Observational Signature of a Near-extremal Kerr-like Black Hole at the Event Horizon Telescope

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Outline

Introduction

- The Event Horizon Telescope (EHT) will announce the first image of a black hole in 2019. EHT website.
- The observational signature of a near-extremal Kerr black hole (predicted by General Relativity) has been theoretically studied in Gralla, Lupsasca and Strominger, 2017
- GR is supposed to be modified, what about the signatures in alternative gravitational theories?
- I will introduce a generalization to one of these alternative theories: the MOG theory Moffat, 2005 -a modified gravity theory without invoking dark matter.

The Kerr-MOG spacetime

- The rotating solution is given by the Kerr-MOG metric

$$ds^2 = -\frac{\Delta\Sigma}{\Xi}dt^2 + \frac{\Sigma}{\Delta}dr^2 + \Sigma d\theta^2 + \frac{\Xi \sin^2\theta}{\Sigma}(d\phi - \omega dt)^2, \quad (1)$$

where

$$\Delta = r^2 - 2GMr + a^2 + \alpha G_N G M^2, \qquad \Sigma = r^2 + a^2\cos^2\theta, \quad (2)$$

$$\Xi = (r^2 + a^2)^2 - \Delta a^2 \sin^2\theta, \qquad \omega = \frac{a(a^2 + r^2 - \Delta)}{\Xi}, \quad (3)$$

where α is the modified parameter and $G = (1+\alpha)G_N$ is called as an enhanced gravitational constant.

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- For simplicity, we set

$$G_N = 1, \qquad M_\alpha \equiv M_{\mathrm{ADM}} = (1+\alpha)M, \qquad \beta^2 = \frac{\alpha}{1+\alpha}M_\alpha^2, \quad (4)$$

thus

$$\Delta = r^2 - 2M_\alpha r + a^2 + \beta^2. \quad (5)$$

The Kerr-MOG spacetime

- Mass dependent gravitational charge

$$K = \sqrt{\alpha G_N} M. \tag{6}$$

- The event horizon is obtained for $\Delta = 0$,

$$r_{\pm} = M_{\alpha} \pm \sqrt{M_{\alpha}^2 - (a^2 + \beta^2)}. \tag{7}$$

- The extremal condition

$$a^2 + \beta^2 = M_{\alpha}^2. \tag{8}$$

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- Note that the quantities under the square roots of (6) and (7) should be nonnegative, thus we obtain physical bounds on α as

$$0 \leq \alpha \leq \frac{M_\alpha^2}{a^2} - 1. \tag{9}$$

Orbiting emitter

We assume the emitter ("hot spot") is on a circular orbit with radius r_s at the equatorial plane. The angular velocity is

$$\Omega_s = \frac{d\phi}{dt} = \pm \frac{\Gamma(r_s)}{r_s^2 \pm a\Gamma(r_s)}, \tag{10}$$

where

$$\Gamma^2(r) = M_\alpha r - \beta^2. \tag{11}$$

- There are four conserved quantities along each photon trajectory: the invariant mass $\mu^2 = 0$, the total energy E, the angular momentum L and the Carter constant Q.
- Introducing two rescaled quantities,

$$\hat{\lambda} = \frac{L}{E}, \qquad \hat{q} = \frac{\sqrt{Q}}{E}. \tag{12}$$

- Using Hamilton-Jacobi method, we can obtain the equations,

$$\int_{r_s}^{r_o} \frac{dr}{\pm\sqrt{\mathcal{R}(r)}} = \int_{\theta_s}^{\theta_o} \frac{d\theta}{\pm\sqrt{\Theta(\theta)}}, \tag{13}$$

$$\Delta\phi = \phi_o - \phi_s = \int_{r_s}^{r_o} \frac{a(2M_\alpha r - \beta^2 - a\hat{\lambda})}{\pm\Delta\sqrt{\mathcal{R}(r)}} dr + \int_{\theta_s}^{\theta_o} \frac{\hat{\lambda}\csc^2\theta}{\pm\sqrt{\Theta(\theta)}} d\theta, \tag{14}$$

