2 \mathcal{R} \quad \langle \delta(\vec{k}_1) \delta(\vec{k}_2) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_m(k_1)$$

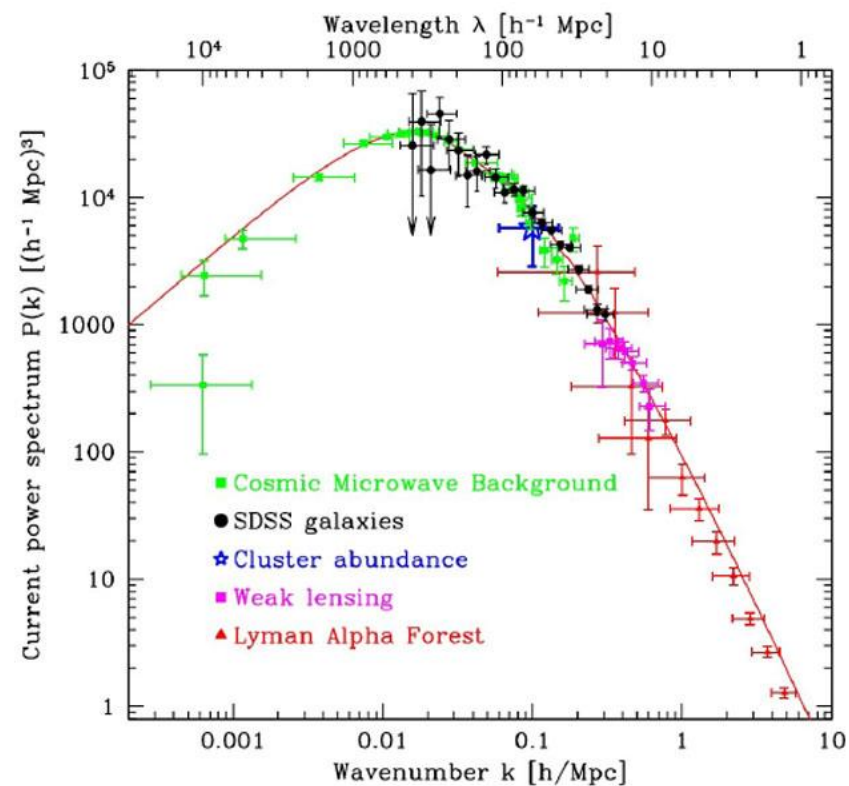
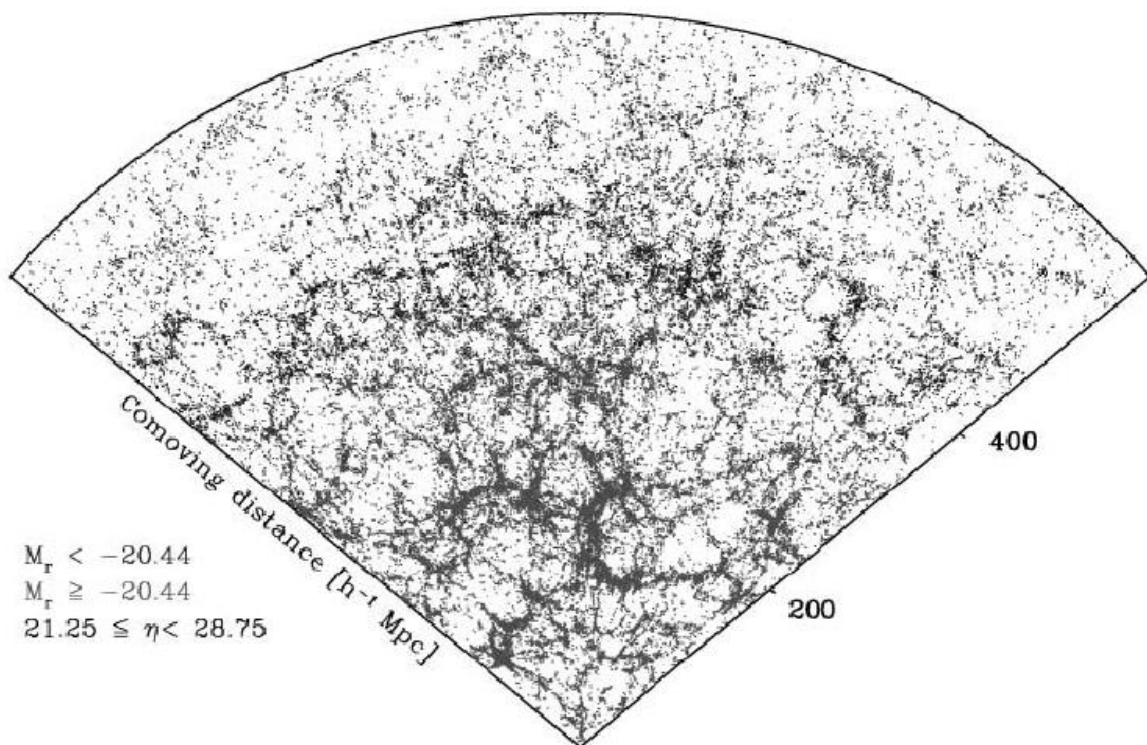
matter power spectrum



Large Scale Structure

$$\delta := \frac{\delta\rho}{\rho} \propto \nabla^2 \mathcal{R} \quad \langle \delta(\vec{k}_1) \delta(\vec{k}_2) \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_m(k_1)$$

$$\delta_{\text{galaxies}} = f(\delta), \text{ linear bias} \rightarrow \delta_g = b(z)\delta$$



This concludes the lightning tour, if you want more reading/
detailed references, ask me!

