Neutrinos in Cosmology and Astronomy

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Outline

β decays

- β^- involves the production of an electron

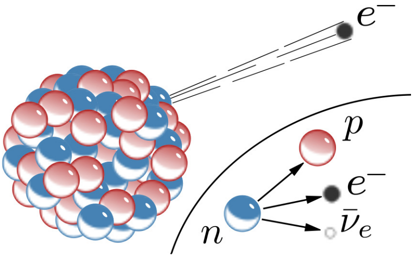
$$^{14}_{6}\mathrm{C} \to ^{14}_{7}\mathrm{N} + e^- + \dots$$

- β^+ involves the production of a positron

$$^{23}_{12}\mathrm{Mg} \to ^{23}_{11}\mathrm{Na} + e^+ + \dots$$

- unlike α and γ decays, in 1914 Chadwick established that the electrons emitted in β decays have a continuous spectrum
- In 1930 Pauli postulated the existence a new neutral particle, neutron, with mass of the same order of magnitude as the electron mass and maybe penetrating power equal or ten times bigger than a γ ray
- In 1934 Fermi introduced a new particle 'neutrino'

β^- *decay*



- Fermi: β-decay is the decay of neutron

$$n \to p + e^- + \bar{\nu}_e$$

inside the nucleus

- Two papers by E. Fermi:

An attempt of a theory of beta radiation. 1. (In German) Z.Phys. 88 (1934) 161-177
DOI: 10.1007/BF01351864
Trends to a Theory of beta Radiation. (In Italian) Nuovo Cim. 11 (1934) 1-19
DOI: 10.1007/BF02959820

- Electron capture:

$$^{22}_{11}\mathrm{Na} + e^- \to {}^{22}_{10}\mathrm{Ne} + \nu_e$$

- The process $p + e^- \to n + \nu_e$ inside a nucleus

β^+-decay

- Also observed was β^+-decay

$$^{22}_{11}\mathrm{Na} \to {}^{22}_{10}\mathrm{Ne} + e^+ + \nu_e$$

- Formally β^+ decay would come from the

$$p \xrightarrow{?} n + e^+ + \nu_e$$

but mass of proton $m_p < m_n$ (mass of neutron)?!...

...possible if neutrons are not free (nuclear binding energy)

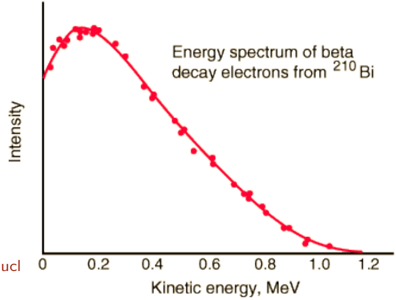
- This reaction is the main source of solar neutrinos:

$$4p \to {}^4\mathrm{He} + 2e^+ + 2\nu_e + 26.7\ \mathrm{MeV}$$

Fermi theory of weak interactions I

- The emitted electrons are (quasi) relativistic (kinetic energies from keV to few MeV). Neutrinos are relativistic
- Typical nuclear size $r_{\rm nucl} \sim 10^{-13}$ cm.
- Typical wavelengths:

$$\frac{\hbar}{p_e} \sim \frac{\hbar}{p_{\bar{\nu}_e}} \sim \frac{\hbar}{100\,\mathrm{keV}} \sim 2\times 10^{-10}\,\mathrm{cm} \gg r_{\rm nucl}$$



(where $\hbar = 6.6 \times 10^{-19}$ keV·sec)

Fermi theory of weak interactions II

- **Proposal of Fermi:** β-decay is the decay of neutron inside a nucleus

$$n \to p + e^- + \boldsymbol{\nu} \tag{1}$$

- Electron capture ${}^{22}_{11}\mathrm{Na} + e^- \to {}^{22}_{10}\mathrm{Ne}$ is then interpreted as

$$p + e^- \to n + \boldsymbol{\nu} \tag{2}$$

First theory of weak interactions

$$\mathcal{L}_{\mathrm{Fermi}} = \frac{\boldsymbol{G_F}}{\sqrt{2}}[\bar{p}(x)\Gamma n(x)][\bar{e}(x)\Gamma\nu(x)] \tag{3}$$

- G_F – Fermi coupling constant. Value determined experimentally to be $G_F \approx 1.16 \times 10^{-5}\ \mathrm{GeV}^{-2} \approx 1/(300\ \mathrm{GeV})^2$

Proton-antineutrino scattering I

- Let us compute the cross-section in the Fermi theory of the process

$$p + \bar{\nu}_e \to n + e^+$$

— the process that has been used to detect the neutrino for the first time!

- Dimension of cross-section $[\sigma] = L^2 = E^{-2}$, dimension of the Fermi constant: $[G_F] = E^{-2}$. So at high energies we expect

$$\sigma \propto G_F^2 E^2 \tag{4}$$

- For $E_\nu \sim 1$ MeV (i.e. $E_\nu \ll m_p \approx 1$ GeV) we find that

$$\sigma_{tot}(1\,\mathrm{MeV}) \approx 3 \times 10^{-17}\,\mathrm{GeV}^{-2} \approx 10^{-44}\,\mathrm{cm}^2 \tag{5}$$

using $1\,\mathrm{barn} \equiv 10^{-24}\,\mathrm{cm}^2 = 2.57 \times 10^3\,\mathrm{GeV}^{-2}$

- for comparison: photon scattering on non-relativistic electron (**Thomson cross-section**) is $\sigma_{Thomson} \sim 10^{-24}\,\mathrm{cm}^2$
- In 1934 Bethe & Peierls did the estimate of the cross-section (5)

Proton-antineutrino scattering II

- In 1942 Fermi had build the first nuclear reactor – source of large number of (anti)neutrinos ($\sim 10^{13}\mathrm{neutrinos/sec/cm^2}$) $n \to p + e^- + \bar{\nu}$
- Flux of neutrinos from e.g. a nuclear reactor can initiate β^+ decays in protons of **water** $\bar{\nu} + p \to n + e^+$
- Event rate:

$$R = \text{Flux} \times \sigma_{tot} \times N_{\text{protons}} = 10^{13}\,\frac{1}{\mathrm{sec}\cdot\mathrm{cm}^2}10^{-44}\,\mathrm{cm}^2 \times 10^{23} \approx 10^{-8}\mathrm{sec}^{-1} \quad (6)$$

- Positrons annihilate and two γ-rays and neutron were detected!
- The existence of anti-neutrino has been experimentally confirmed by Cowan & Reines in 1956