$$\Delta t = t_o - t_s = \int_{r_s}^{r_o} \frac{\left[r^4 + a^2(r^2 + 2M_\alpha r - \beta^2) - a(2M_\alpha r - \beta^2)\hat{\lambda}\right]}{\pm\Delta\sqrt{\mathcal{R}(r)}} dr + \int_{\theta_s}^{\theta_o} \frac{a^2\cos^2\theta}{\pm\sqrt{\Theta(\theta)}} d\theta, \tag{15}$$

where

$$\mathcal{R}(r) = (r^2 + a^2 - a\hat{\lambda})^2 - \Delta\left[\hat{q}^2 + (a - \hat{\lambda})^2\right], \tag{16}$$

$$\Theta(\theta) = \hat{q}^2 + a^2\cos^2\theta - \hat{\lambda}^2\cot^2\theta. \tag{17}$$

- Each photon trajectory can be labeled by a pair of conserved quantities $(\hat{\lambda}, \hat{q})$, which connects the star $(t_s, r_s, \theta_s, \phi_s)$ to an observer $(t_o, r_o, \theta_o, \phi_o)$.
- We decouple the coordinates t_s and ϕ_s by using $\phi_s = \Omega_s t_s$, and made the following choice, $\theta_s = \pi/2$, $r_o \to \infty$ and $\phi_o = 2\pi N$.
- For given choice of r_s and θ_o, solving these equations gives the time-dependent trajectories $[\hat{\lambda}(t_o), \hat{q}(t_o)]$ corresponding to a track of the emitter's image.
- Plugging $\hat{\lambda}$ and $\hat{q}$ into the functions of observables: position $[x(\hat{\lambda}, \hat{q}], y(\hat{\lambda}, \hat{q}))$, redshift $g(\hat{\lambda}, \hat{q})$ and flux $F_o(\hat{\lambda}, \hat{q})$.

- Set $M_\alpha = 1$ and introduce dimensionless coordinate $R = r - 1$,
- The near-extremal condition ($\epsilon \ll 1$)

$$a^2 + \beta^2 = 1 - \epsilon^3, \qquad \beta^2 = \alpha/(1+\alpha). \tag{18}$$

- We use a as modified parameter instead of α (to avoid square root),

$$\alpha = 1/a^2 - 1 + \mathcal{O}(\epsilon^3). \tag{19}$$

- The emitter is located on (or near) ISCO $R_s = \epsilon \bar{R} + \mathcal{O}(\epsilon^2)$, where

$$\bar{R} \geq \bar{R}_{\rm ISCO} = \left(2a^2/(2a^2-1)\right)^{1/3}. \tag{20}$$

- Introducing new quantities λ and q to track the small corrections

$$\hat{\lambda} = \frac{1+a^2}{a}(1-\epsilon\lambda), \qquad \hat{q} = \sqrt{4 - \frac{1}{a^2} - q^2}. \tag{21}$$

Near-extremal solutions

- $r - \theta$ equation: $\int_{r_s}^{r_o} \frac{dr}{\pm\sqrt{\mathcal{R}(r)}} = \int_{\theta_s}^{\theta_o} \frac{d\theta}{\pm\sqrt{\Theta(\theta)}}$,
  - Introducing a separation scaling of ϵ^p ($\epsilon \ll \epsilon^p \ll 1$) with $p \in (0,1)$ and split the radial integral into two pieces (set $M_\alpha = 1$),

$$I_r = \int_{\epsilon \bar{R}}^{\epsilon^p C} \frac{dR}{\sqrt{\mathcal{R}}} + \int_{\epsilon^p C}^{R_o} \frac{dR}{\sqrt{\mathcal{R}}}. \tag{22}$$

  - Performing the radial integral by using matched asymptotic expansion method.
  - The angular integral is given by elliptic integral.
  - From the $r - \theta$ equation, we can write λ in terms of q, i.e., we get a function $\lambda(q)$.

Near-extremal solutions

- Δt and $\Delta\phi$ equation: $\Delta\phi - \Omega_s \Delta t = -\Omega_s t_o + \phi_o$ (set $\phi_o = 2\pi N$),
  - We introduce a dimensionless time coordinate $\hat{t}_o$,

$$\hat{t}_o = \frac{t_o}{T_s} = \frac{t_o \Omega_s}{2\pi} = \frac{a t_o}{2(1+a^2)\pi M_\alpha} + \mathcal{O}(\epsilon).$$

  - Plugging $\lambda(q)$ into this equation gives functions of $q(\hat{t}_o)$ and $\lambda(\hat{t}_o)$.

