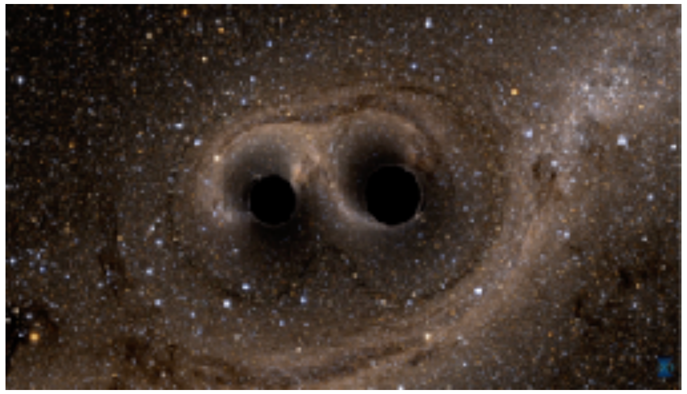
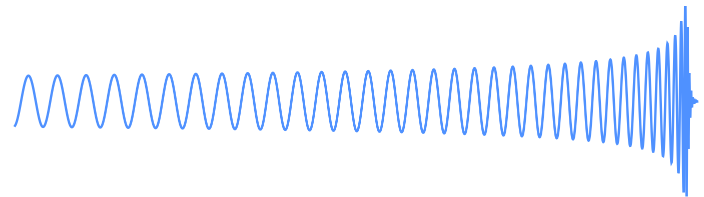
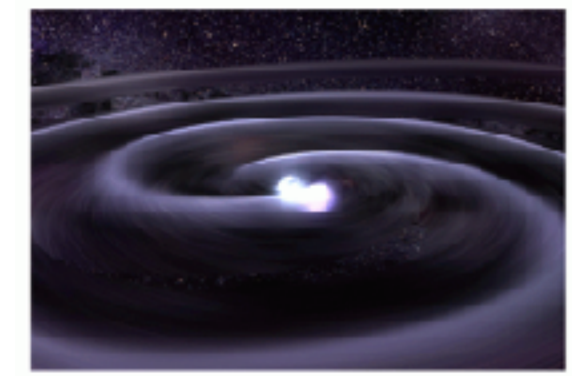


credit: SXS





credit: NASA

Gravitational waves

Tanja Hinderer

University of Amsterdam

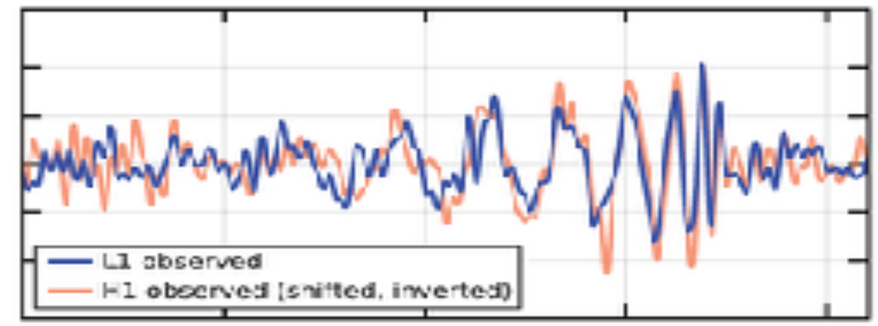
Nordic winter school

Jan 2-7 , 2019

Recent discoveries

2016: first detection of a gravitational-wave (GW) transient announced,

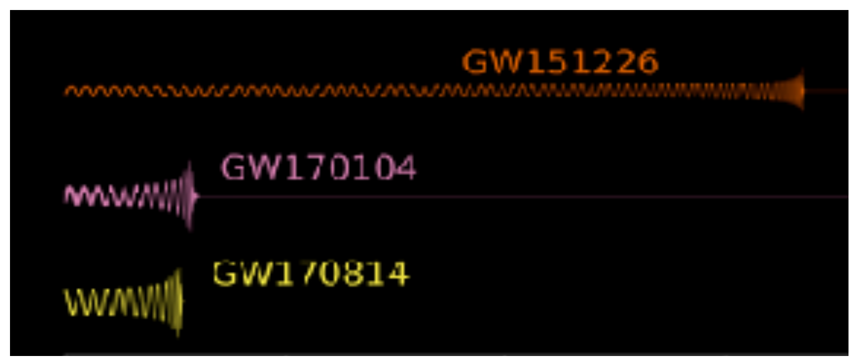
observation of merging binary black holes

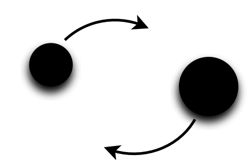


credit: LVC

Opened a completely new window onto the universe

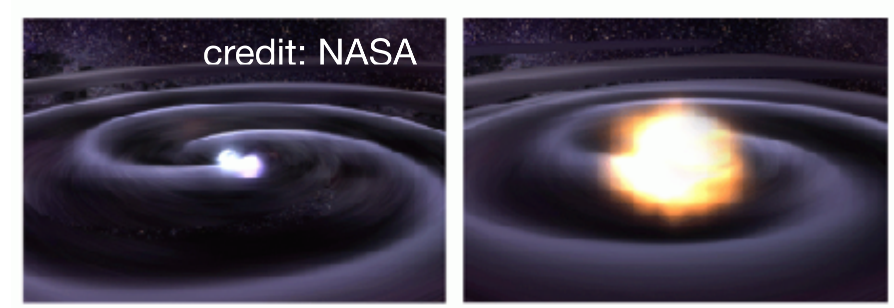
2017:

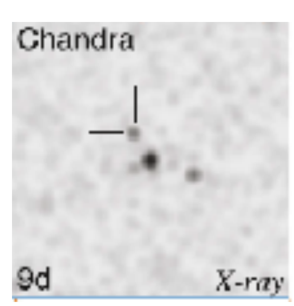




Aug 2017: GW and multi-messenger signals from binary neutron stars

New era of gravitational-wave astronomy

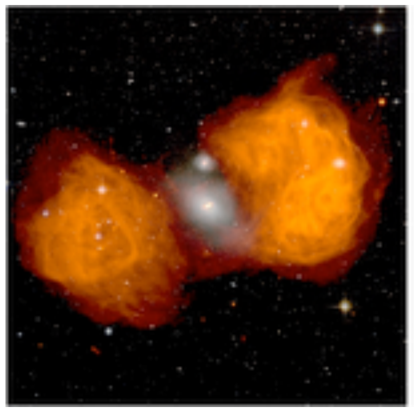
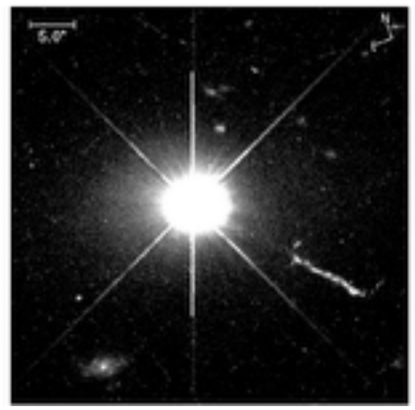
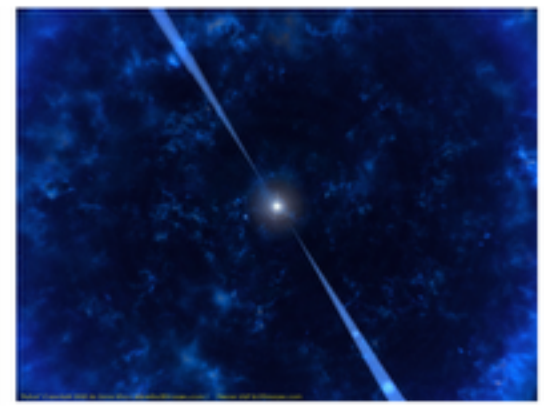
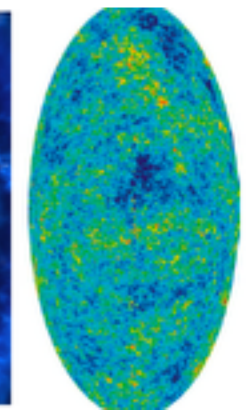




Examples of new windows onto the universe

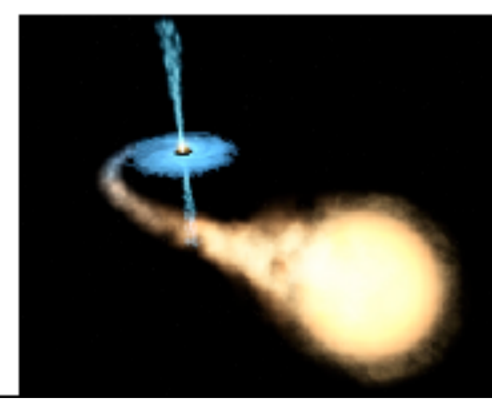
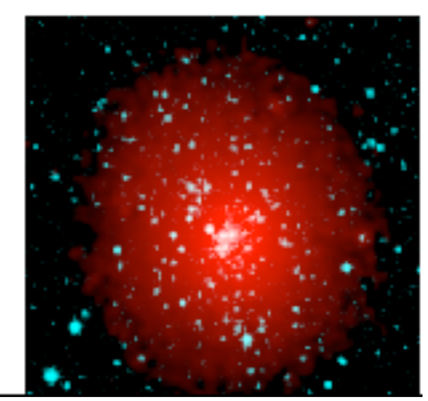
radio window (1940s & 50s): 10^4 x lower frequency than optical