Muon and muon neutrino

- The muon has been discovered in 1936 when studying cosmic rays
- Its mass $m_\mu \approx 104$ MeV is smaller than the mass of any known particle except the electron and neutrino. Therefore, naively the decay channels of μ are

$$\mu \to e + \gamma, \quad \mu \to e + \nu \tag{7}$$

- However, experimentally was observed only the 3-body decay

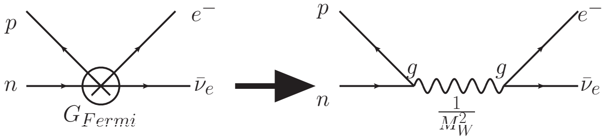
$$\mu \to e + \bar{\nu}_e + \nu \tag{8}$$

- Non-observation of the decays (7) can be naturally explained if one assumes the conservation of the electron and muon lepton numbers, thus associating the second neutrino in the reaction (8) with the new type of the neutrino — the muon neutrino ν_μ
- The decay (8) can be well described using the Lagrangian

$$\mathcal{L}_{\mu e} = -\frac{G_F}{\sqrt{2}}[\bar{\nu}_\mu \gamma_\alpha \mu][\bar{e}\gamma^\alpha \nu_e] \tag{9}$$

From β-decay to weak interactions. Fast forward

- Fermi theory predicted that if electron and neutrino collide at high energies, the probability of such a process can "exceed 1"
- **Prediction** of massive vector bosons — weak interaction is mediated by some particle (as all other interactions)



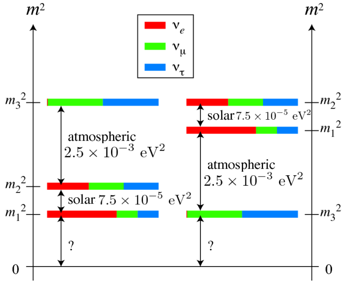
- Predictions of neutral currents (**Glashow 1961**)
- Building of electroweak theory. New gauge symmetries. Higgs mechanism (**1964 – 1968**)
- Proposals for unification of electromagnetic and weak forces ; (**Weinberg 1967; Salam 1968**)
- Discovery of neutral currents (**1974**)
- Discovery of Z and W bosons at LEP@CERN (**1980s**)

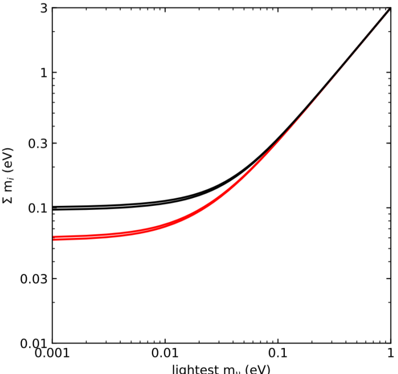
Neutrino oscillations

- in the 60s Davis et al observed a deficit of solar neutrinos
- **Atmospheric** evidence: observed disappearence of ν_μ and $\bar{\nu_\mu}$
  $\nu_\mu \to \nu_\tau$ with quasi-maximal mixing angle and $\Delta m^2 \sim 10^{-3}\,\mathrm{eV}^2$
- **Solar** evidence: observed disappearence of ν_e
  $\nu_e \to \nu_{\mu,\tau}$ with large but not-maximal mixing angle and $\Delta m^2 \sim 10^{-4}\,\mathrm{eV}^2$
- interpreted as eigenstate of flavor are not eigenstate of the mass
- in the simplified case of two masses and flavors in void

$$\nu = e^{ip_1 x} \cos\theta|\nu_1\rangle + e^{ip_2} \sin\theta|\nu_2\rangle$$

$$P(\nu_{e_5} \to \nu_\mu) = \sin^2 2\theta \sin^2 \frac{\Delta m_{12}^2 L}{4E}$$





Summary on neutrino properties

- there are 3 neutrinos (for each generation): ν_e, ν_μ, ν_τ
- neutrinos are stable
- neutrinos are electrically neutral
- neutrinos have tiny masses (much smaller than mass of the electron)
- neutrinos participate in weak interactions
- neutrinos oscillate in masses

How neutrinos are produced in the early Universe?

Outline

Friedmann-Lemaitre-Robertson-Walker metric I

- Universe is homogeneous and isotropic

$$ds^2 = dt^2 - a^2(t)\gamma_{ij}dx^i dx^j$$

where γ_{ij} is the metric of the unit three-sphere

- we define the redshift as

$$z(t) = \frac{a_0}{a(t)} - 1$$

Drop of temperature with expansion I

- $$f(\mathbf{X},\mathbf{p})d^3\mathbf{X}d^3\mathbf{p}$$
- $d^3\mathbf{X}$ and $d^3\mathbf{p}$ $\rightarrow$ the physical volume element and the physical momenta
- $d^3\mathbf{x}$ and $d^3\mathbf{k}$ are the integration elements in comoving coordinate and in comoving momenta
- homogeneous gas $f(\mathbf{X},\mathbf{p},t) = f(\mathbf{p},t)$
- the coordinate momenta of free particle k are independent of time $\rightarrow$ $f(\mathbf{k},t) = f(\mathbf{k})$ time-independent
- number of particles in comoving space-phase is independent of time $f(\mathbf{k})d^3\mathbf{x}d^3\mathbf{k} = const$
- The comoving phase space volume coincides with the physical one

$$d^3\mathbf{X}d^3\mathbf{p} = d^3\left(\frac{\mathbf{x}}{a}\right)d^3\left(a\mathbf{k}\right) = d^3\mathbf{x}d^3\mathbf{k}$$

- Hence, $f(\mathbf{p},t) = f(\mathbf{k}) = f\left(a(t)\cdot\mathbf{p}\right)$

Drop of temperature with expansion II

- if f is known at some time $f(\mathbf{p}, t_i) = f_i(\mathbf{p})$

$$f(\mathbf{p}, t) = f_i\left(\frac{a(t)}{a_i}\mathbf{p}\right)$$

- if photons, $f_i(\mathbf{p}) = f_{Pl}\left(\frac{|\mathbf{p}|}{T_i}\right) = \frac{1}{\pi^3}\frac{1}{e^{|\mathbf{p}|/T_i}-1}$
-

$$\begin{aligned} f(\mathbf{p}, t) &= f_{Pl}\left(\frac{a(t)|\mathbf{p}|}{a_i T_i}\right) = f_{Pl}\left(\frac{|\mathbf{p}|}{T_{eff}(t)}\right) \\ T_{eff} &= \frac{a_i}{a(t)} T_i \propto \frac{1}{a(t)} \end{aligned} \tag{10}$$

- it applies to photons – also to particle have mass but are relativist at decoupling, this is the case of **hot dark matter**

Neutrinos in primordial plasma I

- In the Early universe weak reactions are initially in thermal equilibrium, neutrinos can be created and annihilated through processes like $e^+ + e^- \to \nu_\alpha + \bar{\nu}_\alpha$, $e^- + \nu \to e^- + \nu$
- As Universe expands, its temperature drops down as $T \sim 1/a$. At some moment reaction rates drop too much and neutrinos go out from equilibrium

What is a neutrino reaction rates?

