# Nordic 2 – GRQFT – Donoghue

Note Title — Jan 4, 2019 ~~12/27/2018~~

## Review from first lecture:

Equivalence principle $\Longrightarrow$ focus on $T_{\mu\nu}$ as source

$\Longrightarrow$ gauge $t$, $\vec{x}$ translation invariance

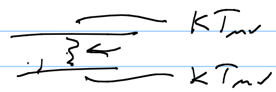
$dx^{\mu} \to dx'^{\mu} = \Lambda^{\mu}{}_{\nu}(x)\, dx^{\nu}$

For invariance introduce new field $g_{\mu\nu}(x) \to g'_{\mu\nu}(x') = \Lambda^{-1}{}_{\mu}{}^{\alpha}(x)\, g_{\alpha\beta}(x)\, \Lambda^{-1}{}_{\nu}{}^{\beta}$

First success – Invariant interaction for scalar $\Longrightarrow \dfrac{\delta S_m}{\delta g^{\mu\nu}} = \dfrac{\sqrt{-g}}{2} T_{\mu\nu}$

Second success – also Schroedinger eq in NR limit

$$i\frac{\partial}{\partial t}\psi = \left[-\frac{\nabla^2}{2m} + m\phi_g\right]\psi$$

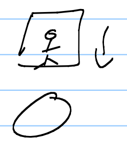
We are on our way to 


## Covariant Derivative

| With indices | Without |
|---|---|
| Consider Vector field $V^{\mu}$ | |
| $V'^{\mu}(x') = \Lambda^{\mu}{}_{\nu} V^{\nu}$ | $V' = \Lambda(x) V$ |
| Want $D_{\mu} V^{\nu} \to D'_{\mu} V'^{\nu} = \Lambda^{-1}{}_{\mu}{}^{\sigma} \Lambda^{\nu}{}_{\rho} D_{\sigma} V^{\rho}$ | $\underline{D'V'} = \Lambda\Lambda\, \underline{DV}$ |
| Following gauge $D_{\mu} V^{\nu} = \partial_{\mu} V^{\nu} + \Gamma_{\mu\rho}{}^{\nu} V^{\rho}$ | $D_{\mu} = \partial_{\mu} + \Gamma$ |
| Gauge trans $\Gamma'_{\mu\nu}{}^{\lambda} = \Lambda^{-1}{}_{\mu}{}^{\mu'} \Lambda^{-1}{}_{\nu}{}^{\nu'} \Lambda^{\lambda}{}_{\lambda'} \left(\Gamma_{\mu'\nu'}{}^{\lambda'} + \Lambda^{-1}{}_{\sigma}{}^{\lambda'} \partial_{\mu} \Lambda^{\sigma}{}_{\nu'}\right)$ | $\Gamma' = (\Gamma + \Lambda^{-1}\partial_{\mu}\Lambda)$ |

Solution $\Gamma_{\mu\nu}{}^{\lambda} = \frac{1}{2} g^{\lambda\sigma}\left(\partial_{\mu} g_{\nu\sigma} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu}\right)$ *

Note: Can be derived from EP



$\eta_{\mu\nu} \qquad \partial_\alpha \eta_{\mu\nu} = 0 \qquad \Gamma$

other frame $\quad D_\alpha g_{\mu\nu} = 0$

$$D_\alpha g_{\mu\nu} - D_\mu g_{\nu\alpha} - D_\nu g_{\alpha\mu} = 0 \Rightarrow \Gamma_{\mu\nu}{}^{\lambda}$$

assume $\Gamma_{\mu\nu}{}^{\lambda} = \Gamma_{\nu\mu}{}^{\lambda}$

Now follow gauge theory $\qquad [D_\mu, D_\nu]\psi = ig F_{\mu\nu}\psi$

$$[D_\mu, D_\nu] V^\beta = R_{\mu\nu\alpha}{}^{\beta} V^\alpha$$

which automatically transforms covariantly

$$R_{\mu\nu\alpha}{}^{\beta} = \partial_\mu \Gamma_{\nu\alpha}{}^{\beta} - \partial_\nu \Gamma_{\mu\alpha}{}^{\beta} + \Gamma_{\mu\rho}{}^{\beta}\Gamma_{\nu\alpha}{}^{\rho} - \Gamma_{\nu\rho}{}^{\beta}\Gamma_{\mu\alpha}{}^{\rho}$$

$$R_{\mu\alpha} = R_{\mu\nu\alpha}{}^{\nu} \quad , \quad R = g^{\mu\alpha} R_{\mu\alpha}$$

invariant

For EFT $\quad R \sim \partial^2 g$

$$R'_{\mu\alpha} = \Lambda^{-1}{}_{\mu}{}^{\mu'}(x)\, \Lambda^{-1}{}_{\alpha}{}^{\alpha'} R_{\mu'\alpha'}$$