radio galaxies, quasars, pulsars, CMB, …

x-ray window (1960s & 70s): 10^3 x higher frequency than optical

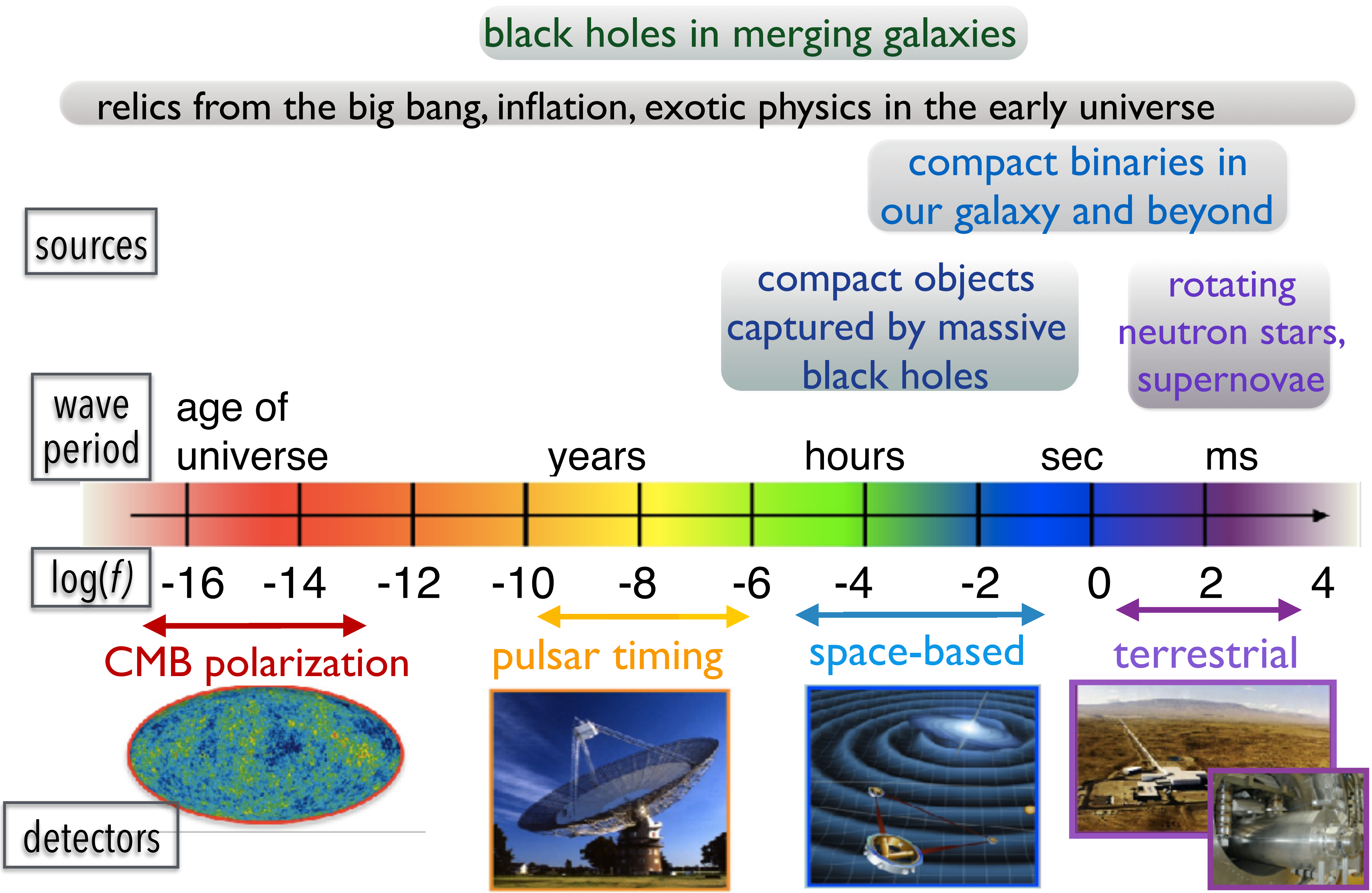
black hole jets, accreting neutron stars, hot intergalactic gas, …

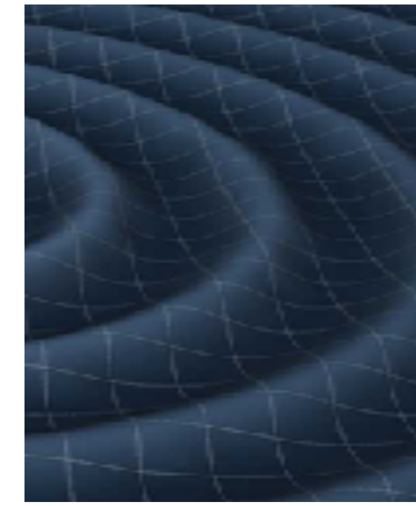
GWs are far more radically new than radio or x-rays :

completely different form of radiation

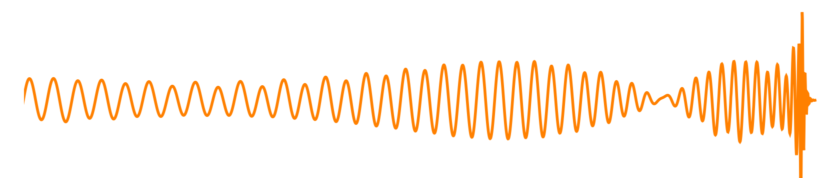
The GW spectrum: over 22 decades in frequency



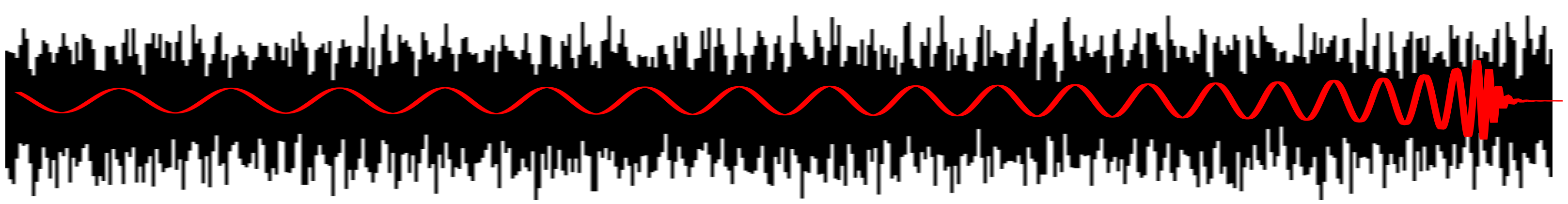
Plan for the lectures

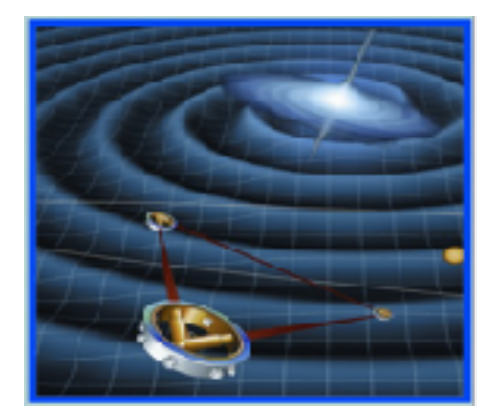


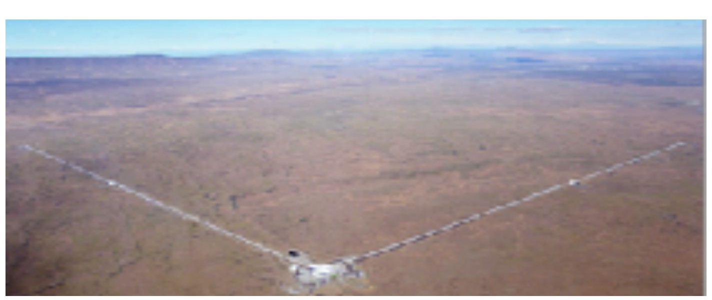
- Today: The basic physics of GWs
  - *Lecture 1:* properties, effects, detection of GWs
  - *Lecture 2:* generation of GWs
- Tomorrow: Identifying and interpreting GW signals



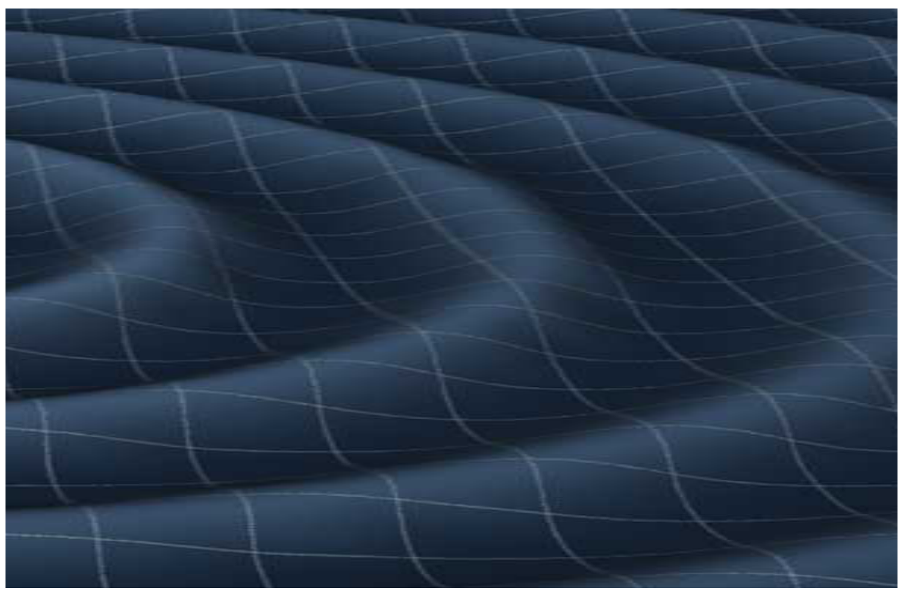
  - *Lecture 3:* GWs from merging black holes
  - *Lecture 4:* other GW sources, outlook







GWs in linearized gravity and their properties



688 Sitzung der physikalisch-mathematischen Klasse vom 22. Juni 1916

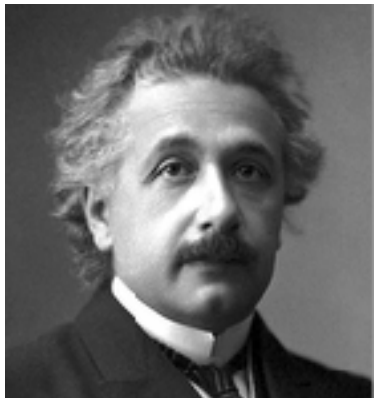
Näherungsweise Integration der Feldgleichungen der Gravitation.

Von A. Einstein.

Bei der Behandlung der meisten speziellen (nicht prinzipiellen) Probleme auf dem Gebiete der Gravitationstheorie kann man sich damit begnügen, die $g_{\mu\nu}$ in erster Näherung zu berechnen. Dabei bedient man sich mit Vorteil der imaginären Zeitvariable $x_4 = it$ aus denselben Gründen wie in der speziellen Relativitätstheorie. Unter »erster Näherung« ist dabei verstanden, daß die durch die Gleichung

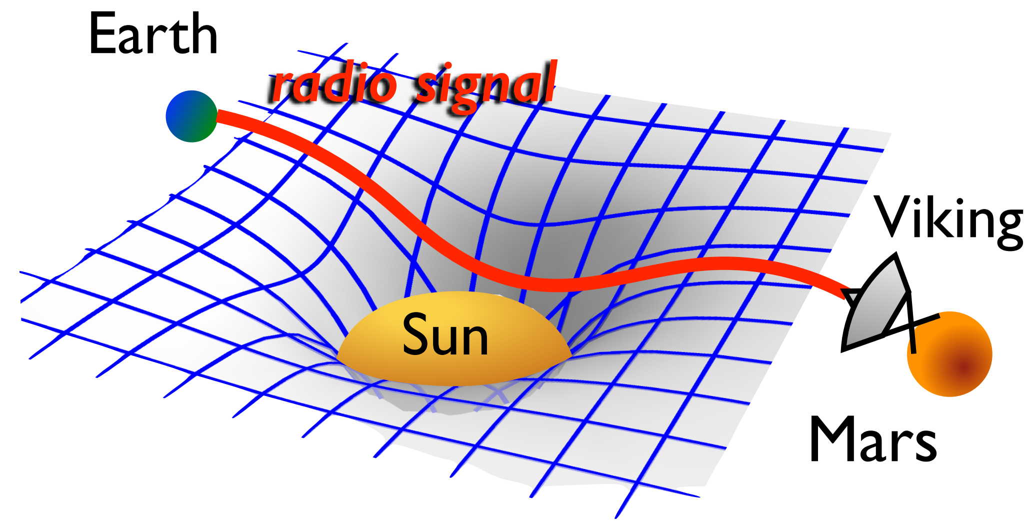
$$g_{\mu\nu} = -\delta_{\mu\nu} + \gamma_{\mu\nu} \qquad (1)$$

