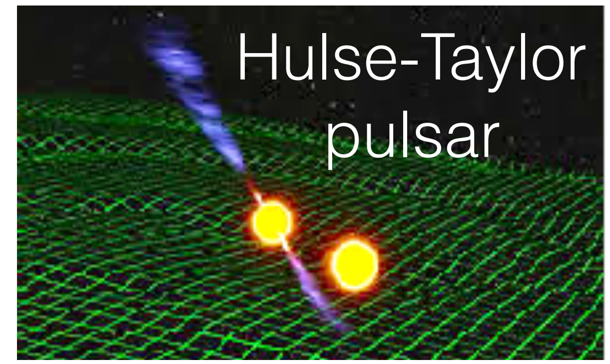
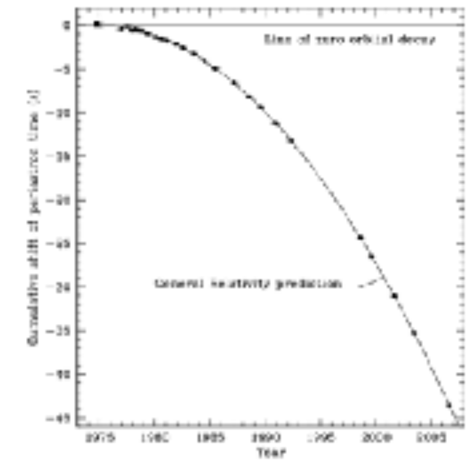
GWs from merging black holes

Credit:SXS

Last time: quadrupole radiation, Newtonian source

Daniel Kennefick: Controversies in the History of the Radiation Reaction problem in General Relativity   arXiv: gr-qc/9704002



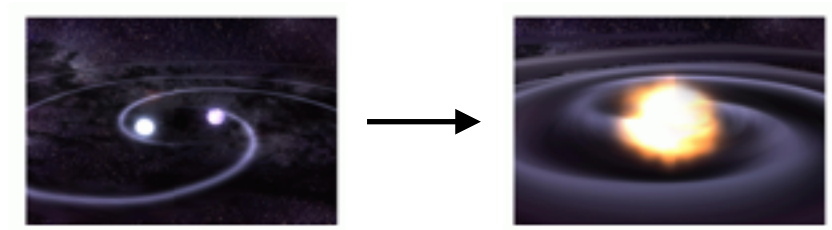


$$P_{\mathrm{GW}} = \frac{1}{5}\,\frac{G}{c^5}\,\langle \frac{d^3Q_{ij}}{dt^3}\frac{d^3Q_{ij}}{dt^3} \rangle$$

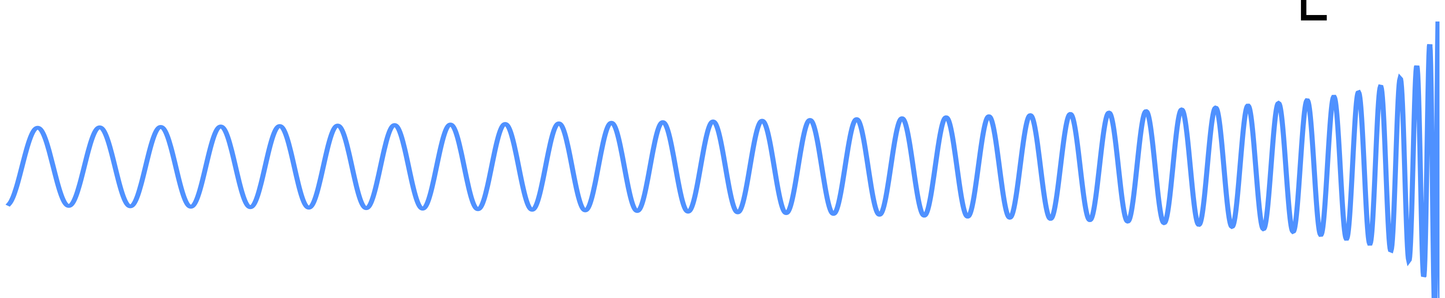
$$Q^{ij} = \int_{\mathrm{source}} d^3x\,\rho\left(x^i x^j - \frac{1}{3}\delta^{ij}|\mathbf{x}|^2\right)$$

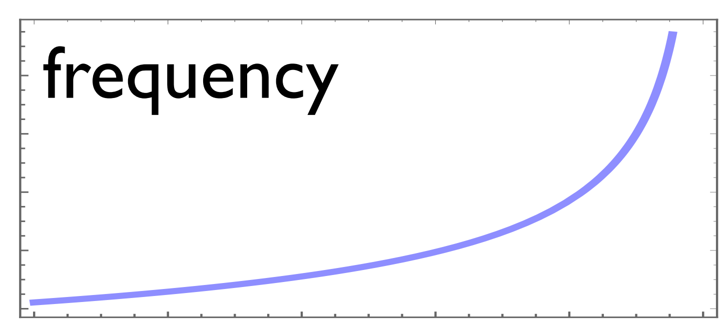
$$h_{ij} = \frac{2}{D}\ddot{Q}_{ij}$$

D=Distance to source



$$h \sim -\frac{\mathcal{M}}{2D}\left(\frac{t_{\mathrm{merge}} - t}{5\mathcal{M}}\right)^{-1/4} \cos\left[2\varphi_{\mathrm{merge}} - 2\left(\frac{t_{\mathrm{merge}} - t}{5\mathcal{M}}\right)^{5/8}\right]$$



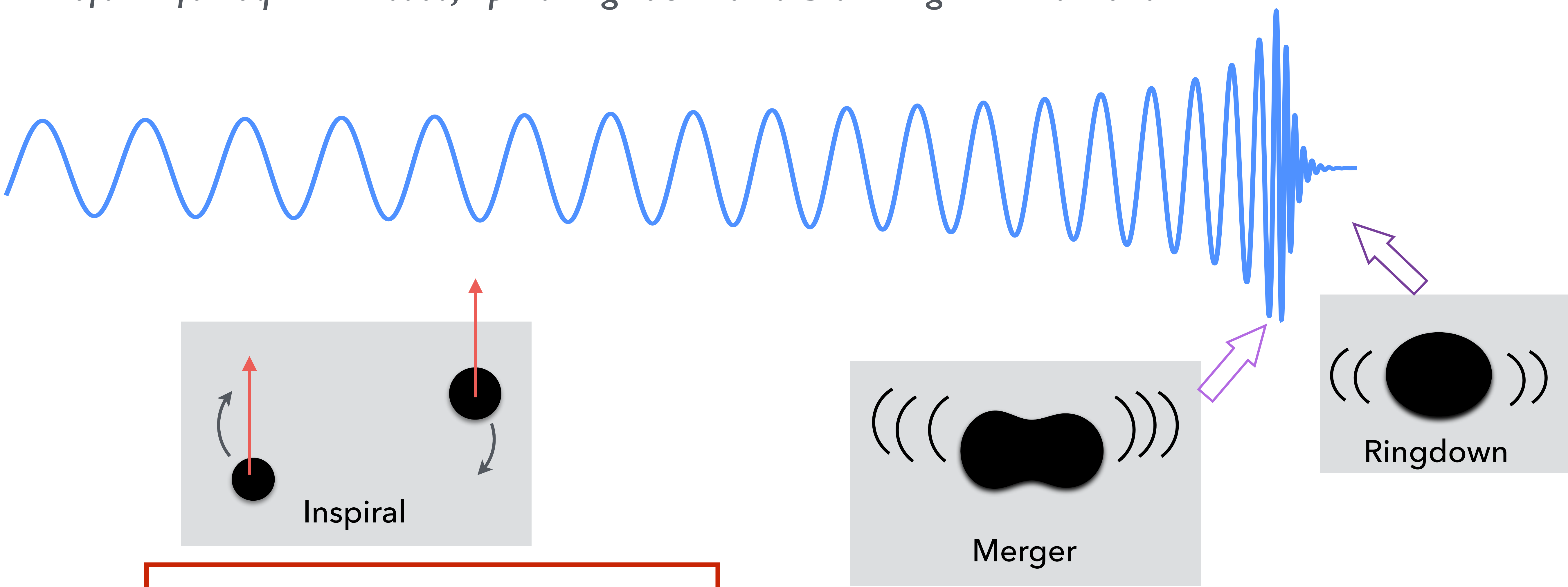


Chirp mass $\mathcal{M} = \mu^{3/5} M^{2/5}$

Signal from binary black holes (BHs)

- Black holes: only warped spacetime
- characterized entirely by mass and spin

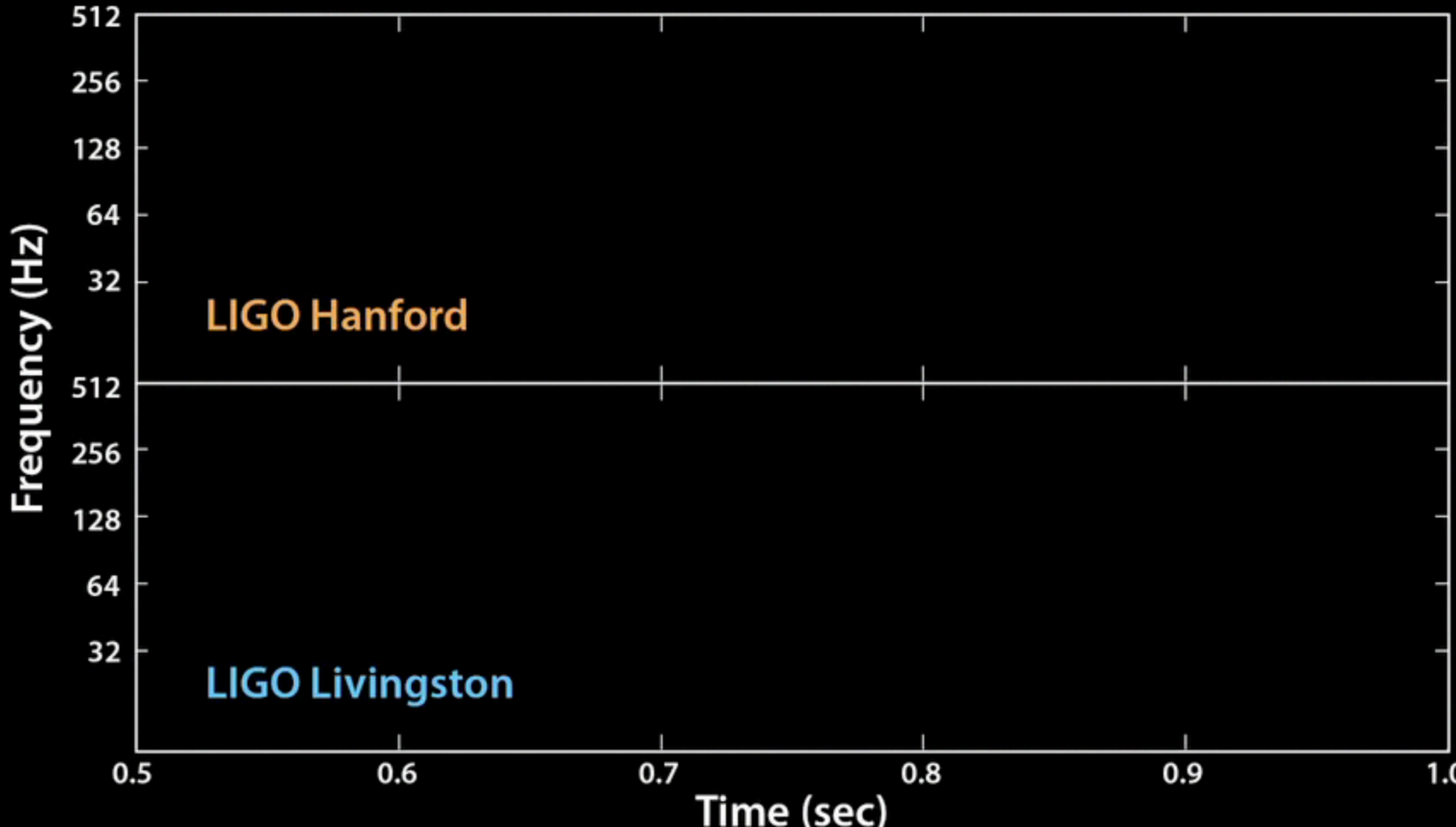
Waveform for equal masses, Spins aligned with orbital angular momentum



Per orbit ~0.05 seconds

Details of the phase evolution encode the parameters of the system

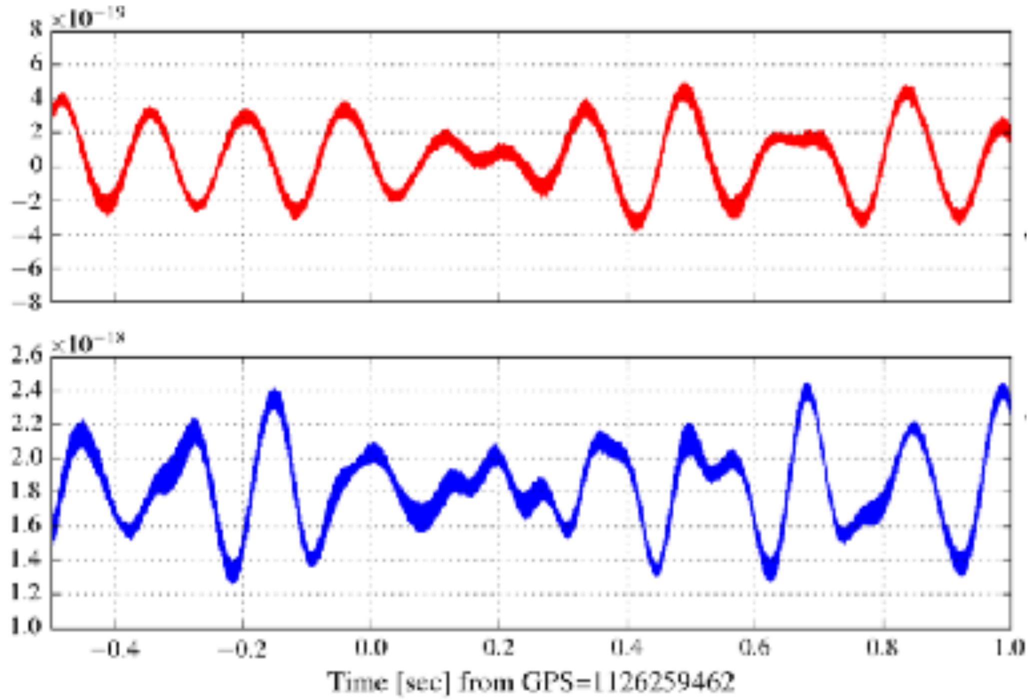
14. September 2015, 10:45:45 CET



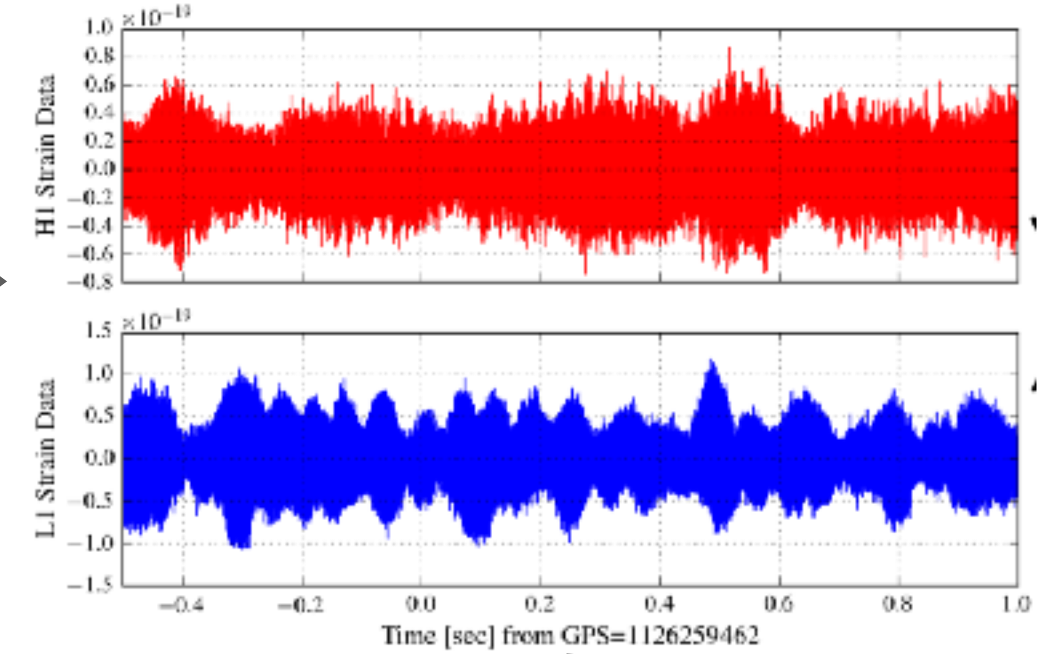
Credit:LSC

What exactly was measured?