## Gravitational action

$$S_g = \int d^4x \sqrt{-g}\left[-\Lambda + \frac{2}{K^2}R + \underbrace{c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu}}_{\partial^4 g \Rightarrow \text{small}}\right]$$

$\Lambda \sim 10^{-122} M_p^4$

Note $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} = 4R_{\mu\nu}R^{\mu\nu} - R^2$ + total deriv (in 4d)

$c_i \lesssim 10^{+65}$

Keep $R$

$S = S_g + S_m$

$(\partial^2 + c_i \partial^4)h = T_{\mu\nu}$

$\delta S = 0 \Rightarrow \frac{4}{K^2}(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) = T_{\mu\nu}$

$K^2 = 32\pi G$

## Basic Facts

Weak field limit $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$\{\partial_\mu \partial_\lambda h^\lambda{}_\nu \quad - \ldots \quad - \Box h^\lambda{}_\lambda\} = 16\pi G T_{\mu\nu}$

not invertable

gauge inv. $x'^\mu = x^\mu + \xi^\mu$

$h'_{\mu\nu} = h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$

Gauge fixing $\partial_\mu h^{\mu\nu} - \frac{1}{2}\partial^\nu h^\lambda{}_\lambda = 0$ (harmonic)

$\Longrightarrow \Box(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^\lambda{}_\lambda) = -16\pi G T_{\mu\nu}$

$$D_{\mu\nu\alpha\beta} = \frac{1}{q^2}\frac{1}{2}\left[\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha} - \eta_{\mu\nu}\eta_{\alpha\beta}\right]$$

Point mass

$$h_{\mu\nu} = \begin{pmatrix} 2\phi_g & & & 0 \\ & 2\phi_g & & \\ & & 2\phi_g & \\ 0 & & & 2\phi_g \end{pmatrix} \qquad \phi_g = \frac{-GM}{r}$$

# Feynman rules:

### A.1 Scalar propagator

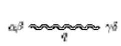
The massive scalar propagator is:



$$= \frac{i}{q^2 - m^2 + i\epsilon}$$

### A.2 Graviton propagator

The graviton propagator in harmonic gauge can be written in the form:



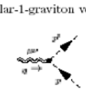
$$= \frac{i\mathcal{P}^{\alpha\beta\gamma\delta}}{q^2 + i\epsilon}$$

where

$$\mathcal{P}^{\alpha\beta\gamma\delta} = \frac{1}{2}\left[\eta^{\alpha\gamma}\eta^{\beta\delta} + \eta^{\beta\gamma}\eta^{\alpha\delta} - \eta^{\alpha\beta}\eta^{\gamma\delta}\right]$$

### A.3 2-scalar-1-graviton vertex

The 2-scalar-1-graviton vertex is discussed in the literature. We write it as:



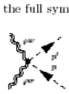
$$= \tau_1^{\mu\nu}(p, p', m)$$

where

$$\tau_1^{\mu\nu}(p, p', m) = -\frac{i\kappa}{2}\left[p^\mu p'^\nu + p^\nu p'^\mu - \eta^{\mu\nu}\left((p \cdot p') - m^2\right)\right]$$

### A.4 2-scalar-2-graviton vertex

The 2-scalar-2-graviton vertex is also discussed in the literature. We write it here with the full symmetry of the two gravitons:



$$= \tau_2^{\eta\lambda\rho\sigma}(p, p', m)$$

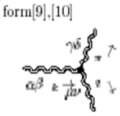
$$\tau_2^{\eta\lambda\rho\sigma}(p, p') = i\kappa^2\Bigg[\left\{I^{\eta\lambda\alpha\delta}I^{\rho\sigma\beta}{}_{\delta} - \frac{1}{4}\left\{\eta^{\eta\lambda}I^{\rho\sigma\alpha\beta} + \eta^{\rho\sigma}I^{\eta\lambda\alpha\beta}\right\}\right\}(p_\alpha p'_\beta + p'_\alpha p_\beta) - \frac{1}{2}\left\{I^{\eta\lambda\rho\sigma} - \frac{1}{2}\eta^{\eta\lambda}\eta^{\rho\sigma}\right\}\left[(p \cdot p') - m^2\right]\Bigg] \tag{61}$$

with

$$I_{\alpha\beta\gamma\delta} = \frac{1}{2}(\eta_{\alpha\gamma}\eta_{\beta\delta} + \eta_{\alpha\delta}\eta_{\beta\gamma}).$$