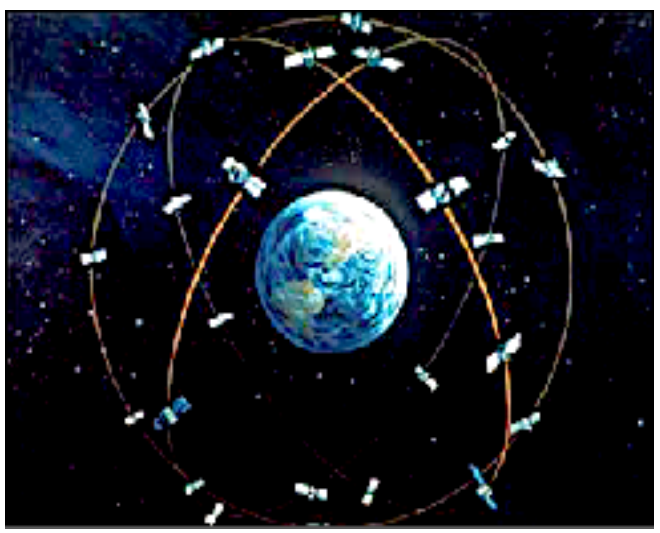
Recall: geometric view of the nature of gravity



Matter & energy curve space …

250 μs delay of radio signals for roundtrip to Mars measured in 1976





Credit: US DoD

… and warp time

time flows slower on Earth than at the GPS satellites

after one day: 17 μs difference,

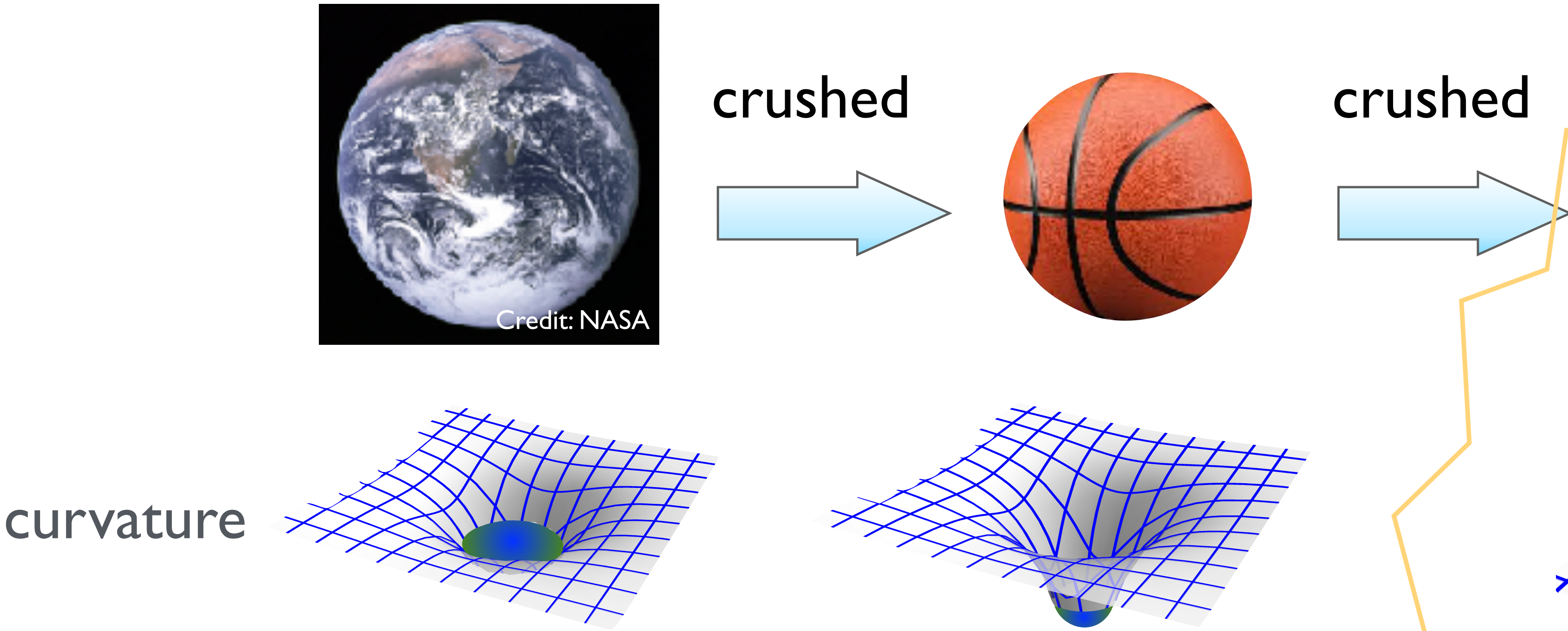
affects positioning by 5 km

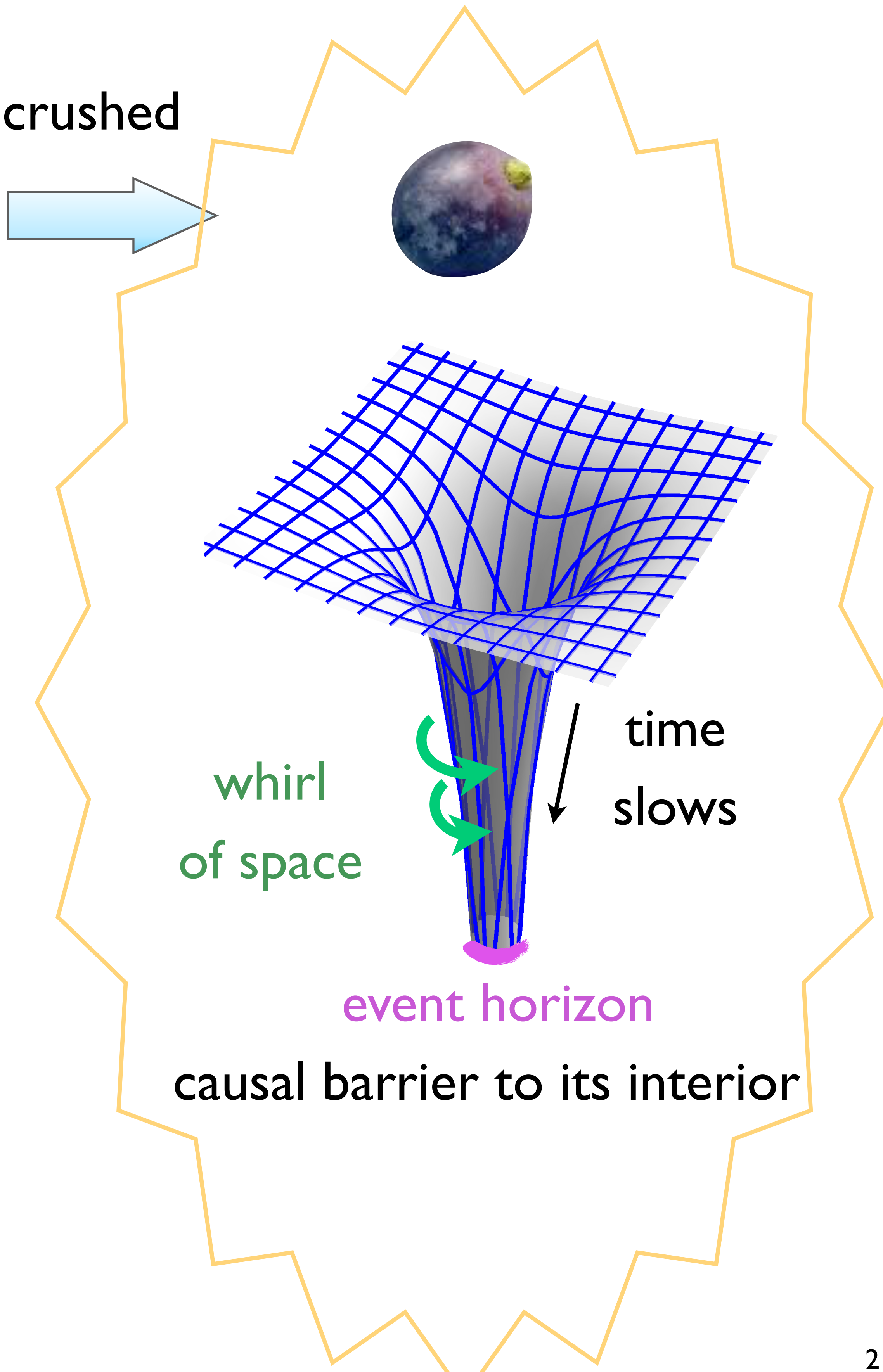
gravity is a manifestation of curvature

objects move along straight paths in curved spacetime geometry

Metric g_{ab} : distance rule *Indices a,b run over (t,x,y,z), will also use Greek letters*

Extremes of curvature: black holes





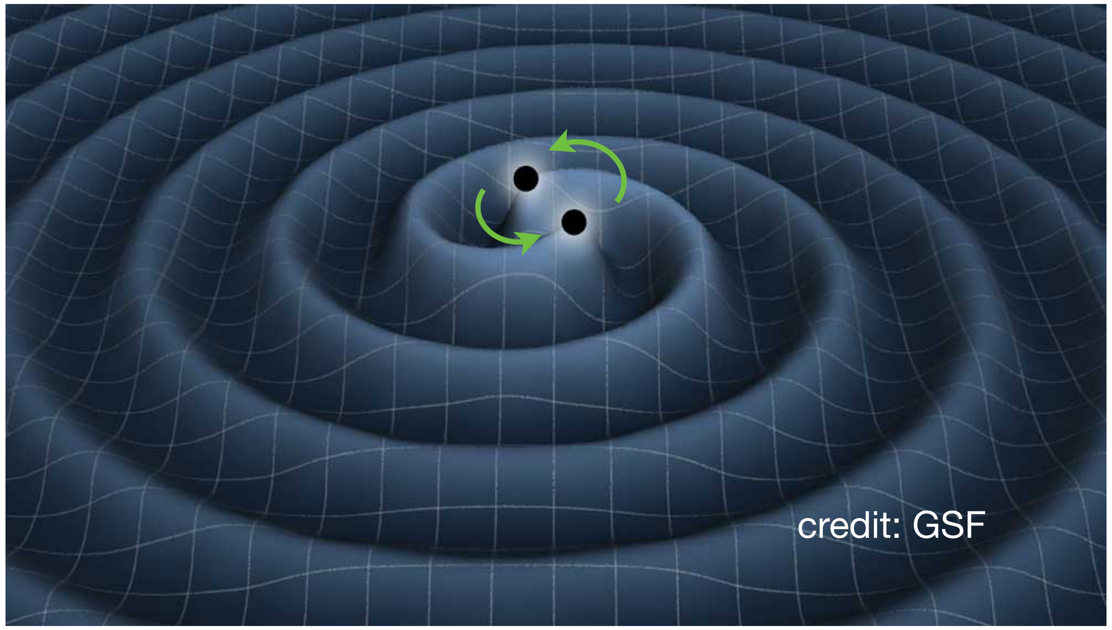
Earth compressed to ≲1cm radius:

collapses to a black hole

- region of immense spacetime curvature
- no surface
- described entirely by its mass and spin

