General Relativity as a QFT/EFT

Jan 3, 2019

Nordic Winter School 2019

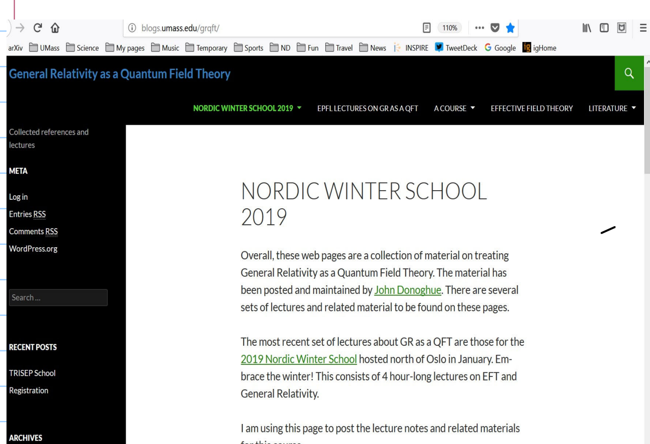
JOHN DONOGHUE

Main goal: How to think of GR as part of our "core theory" $SM+GR+QFT$

– plus QFT/EFT topics

$$Z = \int_{\Lambda} [d\phi\, dA\, d\psi\, dg_{\mu\nu}] \exp i \int d^4x \sqrt{-g} \Big[-\frac{1}{4}F^2 + \bar{\psi} i \not{D} \psi + \partial_\mu \phi^* \partial^\mu \phi - V(\phi) - \Lambda - \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \dots \Big]$$

Web page: Google GRQFT



Lecture 1 Gravity as a field theory in parallel with the Standard Model

Lecture 2 Key effective field theory ingredients.

Lecture 3

Lecture 4

Some exercises are found here.

Some pedagogic references:

The effective field theory treatment of quantum gravity. JFD – for those that know GR best but EFT less.

Introduction to the effective field theory description of gravity. JFD – for those who know EFT best but GR less.

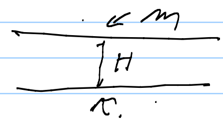
Quantum gravity in everyday life: General relativity as an effective field theory Cliff Burgess – Living reviews

Gravity from a particle physics perspective

- other interactions from gauging symmetries
- lets do the same for gravity
- de-emphasize technology

$$V(r) = -G\frac{m_1 m_2}{r}$$

Higgs $\mathcal{L} = -m\left(1 + \frac{H}{v}\right)\bar{\psi}\psi$



$$-i\mathcal{M} = -i\frac{m_1}{v}\,\frac{i}{q^2 - m_H^2}\left(-i\frac{m_2}{v}\right)$$

$$\Longrightarrow V = -\underbrace{\left(\frac{1}{4\pi v^2}\right)}_{G?}\frac{m_1 m_2}{r}\,e^{-m_H r} \quad \leftarrow m_H = 0$$

Doesn't Work

1) $m_q \ll m_p = \langle p | T^\mu{}_\mu | p\rangle = \langle p | \beta F^2 + \underbrace{m_u \bar{u}u + m_d \bar{d}d}_{30\,\mathrm{MeV}} | p\rangle$

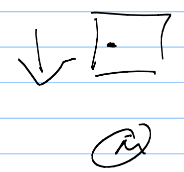
2) Binding energies

Equiv. Princ.

$F = m_g g = m_I a$ — ingredient

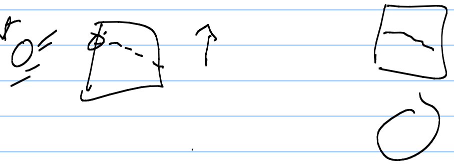
$\uparrow a$

Free fall



No grav

Light



Not scalars $\quad \phi F_{\mu\nu} F^{\mu\nu} = \phi (E^2 - B^2) = 0$ on shell

Energy + Momentum

$T_{\mu\nu}$ = $E + p$ tensor

$H = \int d^3x \, T_{00} \quad , \quad \vec{P}_i = \int d^3x \, T_{0i}$

$\partial^\mu T_{\mu\nu} = 0$

E, p associated with t, x translations

Noether = $T_{\mu\nu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \partial_\nu \phi - \eta_{\mu\nu} \mathcal{L}$

Example $\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \, \partial^\mu \phi - m^2 \phi^2)$

$T_{\mu\nu} = \partial_\mu \phi \, \partial_\nu \phi - \frac{1}{2} \eta_{\mu\nu} (\partial_\lambda \phi \, \partial^\lambda \phi - m^2 \phi)$

To get a source?