- Recall: weak interaction strength is **Fermi coupling constant** $G_F \approx 10^{-5}\ \text{GeV}^{-2}$
- the interaction rate

$$\Gamma_{ee\to\nu\bar{\nu}} = n_e(T) \times \sigma_{\text{Weak}}$$

where from dimentional arguments

$$\sigma_{\text{Weak}} \propto G_F^2 \times E_e^2$$

What is the typical energy of electrons in this reaction?

Neutrinos in primordial plasma II

- At temperatures $T \gg m$ (relativistic) electron distribution function is

$$f_e(p) = \frac{1}{(2\pi)^3} \frac{1}{e^{E(p)/T} + 1}$$

- Number density of the electrons

$$n_e(T) = 4 \int \frac{d^3 p}{(2\pi)^3} \frac{1}{e^{E(p)/T} + 1} \propto T^3$$

- Average energy of the electron $E_e = c \times \langle p \rangle$ i.e

$$E_e = \frac{4}{n_e(T)} \int \frac{d^3 p}{(2\pi)^3} \frac{p}{e^{E(p)/T} + 1} \sim T$$

- As a result $E_e \sim T$
- Reaction rate $\Gamma_{ee \to \nu\bar{\nu}} \sim G_F^2 T^5$

Neutrinos in primordial plasma III

- Compare the characteristic interaction time $\Gamma^{-1}_{ee\to\nu\bar{\nu}}$ with the age of the Universe $t_{\text{Univ}} = 1/H(T)$. To establish equilibrium we need

$$\Gamma^{-1}_{ee\to\nu\bar{\nu}} \ll t_{\text{Univ}}$$

or

$$\Gamma_{ee\to\nu\bar{\nu}} \gg H(T)$$

At what temperatures neutrinos are in equilibrium?

- We need to estimate $H = \frac{8\pi}{3} G\rho$
- then we need to estimate ρ in radiation domination

g_* in Standard Model I

- In general, in the expanding Universe particles that are in thermal equilibrium have either Fermi-Dirac or Bose-Einstein distributions
- At temperature $T \gg m$ we are in the relativistic regime
- Energy density for the Bose/Fermi distributions

$$\rho_i = \frac{g_i}{2\pi^2}\int \frac{E^3}{e^{E/T} \mp 1} dE = \begin{cases} g_i \frac{\pi^2}{30} T^4 & -\text{Bose} \\ \frac{7}{8} g_i \frac{\pi^2}{30} T^4 & -\text{Fermi} \end{cases}$$

- The Friedmanns equation for RD epoch can be written as:

$$H^2(T) = \frac{8\pi G_N}{3} \underbrace{g_*(T)\frac{\pi^2}{30} T^4}_{\rho_{\rm rad}}$$

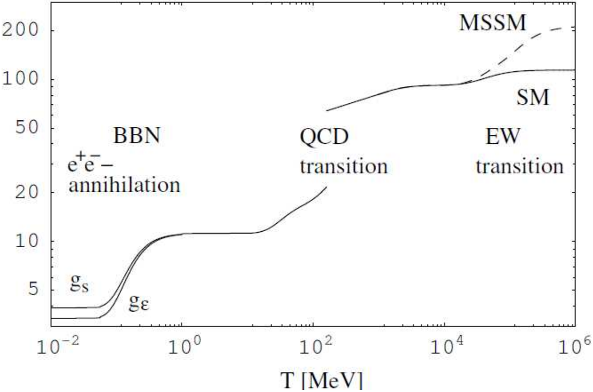
- Here g_* – effective number of relativistic degrees of freedom

$$g_* \equiv \sum_{\text{boson species}} g_i + \frac{7}{8} \sum_{\text{fermion species}} g_i$$

where the sum goes over **relativistic species only** (massless particles, or massive particles that have $\langle p \rangle \gtrsim m$ at temperature T)

g_* in Standard Model II

- As a result, $2 \lesssim g_* \lesssim 110$ for Standard Model:



at what temperature neutrinos come out of equilibrium?

- Given $H = \sqrt{\frac{8\pi}{3} G_N \rho}$
- Given $\rho_{\rm RD} = g_*(T) \frac{\pi^2}{30} T^4$
- Given $g_*(T \sim \text{ MeV}) \sim 10$
- Then neutrinos decouple when temperature satisfies

$$\Gamma \sim H$$

that can be rewritten as

$$G_F^2 T^5 = T^2 \sqrt{\frac{8\pi^3}{90} G_N g_*(T)}$$

this implies a temperature

$$T_{\nu,f} \sim 2 - 3 \text{ MeV}$$

Neutrino in the early Universe: summary

- Neutrinos are produced in the early Universe and are in thermal equilibrium in plasma at $T \gtrsim T_{\text{dec}} \sim 1$ MeV
- As all equilibrium ultra-relativistic particles their average energy is $\langle E_\nu \rangle \sim T$, their number density is $\sim T^3$
- Their interaction rate with other particles $\Gamma_\nu \sim G_F^2 T^5$
- The assumption that existed temperatures of the order of a few MeV leads to the conclusion that the present Universe contains the gas of relic neutrinos analogous to the CMB

Relic neutrinos today I

- We saw that at $T_{\rm dec} \sim 1$ MeV particle go out of thermal equilibrium – **freeze-out**
- Their concentration at the time of decoupling is given by

$$n(T_{dec}) = \frac{3}{4}\frac{2\zeta(3)}{\pi^2} T_{dec}^3 \tag{11}$$

What happens below $T_{\rm dec}$?

- The number of neutrinos does not change (particles are no longer produced or destroyed)
- Therefore, the comoving number density is conserved:

$$n_{\rm co}(T < T_{dec}) = n_{\rm co}(T_{dec}) \propto T_{dec}^3 \tag{12}$$

- Average momentum today is $\sim 10^{-3}$ eV The average momentum of decoupled particles changes with time (redshifts). Average momentum at the time of decoupling was ~ 1 MeV

Relic neutrinos today II

- As a result **today** in the Universe there are lots (about $3 \times 112\,\mathrm{cm}^{-3}$) neutrinos (**exercise:** reproduce this number). Is it a lot or not?
- The number density of CMB photons is about $410\,\mathrm{cm}^{-3}$ so we have as much relic neutrino as photons

What is the current temperature of νs?