Gravitational waves (GWs) in brief

- Accelerating masses generate dynamical distortions in curvature
- Interact very weakly with matter:
  propagate through the universe without significant attenuation, dispersion, scattering
- Carry an enormous amount of energy
- But they deform space only very slightly

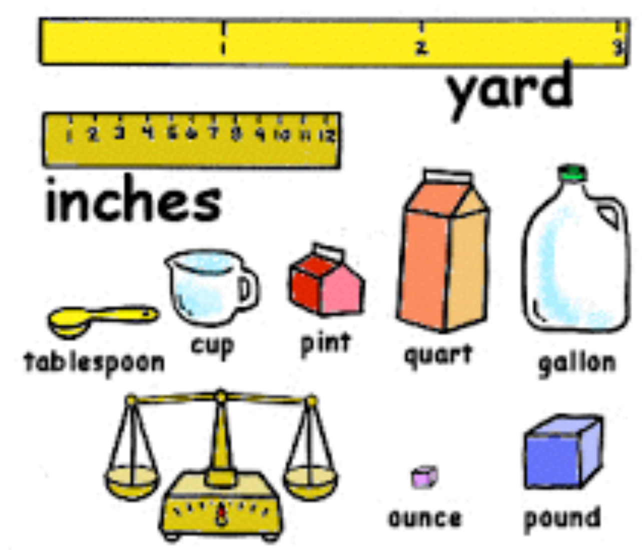


Will now consider these properties
in more concrete detail

Conventions

will mainly use geometric units **G=c=1**

mass, length & time all have units of length



Credit:insynco

Useful conversions:

time [cm] ~ c time [s]~ ($3\mathrm{x}10^{10}$ cm) (time [s])

mass [cm] = (G/c^2) mass [g] ~ (0.742×10^{-28} cm) (mass [g])

Also note: Repeated indices on tensors summed over

Mathematical description of curvature

Einstein field equations:

$$\boxed{G_{ab} = 8\pi T_{ab}}$$

Einstein tensor

sources


778 Gesamtsitzung vom 4. November 1915

Zur allgemeinen Relativitätstheorie.

Von A. Einstein.

844 Sitzung der physikalisch-mathematischen Klasse vom 25. November 1915

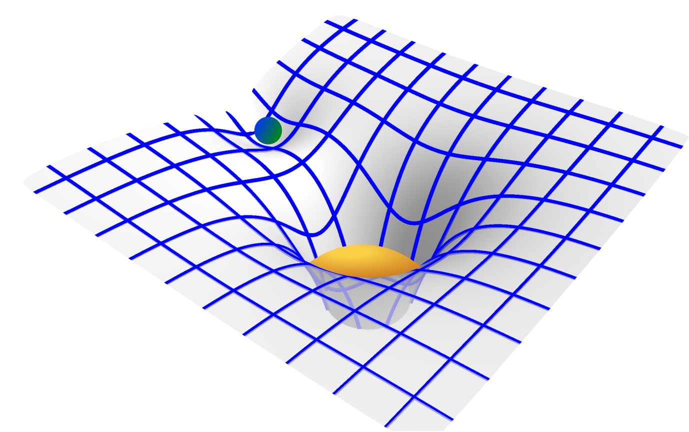
Die Feldgleichungen der Gravitation.

Von A. Einstein.

In zwei vor kurzem erschienenen Mitteilungen[1] habe ich gezeigt, wie man zu Feldgleichungen der Gravitation gelangen kann, die dem Postulat allgemeiner Relativität entsprechen, d. h. die in ihrer allgemeinen

$$G_{ab} = R_{ab} - \frac{1}{2} g_{ab} g^{cd} R_{cd}$$

$$R_{ab} = R_{acb}{}^{c}$$ ← Riemann curvature tensor, two derivatives of $\boldsymbol{g_{ab}}$

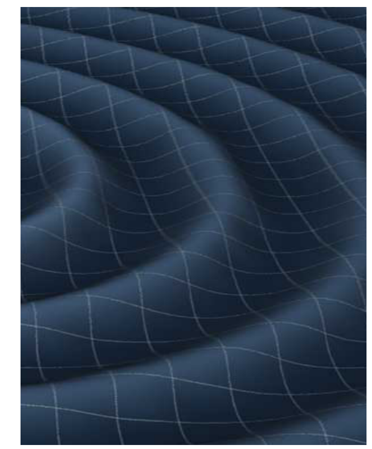


Bianchi identity for covariant derivatives ∇_f of $\boldsymbol{R_{abcd}}$ implies:

$$\nabla_a G^{ab} = 0$$

Setup for computing GWs far from their source

Spacetime divided into different zones, different calculational strategies in each



credit: LIGO

first consider: - wave zone far from the source

- region of spacetime that is nearly flat

$$g_{ab} = \eta_{ab} + h_{ab}$$

Minkowski metric (flat space)

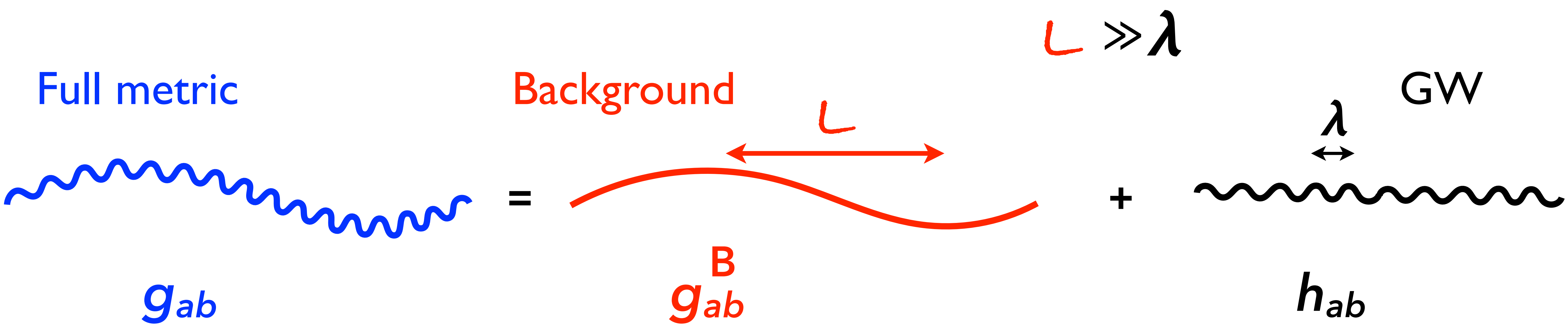
$$\eta_{ab} = \mathrm{diag}(-1,1,1,1)$$

$$|h_{ab}| \ll 1$$

Small deviation away from flat space

Remark: GWs in general relativity

- Discussion of GWs is simplest in linearized gravity
- But can also define GWs in curved spacetime (in wave zone) if there is a separation of scales in variations of curvature



GWs = rapidly varying part of spacetime curvature and metric

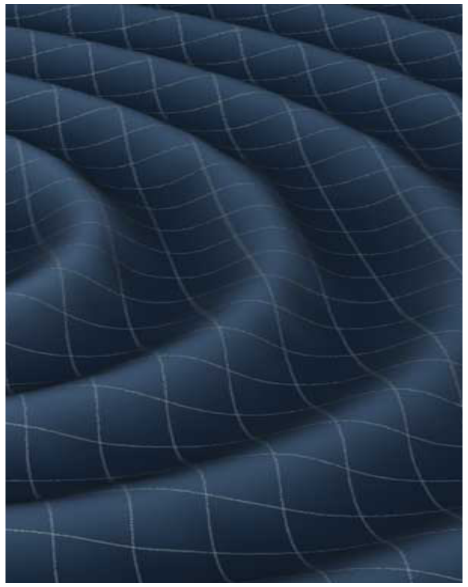
Similar definition used for waves in plasmas, fluids, solids

Linearized gravity

$$g_{ab} = \eta_{ab} + h_{ab} \qquad |h_{ab}| \ll 1 \qquad \eta_{ab} = \mathrm{diag}(-1,1,1,1)$$

- Introduce the trace-reversed metric perturbation

$$\bar{h}_{ab} = h_{ab} - \frac{1}{2}\eta_{ab}\eta^{cd}h_{cd}$$



- Choose harmonic gauge:

$$\frac{\partial}{\partial x^a}\bar{h}^{ab} = 0$$

- Compute curvature quantities to obtain the linearized field equations

$$\Box\bar{h}_{ab} = -\cancel{16\pi T_{ab}} \qquad \Box = \eta^{ab}\frac{\partial}{\partial x^a}\frac{\partial}{\partial x^b} = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

in vacuum

c=1 units

Recall plane wave solutions for a scalar function

- Wave eq. for a *scalar* function **f** : $\Box f = 0$

$$\Box = -\frac{\partial^2}{\partial t^2} + \nabla^2$$

$$\vec{k}\cdot\vec{x} = -k^t t + \delta_{ij} k^i x^j$$
$$= -k^t t + \mathbf{k}\cdot\mathbf{x}$$

- Ansatz: $f = a e^{i\vec{k}\cdot\vec{x}}$

$$0 = \Box f = -\vec{k}\cdot\vec{k}\; f$$

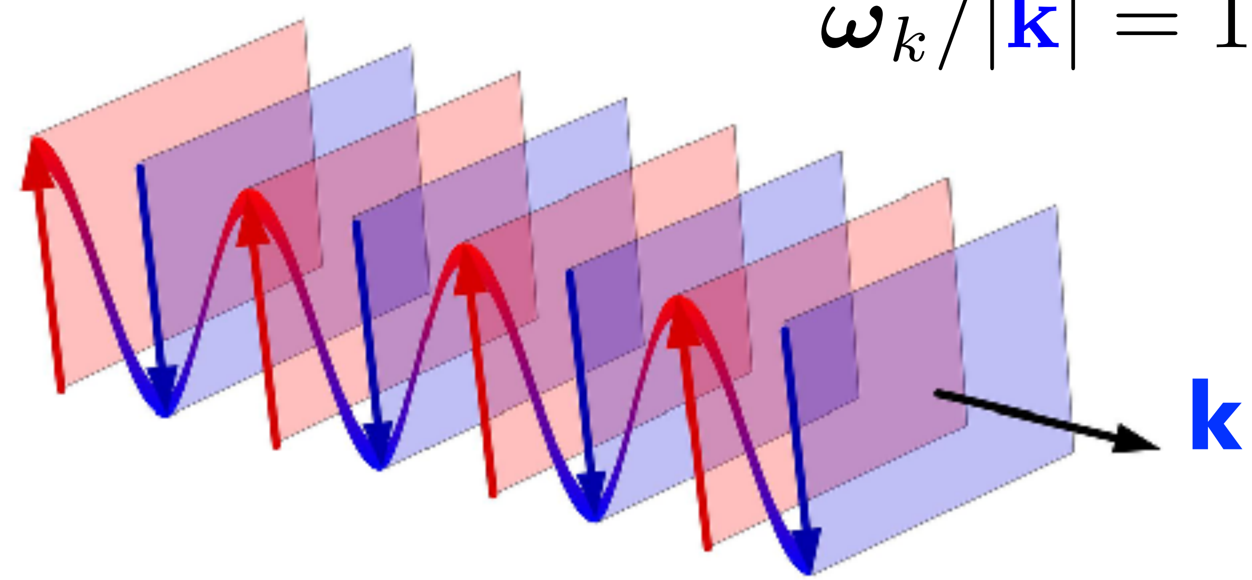
- Implies: $0 = \vec{k}\cdot\vec{k} = -(k^t)^2 + \mathbf{k}\cdot\mathbf{k}$ or $k^t = |\mathbf{k}| \equiv \boldsymbol{\omega}_k$