Data output **H1** and **L1** strains

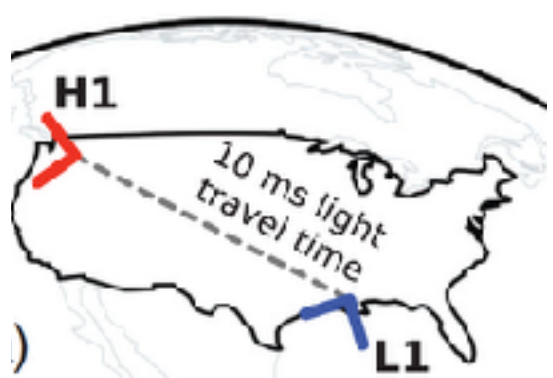


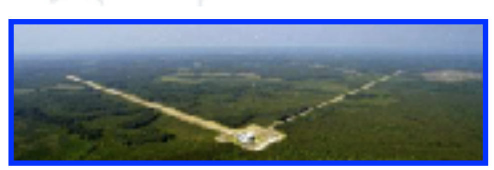
Highpass filter

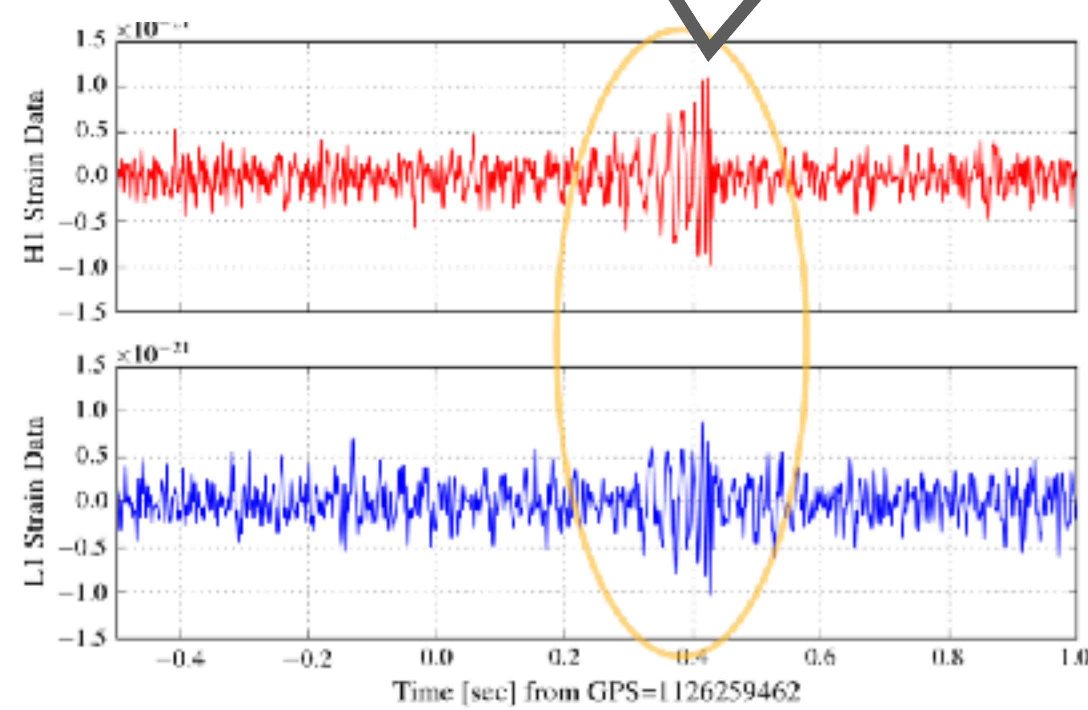


[credit: Harry & LSC]

whitening



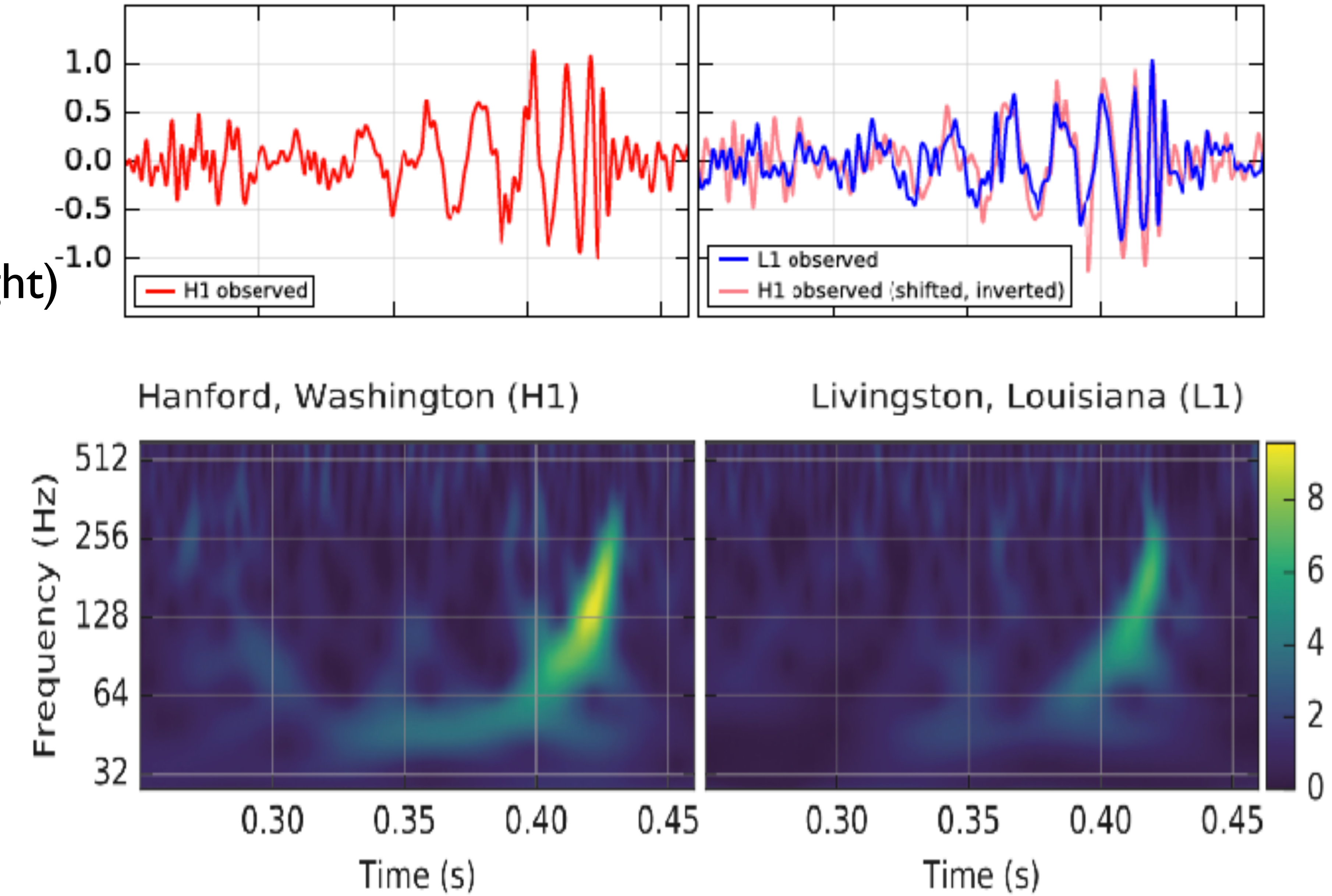




What was observed?

- Coincident detection

(~ 7 ms difference,
GWs travel at the speed of light)



- Very loud signal

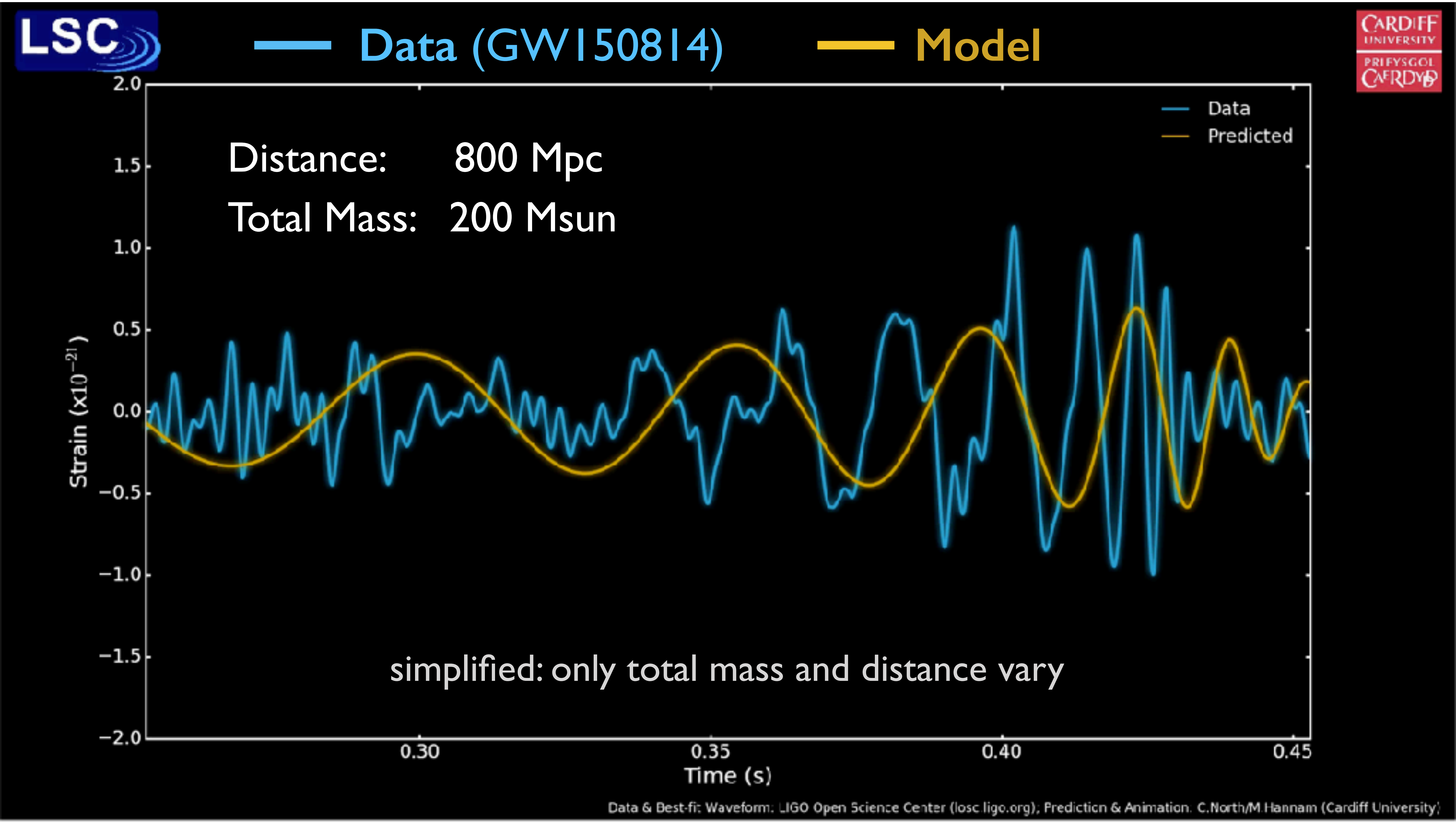
- Characteristic of a binary coalescence

Chirp signal

from 35 Hz to ~300 Hz in 0.2 sec

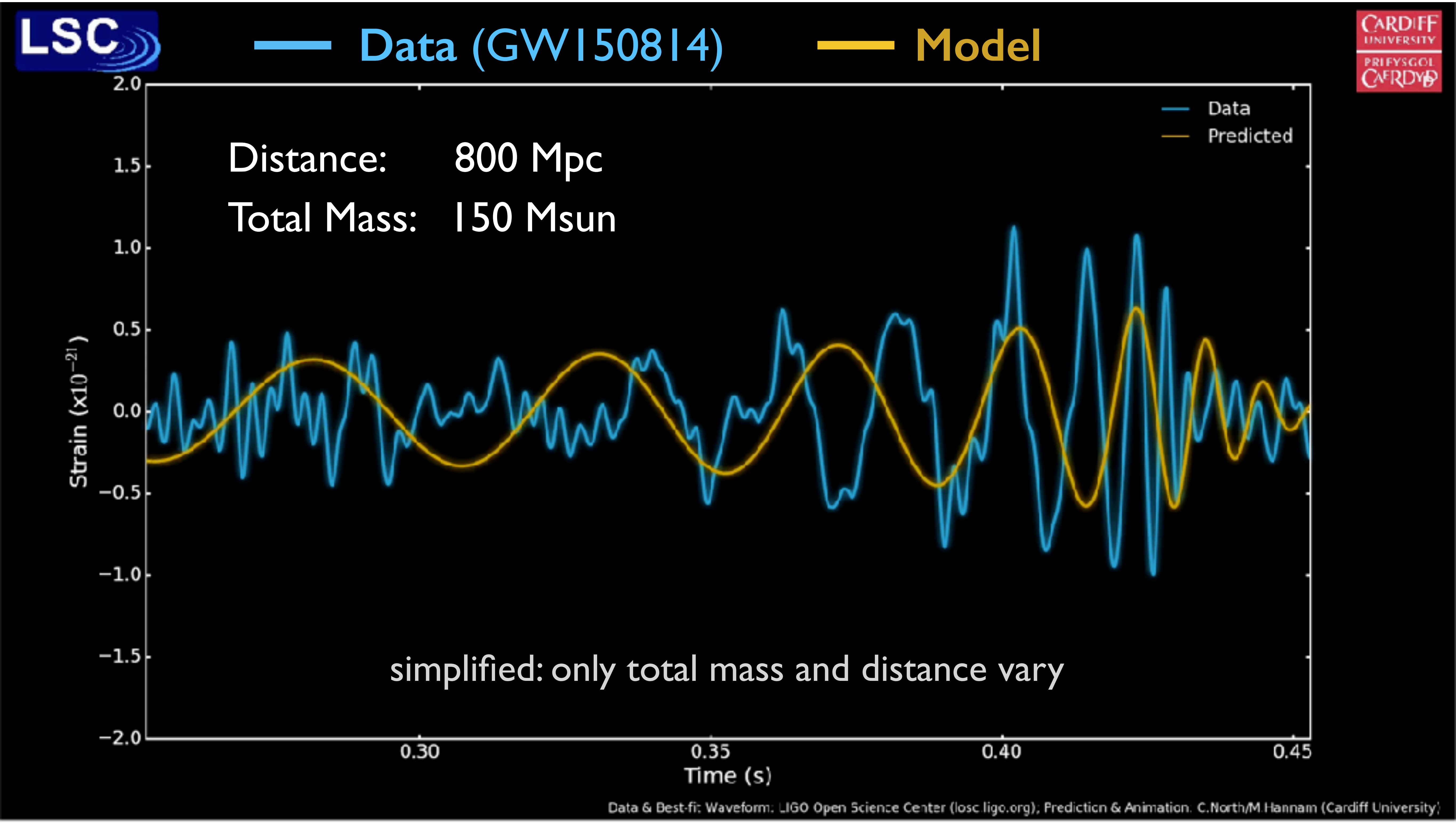
[Abbott et al (LSC) 2016]

Interpreting GW signals



Newtonian: $h \sim -\frac{\mathcal{M}}{2D}\left(\frac{t_{\mathrm{merge}}-t}{5\mathcal{M}}\right)^{-1/4} \cos\left[2\varphi_{\mathrm{merge}} - 2\left(\frac{t_{\mathrm{merge}}-t}{5\mathcal{M}}\right)^{5/8}\right]$

Interpreting GW signals



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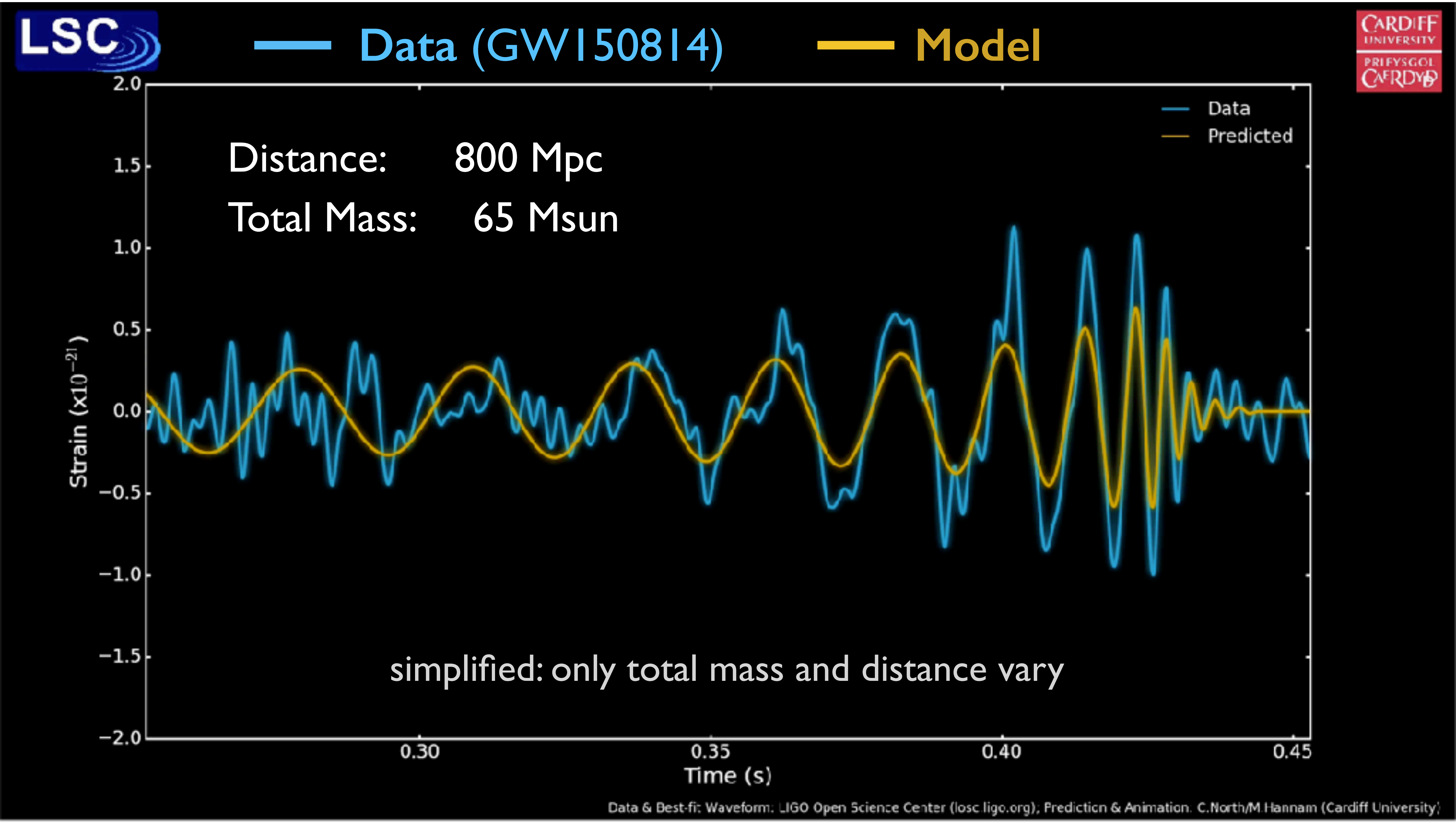