- at neutrinos freeze-out there cosmic plasma contained a lot of relativistic electrons and positrons.
- after the temperature drops below the electron mass, e^- and e^+ annihilate away, injecting energy into the photon component.
- using entropy conservation $g_*(T)a^3T^3 = const$

$$\frac{T_{\gamma,0}}{T_{\nu,0}} = \left(\frac{g_*(T_{\nu,f})}{g_*(T_{\gamma,f})}\right)^{1/3} = \left(\frac{11}{4}\right)^{1/3} \simeq 1.4 \quad (13)$$

$$T_\nu(t_0) \simeq 1.95\,\mathrm{K} \quad (14)$$

Relic neutrinos today III

How much do νs contribute to current budget of energy density?

- If neutrinos were **massless**:

$$\Omega_\nu = \frac{\rho_{\nu,0}}{\rho_c} = 2\frac{7}{8}\frac{\pi^2}{30}\frac{T^4_{\nu,0}}{\rho_c} \sim 10^{-5}$$

  for each neutrino plus its anti-neutrino.

- massless neutrinos $\rightarrow$ neutrinos would be negligible in today energy budget
- If neutrinos **have a mass** $m_\nu > T_{\nu,0}$:

$$\rho_{\nu,0} = m_\nu n_{\nu,0} \quad (15)$$

$$\Omega_\nu \sim \left(\frac{m_\nu}{1\,\mathrm{eV}}\right) \cdot 0.01 h^{-2} \quad (16)$$

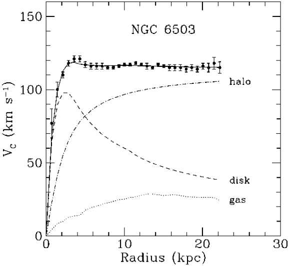
- we can get a cosmological bound on the mass of neutrinos if we require they do not exceed the total amount of matter in the Universe

$$\Sigma_i m_{\nu i} < 100 \cdot h^2 \Omega_M \,\mathrm{eV} < 11\,\mathrm{eV}$$

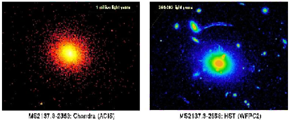
Outline

Many astrophysical observations point to dark matter existence:

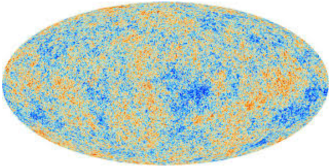
- Galactic scale



- Galaxy Cluster scale

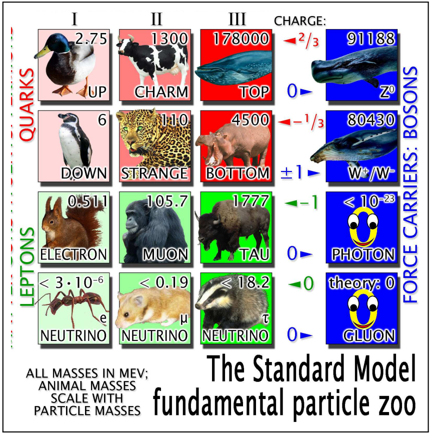


- Cosmology scale



Properties of Dark Matter particles I

- DM particle should be: **massive** (relativistic particles do not cluster)
- If DM particles ever were relativistic – they should have **slow down early** in the history of the Universe
- DM particles should be **neutral** (not to interact with photons)
- DM particles should be **stable** or have cosmologically **long lifetime**



Any candidates in the Standard Model?

Energy density of relic neutrinos I

- After neutrino decouple, their comoving number density does not change
- Their momentum decreases

$$\langle p_\nu \rangle \propto T \tag{17}$$

- If neutrinos are massive, at some moment $\langle p_\nu(T) \rangle$ becomes **smaller than** m_ν – neutrinos become non-relativistic
- Their number does not change, but their energy density changes from $\rho_\nu \propto T^4$ to

$$\rho_\nu = \underbrace{\left(\sum m_\nu\right) \times n_{\rm dec}}_{\text{number density of non-relativistic particles}} \quad \text{or numerically} \quad \Omega_\nu h^2 \equiv \frac{\rho_\nu}{\rho_{\rm crit}} \approx \frac{\sum m_\nu}{94 \text{ eV}} \tag{18}$$

$$\Omega_\nu h^2 < 0.12 \rightarrow \Sigma m_\nu < 11 eV$$

Neutrino Dark Matter? I

Tremaine-Gunn bound

In 1979 when S. Tremaine and J. Gunn published in Phys. Rev. Lett. a paper *"Dynamical Role of Light Neutral Leptons in Cosmology"*

- The smaller is the mass of Dark matter particle, the larger is the number of particles in an object with the mass M_{gal}
- Average **phase-space density** of **any fermionic** DM should be **smaller** than density of **degenerate Fermi gas**
- The density of degenerate Fermi gas is given by

$$\Pi_{deg} = \frac{1}{(2\pi\hbar)^3} \tag{19}$$

- The mass density of non-relativistic degenerate fermions is given by

$$\rho_{deg} = m \times \underbrace{\int d^3\boldsymbol{x} d^3\boldsymbol{p} \frac{1}{(2\pi\hbar)^3}}_{\text{number density}} = m \times \int d^3\boldsymbol{x} d^3\boldsymbol{v} \frac{m^3}{(2\pi\hbar)^3} \tag{20}$$

where we took into account that fermions are non-relativistic and so $\boldsymbol{p} = m\boldsymbol{v}$

Neutrino Dark Matter? II

Tremaine-Gunn bound

- Thus, the mass density in the **velocity** (rather than momentum) space is given by

$$\Pi_{deg} = \frac{m^4}{(2\pi\hbar)^3} \tag{21}$$

- If dark matter is made of fermions – **its mass is bounded from below**
- Indeed, a galaxy with mass M_{gal} and size R_{gal} has average matter density

$$\rho_{matter} = \frac{M_{gal}}{\frac{4\pi}{3} R_{gal}^3} \tag{22}$$

- It occupies the volume of velocity space with $|\boldsymbol{v}| < v_\infty$, where v_∞ is the **escape velocity** – velocity that is sufficient for a particle to break gravitational attraction of the galaxy and leave it

Neutrino Dark Matter? III

Tremaine-Gunn bound

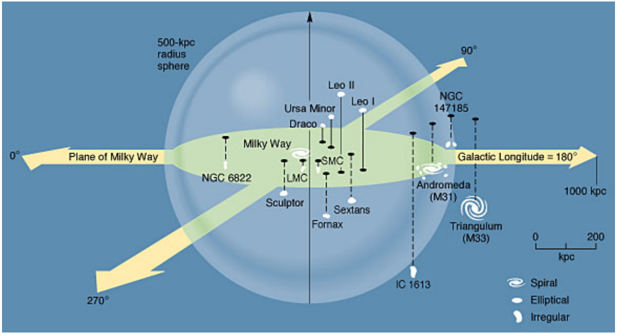
- Average phase-space density of any system of fermions should be **lower** than the phase-space density of degenerate gas (21)

$$\underbrace{\frac{M_{gal}}{\frac{4\pi}{3}R_{gal}^3}}_{\text{volume of space}} \underbrace{\frac{1}{\frac{4\pi}{3}v_\infty^3}}_{\text{volume of } \textbf{velocity} \text{ space}} \leq \frac{2{m_{\rm DM}}^4}{(2\pi\hbar)^3} \tag{23}$$