- General solution: superposition

$$f(x) = \int d^3k\, a(k) e^{-i\boldsymbol{\omega}_k t + i\mathbf{k}\cdot\mathbf{x}}$$

wave traveling with speed

$$\boldsymbol{\omega}_k/|\mathbf{k}| = 1$$



Similarly for the metric perturbation

$$\Box f = 0 \quad \longleftrightarrow \quad \Box \bar{h}_{ab} = 0$$

- General solution:

$$\bar{h}_{ab}(x) = \mathrm{Re} \int d^3k\, A_{ab}(k) e^{-i\omega_k t + i\mathbf{k}\cdot\mathbf{x}} \qquad \omega_k = |\mathbf{k}|$$

complex coefficients

- Constraints on A_{ab} from harmonic gauge condition, otherwise still arbitrary:

$$\frac{\partial}{\partial x^a}\bar{h}_{ab} = 0 \quad \longrightarrow \quad k^a A_{ab} = 0 \qquad k^a = (\omega, \mathbf{k})$$

Further specialization: transverse-traceless (TT) gauge

- Harmonic gauges ($\frac{\partial}{\partial x^a}\bar{h}^{ab} = 0$) still leave remaining freedom:

coordinate transformations of the form

gauge vector

$$x^a_{\text{new}} = x^a_{\text{old}} + \xi^a(x) \qquad \text{with} \qquad \Box\xi^a = 0$$

then: $$h^{\text{new}}_{ab} = h^{\text{old}}_{ab} - \frac{\partial \xi_a}{\partial x^b} - \frac{\partial \xi_b}{\partial x^a}$$

- Use this freedom to make h_{ab} purely spatial and traceless:

$$h_{tt} = h_{ti} = 0 \qquad \eta^{ab} h_{ab} = 0$$

so $$\bar{h}_{ij} = h_{ij}$$

[TT gauge only possible in globally vacuum spacetimes]

Further specialization: TT gauge cont.

- Had further gauge specialization: $h_{tt} = h_{ti} = 0$ $\qquad \bar{h}_{ij} = h_{ij}$

- Use this in the harmonic gauge condition: $\frac{\partial}{\partial x^a}\bar{h}^{ab} = 0$

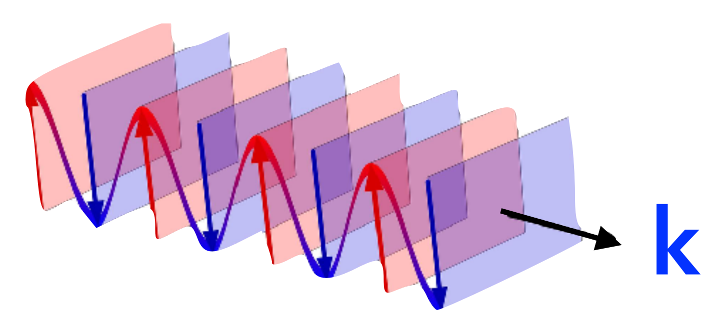
$$0 = \partial_a \bar{h}^{at} = \cancel{\partial_t \bar{h}^{tt}} + \cancel{\partial_i \bar{h}^{ti}} \quad ✓$$

$$\bar{h}_{ab} \sim A_{ab} e^{-i\omega_k t + i k_j x^j}$$

$$0 = \partial_a \bar{h}^{ak} = \cancel{\partial_t \bar{h}^{tk}} + \partial_j \bar{h}^{jk} \longleftrightarrow \boxed{0 = i k_j \, h^{jk}}$$

Waves are transverse

``Transverse-traceless" (TT) gauge



- No further remaining freedom, gauge is now completely fixed

Properties of plane wave solution in TT gauge

- Two independent components: $\underbrace{10}_{\text{Symmetric 4x4 matrix}} - \underbrace{3}_{h_{ti}=0} - \underbrace{3}_{\text{transverse}} - \underbrace{1}_{\text{traceless}} - \underbrace{1}_{\text{purely spatial}}$ = 2 polarizations

- specialize harmonic-gauge solution to TT gauge & propagation along z-direction:

$$\bar{h}_{ab} \sim A_{ab} e^{-i\omega t + i k_j x^j} \qquad \mathbf{k} = (0, 0, \omega) \qquad k^j A_{jk} = 0$$

- Final form:

$$h_{ab}^{\mathrm{TT}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & a_+ & a_\times & 0 \\ 0 & a_\times & -a_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{-i\omega(t-z)}$$

Common notation for the two polarizations: + and x

Properties of the waves

- The solution is often written in the general form

$$h_{ab}^{\mathrm{TT}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+(t-z) & h_\times(t-z) & 0 \\ 0 & h_\times(t-z) & -h_+(t-z) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- h^{TT} contains only physical information about radiation, all gauge modes eliminated

- linearized curvature tensor:

$$R_{abcd} = \frac{1}{2}\left(h_{ad,bc} + h_{bc,ad} - h_{ac,bd} - h_{bd,ac}\right) \qquad h_{ab,cd} = \frac{\partial^2}{\partial x^d \partial x^c} h_{ab}$$

Independent components simply related to ***h_+, h_x*** :

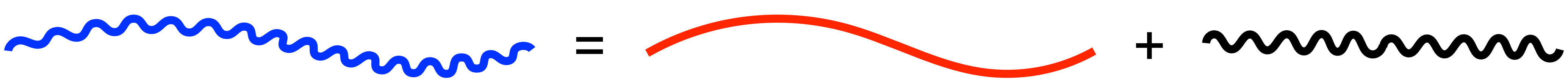
$$R_{txtx} = -R_{tyty} = -\frac{1}{2}\ddot{h}_+ \qquad R_{txty} = R_{tytx} = -\frac{1}{2}\ddot{h}_\times$$

remaining components from Riemann symmetries *overdot=time derivative*

What is the energy carried by GWs?

Need nonlinear order in h_{ab} to see how GWs produce spacetime curvature

- Consider split: background + wave: $g_{ab} = g^{\mathrm{B}}_{ab} + h_{ab}$



- Collect terms in Einstein tensor with different powers of *h*:

$$G_{ab} = G^{\mathrm{B}}_{ab} + G^{(1)}_{ab} + G^{(2)}_{ab}$$

Linear in h *Quadratic in h*

- Einstein equations with generic source: $G^{\mathrm{B}}_{ab} + G^{(1)}_{ab} + G^{(2)}_{ab} = 8\pi T_{ab}$

Average over wavelengths to get meaningful coarse-grained quantities:

$$G^{\mathrm{B}}_{ab} + \langle G^{(2)}_{ab} \rangle = 8\pi \langle T_{ab} \rangle$$

Energy carried by GWs

Rearrange:

$$G^{\mathrm{B}}_{ab} = 8\pi\langle T_{ab}\rangle - \langle G^{(2)}_{ab}\rangle$$

$$= 8\pi\left(\langle T_{ab}\rangle + T^{\mathrm{GW}}_{ab}\right)$$

Terms quadratic in *h*

for $g_{ab} = g^{\mathrm{B}}_{ab} + h_{ab}$

Credit:toony

$$\boxed{T^{\mathrm{GW}}_{ab} = -\frac{1}{8\pi}\langle G^{(2)}_{ab}\rangle}$$

Effective stress-energy of GWs

For TT gauge metric: $$T^{\mathrm{GW}}_{ab} = -\frac{1}{32\pi}\langle\frac{\partial h^{ij}}{\partial x^a}\frac{\partial h_{ij}}{\partial x^b}\rangle$$

Average over GW period

Energy carried by GWs

For a plane wave (TT gauge) propagating in the z-direction:

Energy density: $$T_{00}^{\mathrm{GW}} = \frac{1}{16\pi}\langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle \qquad \left(\times \frac{c^4}{G}\right)$$

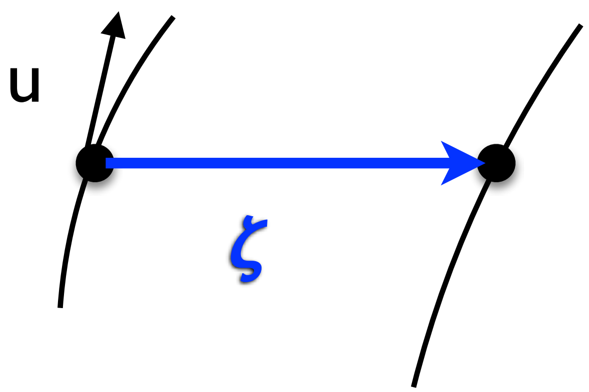
Energy flux: $$T_{0z}^{\mathrm{GW}} = -\frac{1}{16\pi}\langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle \qquad \left(\times \frac{c^3}{G}\right)$$

Momentum density: $$T_{zz}^{\mathrm{GW}} = \frac{1}{16\pi}\langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle \qquad \left(\times \frac{c^2}{G}\right)$$

All other independent components of T^{GW} vanish.

equivalent to T^{ab} of a beam of massless particles (e.g. photons): gravitons

The effect of a propagating GW



- Recall from GR: geodesic deviation measures relative acceleration of two nearby geodesics due to curvature

ζ: deviation vector

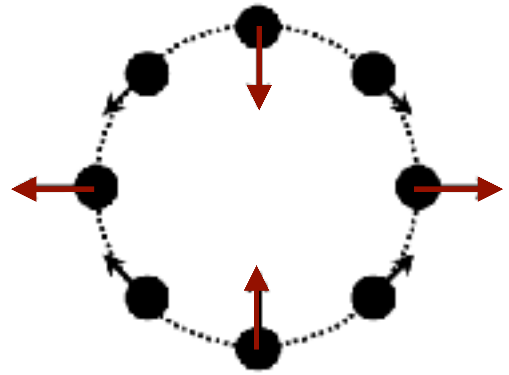
$$\frac{D^2\zeta^\alpha}{d\tau^2} = -R_{\mu\sigma\nu}{}^{\alpha} u^\mu u^\nu \zeta^\sigma$$

- Geodesic deviation in linearized gravity, freely falling frame:

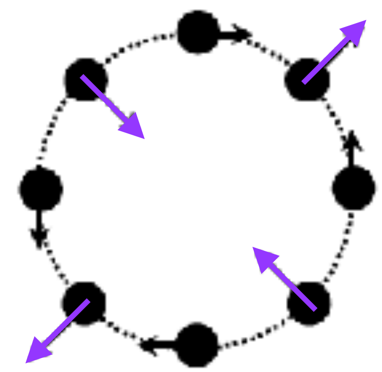
$$\frac{d^2\zeta^i}{dt^2} = -R_{tjt}{}^{i}\zeta^j$$

with $\quad R_{txtx} = -R_{tyty} = -\frac{1}{2}\ddot{h}_+ \qquad R_{txty} = R_{tytx} = -\frac{1}{2}\ddot{h}_\times$

Effect of a GW on a ring of test masses:

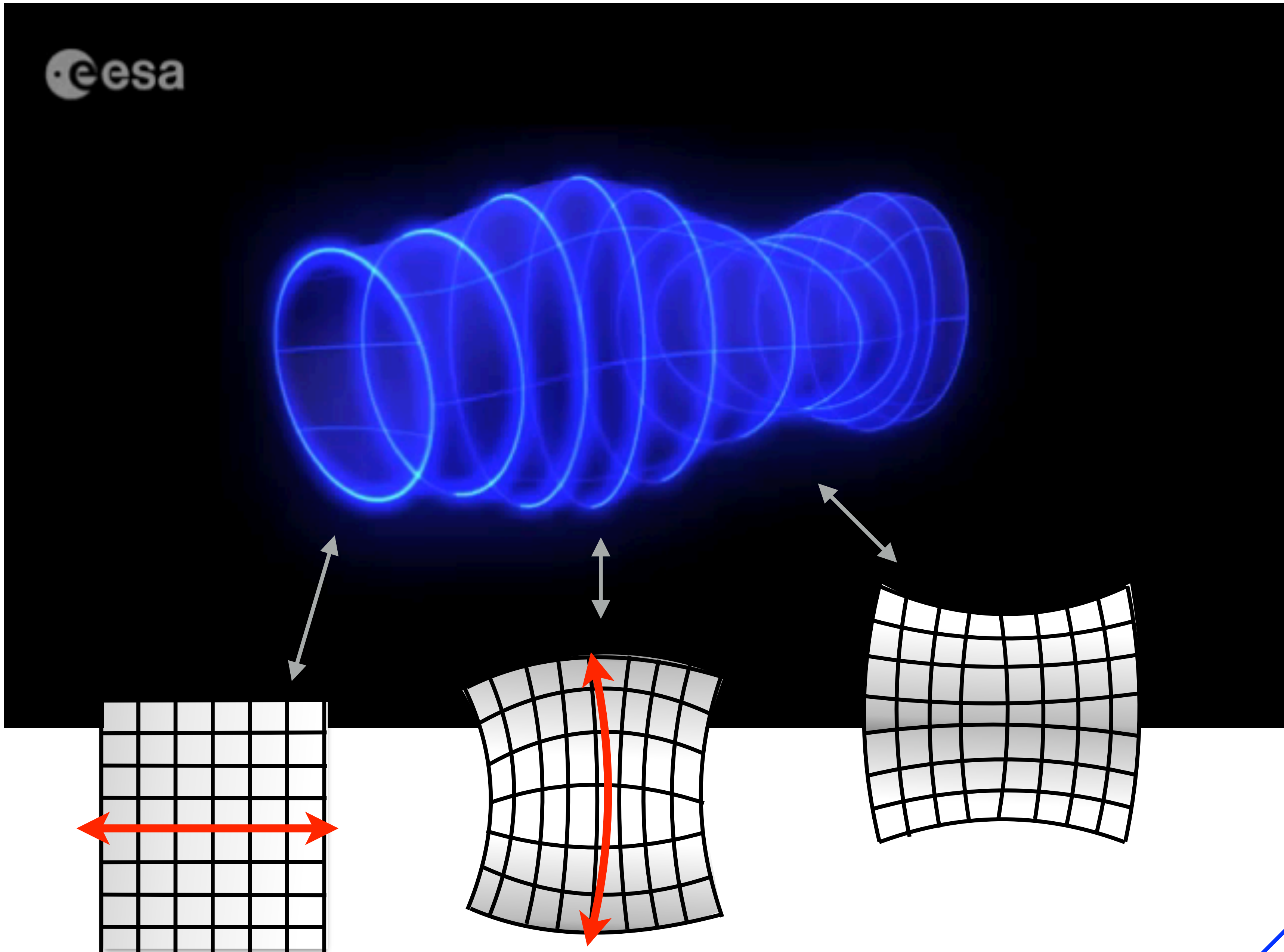


plus-polarization h_+



cross-polarization h_x

The effect of a propagating GW cont.



Stretching and squeezing of space

relative length
L

L+ΔL

Wave amplitude, mix of two polarizations

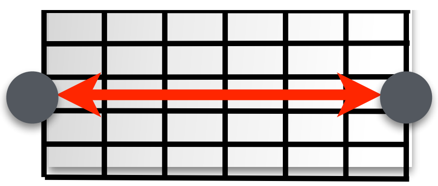
$$\boxed{\frac{\Delta \mathrm{L}}{\mathrm{L}} \sim h(t)}$$

``strain"

Interaction of GWs with a detector

Consider line element with GW propagating along z (TT gauge):

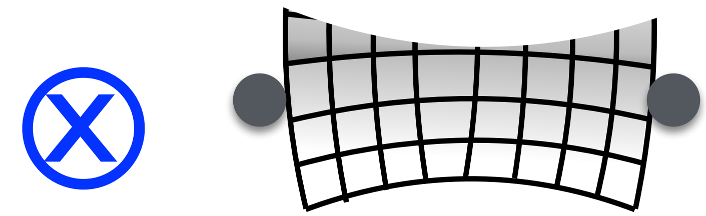
$$ds^2 = -dt^2 + [1 + h_+(t-z)]\, dx^2 + [1 - h_+(t-z)]\, dy^2 + 2h_\times(t-z)dxdy + dz^2$$



(0,0,0) (0,0,L)

Test masses initially at rest

$u^a{}_0=(1,0,0,0)$



$x^i= x^i{}_0+\delta x^i$ $u^a= u^a{}_0+\delta u^a$

Geodesic evolution of mass' positions to linear order:

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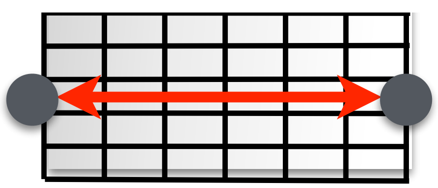

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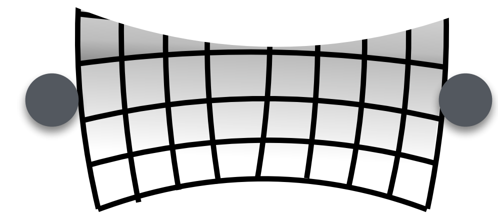
$\otimes$



$x^i = x^i{}_0 + \delta x^i$ $u^a = u^a{}_0 + \delta u^a$

Geodesic evolution of mass' positions to linear order:

$$\frac{d^2 x_0^i}{d\tau^2} + \Gamma^i_{0\ ab} u_0^a u_0^b + \left[\frac{d^2 \delta x^i}{d\tau^2} + u_0^a u_0^b \delta \Gamma^i{}_{ab} + 2\Gamma^i_{0\ ab} u_0^a \delta u^b\right] = 0$$

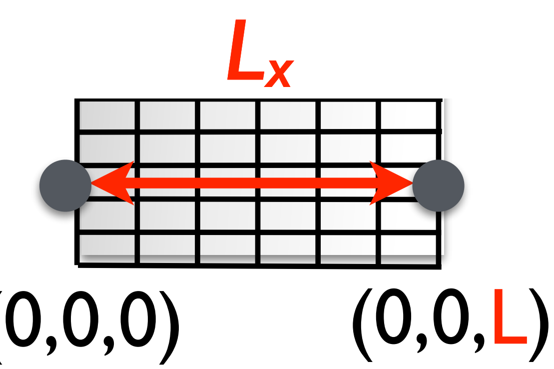
For this metric: $\Gamma^i{}_{tt} = \frac{1}{2} g^{ij}\left(2\partial_t g_{jt} - \partial_j g_{tt}\right) = 0$ so $\boxed{\frac{d^2 x^i}{d\tau^2} = 0 = \frac{d^2 x^i}{dt^2}}$

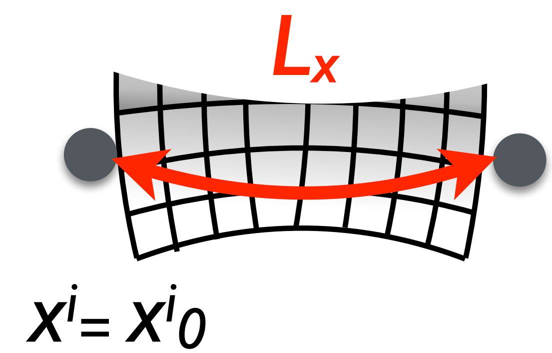
in TT gauge: Coordinate location of test masses is unchanged by GWs

to linear order in GW amplitude

Interaction of GWs with a detector

Must use gauge-invariant quantities to determine physical effects: e.g. proper separation





distance between masses (along x, at y=z=0):

$$L_x = \int_0^L \sqrt{g_{xx}}dx = \int_0^L \sqrt{1+h_+}dx \approx \int_0^L \left[1+\frac{1}{2}h_+(t-z)\right]dx \approx L\left[1+\frac{1}{2}h_+\right]$$