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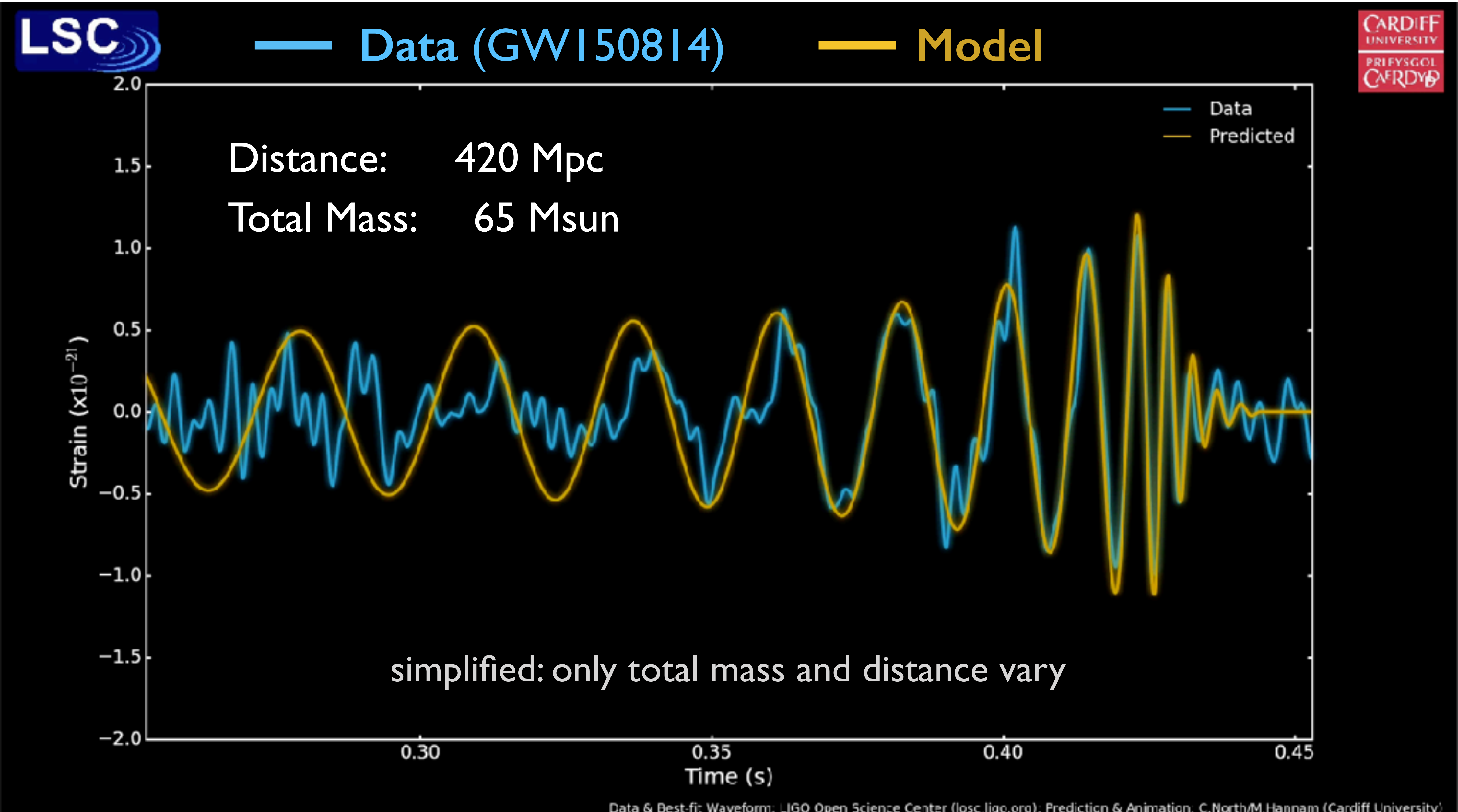

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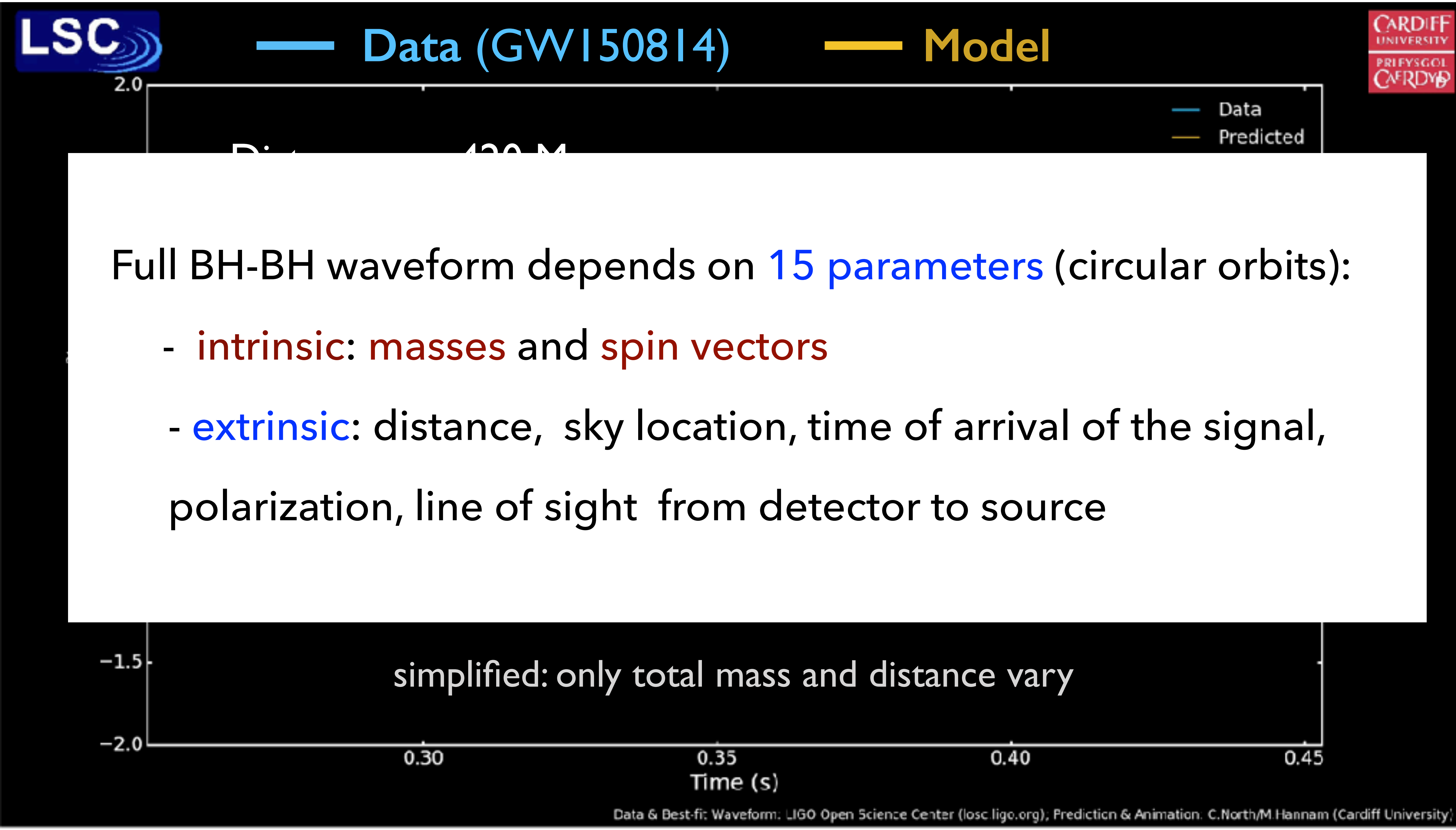


Full BH-BH waveform depends on 15 parameters (circular orbits):

- intrinsic: masses and spin vectors
- extrinsic: distance, sky location, time of arrival of the signal, polarization, line of sight from detector to source

Newtonian: $$h \sim -\frac{\mathcal{M}}{2D}\left(\frac{t_{\rm merge}-t}{5\mathcal{M}}\right)^{-1/4}\cos\left[2\varphi_{\rm merge}-2\left(\frac{t_{\rm merge}-t}{5\mathcal{M}}\right)^{5/8}\right]$$

Challenges for computing templates

- must solve for the dynamical spacetime of the binary system.

	field equations:	equations of motion:
Newtonian gravity	*Gravitational potential* *Mass density* $\nabla^2 \Phi = 4\pi G \rho$	$\ddot{x}^i = -\frac{\partial \Phi}{\partial x^i}$
General Relativity	*dynamical spacetime geometry* *sources* $G_{\mu\nu}[g_{\alpha\beta}] = \frac{8\pi G}{c^4} T_{\mu\nu}$	$\nabla^\nu G_{\mu\nu}[g_{\alpha\beta}] = 0$

$G_{\mu\nu}$: highly nonlinear differential operator

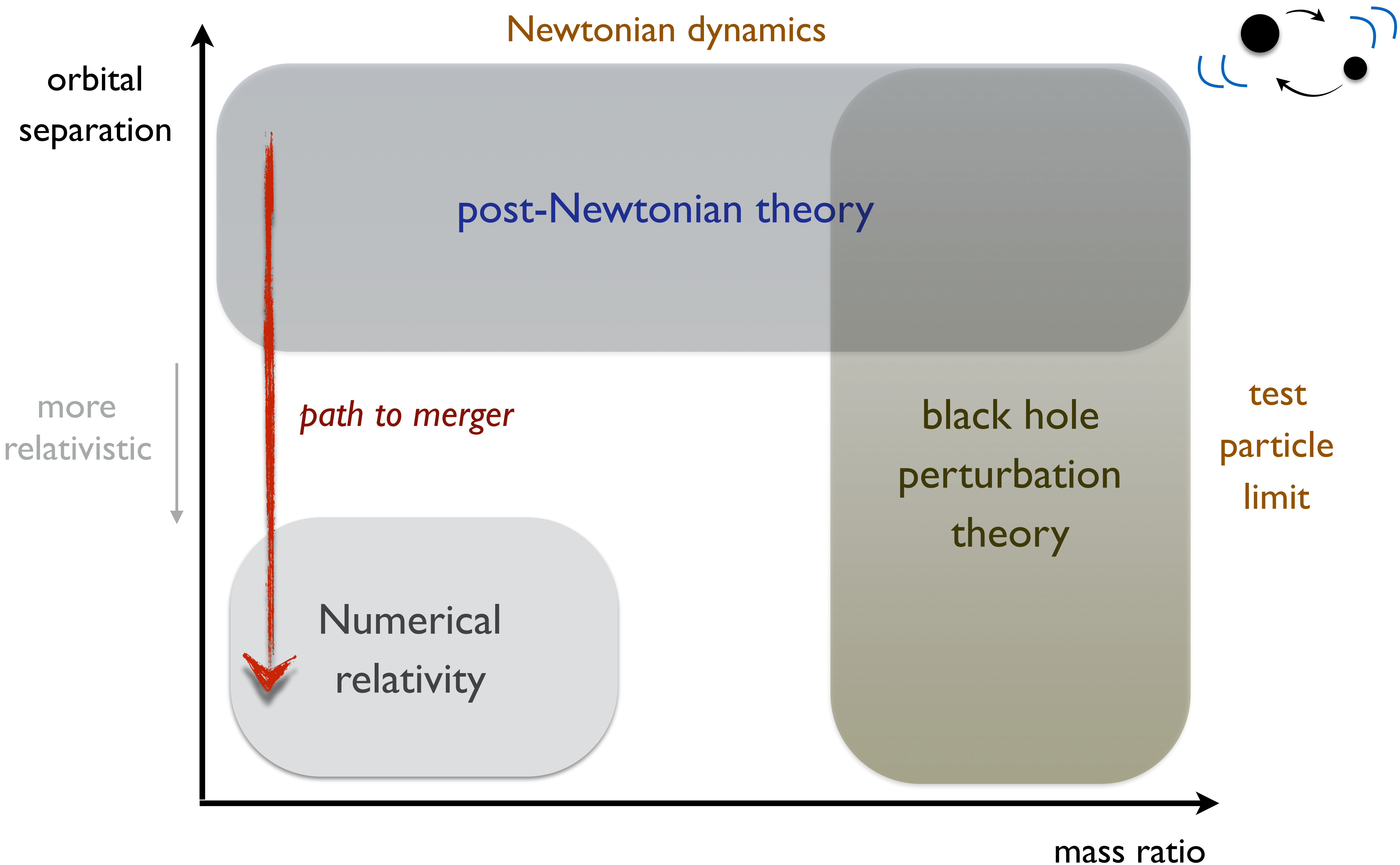
$T_{\mu\nu}$: density, pressure, flow of energy / momentum, …

$\nabla^\nu G_{\mu\nu}[g_{\alpha\beta}] = 0$: complicated differential eqs.