Mass of any fermionic dark matter is **bounded from below**

Tremaine-Gunn bound and observations I

- Let us put the numbers for M_{gal}, R_{gal} and v_∞ from observations
- Objects with highest phase-space density – dwarf spheroidal galaxies – lead to the **lower bound** on the fermionic DM mass $M_{\rm DM} \gtrsim 300 - 400$ **eV** **[0808.3902]**



Tremaine-Gunn bound and observations II

- **However**, as we have seen if you compute contribution to DM density from massive active neutrinos ($m_\nu \lesssim$ MeV), you get

$$\Omega_{\nu\ \mathrm{DM}} h^2 = \boxed{\frac{\sum m_\nu [\mathrm{eV}]}{94\ \mathrm{eV}}} \lesssim 0.12$$

- Using minimal mass of 300 eV you get $\Omega_{\mathrm{DM}} h^2 \sim 3$ (**wrong by about a factor of 30!**)
- Sum of masses to have the correct abundance $\boxed{\sum m_\nu \approx 11\ \mathrm{eV}}$ at most

Tremaine-Gunn bound and observations III

- Massive Standard Model neutrinos cannot be simultaneously "astrophysical" and "cosmological" dark matter: to account for the missing mass in galaxies **and** to contribute to the cosmological expansion
- Relic neutrinos cannot expain all dark matter and are only small part of it. How do they behave, are they different from the rest?

Outline

Estimating effective number of relativistic species I

- for the purpose of studying the impact of neutrinos on structure formation, we estimate the contribution of neutrinos to radiation energy density
- we remind the energy density for relativistic species

$$\rho_i = \frac{g_i}{2\pi^2}\int \frac{E^3}{e^{E/T}\mp 1}dE = \begin{cases} g_i \frac{\pi^2}{30} T^4 & -\text{Bose} \\ \frac{7}{8} g_i \frac{\pi^2}{30} T^4 & -\text{Fermi} \end{cases}$$

- we remind the temperature of freeze-out for neutrinos

$$\frac{T_{\gamma,0}}{T_{\nu,0}} = \left(\frac{11}{4}\right)^{1/3}$$

Estimating effective number of relativistic species II

- after neutrino decoupling and after the annihilation of e^- and e^+

$$\rho_R = \rho_\gamma \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} 3\right)$$

- this result holds if
  - there are only three light neutrino species and no other relativistic particles
  - neutrino distributions are standard Fermi-Dirac functions
  - we trust the instantaneous decoupling limit

Estimating effective number of relativistic species III

- for a general parametrization

$$\rho_R = \rho_\gamma \left(1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\text{eff}}\right)$$

- precise simulations of neutrino decoupling and $e^\pm$ annihilation, taking into account flavor oscillations $\rightarrow$ precise predictions for the actual phase-space distribution of relic neutrinos.
- The most recent analysis, that includes the effect of neutrino oscillations with the present values of the mixing parameters gives

$$N_{\text{eff}} = 3.045$$

Transition to not-relativistic regime I

- estimating the time of transition from relativistic to not relativistic regime

$$\begin{aligned} p &= 3.15\,T_\nu \\ T_\nu(z) &= \left(\frac{4}{11}\right)^{1/3} T_0(1+z) \\ &= 1.68\times 10^{-4}(1+z)\,\mathrm{eV} \\ p(z_\nu) &= T(z_\nu) \end{aligned}$$

$$\rightarrow 1+z_{\rm nu} \simeq 1900\left(\frac{m_\nu}{eV}\right)$$

- until they are relativistic, neutrinos enhance the density of radiation, their effect can be described by $N_{\rm eff}$

$$\rho \propto N_{\rm eff}$$

- after they become not relativistic, neutrinos effects can be described by $\Sigma_\nu m_\nu$.

$$\rho \propto \Sigma_\nu m_\nu$$

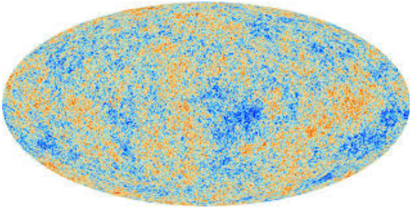
- in both regimes we have to distinguish between the background effects and the effect on perturbations

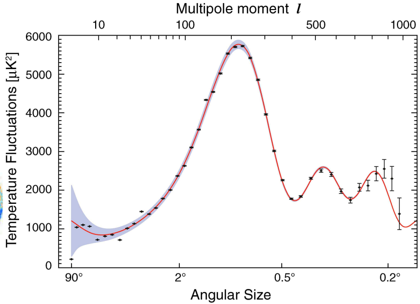
Cosmological observable: CMB

Scales probed by **CMB experiments** (**linear** regime of perturbation growth)

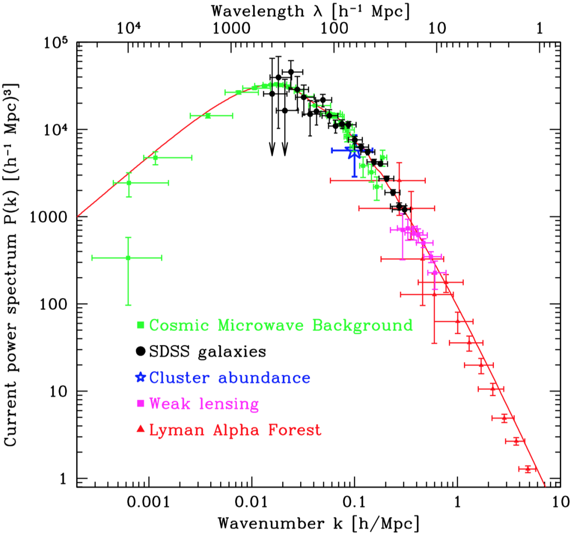
$$k \simeq \ell \times \frac{H_0}{2} = \frac{\ell}{6000} \frac{h}{\text{Mpc}} \tag{24}$$