### A.5 3-graviton vertex

The 3-graviton vertex can be derived via the background field method and has the form[9],[10]

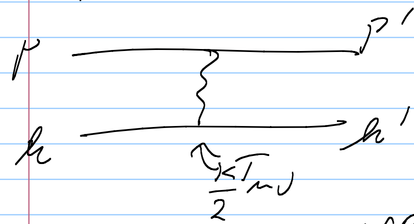


$$= \tau 3^{\mu\nu}_{\alpha\beta\gamma\delta}(k, q)$$

where

$$\begin{aligned}
\tau 3^{\mu\nu}_{\alpha\beta\gamma\delta}(k,q) = -\frac{i\kappa}{2} \times \Bigg(&\mathcal{P}_{\alpha\beta\gamma\delta}\left[k^\mu k^\nu + (k-q)^\mu(k-q)^\nu + q^\mu q^\nu - \frac{3}{2}\eta^{\mu\nu}q^2\right] \\
&+ 2q_\lambda q_\sigma\left[I_{\alpha\beta}{}^{\sigma\lambda}I_{\gamma\delta}{}^{\mu\nu} + I_{\gamma\delta}{}^{\sigma\lambda}I_{\alpha\beta}{}^{\mu\nu} - I_{\alpha\beta}{}^{\mu\sigma}I_{\gamma\delta}{}^{\nu\lambda} - I_{\gamma\delta}{}^{\mu\sigma}I_{\alpha\beta}{}^{\nu\lambda}\right] \\
&+ \left[q_\lambda q^\mu\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\nu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\nu\lambda}\right) + q_\lambda q^\nu\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\mu\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\mu\lambda}\right)\right. \\
&\left. - q^2\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\mu\nu} - \eta_{\gamma\delta}I_{\alpha\beta}{}^{\mu\nu}\right) - \eta^{\mu\nu}q_\sigma q_\lambda\left(\eta_{\alpha\beta}I_{\gamma\delta}{}^{\sigma\lambda} + \eta_{\gamma\delta}I_{\alpha\beta}{}^{\sigma\lambda}\right)\right] \\
&+ \Big[2q_\lambda\left(I_{\alpha\beta}{}^{\lambda\sigma}I_{\gamma\delta\sigma}{}^{\nu}(k-q)^\mu + I_{\alpha\beta}{}^{\lambda\sigma}I_{\gamma\delta\sigma}{}^{\mu}(k-q)^\nu - I_{\gamma\delta}{}^{\lambda\sigma}I_{\alpha\beta\sigma}{}^{\nu}k^\mu - I_{\gamma}\right. \\
&+ q^2\left(I_{\alpha\beta\sigma}{}^{\mu}I_{\gamma\delta}{}^{\nu\sigma} + I_{\alpha\beta}{}^{\nu\sigma}I_{\gamma\delta\sigma}{}^{\mu}\right) + \eta^{\mu\nu}q_\sigma q_\lambda\left(I_{\alpha\beta}{}^{\lambda\rho}I_{\gamma\delta\rho}{}^{\sigma} + I_{\gamma\delta}{}^{\lambda\rho}I_{\alpha\beta\rho}{}^{\sigma}\right. \\
&+ \Big\{(k^2 + (k-q)^2)\left[I_{\alpha\beta}{}^{\mu\sigma}I_{\gamma\delta\sigma}{}^{\nu} + I_{\gamma\delta}{}^{\mu\sigma}I_{\alpha\beta\sigma}{}^{\nu} - \frac{1}{2}\eta^{\mu\nu}\mathcal{P}_{\alpha\beta\gamma\delta}\right] \\
&- \left(I_{\gamma\delta}{}^{\mu\nu}\eta_{\alpha\beta}k^2 + I_{\alpha\beta}{}^{\mu\nu}\eta_{\gamma\delta}(k-q)^2\right)\Big\}\Bigg)
\end{aligned} \tag{62}$$