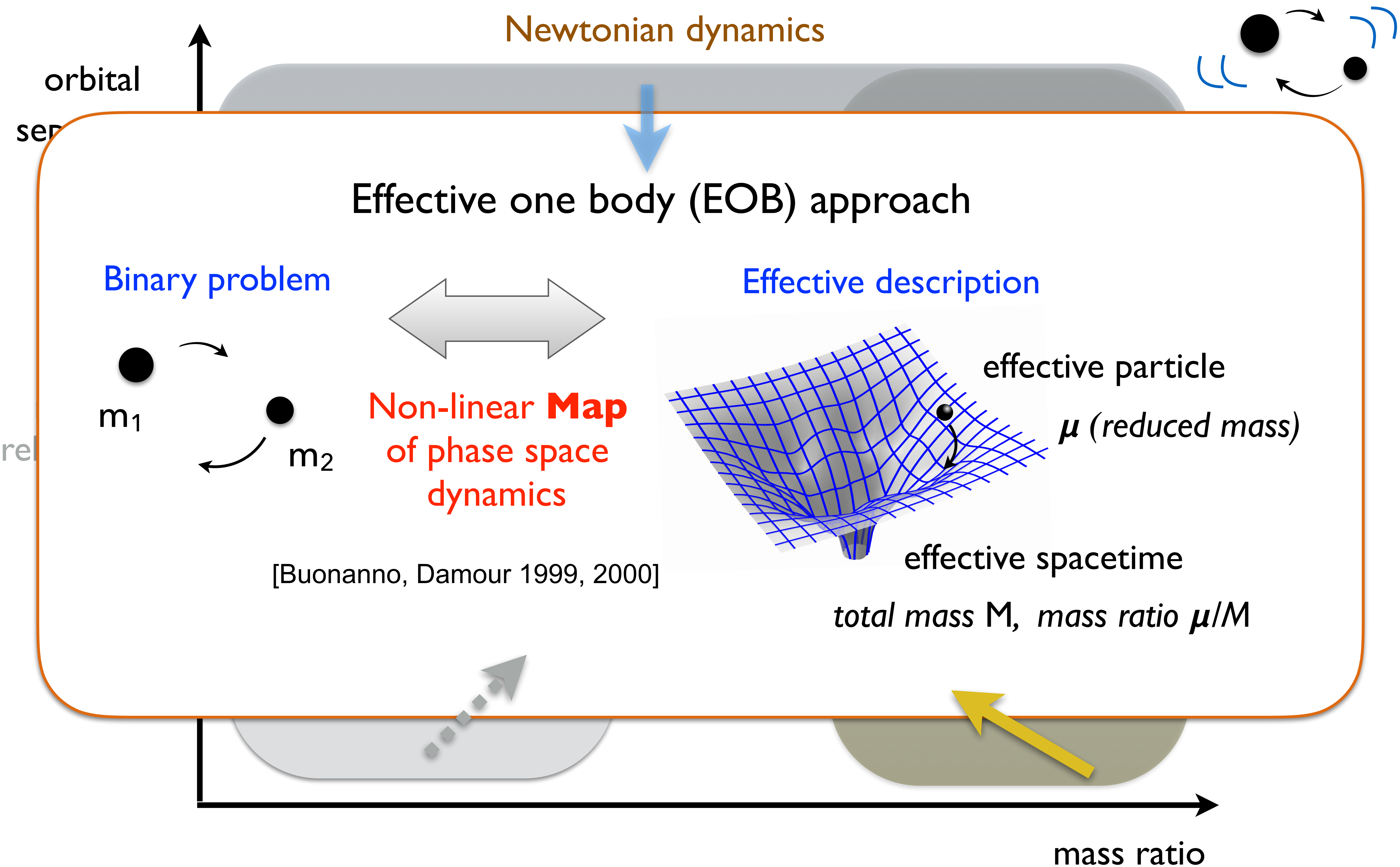
6 coupled eqs. in 6 variables,

gauge conditions, well-posed initial value formulation, horizons/singularities …

Approaches to computing templates



Complete waveform model for comparable-mass binaries



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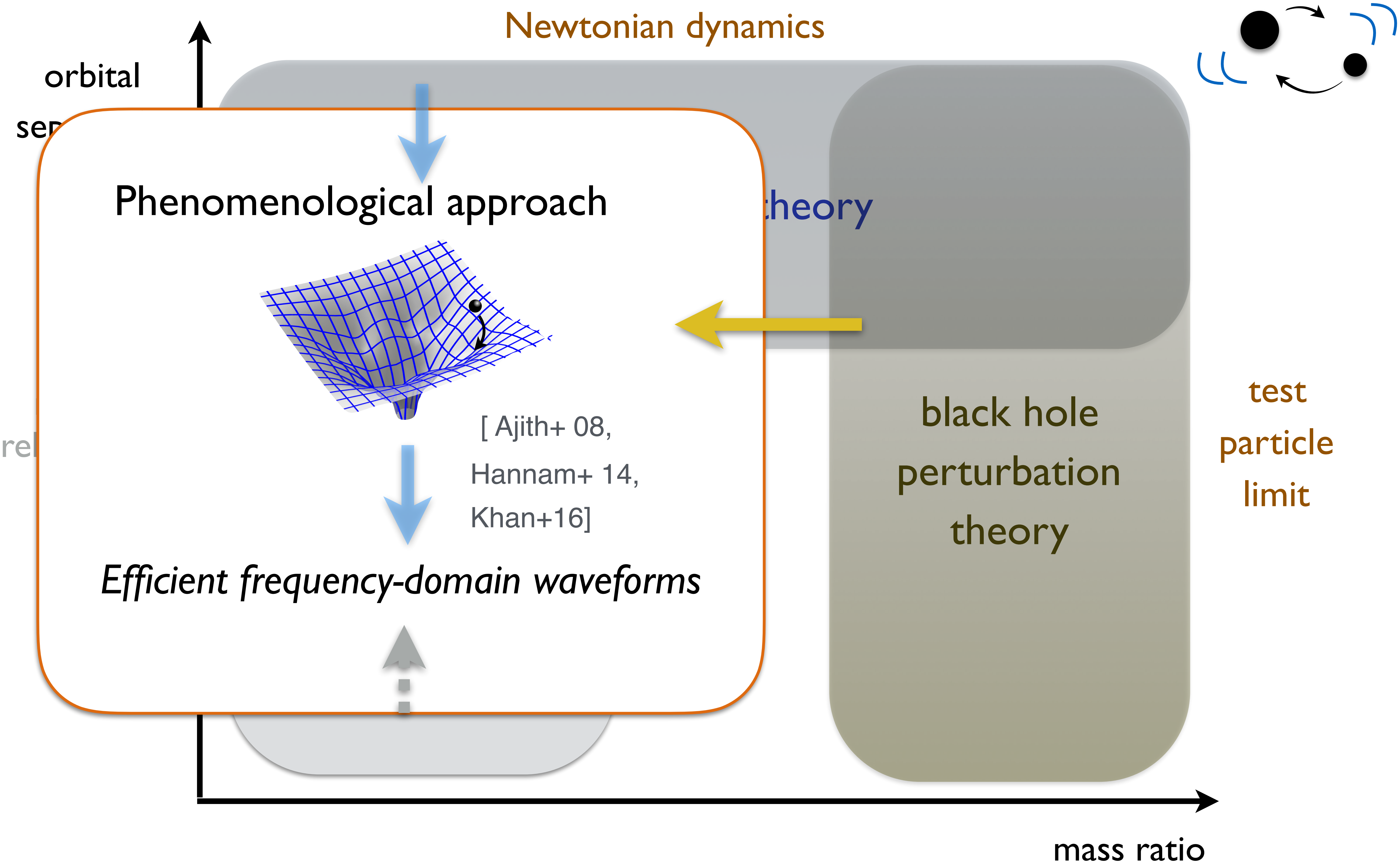
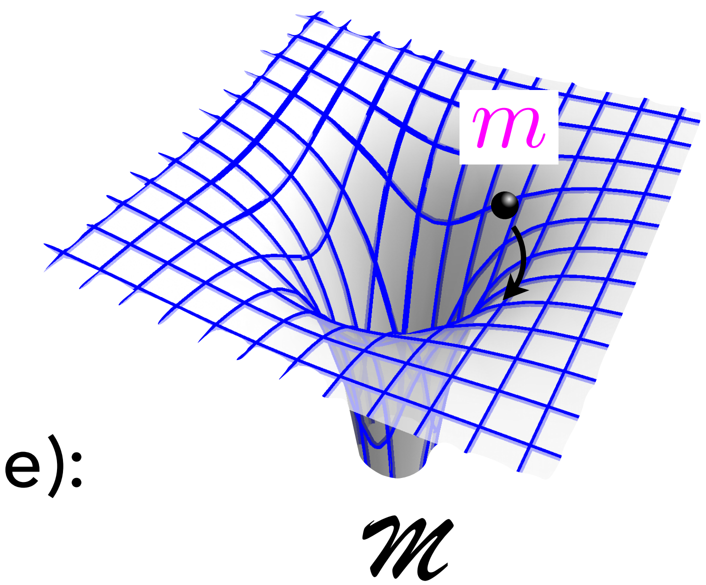


Setting up the effective description

- Exact strong-field dynamics: particle in Schwarzschild



$$g_{\mu\nu}dx^\mu dx^\nu = -A\,dT^2 + B\,dR^2 + R^2 d\Omega^2$$

- Hamiltonian on 8-d phase space (conjugate to proper time):

$$\mathcal{H} = \frac{1}{2}g^{\mu\nu}P_\mu P_\nu = -\frac{1}{2}m^2$$

for timelike geodesics ($g^{\mu\nu}u_\mu u_\nu = -1$)

- Want global evolution parameter: solve for energy (=$-P_t/m$) to get a Hamiltonian H_{geodesic} conjugate to coordinate time

$$\frac{H_{\text{geodesic}}}{m} = \sqrt{A\left(1 + \frac{P_R^2}{m^2 B} + \frac{P_\phi^2}{m^2 R^2}\right)}$$

Setting up the effective description

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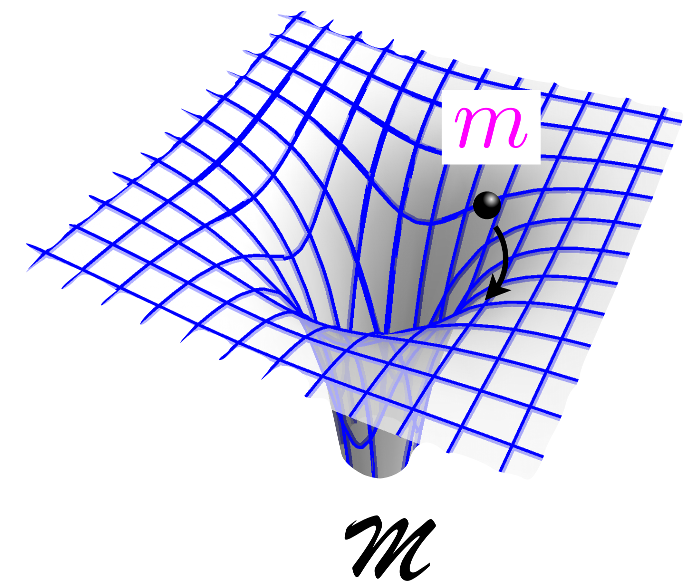


$$g_{\mu\nu}dx^\mu dx^\nu = -A\,dT^2 + B\,dR^2 + R^2 d\Omega^2$$

- Hamiltonian on 6-d phase space:

$$\frac{H_{\rm geodesic}}{m} = \sqrt{A\left(1 + \frac{P_R^2}{m^2 B} + \frac{P_\phi^2}{m^2 R^2}\right)}$$

- Assume effective dynamics for finite mass ratio $m/\mathcal{M}$ are “smooth deformations” of the test particle limit so e.g.

$$A^{\rm eff} = 1 - \frac{2\mathcal{M}}{R} + \frac{m}{\mathcal{M}}\,\delta A$$

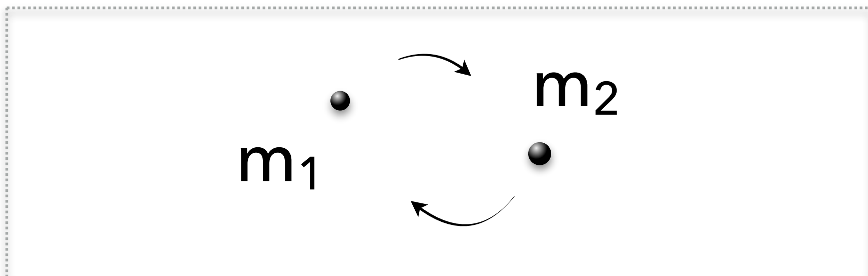
Correction to be determined

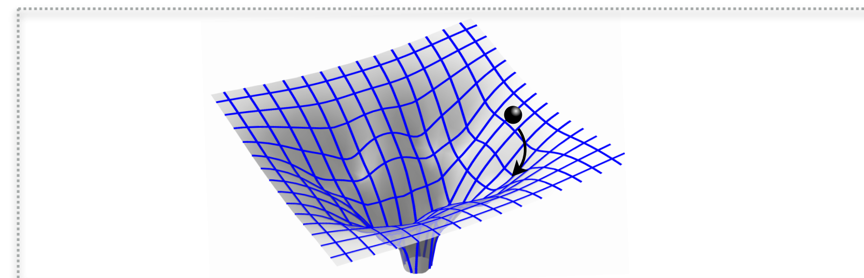
- Newtonian limit of two-body map to effective reduced-mass motion requires:

$$\mathcal{M} \leftrightarrow M = m_1 + m_2 \qquad\qquad m \leftrightarrow \mu = m_1 m_2/M$$

Mapping to the effective description

- Constants of motion in both cases: energy E and angular momentum L
- invariant quantities are the action variables





$$J_r(E,L) = \frac{1}{2\pi}\oint p_r\, dr$$

$$J_\phi(E,L) = L$$

Identify

$$J_r(E^{\rm eff}, L^{\rm eff}) = \frac{1}{2\pi}\oint P_R\, dR$$

$$J_\phi(E^{\rm eff}, L^{\rm eff}) = L^{\rm eff}$$

$$L_{\rm eff} = L$$

$$E_{\rm eff} = \frac{E^2 - m_1^2 - m_2^2}{2M}$$

Energy map, c.f. scattering calculations

EOB idea inspired by similar analysis in QED (positronium - hydrogen)

[Brezin, Itzykson, Zinn-Justin (1970)]

Mapping to the effective description cont.

denote $\nu = \mu/M \in [0, 1/4]$ "symmetric mass ratio"

test particle limit (→ 0) *equal-mass case* (→ 1/4)

- From energy map:

$$H_{\mathrm{EOB}} = M\sqrt{1 + 2\nu\left(\frac{H_{\mathrm{eff}}}{\mu} - 1\right)}$$

- Explicit canonical transformations of PN results lead to compact representation, up to 2PN order:

$$\frac{H_{\mathrm{eff}}^2}{\mu^2} = A\left(1 + \frac{p_r^2}{\mu^2 B} + \frac{p_\phi^2}{\mu^2 r^2}\right)$$

$$A = 1 - \frac{2M}{r} + 2\nu\frac{M^3}{r^3}$$

$$B = 1 + \frac{2M}{r} + (4 - 6\nu)\frac{M^2}{r^2}$$

c.f. post-Newtonian result at 2PN

$$\hat{H}[\mathbf{r},\mathbf{p}] = \hat{H}_{\mathrm{N}}(\mathbf{r},\mathbf{p}) + \frac{1}{c^2}\hat{H}_{1\mathrm{PN}}(\mathbf{r},\mathbf{p}) + \frac{1}{c^4}\hat{H}_{2\mathrm{PN}}(\mathbf{r},\mathbf{p})$$

$$\hat{H}_{\mathrm{N}}(\mathbf{r},\mathbf{p}) = \frac{p^2}{2} - \frac{1}{r},$$

$$\hat{H}_{1\mathrm{PN}}(\mathbf{r},\mathbf{p}) = \frac{1}{8}(3\nu-1)p^4 - \frac{1}{2}\left[(3+\nu)p^2 + \nu p_r^2\right]\frac{1}{r} + \frac{1}{2r^2},$$

$$\begin{aligned}\hat{H}_{2\mathrm{PN}}(\mathbf{r},\mathbf{p}) = {} & \frac{1}{16}(1-5\nu+5\nu^2)p^6 \\ & + \frac{1}{8}[(5-20\nu-3\nu^2)p^4 - 2\nu^2 p_r^2 p^2 - 3\nu^2 p_r^4]\frac{1}{r} \\ & + \frac{1}{2}[(5+8\nu)p^2 + 3\nu p_r^2]\frac{1}{r^2} - \frac{1}{4}(1+3\nu)\frac{1}{r^3},\end{aligned}$$

[Schaefer & Jaranowski, Living Reviews in Relativity 21:7 (2018)]

Beyond 2PN results

- For 3PN+: H_{eff} includes non-geodesic terms $O(p_r^4, p_r^6)$

- Results for potentials A, B are re-summed (written as non-analytic functions)

- Additional terms included with coefficients calibrated to numerical relativity

[Buonanno, Damour, Nagar, Pan, Taracchini, Barausse, many others]

Beyond the conservative dynamics

- Include GW dissipation as radiation reaction forces in equations of motion:

$$\frac{dP_i}{dt} = \{P_i, H^{\rm EOB}\} + \mathcal{F}_{\rm rr}$$

- Waveforms: PN results for dominant mode of GW strain amplitudes

$$h_{22}^{\rm PN}(t) = -\frac{8\pi}{5}\frac{\eta M}{\mathcal{R}} v^2\ e^{-2i\Phi}\left\{1 - \left(\frac{107}{42} - \frac{55}{42}\eta\right) v^2 + \left[2\pi + 12 i \log\left(\frac{v}{v_0}\right)\right] v^3 + \dots\right\}$$

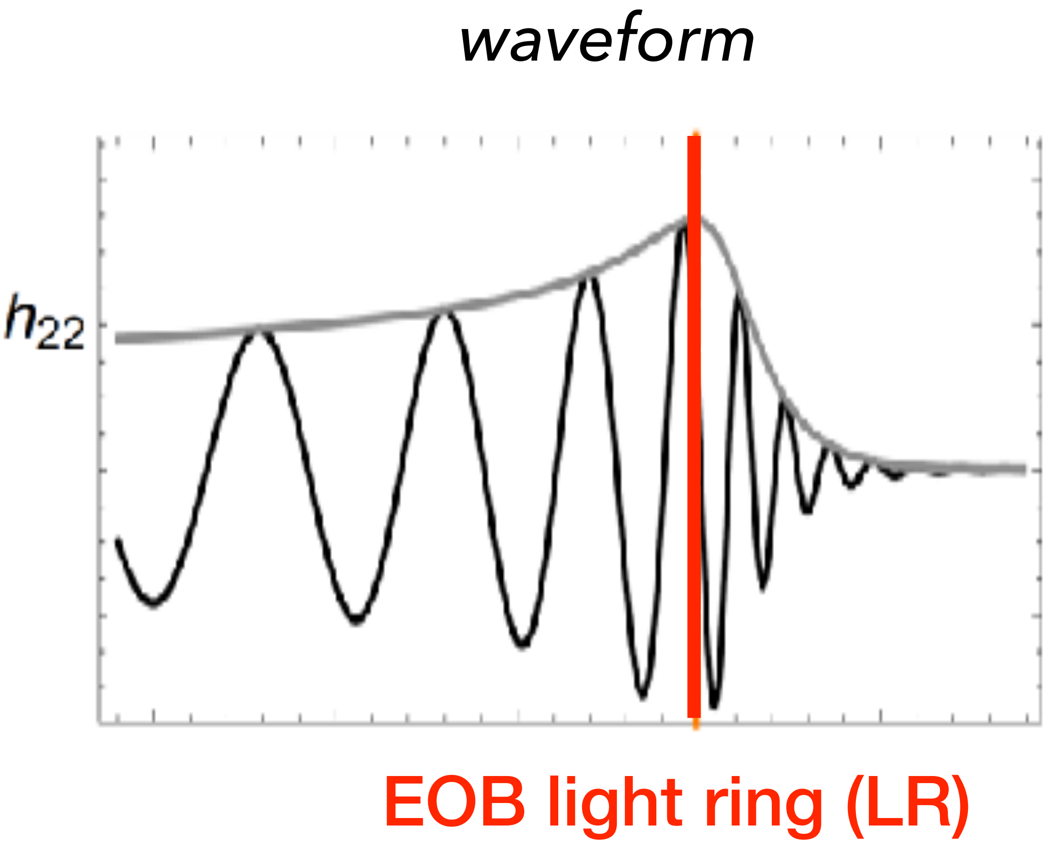
Hereditary effects, e.g. GWs scattered off space-time curvature