Gauge theories current as source

global $\psi \to e^{-i\theta}\psi \quad \Rightarrow J_\mu = \bar{\psi}\gamma_\mu\psi$

gauge $\psi \to e^{-i\theta(x)}\psi$

$$D_\mu\psi = (\partial_\mu + ieA_\mu)\psi \to e^{i\theta(x)} D_\mu\psi$$

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi \longrightarrow \ldots\ \underbrace{eA_\mu\bar{\psi}\gamma^\mu\psi}$$

also $[D_\mu, D_\nu]\psi = -ieF_{\mu\nu}\psi \qquad \frac{\delta\mathcal{L}}{\partial A_\mu} = e\bar{\psi}\gamma_\mu\psi$

Non Abelian $\quad U = \exp(i\alpha^a\lambda^a)$

$\psi \to U\psi$

$D_\mu\psi \to U D_\mu\psi$

$D_\mu = \partial_\mu + ig\frac{\lambda^a}{2}A^a = \partial_\mu + ig\underline{A}_\mu$

$[D_\mu, D_\nu]\psi = -ig\underline{F}_{\mu\nu}\psi \quad \boxed{\underline{A}_\mu \to \underline{A}_\mu' = U\underline{A}_\mu U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1}}$

$\underline{F}_{\mu\nu} = \partial_\mu\underline{A}_\nu - \partial_\nu\underline{A}_\mu + g(\underline{A}_\mu\underline{A}_\nu - \underline{A}_\nu\underline{A}_\mu)$

Local t, x transformations

$$x^\mu \to x^\mu + a^\mu(x) = x'^\mu$$

$$dx'^\mu = \Lambda^\mu{}_\nu \, dx^\nu \qquad \Lambda^\mu{}_\nu = \frac{\partial x'^\mu}{\partial x^\nu}$$

New field

$$d\tau^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu = g'_{\mu\nu}(x')\, dx'^\mu dx'^\nu$$

$$g'_{\alpha\beta} \Lambda^\alpha{}_\mu \Lambda^\beta{}_\nu = g_{\mu\nu} \qquad dx' = \Lambda\, dx$$

$$g'_{\alpha\beta} = \Lambda^{-1}{}_\alpha{}^\mu \Lambda^{-1}{}_\beta{}^\nu \, g_{\mu\nu} \qquad g' = \Lambda^{-1} g \Lambda^{-1}$$

$$g^{\mu\nu} g_{\nu\alpha} = \delta^\mu_\alpha$$

Immediate success: $T_{\mu\nu}$ = source

$$S_m = \int d^4x \sqrt{-g}\, \frac{1}{2} \left\{ g^{\mu\nu}_{(x)} \partial_\mu \phi \, \partial_\nu \phi - m^2 \phi^2 \right\}$$

Source:

$$\frac{\delta S_m}{\delta g^{\mu\nu}} = \frac{\sqrt{-g}}{2} \left[\partial_\mu \phi \, \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \left[\partial_\lambda \phi \, \partial^\lambda \phi - m^2 \phi^2 \right] \right]$$

$$= \frac{\sqrt{-g}}{2} T_{\mu\nu} \longleftarrow$$

Preview of second success

Eq of motion $(\Box + m^2)\phi = 0$ $\qquad \Box = \frac{1}{\sqrt{-g}}\partial_\mu(g^{\mu\nu}\sqrt{-g})\partial_\nu$

NR reduction $\phi = e^{-imt}\psi(x,t)$

$\to$ $g_{00} = 1 + 2\phi_g$ $\qquad \phi_g = -\frac{GM_E}{r}$

Get $i\frac{\partial}{\partial t}\psi = \left[-\frac{\nabla^2}{2m} + m\phi_g\right]\psi$ $\quad + \mathcal{O}\left(\frac{\partial^2}{\partial t^2}\psi, \frac{\phi\nabla^2}{m^2}\right)$

Sch Eq in grav Field

$-i[H, \vec{p}] = \dot{\vec{p}} = -m\nabla\phi_g$

Strategy

D_μ of $\quad V \to \Lambda V$

$DV \to \Lambda DV$

$DV = (\partial + \Gamma)V$

$\Gamma \sim (\Gamma + \Lambda^{-1}\partial_\mu\Lambda)$

$\to R = \partial_\mu\Gamma_\nu - \partial_\nu\Gamma_\mu + \Gamma\Gamma - \Gamma\Gamma$

Aside: with Fermions ~ extra technology

$\bar{\psi}\gamma^a \underbrace{e_a^\mu}_{\text{vierbein}}(\partial_\mu + \underbrace{A_\mu}_{\text{spin connection}})\psi$

$\boxed{\text{Read Kibble or EPFL}}$