## Potential




$$-i\mathcal{M} = i\frac{\kappa}{2}\left(p^\mu p'^\nu - - -\right) \, i\frac{P_{\mu\nu\alpha\beta}}{q^2} \, \frac{i\kappa}{2}\left(k'^\alpha k^\beta - -\right)$$

$$NR + F.T. \Rightarrow V = -\frac{G m_1 m_2}{r} \quad !$$

## Ghosts Stories

Vol. XXIV (1963) ACTA PHYSICA POLONICA Fasc. 6 (12)

# QUANTUM THEORY OF GRAVITATION*

By R. P. Feynman

(Received July 3, 1963)

My subject is the quantum theory of gravitation. My interest in it is primarily in the relation of one part of nature to another. There's a certain irrationality to any work in gravitation, so it's hard to explain why you do any of it; for example, as far as quantum effects are concerned let us consider the effect of the gravitational attraction between an electron and a proton in a hydrogen atom; it changes the energy a little bit. Changing the energy of a quantum system means that the phase of the wave function is slowly shifted relative to what it would have been were no perturbation present. The effect of gravitation on the hydrogen atom is to shift the phase by 43 seconds of phase in every hundred times the lifetime of the universe! An atom made purely by gravitation, let us say two neutrons held together by gravitation, has a Bohr orbit of $10^8$ light years. The energy of this system is $10^{-70}$ rydbergs. I wish to discuss here the possibility of calculating the Lamb correction to this thing, an energy, of the order $10^{-120}$. This irrationality is shown also in the strange gadgets of Prof. Weber, in the absurd creations of Prof. Wheeler and other such things, because the dimensions are so peculiar. It is therefore clear that the problem we are working on is not the correct problem; the correct problem is what determines the size of gravitation? But since I am among equally irrational men I won't be criticized I hope for the fact that there is no possible, practical reason for making these calculations.

Tree theorems

This made me investigate the entire subject in great detail to find out what the trouble is. I discovered in the process two things. First, I discovered a number of theorems, which as far as I know are new, which relate closed loop diagrams and diagrams without closed loop diagrams (I shall call the latter diagrams "trees"). The unitarity relation which I have just been describing, is one connection between a closed loop diagram and a tree; but I found a whole lot of other ones, and this gives me more tests on my machinery. So let me just tell you a little bit about this theorem, which gives other rules. It is rather interesting. As a matter of fact, I proved that if you have a diagram with rings in it there are enough theorems altogether, so that you can express any diagram with circuits completely in terms of diagrams with trees and with all momenta for tree diagrams in physically attainable regions and on the mass shell. The demonstration is remarkably easy. There are several ways of demonstrating it; I'll only chose one. Things propagate from one place to another, as I said, with

There is another theory, more well-known to meson physicists, called the Yang-Mills theory, and I take the one with zero mass; it is a special theory that has never been investigated in great detail. It is very analogous to gravitation; instead of the coordinate transformation group being the source of everything, it's the isotopic spin rotation group that's the source of everything. It is a non-linear theory, that's like the gravitation theory, and so forth. At the suggestion of Gell-Mann I looked at the theory of Yang-Mills with zero mass, which has a kind of gauge group and everything the same; and found exactly the same difficulty. And therefore in meson theory it was not strictly unknown difficulty, because it should have been noticed by meson physicists who had been fooling around the Yang-Mills theory. They had not noticed it because they're practical, and the Yang-Mills theory with zero mass obviously does not exist, because a zero mass field would be obvious; it would come out of nuclei right away. So they didn't take the case of zero mass and investigate it carefully. But this disease which I discovered here is a disease which exist in other theories. So at least there is one good thing: gravity isn't alone in this difficulty. This observation that Yang-Mills was also in trouble was of very great advantage to me; it made everything much easier in trying to straighten out the troubles of the preceding paragraph, for several reasons.