- EOB: factorized form, inspired by test-particle limit

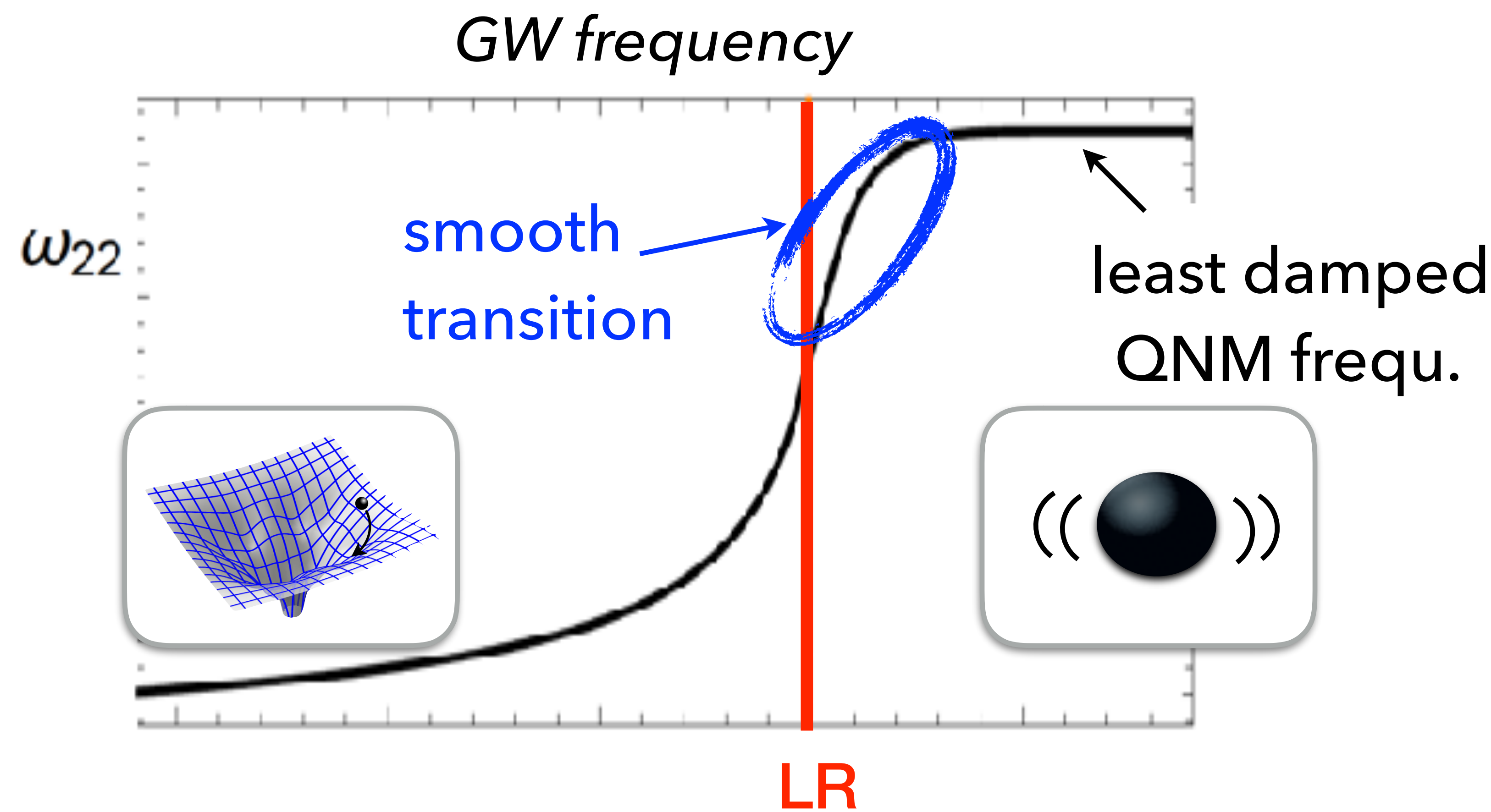
$$h_{22}^{\rm EOB}(t) = h^{\rm Newt} e^{-2i\Phi} \mathcal{S}_{\rm eff}\, \rho^2\, T e^{i\delta}\, h^{\rm NQC}$$

non-quasi-circular correction,
important near merger,
tuned to numerical relativity results

Complete EOB waveforms for black hole binaries

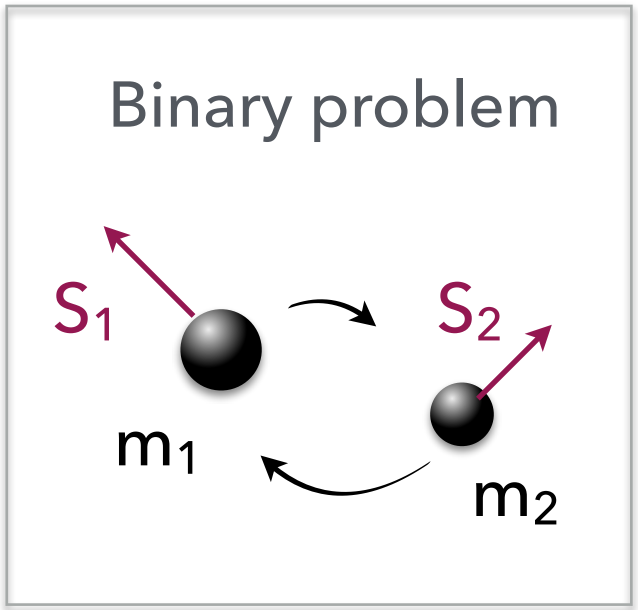


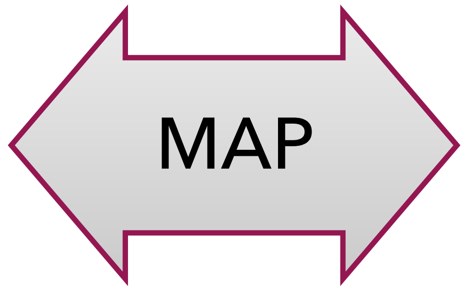
Marginally stable photon orbit



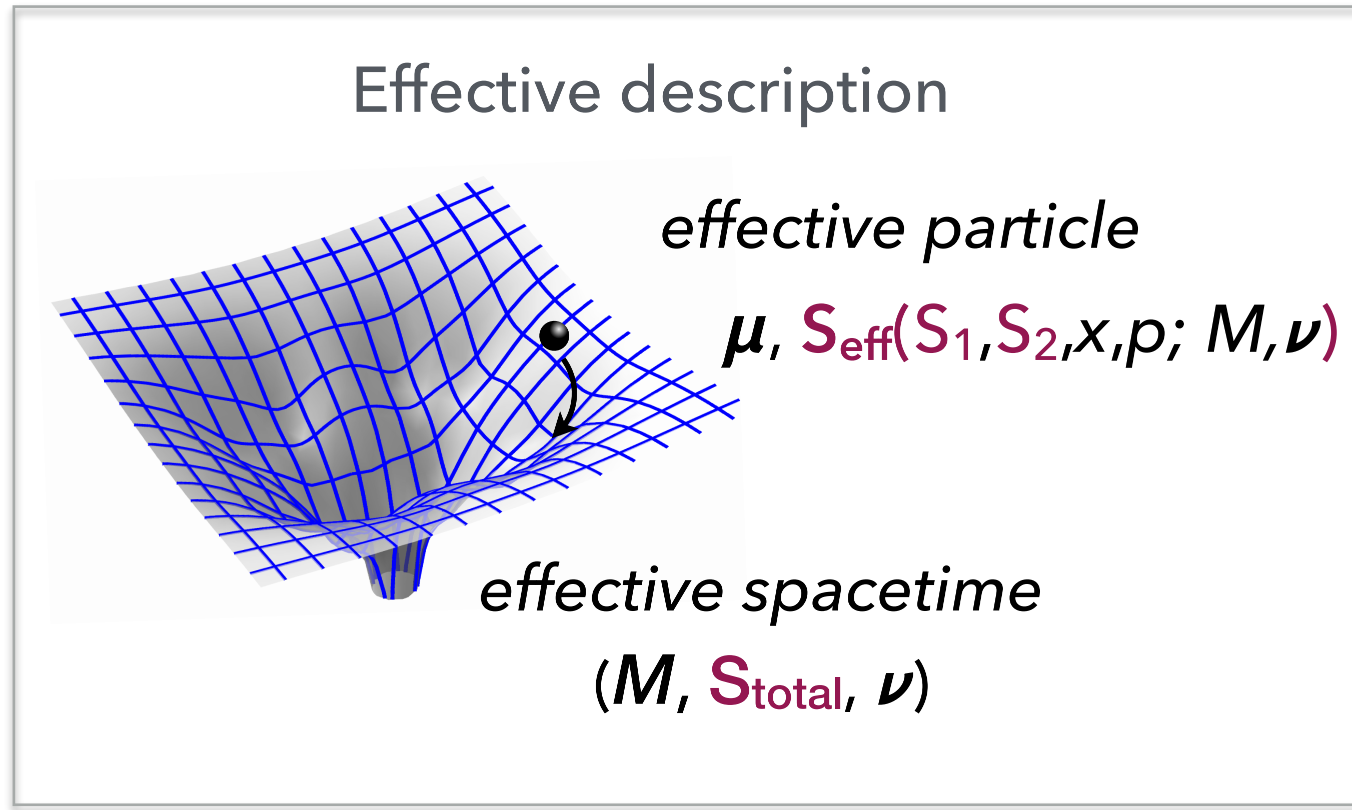
QNM: quasi-normal modes, characteristic frequencies of a perturbed black hole

Including spins

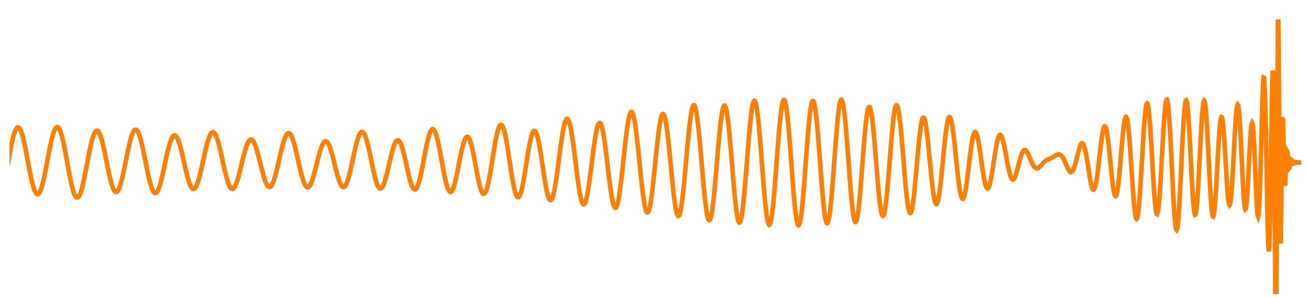




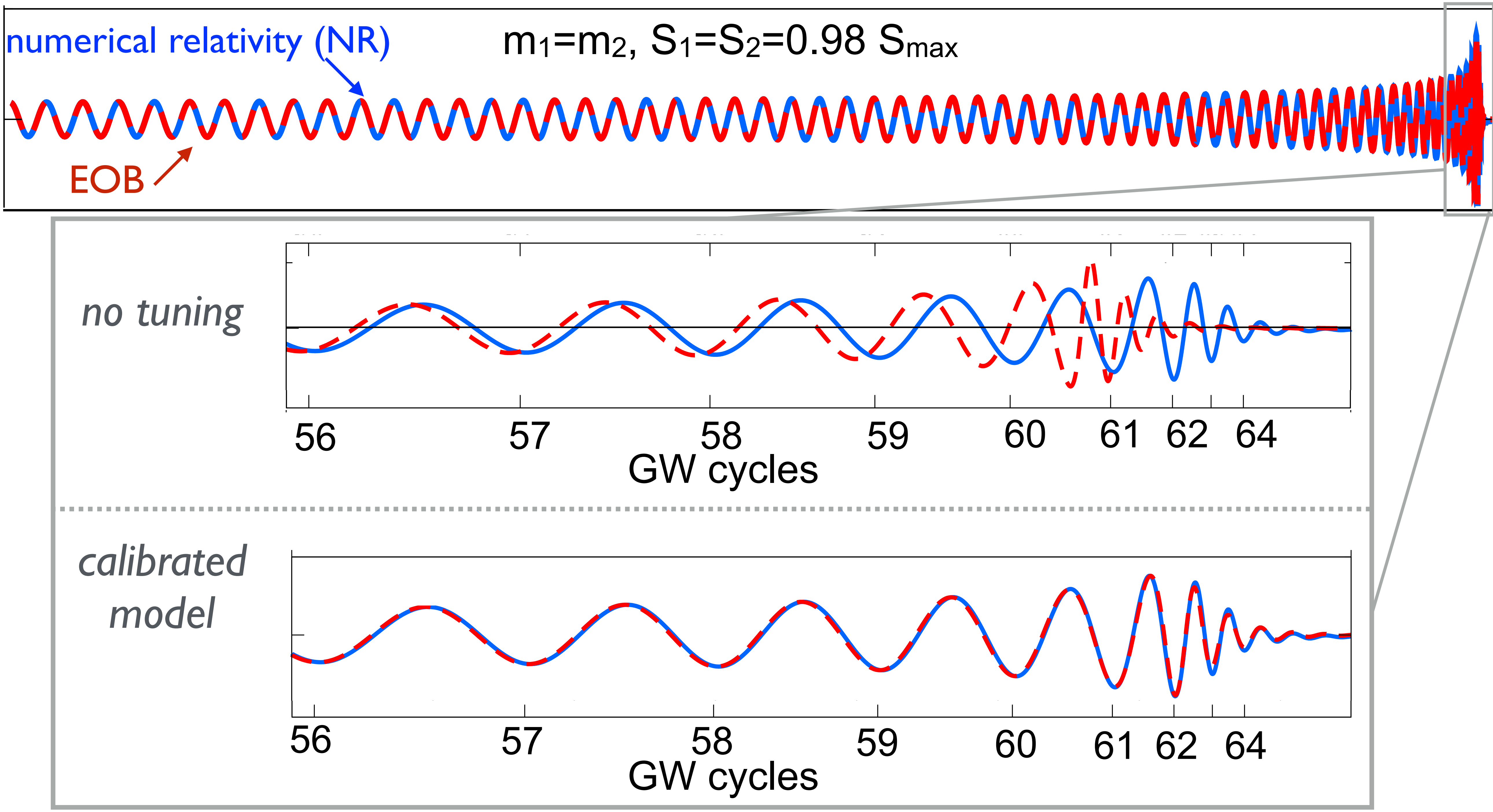
[same energy map as for non-spinning]



[Barausse, Bounanno, 2011]



Performance of EOB waveforms for BHs

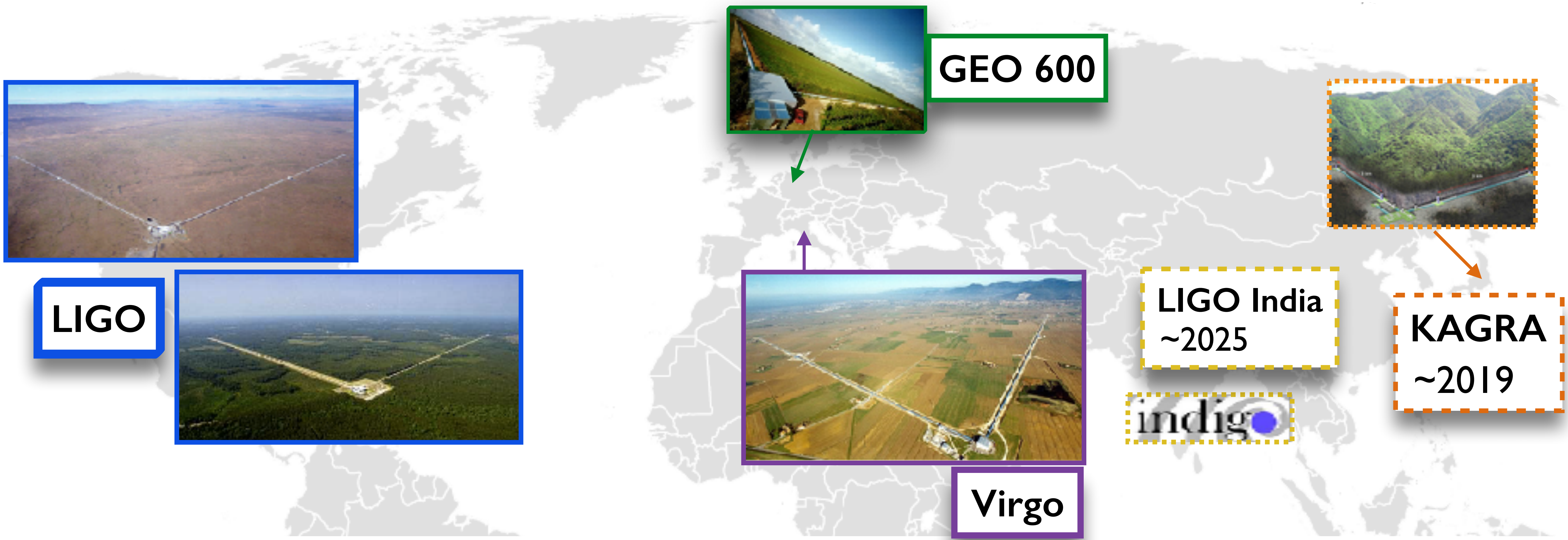


[courtesy A. Taracchini]