Is sensitive up to scales $k \lesssim 0.1\ h/\ \text{Mpc}$



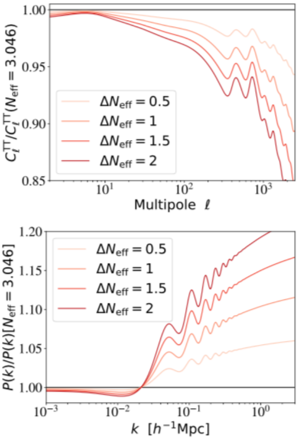


Cosmological observable: matter power spectrum



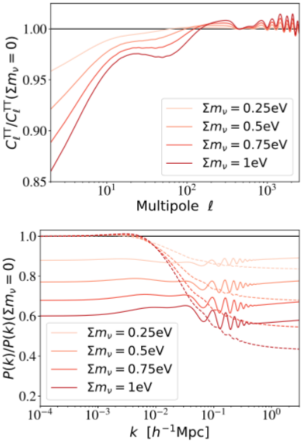
Effect of $N_{\rm eff}$ on the CMB and matter power spectrum

- If the densities of other species are kept fixed, a higher $N_{\rm eff}$ implies a smaller redshift of radiation-to-matter equality
- Instead, we keep the redshift of radiation-to-matter equality $z_{\rm eq}$ and matter-to-Λ equality zΛ and $\omega_b = \Omega_b h^2$ fixed (these quantities are very accurately constrained by CMB)



Effect of $\Sigma_\nu m_\nu$ on the CMB and matter power spectrum

- m_ν modifies the CMB through the angular diameter distance to recombination, $d_A(z_{\rm rec})$, and on the redshift of matter-to-Λ equality.
- Since CMB measures $d_A(z_{\rm rec})$ very well we maintain it fixed



Constraints on $N_{\rm eff}$

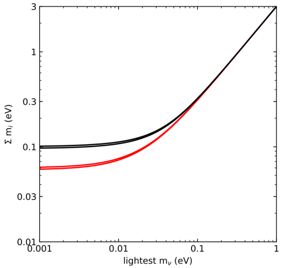
	Model	68%CL	Ref.
CMB alone			
Pl15[TT+lowP]	ΛCDM+$N_{\rm eff}$	3.13 ± 0.32	[29]
Pl15[TT+lowP]	ΛCDM+$N_{\rm eff}$+$\sum m_\nu$	3.08 ± 0.31	[35]
CMB + probes of background evolution			
Pl15[TT+lowP] + BAO	ΛCDM+$N_{\rm eff}$	3.15 ± 0.23	[29]
Pl15[TT+lowP] + BAO	ΛCDM+$N_{\rm eff}$+$\sum m_\nu$	$3.18^{+0.24}_{-0.27}$	[35]
CMB + probes of background evolution + LSS			
Pl15[TT+lowP+lensing] + BAO	ΛCDM+$N_{\rm eff}$	$3.08^{+0.22}_{-0.24}$	[35]
" + BAO + JLA + HST	ΛCDM+$N_{\rm eff}$	3.41 ± 0.22	[31]
" + BAO	ΛCDM+$N_{\rm eff}$+$\sum m_\nu$	3.2 ± 0.5	[29]
Pl15[TT,TE,EE+lowP+lensing]	ΛCDM+$N_{\rm eff}$+5-params.	$2.93^{+0.51}_{-0.48}$	[34]

Constraints on $\Sigma_\nu m_\nu$

	Model	95% CL (eV)	Ref.
CMB alone			
Pl15[TT+lowP]	ΛCDM+$\sum m_\nu$	< 0.72	[29]
Pl15[TT+lowP]	ΛCDM+$\sum m_\nu$+$N_{\rm eff}$	< 0.73	[35]
Pl16[TT+SimLow]	ΛCDM+$\sum m_\nu$	< 0.59	[32]
CMB + probes of background evolution			
Pl15[TT+lowP] + BAO	ΛCDM+$\sum m_\nu$	< 0.21	[29]
Pl15[TT+lowP] + JLA	ΛCDM+$\sum m_\nu$	< 0.33	[35]
Pl15[TT+lowP] + BAO	ΛCDM+$\sum m_\nu$+$N_{\rm eff}$	< 0.27	[35]
CMB + probes of background evolution + LSS			
Pl15[TT+lowP+lensing]	ΛCDM+$\sum m_\nu$	< 0.68	[29]
Pl15[TT+lowP+lensing] + BAO	ΛCDM+$\sum m_\nu$	< 0.25	[35]
Pl15[TT+lowP] + $P(k)_{\rm DR12}$	ΛCDM+$\sum m_\nu$	< 0.30	[50]
Pl15[TT,TE,EE+lowP] + BAO+ $P(k)_{\rm WZ}$	ΛCDM+$\sum m_\nu$	< 0.14	[52]
Pl15[TT,TE,EE+lowP] + BAO+ $P(k)_{\rm DR7}$	ΛCDM+$\sum m_\nu$	< 0.13	[52]
Pl15[TT+lowP+lensing] + Lyα	ΛCDM+$\sum m_\nu$	< 0.12	[48]
Pl16[TT+SimLow+lensing] + BAO	ΛCDM+$\sum m_\nu$	< 0.17	[48]
Pl15[TT+lowP+lensing] + BAO	ΛCDM+$\sum m_\nu$+Ω_k	< 0.37	[35]
Pl15[TT+lowP+lensing] + BAO	ΛCDM+$\sum m_\nu$+w	< 0.37	[35]
Pl15[TT+lowP+lensing] + BAO	ΛCDM+$\sum m_\nu$+$N_{\rm eff}$	< 0.32	[29]
Pl15[TT,TE,EE+lowP+lensing]	ΛCDM+$\sum m_\nu$+5-params.	< 0.66	[34]

Future constraints

- Direct detection: PTOLEMY
- New measurements of matter power spectrum shape:
  DES, eBOSS, LSST, Euclid, DESI, WFIRST
- new CMB data:
  ground based: CMB-S4
  satellite: LateBird , CORE
- New measurements of Baryon Acoustic Oscillation (BAO) scale
  DES, eBOSS, LSST, DESI, Euclid, WFIRST
- 21cm Hydrogen-line surveys
  HERA, SKA
- if ΛCDM model and if neutrinos are standard, the total neutrino mass should be detected at the level of 3-4σ
- their effects on the matter power spectrum can be as low as the 5% level $\rightarrow$ exquisite control of systematic errors will be crucial



Outline

Hot dark matter I

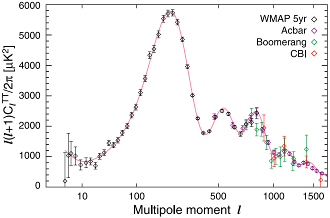
- Next blow to neutrino DM came around 1983–1985 when M. Davis, G. Efstathiou, C. Frenk, S. White, *et al.* "*Clustering in a neutrino-dominated universe*"
- They argued that structure formation in the neutrino dominated Universe (with masses around 100 eV would be incompatible with the observations)

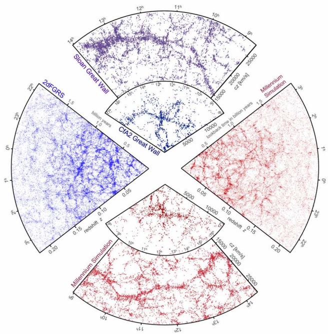
http://www.adsabs.harvard.edu/abs/1983ApJ...274L...1W

Abstract

The nonlinear growth of structure in a universe dominated by massive neutrinos using initial conditions derived from detailed linear calculations of earlier evolution has been simulated . **The conventional neutrino-dominated picture appears to be ruled out.**

But dark matter is cold?