Well, what then, now you have the difficulty; how do you cure it? Well I tried the following idea: I assumed the tree theorem to be true, and used it in reverse. If every closed ring diagram can be expressed as trees, and if trees produce no trouble and can be computed, then all you have to do is to say that the closed loop diagram is the sum of the corresponding tree diagrams, that it should be. Finally in each tree diagram for which a graviton line has been opened, take only real transverse graviton to represent that term. This then serves as the definition of how to calculate closed-loop diagrams; the old rules, involving a propagator $1/k^2+i\varepsilon$ *etc.* being superseded. The advantage of this is, first, that it will be gauge invariant, second, it will be unitary, because unitarity is a relation between a closed diagram and an open one, and is one of the class of relations I was talking about, so there's no difficulty. And third, it's completely unique as to what the answer is; there's no arbitrary fiddling around with different gauges and so forth, in the inside ring as there was before. So that's the plan.

FDFP ghosts

when I made it gauge invariant. But then secondly, you must subtract from the answer, the result that you get by imagining that in the ring which involves only a graviton going around, instead you calculate with a different particle going around, an artificial, dopey particle is coupled to it. It's a vector particle, artificially coupled to the external field, so designed as to correct the error in this one. The forms are evidently invariant,

DeWitt: Because of the interest of the tricky extra particle that you mentioned at the end, and its possible connection, perhaps, with some work of Dr Białynicki-Birula, have you got far enough on that so that you could repeat it with just a little more detail? The structure of it and what sort of an equation it satisfies, and what is its propagator? These are technical points, but they have an interest.

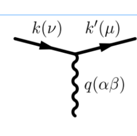
Feynman: Give me ten minutes. And let me show how the analysis of these tree diagrams, loop diagrams and all this other stuff is done mathematical way. Now I will show you that I too can write equations that nobody can understand. Before I do that I should

## The Ghost Lagrangian

- derived later (we need the background field method)
- "fermionic vector" $\eta^\mu$

$$\mathcal{L}_{ghost} = \bar{\eta}^\mu \left[ g_{\mu\nu} D^2 + R_{\mu\nu} \right] \eta^\nu$$

New Feynman rule:




$$= -\frac{i\kappa}{2}\left[\eta_{\mu\nu}k_\alpha k'_\beta + \eta_{\mu\nu}k_\beta k'_\alpha - \eta_{\mu\alpha}q_\beta k'_\nu - \eta_{\mu\beta}q_\alpha k'_\nu\right].$$

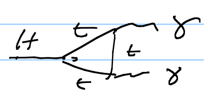
# Background Field Method

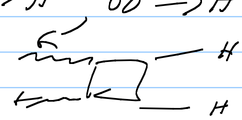
- constructing $\mathcal{L}_{eff}$
- renorm.
-

} maintains gauge invariance

---

Example $GG \to H$, $\gamma\gamma \to H$, $GG \to HH$ + -- $\boxed{m_t >> M_H}$




$$\mathcal{L}_t = -\underbrace{m_t\left(1+\frac{H}{v}\right)}_{m_t(H)}\bar{t}t$$

$$\Pi(q) = \frac{e_0^2}{12\pi^2}\left[\frac{1}{\epsilon} + \ln(4\pi) - \gamma - 6\int_0^1 dx\, x(1-x)\ln\left(\frac{m^2 - q^2 x(1-x)}{\mu^2}\right) + \mathcal{O}(\epsilon)\right]$$

$$= \frac{e_0^2}{12\pi^2}\begin{cases} \frac{1}{\epsilon} + \ln(4\pi) - \gamma + \frac{5}{3} - \ln\frac{-q^2}{\mu^2} + \ldots & (|q^2| \gg m^2), \\ \frac{1}{\epsilon} + \ln(4\pi) - \gamma - \ln\frac{m^2}{\mu^2} + \frac{q^2}{5m^2} + \ldots & (m^2 \gg |q^2|). \end{cases}$$

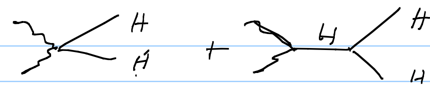
Calculate

$$= -\frac{e^2}{12\pi^2}\ln\left(\frac{M_t^2(H)}{\mu^2}\right)(g_{\mu\nu}q^2 - q_\mu q_\nu)$$

$$\ln\frac{m_t^2}{\mu^2} + \ln\left(1+\frac{H}{v}\right)^2$$

$$\mathcal{L}_{eff} = \frac{\alpha}{18\pi}\ln\left(1+\frac{H}{v}\right)F_{\mu\nu}F^{\mu\nu} + \frac{\alpha_s}{12\pi}\ln\left(1+\frac{H}{v}\right)F^a_{\mu\nu}F^{a\mu\nu} + \frac{\partial H}{m_t}$$