Model tested mainly for mass ratios 1-8

Use of models in data analysis of GWs from BH binaries

Gravitational-wave detector network

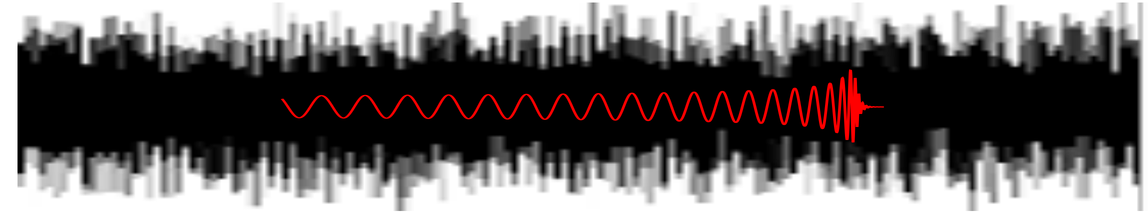


Joined O2 Aug. 1, 2017

Observing runs so far:

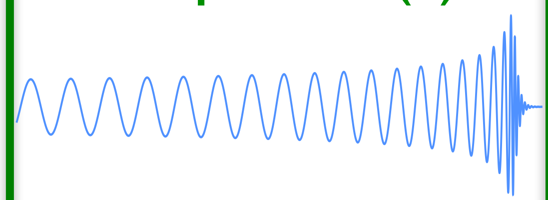
- O1: *Sept. 12, 2015 - Jan 19, 2016*
    - Coincident analysis time: 48.6 days
- O2: *Nov. 30, 2016 - Aug. 25, 2017*
    - Coincident analysis time: 118 days
    - 3-detector coincidence: 15 days

Detecting weak GW signals



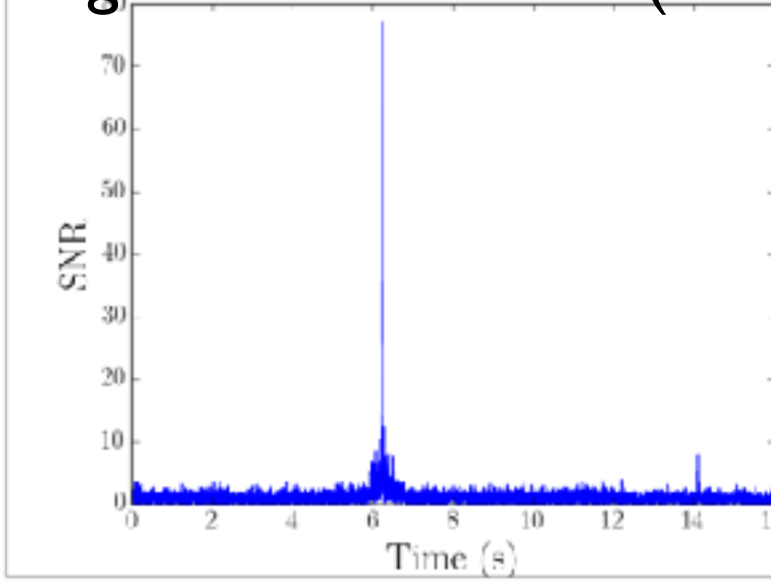
Matched filtering:

template h(t)

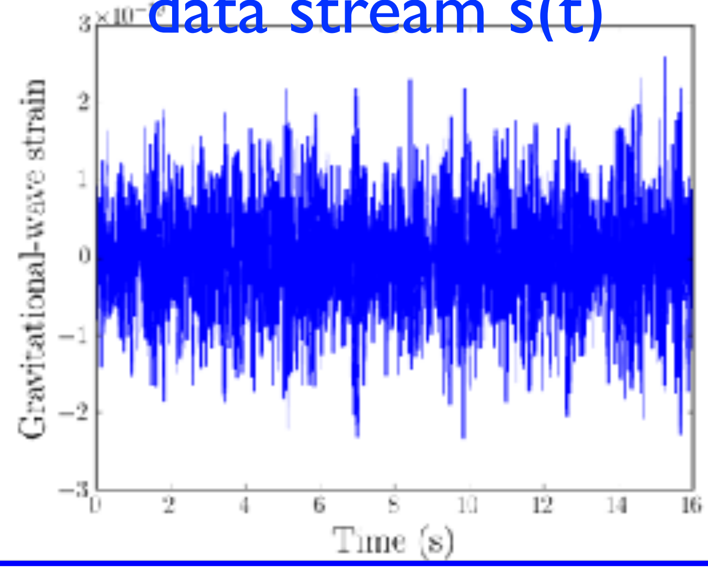


$$(s|h) = 4\,\mathrm{Re} \int df\, \frac{\tilde{s}(f)\, \tilde{h}^*(f)}{S_h(f)}$$

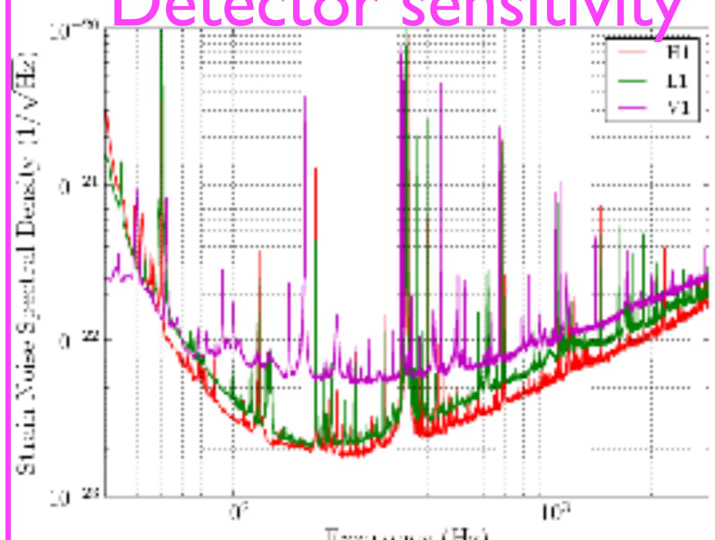
Signal-to-noise ratio (SNR)



data stream s(t)

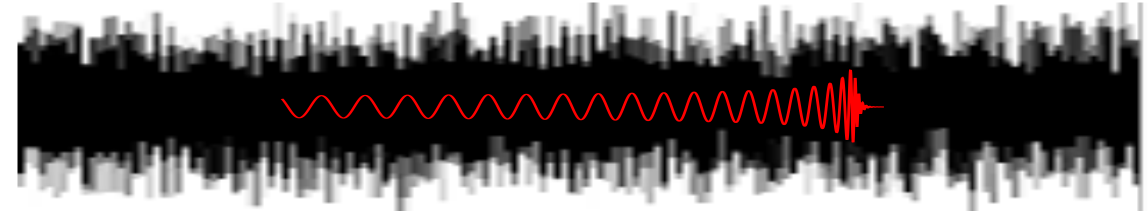


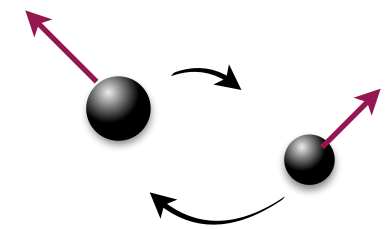
Detector sensitivity



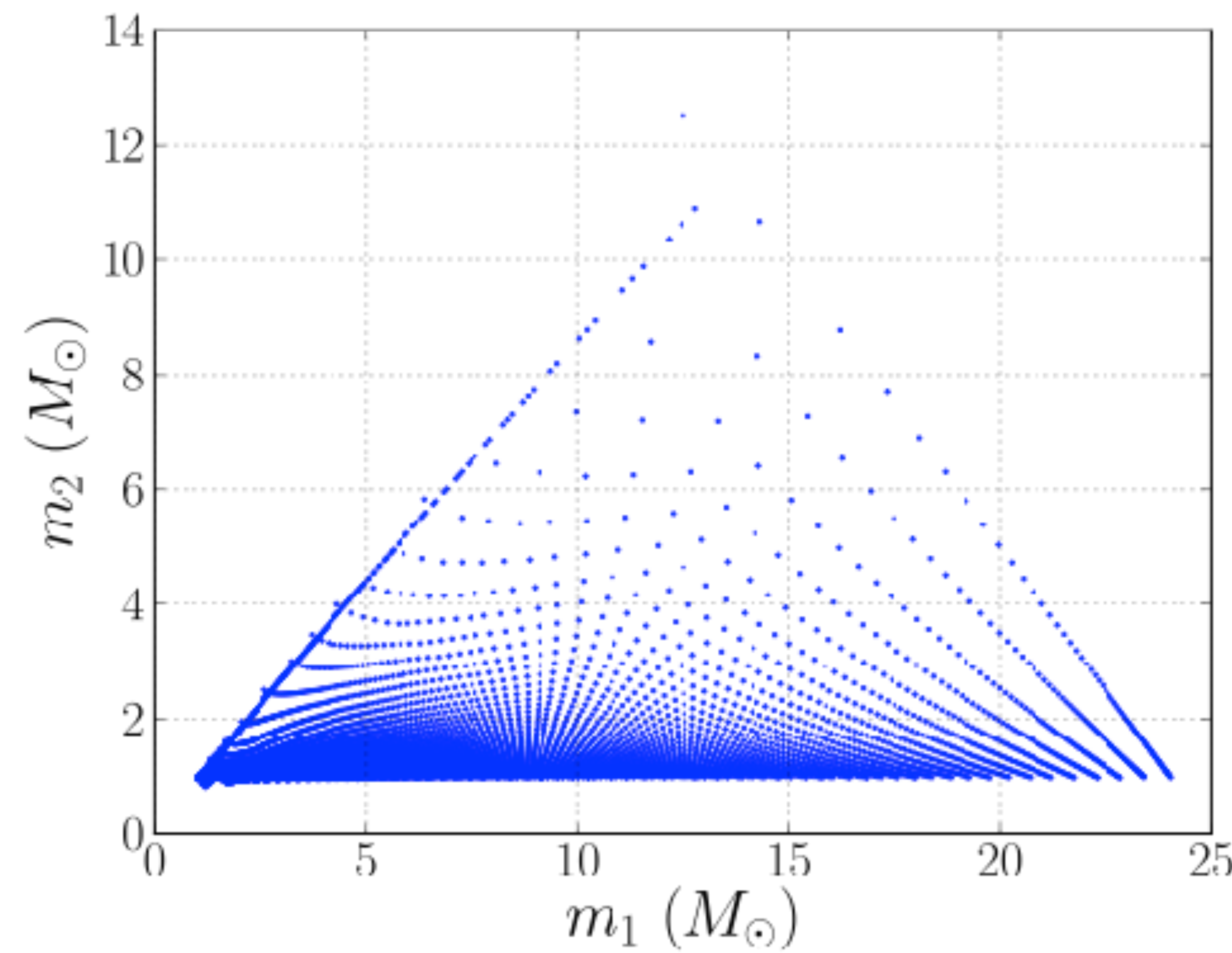
from the LIGO open-science center: http://losc.ligo.org

Detecting GW signals cont.





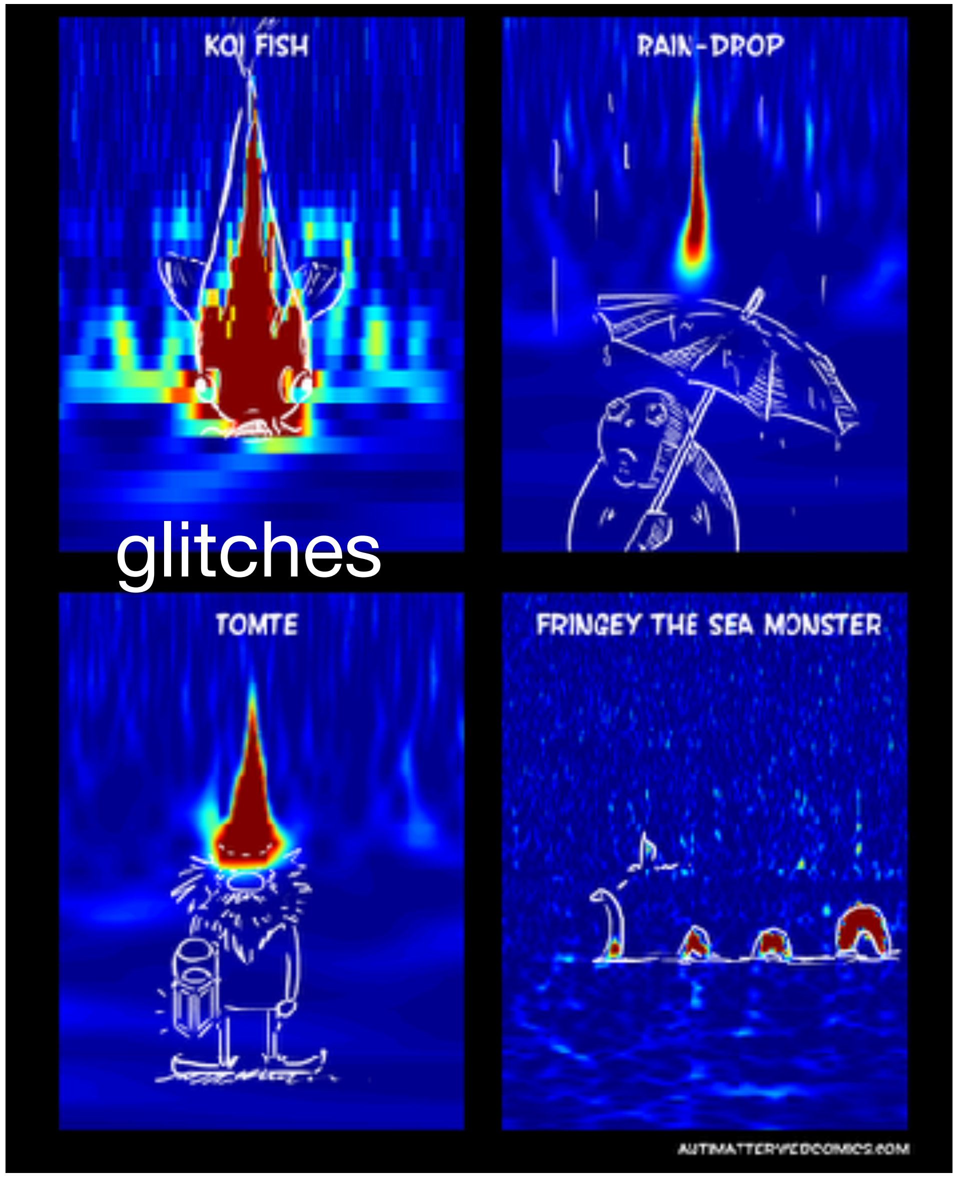
- Signals depend on at least 15 parameters
- Matched-filtering over a >15-D grid of waveforms is computationally prohibitive
  - Maximize over *extrinsic* parameters

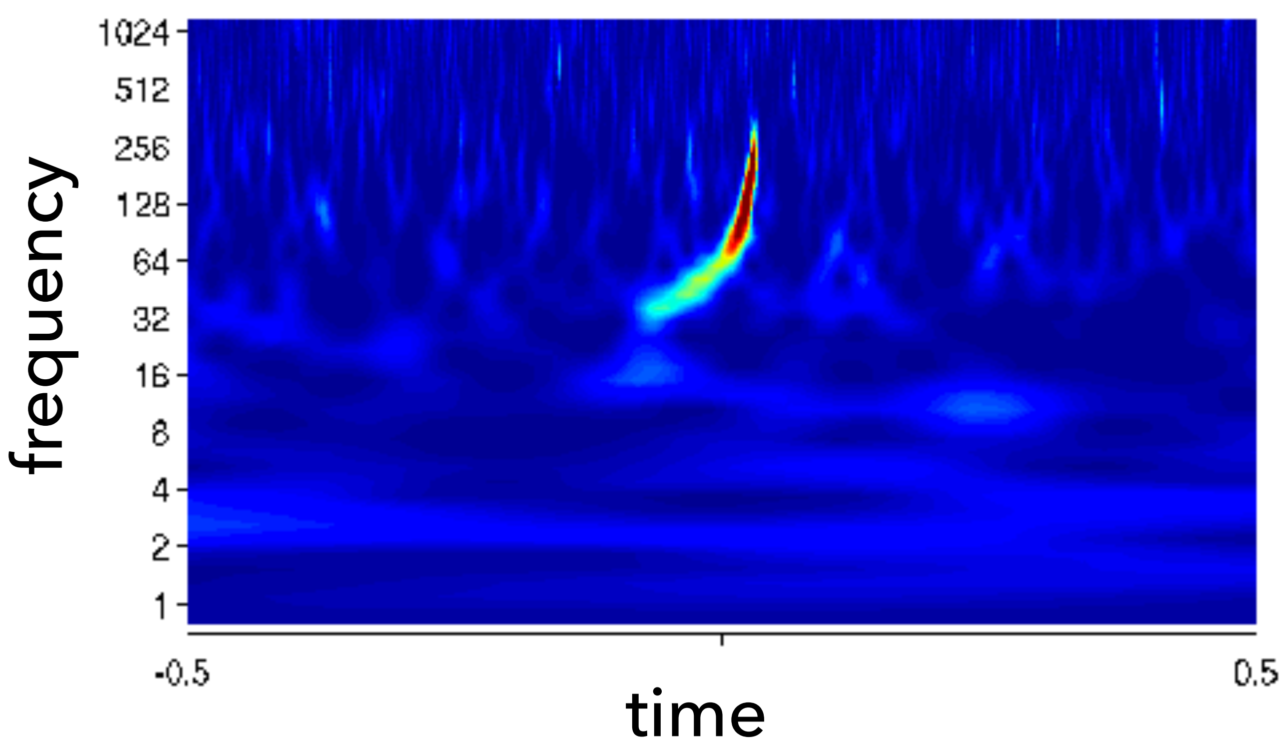


- Template bank: ~ 250 000 waveforms
  - < 3% loss of SNR if between templates
  - Two main search pipelines:
    - PyCBC: total mass 2-500 Msun
    - GstLAL: 2-400 Msun