- *But cosmological model is called* **ΛCDM** *(where "C" stands for "cold")?*
- At cosmological scales (probed mostly by CMB exps.) dark matter particles seem to be **non-interacting**, "cold", . . .
- The correct statement is that DM **became** cold by the time of CMB and that "hot" DM (like **Standard Model neutrinos**) do not contribute significantly to the Universe mass balance at matter-dominated epoch (CMB, LSS, . . .)

The bound on DM primordial velocities

$$\lambda_{FS}^{co} = \int_0^t \frac{v(t')dt'}{a(t')}$$

- The non-zero velocities of DM particles **erase** the primordial spectrum of density perturbations on scales up to the DM particle horizon – **free-streaming length**.

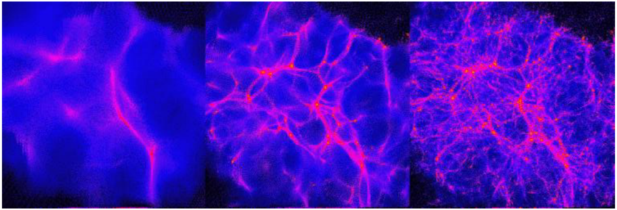
In addition, primordial velocities affect

- **Power-spectrum of density fluctuations** (suppress normalization at large scale);
- **Halo mass function** (number of halos of small mass decreases);
- **Dark matter density profiles** in individual objects.

CDM/WDM/HDM particle physics candidates

In terms of their primordial velocities, the dark matter particles can be divided in tree types:

Hot DM	Warm DM	Cold DM
$t_{nr} > t_{eq}$	$t_{nr} \ll t_{eq}$	$t_{nr} \to 0$



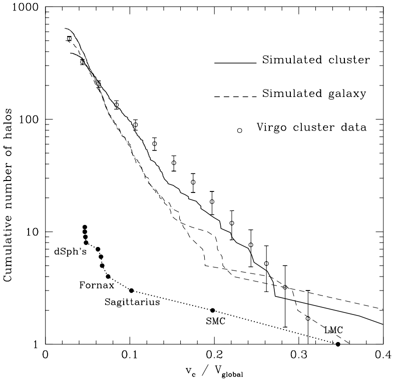
Moore, Macciò'05

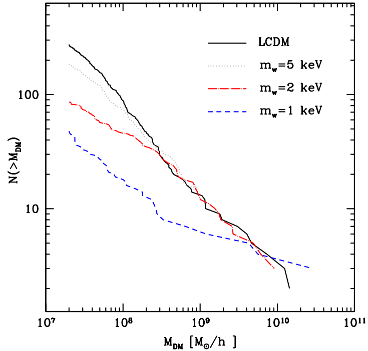
Knowing that dark matter particles were created relativistic (warm) would allow to choose between various candidates

Subhalo mass function

- **Cold** dark matter structures form in a scale-free manner:Diemand et al. 2008
- We see much less satellites than naively expected in CDM
- Where are these "missing satellites"?

Moore et al. '99 Macciò & Fontanot'09





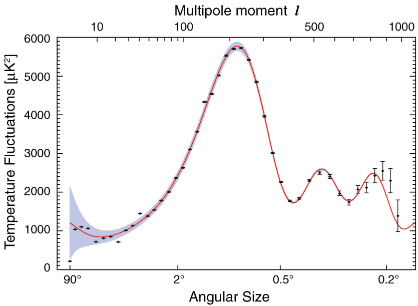
Is suppression of number of substructures due to the **free-streaming**?

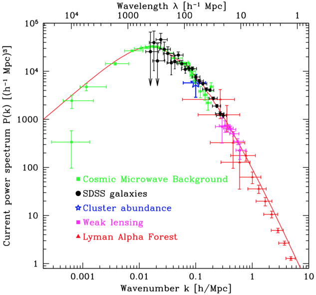
Other effects of primordial velocities

Scales probed by **CMB experiments** (**linear** regime of perturbation growth)

$$k \simeq \ell \times \frac{H_0}{2} = \frac{\ell}{6000}\frac{h}{\mathrm{Mpc}} \tag{25}$$

Is sensitive up to scales $k \lesssim 0.1\ h/\ \mathrm{Mpc}$





Smaller scales ⇒ **non-linear** stage of structure formation.

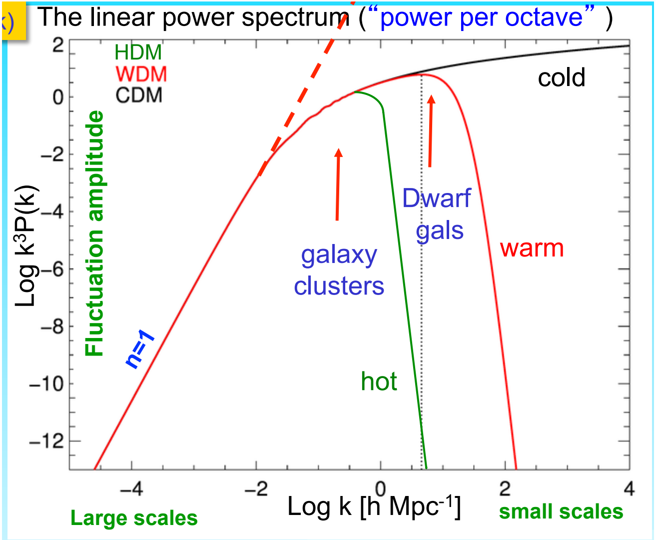
Free-streaming scale

- The free-streaming scale is (roughly)

$$\lambda_{\text{fs}} \approx 1.2\,\text{Mpc}\left(\frac{\langle p_{\text{DM}}\rangle}{\langle p_\nu\rangle}\right)\left(\frac{1\text{ keV}}{M_{\text{DM}}}\right) \tag{26}$$

- Modified number of and properties of structures:
  - At scales **below** the free-streaming λ_{fs} the primordial spectrum of density perturbations is suppressed ("erased") partially or fully (depends on the primordial velocity dispersion spectrum)
  - At scales **larger** than free-streaming, the remaining velocities of dark matter particles do not allow them to fall into "shallow" potential wells (one needs $\frac{v^2}{2} < \frac{GM}{R}$ to confine DM particles). As a result structures in warm DM begin to form **later**, when particles "cool down" because of the expansion

Suppression of power spectrum



What is plotted here is average overdensity at scale k: $k^3 P(k) \sim \left\langle \left| \frac{\delta \rho_k}{\bar{\rho}} \right|^2 \right\rangle$

Warm dark matter affects...