Observational quantities in terms of λ and q:

- The image position

$$x = -\frac{1+a^2}{a}\frac{1}{\sin\theta_o}, \tag{23}$$

$$y = s\sqrt{4 - \frac{1}{a^2} - q^2 + a^2\cos^2\theta_o - \frac{(1+a^2)^2}{a^2}\cot^2\theta_o}. \tag{24}$$

- The image redshift

$$g = \frac{1}{\frac{\sqrt{4a^2-1}}{a} + \frac{2a(1+a^2)}{\sqrt{4a^2-1}}\frac{\lambda}{\bar{R}}}. \tag{25}$$

Observational quantities in terms of λ and q:

- The image flux (relative to the comparable "Newtonian flux") Cunningham & Bardeen, 1972. is given by

$$\frac{F_o}{F_N} = \frac{\sqrt{4a^2 - 1}\epsilon\bar{R}}{2a^2 D_s} \frac{qg^3}{\sqrt{4 - \frac{1}{a^2} - q^2}\sqrt{\Theta_0(\theta_o)}\sin\theta_o} \left|\det\frac{\partial(B,A)}{\partial(\lambda,q)}\right|^{-1},$$

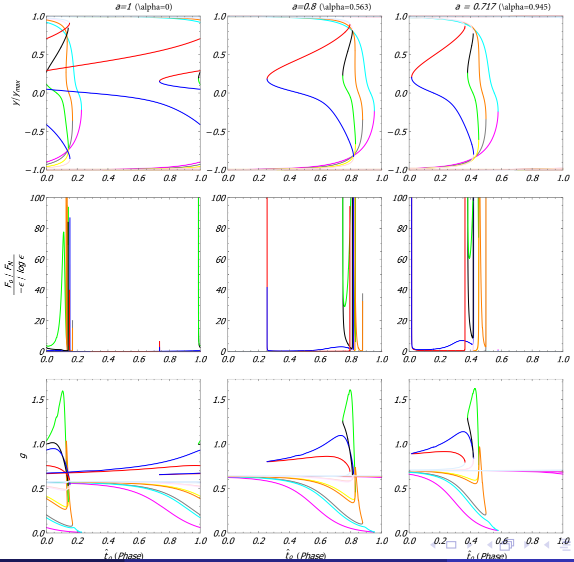
where

$$D_s = \sqrt{q^2\bar{R}^2 + 4(1+a^2)\lambda\bar{R} + (1+a^2)^2\lambda^2}, \quad (26)$$

and A and B are functions associated with the trajectory equations.

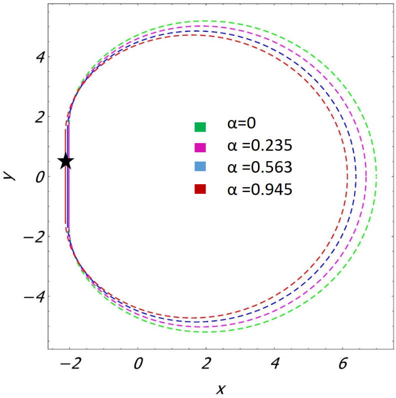
Observational appearance

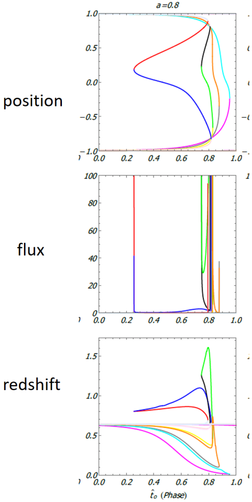
Making a choice of the modified parameter α and parameters ϵ, $\bar{R}$, R_o, θ_o.



Observational appearance

The entire image at EHT





Summary

- We study the observaitonal signature of a near-extremal Kerr-like black hole in the modified gravity theory (MOG), in particular, we study the optical appearance of an emitter orbiting near the BH.
- There are typical signatures away from the Kerr case which may be tested by the Event Horizon Telescope (EHT).
- Outlook: black hole surroundings, such as plasma and accretion disk.
- ...

Thank you for your attention!