distance changes by $$\frac{\Delta L_x}{L_x} \approx \frac{1}{2}\,h_+$$

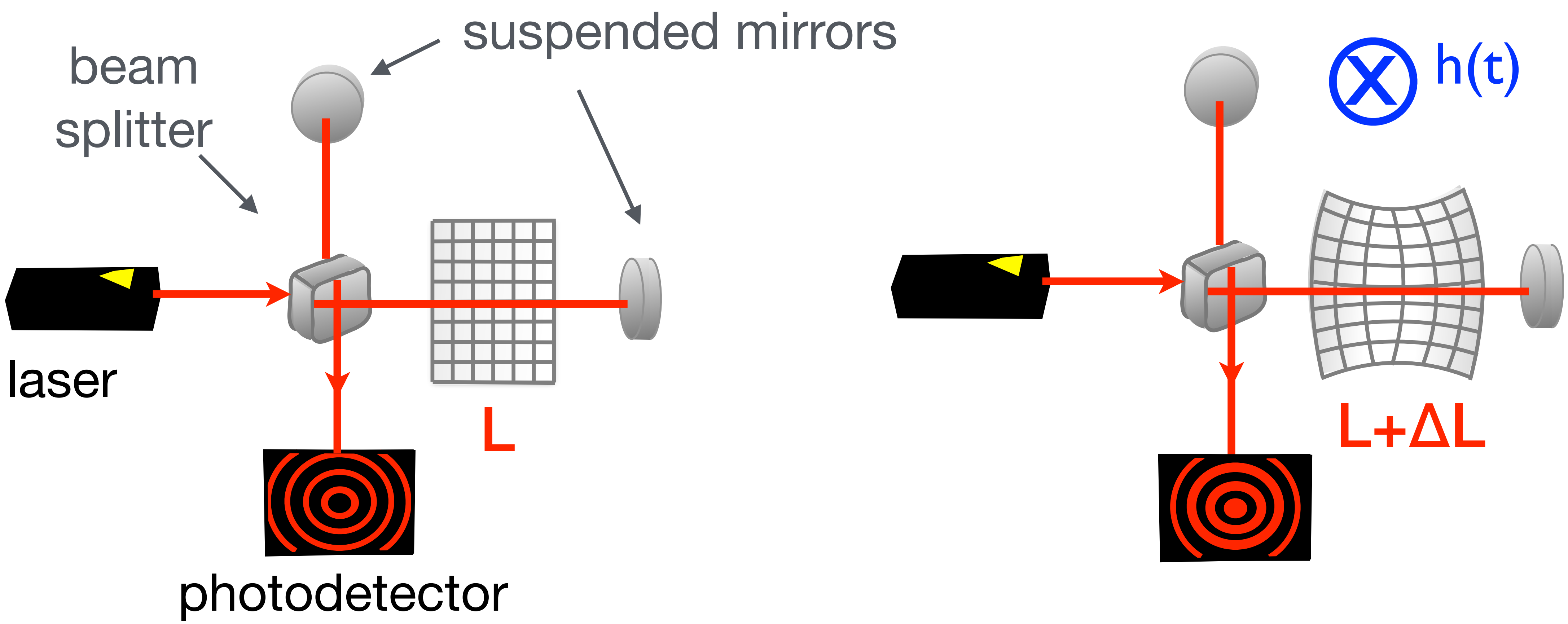
Turning strain into a measurement

First experiments: Resonant mass detectors

J. Weber, University of Maryland, 1966

- Large aluminum cylinder, isolated from vibrations, in vacuum chamber
- Sensitive to GW frequencies close to bar's resonant frequency
- Piezoelectric crystal converts change in length to electric signal
- Similar detectors built around the world but most no longer in use today

Measuring GWs with interferometers



- Laser tracks relative travel time intervals in the arms
- change in intensity due to difference in phase:

$$\Delta\phi = 2\pi f \frac{2\Delta L}{c} = \frac{2\pi f}{c} h(t)\, L$$

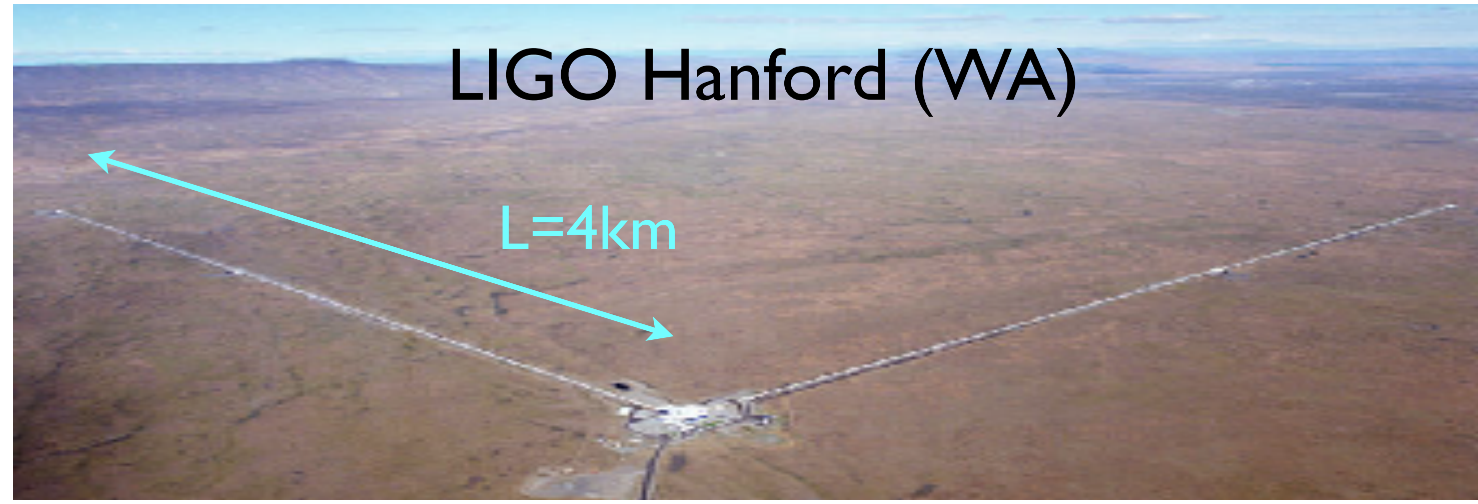
laser frequency

extra roundtrip travel time in the arm

Interferometer detectors

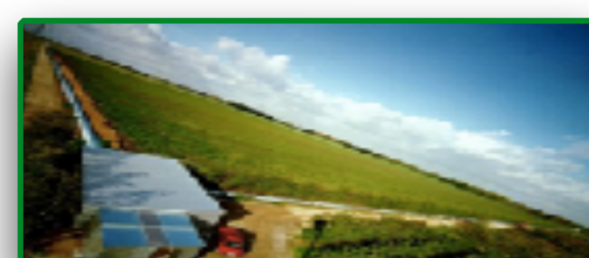
Accuracy: *0.02* fringes

Laser **I**nterferometer **G**W **O**bservatories (USA)



Accuracy: $\Delta L \sim 10^{-18}\ m$

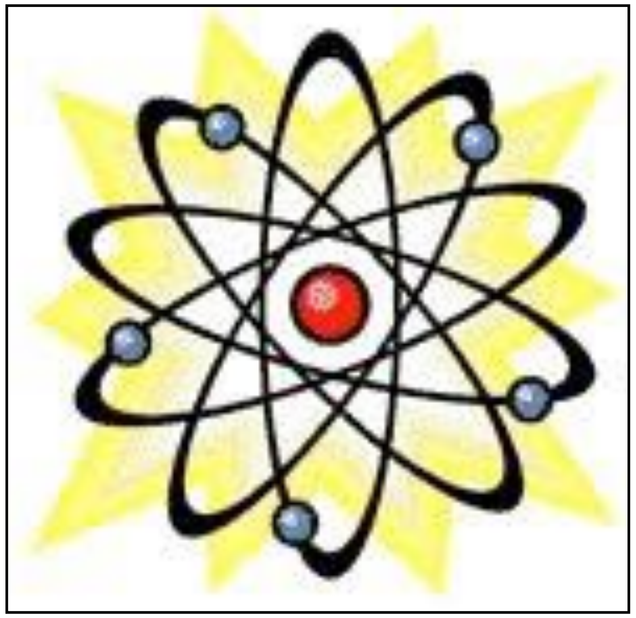
Virgo / EGO (Pisa, Italy)



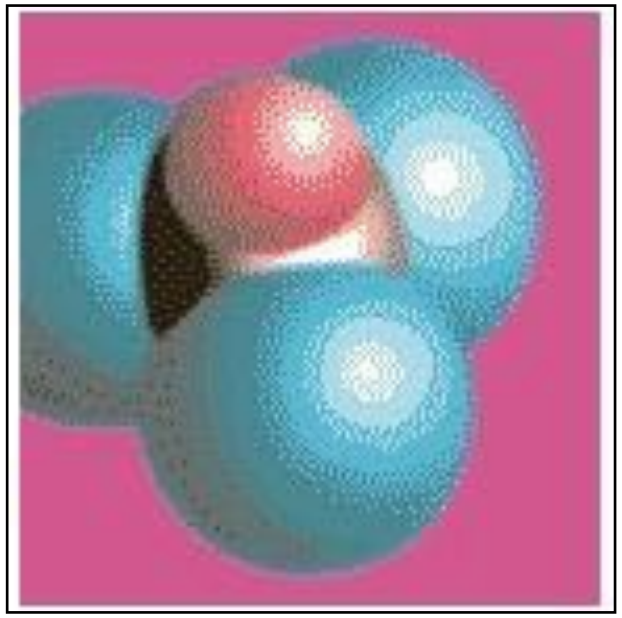
GEO600
(Hannover, Germany)

LIGO measurement accuracy

hair
0.1mm



atom
1/1 000 000 x hair
$=10^{-10}$m



www.ligo.caltech.edu

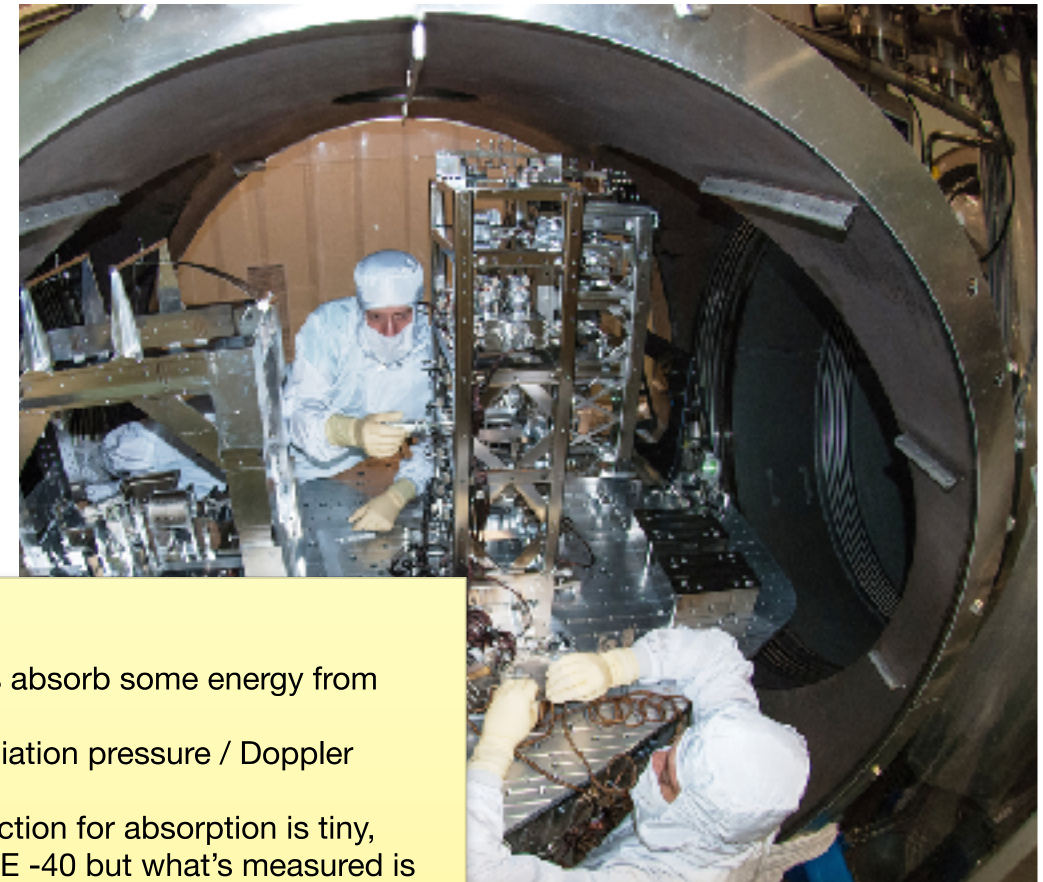
atomic nucleus
1/100 000 x atom
$=10^{-15}$m