Beyond templates: weakly-modeled searches

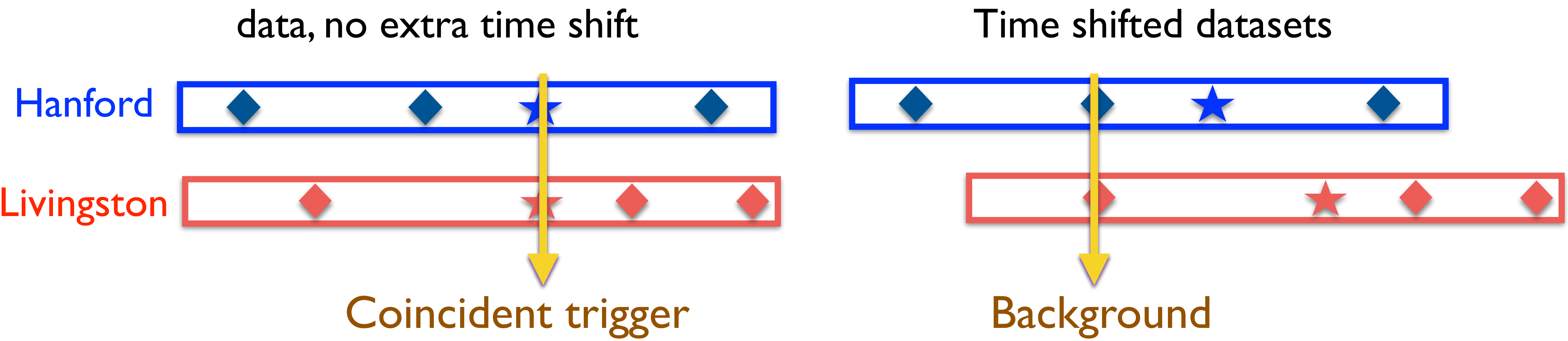
- Fewer assumptions: burst pipeline "cWB" (binaries with Mtotal <100 Msun):
  - Spectrogram of data
  - Look for excess power, features standing out from noise
  - Consistent morphology in both detectors





https://www.zooniverse.org/projects/zooniverse/gravity-spy

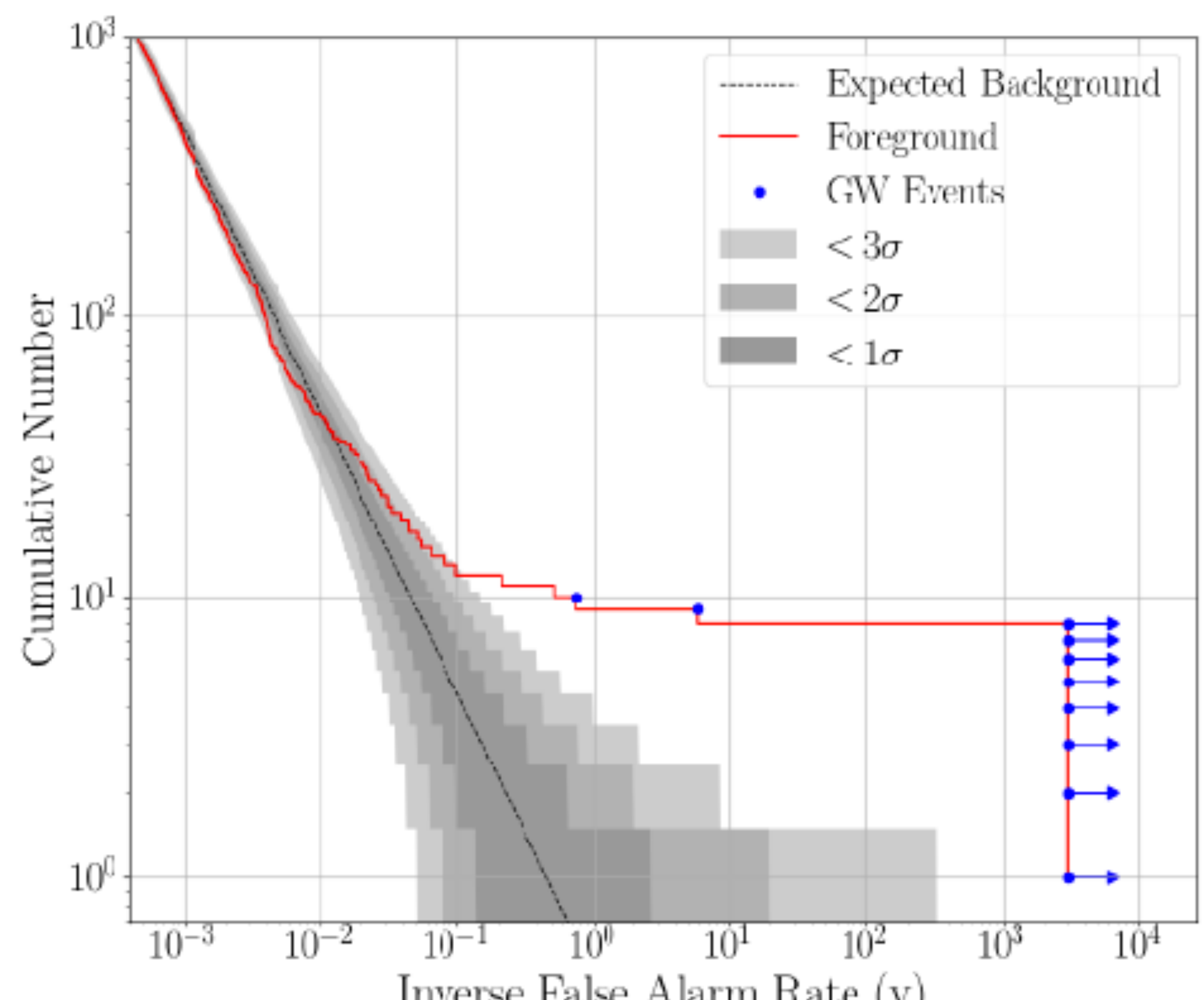
Search significances: estimate of background



False alarm rate (FAR):

$$\frac{\text{(\# similarly ranked background triggers)}}{\text{(length of data)}}$$

~10M time slides / 16 days used to compute estimate of background



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Parameter Estimation

posterior probability of parameters θ given the data d and model M (Bayes' theorem)

$$p(\theta|d, M) \propto p(\theta|M)\, p(d|\theta, M)$$

Prior information

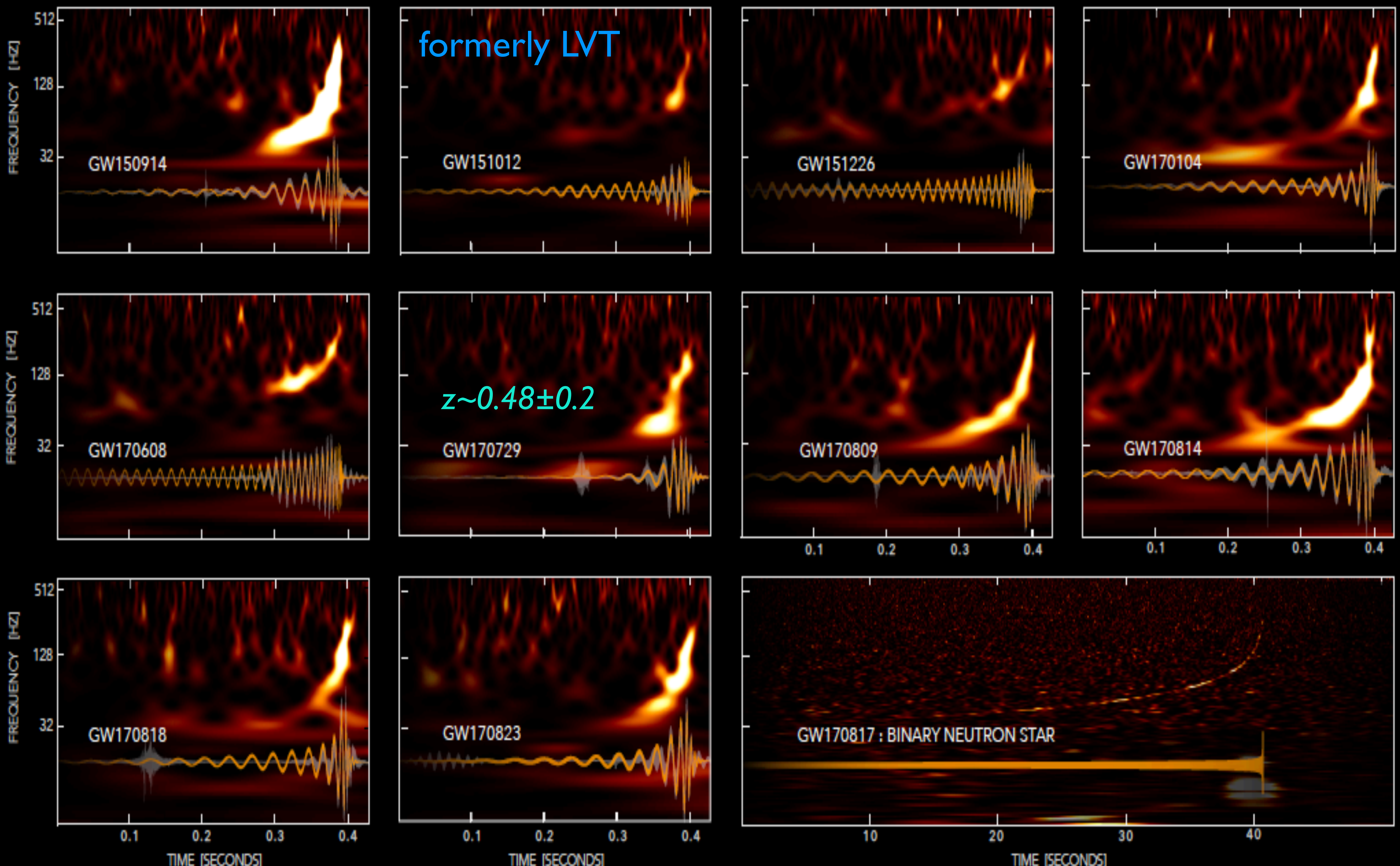
Likelihood function

for stationary Gaussian noise & signal model $h(\theta)$:

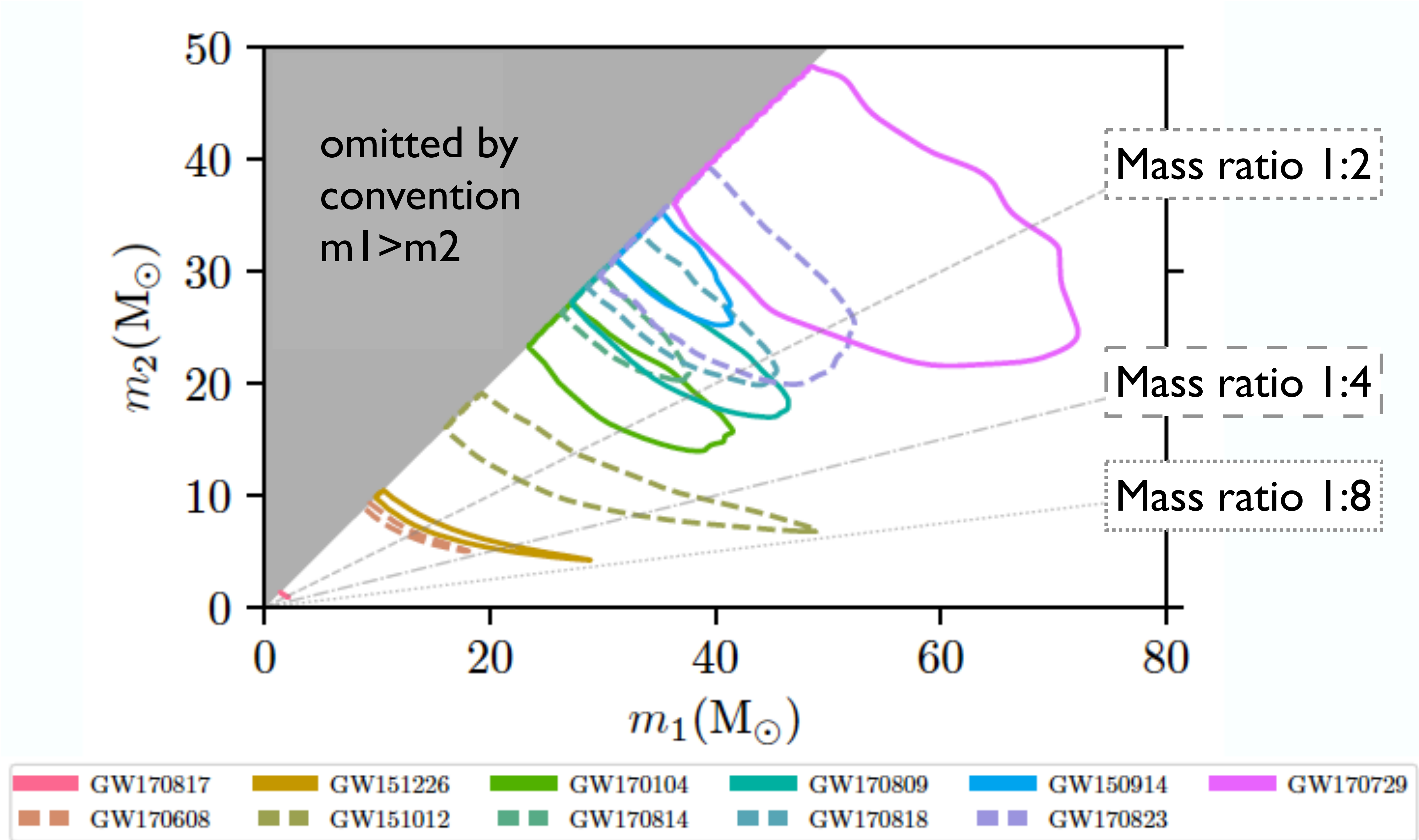
$$\propto \exp\left[-\frac{1}{2}(d-h|d-h)\right]$$

- Numerically sample the posterior distribution
    - LALinference pipelines
    - RIFT / Rapid PE