- **Matter power spectrum** of density fluctuations at scales **below the free-streaming**:

$$\mathbf{P(k)} = \left|\frac{\delta\rho_k}{\rho}\right|^2 = \int d^3\vec{r} e^{ik\cdot(\vec{x}-\vec{x}')} \left\langle \delta(\vec{x})\delta(\vec{x}') \right\rangle$$

- **Halo (subhalo) mass function** (decrease number of halos of small mass)

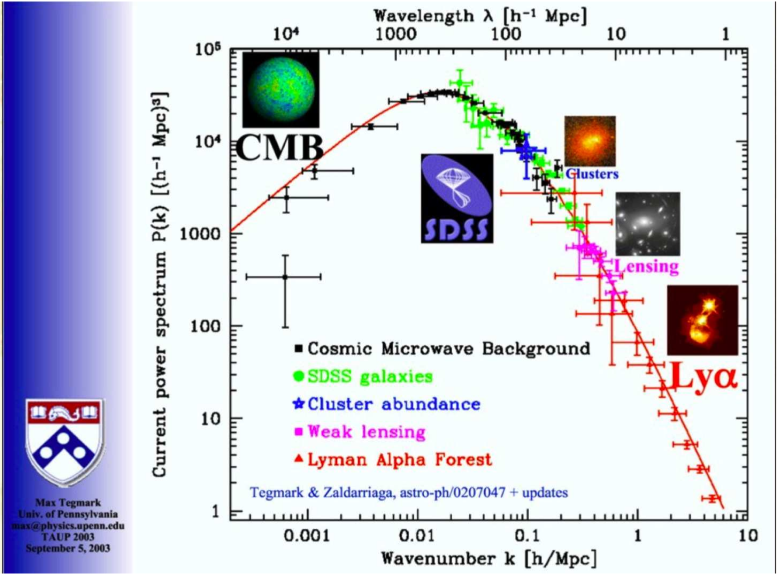
$$N(>M) = \int_M^\infty dM' \frac{dn(M')}{dM}$$

- Density profile (central **core** rather than **cusp**)

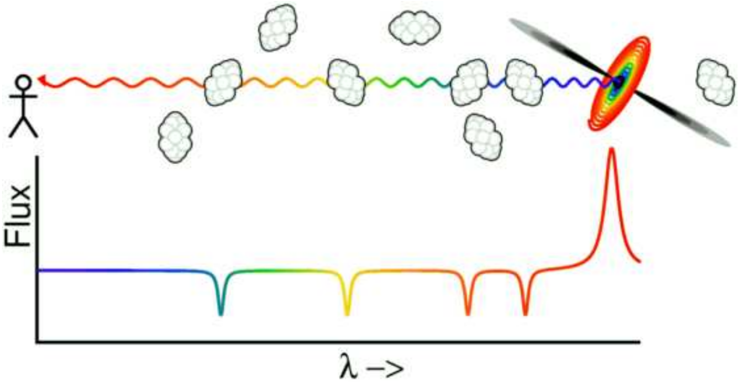
$$\rho(r) = \frac{\rho_0}{(r/r_0)^\alpha (1+(r/r_0)^\beta)^\gamma}$$

- Reionization, first stars, voids...

How to probe power spectrum



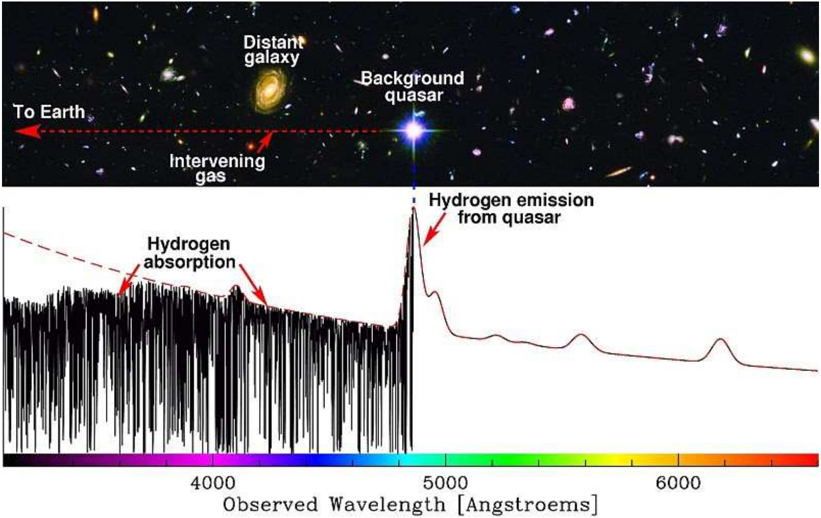
Example: Lyman-α forest



Neutral hydrogen absorption line at $\lambda = 1215.67\text{Å}$
From the Earth observer point of view we see the forest:

$$\lambda = (1+z)1215.67\text{Å}$$

Lyman-α forest and cosmic web



Neutral hydrogen in intergalactic medium is a tracer of overall matter density.
Scales $0.3h/\mathrm{Mpc} \lesssim k \lesssim 3h/\mathrm{Mpc}$ are probed.

The Lyman-α method includes

- Astronomical data analysis of quasar spectra
- Astrophysical modeling of hydrogen clouds
- N-body simulations of DM clustering at non-linear stage
- Solving numerically Boltzmann equations for SM in the early Universe
- Finding global fit to the whole set of cosmological data (CMB, LSS, Ly-α), using Monte-Carlo Markov chains

Main challenge: reliable estimate of systematic uncertainties

Summary

- Neutrino Freeze-out in the early universe
  - estimate of the temperature of freeze-out
  - estimate of relic abundance
- examination of the hypothesis that neutrinos are all DM (FALSE)
- current constraints on neutrino abundances from CMB and LSS
- constraints from future surveys
- Warm dark matter from LSS

References

- Gorbunov and Rubakov, 'Introduction to the Theory of the Early Universe'
- Lesgourgues, Mangano, Miele, Pastor, 'Neutrino Cosmology'
- Review Lesgourgues and Verde 2017
- Strumia and Vissani 2010, arXiv:hep-ph/0606054v3