LIGO measures length to
2×10^{-18}m
= 1/1 000 x nucleus
= 10^{-8} x atom

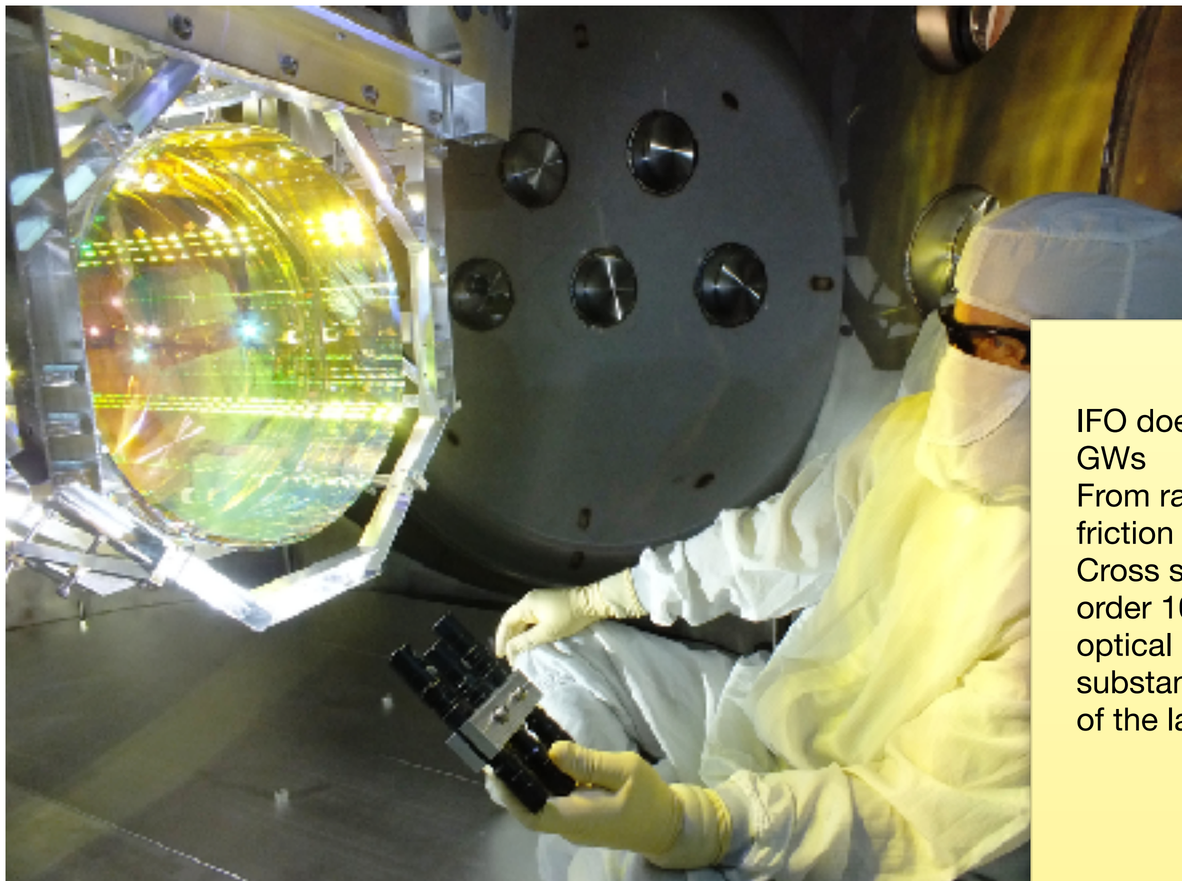
A glimpse inside

Image credit: LIGO

A glimpse inside

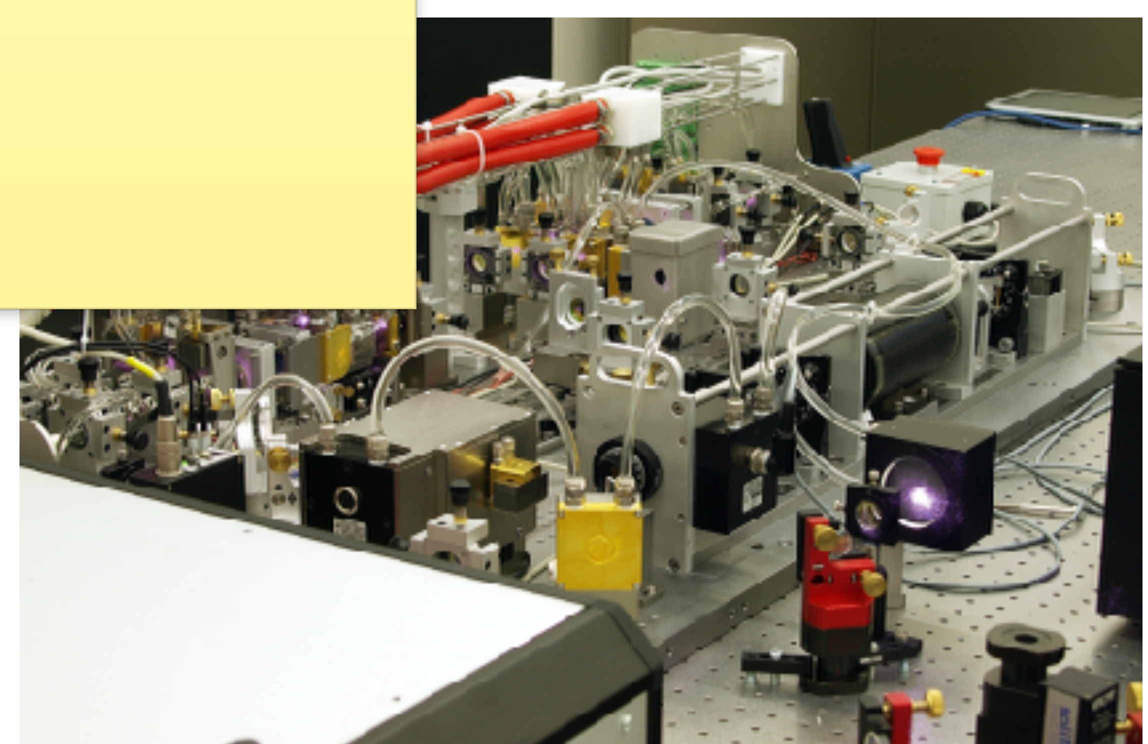


Seismic isolation



Livingston mirror (40 kg)

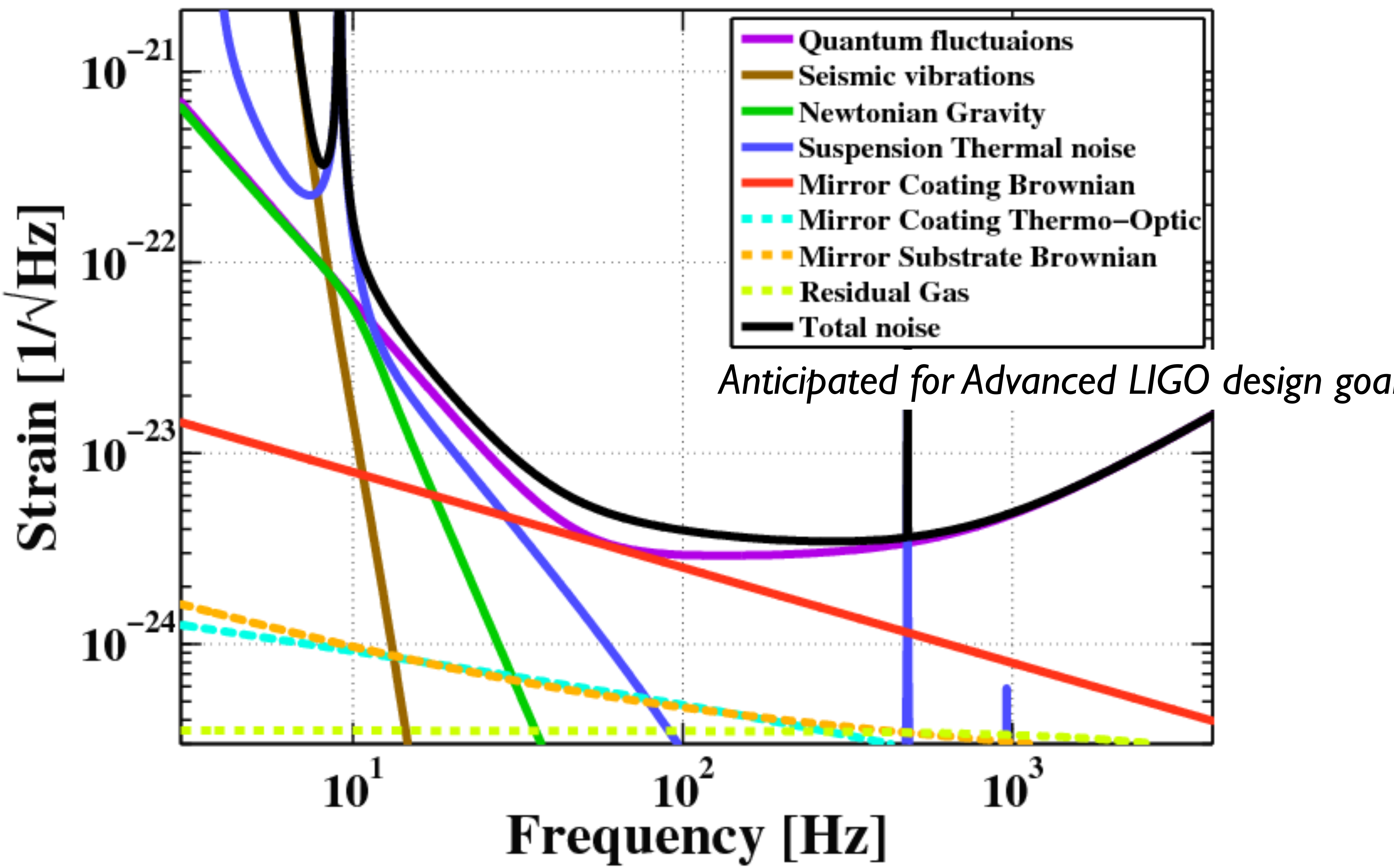
IFO does absorb some energy from GWs
From radiation pressure / Doppler friction
Cross section for absorption is tiny, order 10 E -40 but what's measured is optical output that can be substantially amplified with gain factor of the laser



laser

Image credit: LIGO

Limitations to detector sensitivity due to various noise sources:



R. Adhikari, Rev. Mod. Phys. 2014. arXiv:1305.5188