Gravitational-Wave Transient Catalog-1



results: Masses of observed binaries



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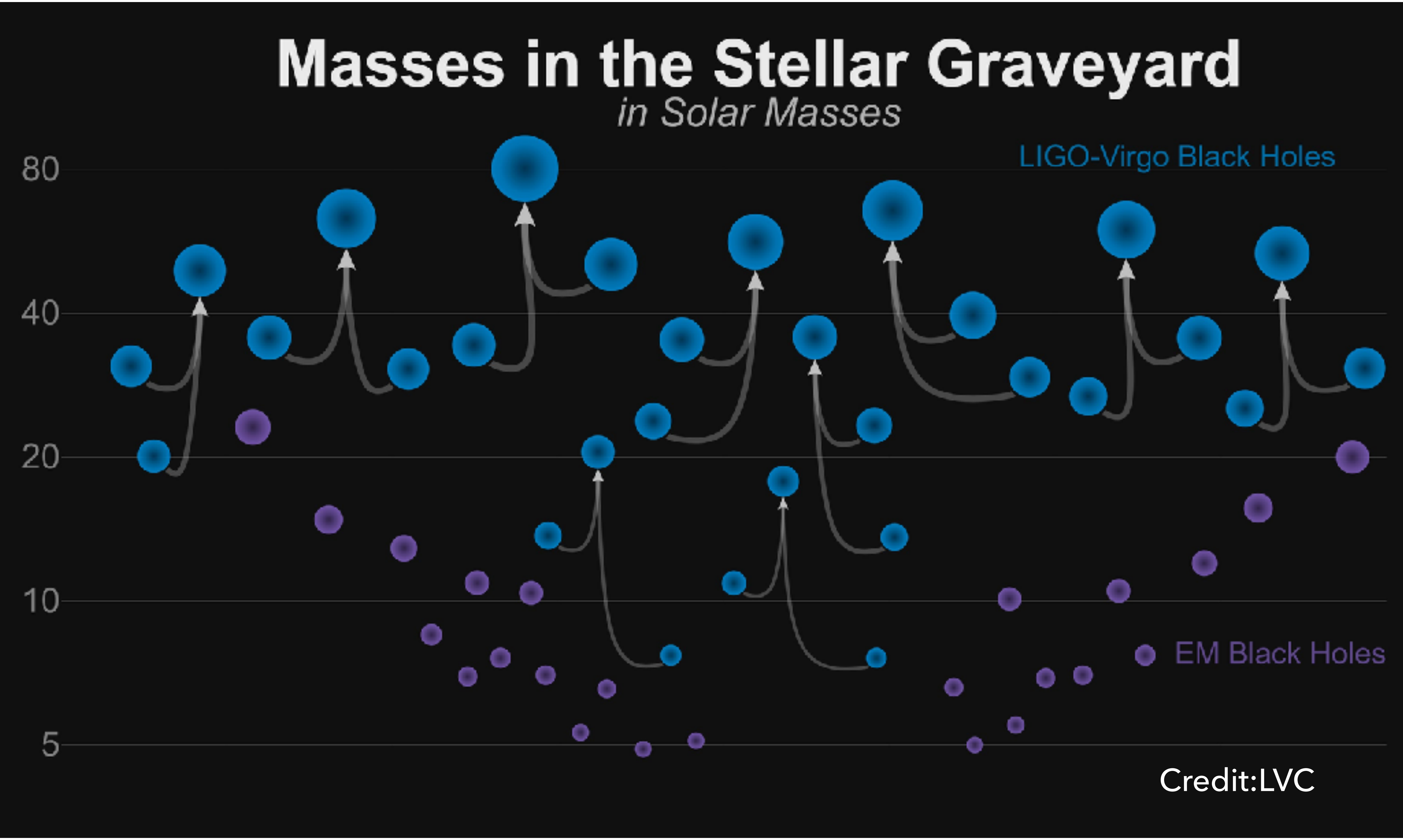
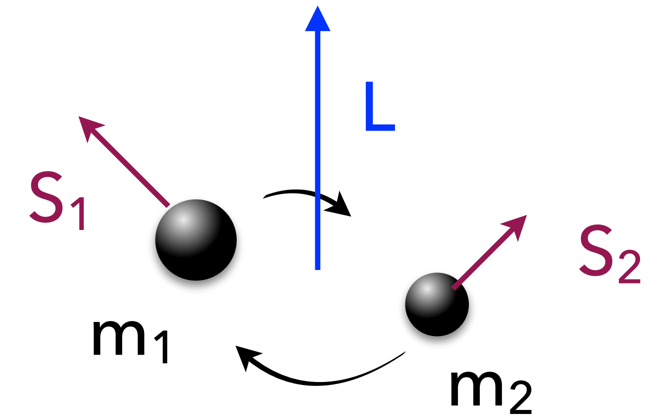
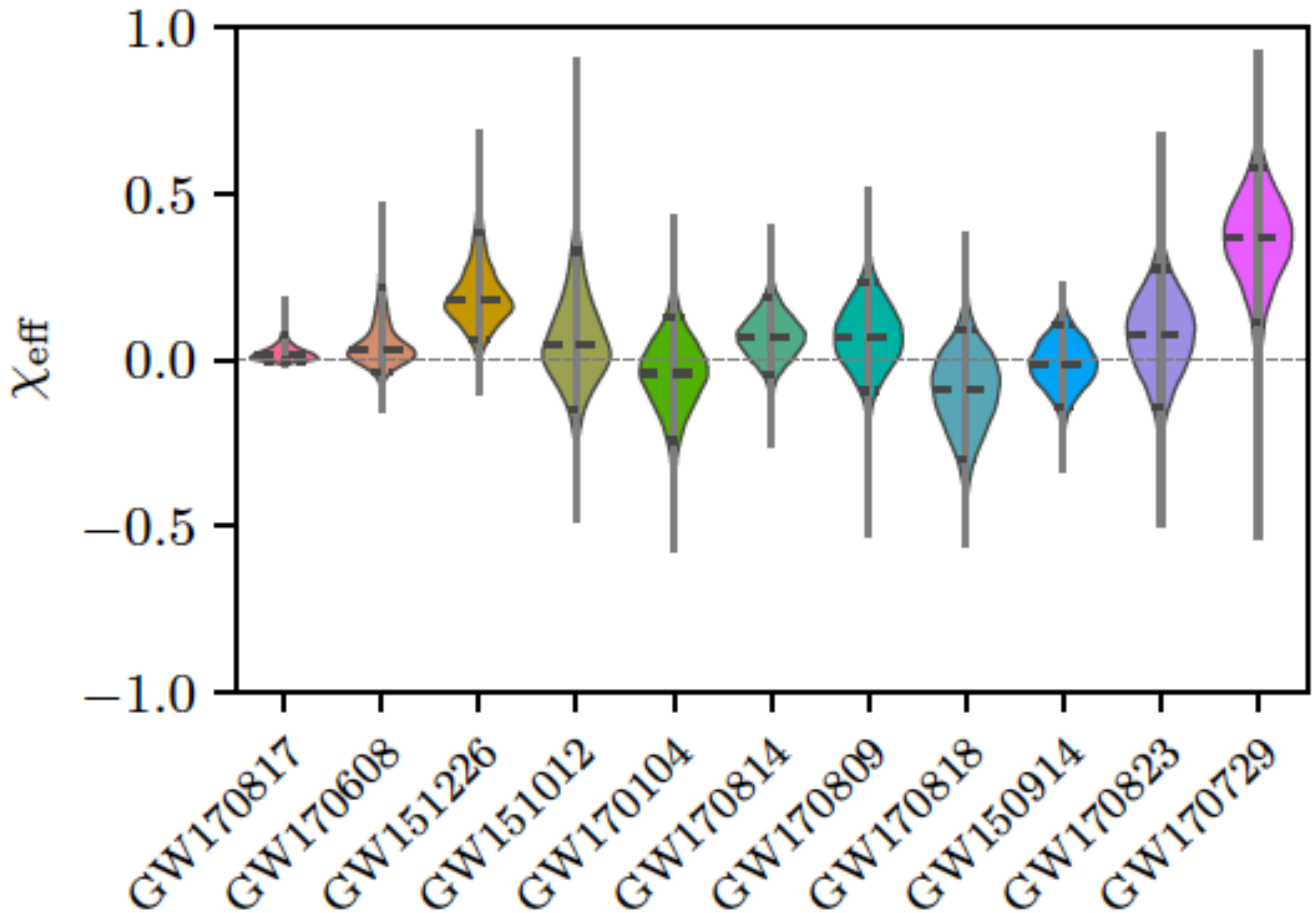

Masses in the Stellar Graveyard
in Solar Masses
LIGO-Virgo Black Holes
EM Black Holes
80
40
20
10
5
Credit:LVC

results: Spins

- Most sensitive to combination of aligned-spin components:

$$\chi_{\rm eff} = \frac{m_1\chi_1 + m_2\chi_2}{m_1 + m_2} \qquad \chi_i = \frac{\vec{S}_i \cdot \hat{L}}{m_I^2}$$





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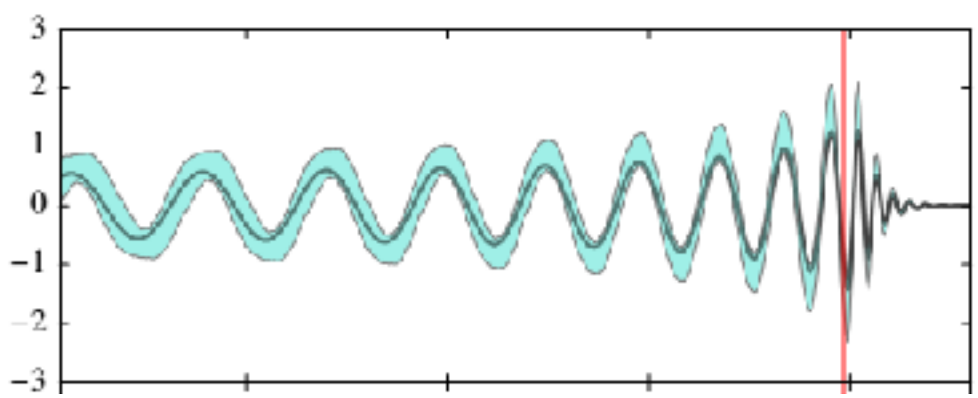
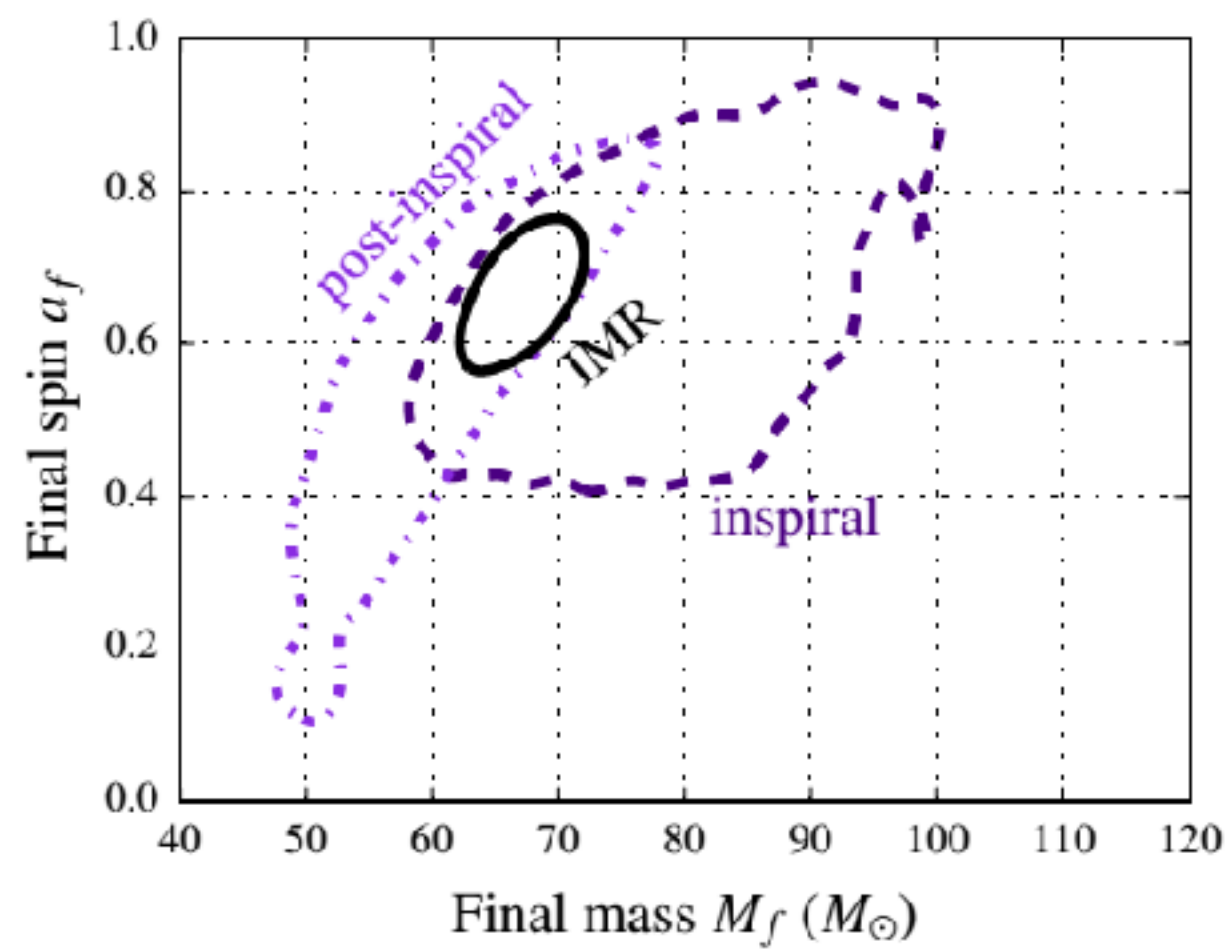
Tests of General Relativity

Several tests of deviations from the GR predictions, example here: GW150914

1. Consistency test of mass and spin estimated before & after merger





2. Pure ringdown of final BH?

Not clear in data, but consistent

Abbott et al., arXiv:1602.03841

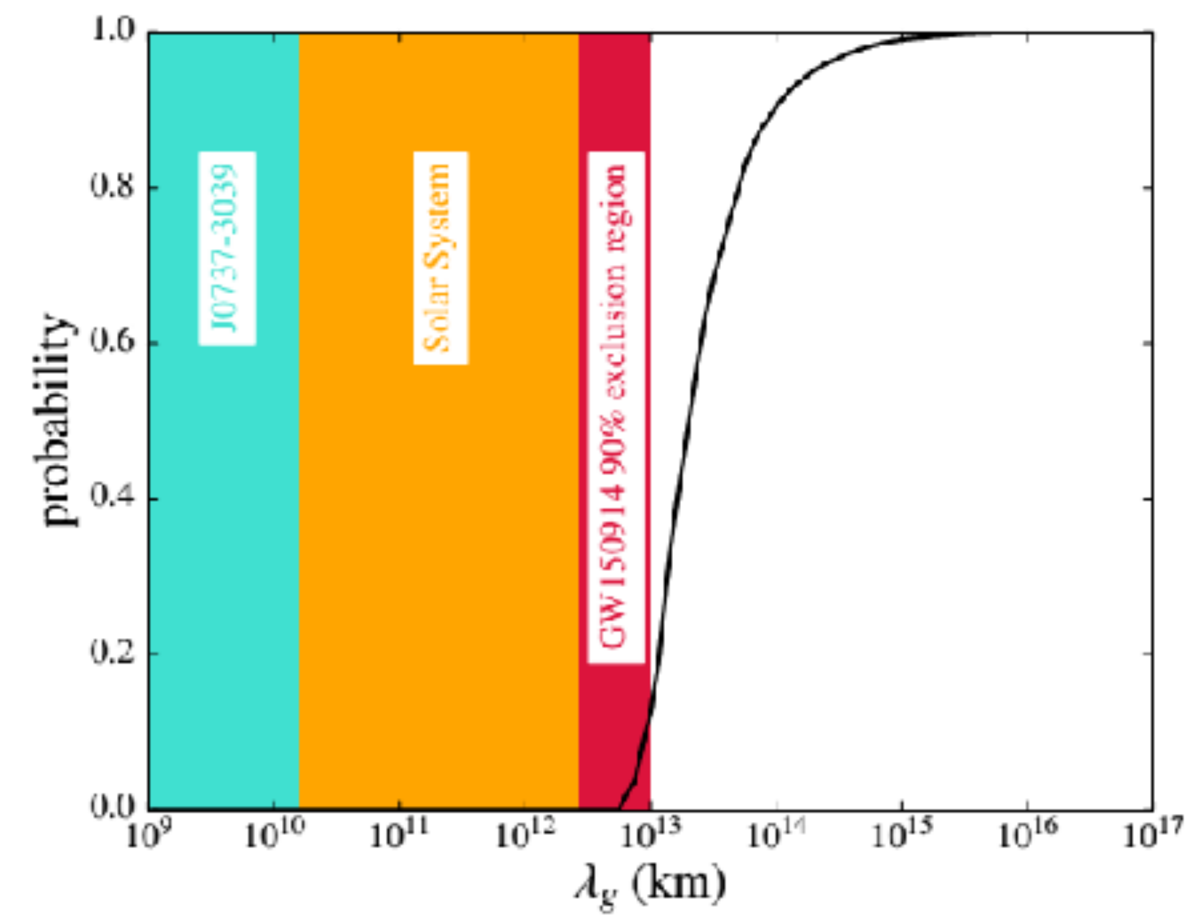
Testing General Relativity II

- parameterized deviations from GR predictions during inspiral phase evolution:

  reasonably consistent with zero

- massive graviton:

  - Would distort waveform due to dispersion
  - Limit on Compton wavelength: > 10^{13}km
  - Implies m_g <1.2 x 10^{-22} eV/c^2



Abbott et al., arXiv:1602.03841

Data and tutorials available at: https://losc.ligo.org/

https://losc.ligo.org/

LIGO

LIGO Open Science Center

LIGO is operated by California Institute of Technology and Massachusetts Institute of Technology and supported by the U.S. National Science Foundation.

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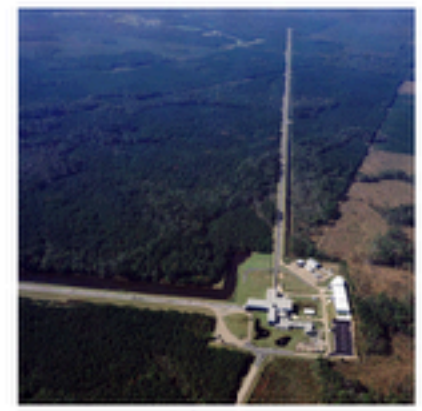
LIGO Hanford Observatory, Washington
(image: C. Gray)



LIGO Livingston Observatory, Louisiana
(image: J. Giaime)

Virgo detector, Italy
(image: Virgo Collaboratio

The LIGO Open Science Center provides data from gravitational-wave observatories, along with access to tutorials and software tools

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