HW $\quad GG \to HH$



$= 0$ at $s = 4m_H^2$

---

<u>Example 2</u> QED with massless scalar

$$\mathcal{L} = (D_\mu \phi)^* (D^\mu \phi)$$

$$\leftarrow \; -\phi^* D_\mu D^\mu \phi = \phi^* \big( \Box \underbrace{+ i\{\partial_\mu, A^\mu\} - A_\mu A^\mu}_{} \big)\phi$$

$\Box + V$

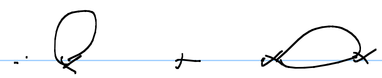
Path integral

$$= N e^{-\mathrm{Tr} \ln D^2} = N e^{-\int d^4x \langle x | \ln D^2 | x \rangle}$$

$$\ln D^2 = \ln (\Box + V) = \underbrace{\ln \Box}_{\text{drop}} + \ln\left(1 + \frac{V}{\Box}\right)$$

$$\frac{V}{\Box} + \frac{V}{\Box}\frac{V}{\Box} +$$

$$\langle x | \frac{1}{\Box} | y \rangle = D_F(x-y)$$



$$\Delta \mathcal{L} = \int d^4x\, d^4y\; F_{\mu\nu}(x)\, \frac{D_F^2(x-y)}{4(d-1)}\, F^{\mu\nu}(y)$$

$$D_F^2(x-y) = F.T.\left[ \frac{-i}{16\pi^2}\left(\frac{1}{\epsilon} + \dots\right) - \ln q^2/\mu^2 \right]$$

$$\Delta \mathcal{L} = \frac{-i}{16\pi^2}\left(\frac{1}{\epsilon} \dots\right)\int d^4x\; F_{\mu\nu} F^{\mu\nu} \quad + b e \int d^4x\, d^4y\; F_{\mu\nu}(x)\, L(x-y)\, F^{\mu\nu}(y)$$

$\propto$ renorm.

F.T. $\ln q^2$

# Appendix B

## Advanced field theoretic methods

### B–1 The heat kernel

When using path integral techniques one must often evaluate quantities of the form

$$H(x,\tau) \equiv \langle x \,| e^{-\tau\mathcal{D}} |\, x\rangle \quad , \tag{1.1}$$

where $\mathcal{D}$ is a differential operator and $\tau$ is a parameter. In this section, we shall describe the *heat kernel* method by which $H(x,\tau)$ is expressed as a power series in $\tau$. For example, if in $d$ dimensions the differential operator $\mathcal{D}$ is of the form

$$\mathcal{D} = \Box + m^2 + V \ , \tag{1.2}$$

where $V$ is some interaction, then the heat kernel expansion for $H(x,\tau)$ is

$$H(x,\tau) = \frac{i}{(4\pi)^{d/2}}\,\frac{e^{-\tau m^2}}{\tau^{d/2}}\left[a_0(x) + a_1(x)\tau + a_2(x)\tau^2 + \dots\right] \ . \tag{1.3}$$

where $a_i(x)$ are coefficients which will be determined below.

$$\langle x|\ln\mathcal{D}|x\rangle = -\int_0^\infty \frac{d\tau}{\tau}\left\langle x\left|e^{-\tau\mathcal{D}}\right|x\right\rangle + C\ , \tag{1.6}$$

where $C$ is a divergent constant having no physical consequences. Substituting Eq. (1.3) into the above yields

$$\langle x|\ln\mathcal{D}|x\rangle - C = -\frac{i}{(4\pi)^{d/2}}\sum_{n=0}^{\infty} m^{d-2n}\ \underline{\Gamma\left(n-\frac{d}{2}\right)a_n(x)}\ \ . \tag{1.7}$$

$a_2$ = divergences

$$\mathcal{D} = d_\mu d^\mu + m^2 + \sigma(x) \qquad \left(d_\mu \equiv \frac{\partial}{\partial x^\mu} + \Gamma_\mu(x)\right)\ ,$$

$$a_0(x) = 1\ , \qquad a_1(x) = -\sigma\ ,$$

$$a_2(x) = \frac{1}{2}\sigma^2 + \underbrace{\frac{1}{12}[d_\mu, d_\nu][d^\mu, d^\nu]}_{F_{\mu\nu}F^{\mu\nu}} + \frac{1}{6}\,[d_\mu,[d^\mu,\sigma]]\ \ .$$