Superstring amplitudes in the CHY formalism

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Introduction

(Super) String Theory as a potential high energy theory

- $10D$ Theory for one-dimensional vibrating objects (open or closed).
- Fermionic d.o.f. obtained through supersymmetry.
- Potentially includes both SM and QG.
- Compactification: $10\ D \rightarrow 4D$.

CHY amplitudes

- Effective way of evaluating tree-level amplitudes for (massless) particles in different theories
- Evaluate diagrammatically or through algorithm
- Diagrammatically: Much lower basis than Feynman diagrams
- Algorithm: Reduces to combinatorial problem

CHY Amplitudes briefly

CHY Integral

$$A_n = \int d\Omega_n \mathbb{I}_L \mathbb{I}_R = \int \frac{\prod_{i=1}^{n} dz_i}{Vol(SL(2,\mathbb{C}))} \delta(E_i) \mathbb{I}_L(z,p,\epsilon) \mathbb{I}_R(z,p,\epsilon) \quad (1)$$

1306.6575, 1307.2199, 1309.0885. Cachazo, He, Yuan.

Different theories : different integrands

Theory	$\mathcal{I}_{\text{left}}$	$\mathcal{I}_{\text{right}}$
Bi adjoint ϕ^3	$\mathtt{PT}(\mathbb{I}_n)$	$\mathtt{PT}(\mathbb{I}_n)$
Yang-Mills	$\mathtt{PT}(\mathbb{I}_n)$	$\mathtt{Pf}$
Einstein gravity	$\mathtt{Pf}$	$\mathtt{Pf}$
non-linear sigma model	$\mathtt{PT}$	$\text{Pf}A$
ϕ^4	$\mathtt{PT}(\mathbb{I}_n)$	$\text{Pf}\ \tilde{A}$

Table: An example of different theories with a CHY integrand.

Yangb-Mills amplitudes in the CHY formalism

$$A_n^{YM}(\beta) = \int d\Omega_n \texttt{PT}(\beta)\texttt{Pf} \quad (2)$$

Parke-Taylor factor

$$\texttt{PT}(\beta) = \frac{1}{z_{1_\beta 2_\beta} z_{2_\beta 3_\beta} \dots z_{n_\beta 1_\beta}} \quad (3)$$

For some ordering β. Pfaffiant Pf incorporates the polarization vector dependence.

Superstring amplitudes in CHY

Superstring amplitudes in CHY

In the two part paper by Mafra, Schlotterer, and Steiberger 1106.2645, 1106.2646, it was shown that the Superstring n-point gluon amplitude could be written as

$$\mathcal{A}_n^{sst}(\beta) = \sum_{\Sigma \in S_{n-3}} A^{SYM}(\Sigma) F_\beta^\Sigma \tag{4}$$

Taking only gluonic final states we can write $SYM \to YM$ and we thus get

$$\mathcal{A}_n^{sst}(\beta) = \sum_{\Sigma \in S_{n-3}} \int d\Omega_n \mathtt{PT}(\Sigma) \mathtt{Pf}\, F_\beta^\Sigma \tag{5}$$

The string-deformed Parke-Taylor factor

It was shown by S. Mizera in 1705.10323

$$\mathtt{PT}_{\alpha'}(\beta) = \sum_{\Sigma \in S_{n-3}} \mathtt{PT}(\Sigma) F_{\beta}^{\Sigma} \tag{6}$$

$$= \sum_{\Sigma \in S_{n-3}} \int_{D(\beta)} d\mu^{\mathsf{KN}} \prod_{a=2}^{n-2} \sum_{b=1}^{a-1} \frac{s_{\Sigma(a)\Sigma(b)}}{z_{\Sigma(a)\Sigma(b)}} \mathtt{PT}(\Sigma)$$

The String Parke-Taylor function

- We can describe any superstring amplitude with gluonic external states using this formula and conventional CHY algorithms.
- How does this behave in the Infinite Tension Limit $\alpha' \to 0$?

F-integrals in the infinite tension limit

KLT decomposition

$$F^{\alpha}(\beta) = \sum_{\gamma \in S_{n-3}} Z^{\alpha}(\beta) \cdot m^{-1}(\beta|\gamma) \tag{7}$$

Z-integrals can be expressed effectively using a scalar bi-adjoint theory with EOM's

$$\Box\Phi = \Phi^2 + {\alpha'}^2 \zeta_2 (\partial^2 \Phi^3 + \Phi^4) + \mathcal{O}({\alpha'}^3) \tag{8}$$

Determine the Wilson coefficients from lower point data to fix higher point data.

Conclusion

- We can find (super) string amplitudes by way of CHY-formalism.
- Introduced gauge invariant string-deformed Parke-Taylor $\mathtt{PT}_{\alpha'}$
  - Can be written effectively in terms of scalar theory
- Shown to work for $n = 4, 5$ and partially for $n = 6$ point functions to order $\mathcal{O}({\alpha'}^2)$.
- Open question: Can we from this construction create a closed formula for the $\mathcal{O}({\alpha'}^k)$- correction on the form:

$$\mathtt{PT}_{\alpha'} = \left(1 + {\alpha'}^2 \sum_{i<j<k<l} s_{jk} s_{li} \frac{z_{ij} z_{kl}}{z_{jk} z_{li}}\right) \mathtt{PT} + \mathcal{O}({\alpha'}^3). \qquad (9)$$