Consistent use of Effective potentials
1810.01416

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Saturday January 5th, 2019

Outline

Effective Potential

Definition

$$V_{\text{eff}}(\phi) = V_0(\phi) + \hbar V_1(\phi) + \hbar^2 V_2(\phi) \ldots$$

1-Loop

$$V_1(\phi) \sim \frac{1}{2} \sum_{\text{d.o.f}} \left(\int \omega_B(\phi) - \int \omega_F(\phi) \right),$$

$$\omega = \sqrt{k^2 + m^2(\phi)}$$

$$V_1(\phi) \sim \sum m^4(\phi) \log(m^2(\phi))$$

Outline

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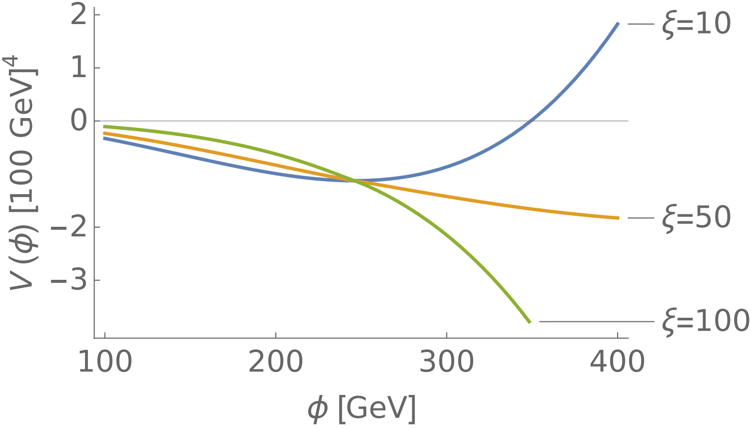
Gauge dependence



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Nielsen identity

Gauge dependence

$$\left(\xi\frac{\partial}{\partial\xi}+C(\phi,\xi)\frac{\partial}{\partial\phi}\right)V_{\text{eff}}=0$$

$$\xi\frac{\partial}{\partial\xi}\phi^{\text{min}}(\xi)=C(\phi^{\text{min}},\xi)$$

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Effective potential

Gauge Dependence

IR divergences

Perturbative expansion

$$\frac{\partial}{\partial \phi} V_{\text{eff}}(\phi, \xi)|_{\phi=\phi^{\text{min}}} = 0$$

$$V_{\text{eff}}(\phi, \xi) = V_0(\phi) + \hbar V_1(\phi, \xi) + \hbar^2 V_2(\phi, \xi) + \ldots$$
$$\phi^{\text{min}} = \phi_0 + \hbar\, \phi_1 + \hbar^2\, \phi_2 + \ldots$$
$$\partial V_0|_{\phi=\phi_0} = 0,\ \phi_1 = -\frac{\partial V_1}{\partial^2 V_0}|_{\phi=\phi_0}, \ldots$$

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"$\hbar$"-expansion

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Minimum

$$\left(V_0(\phi)+\hbar V_1(\phi,\xi)+\hbar^2 V_2(\phi,\xi)+\dots\right)_{\phi^{min}=\phi_0+\hbar\phi_1+\dots}$$

Potential

$$V_{\text{eff}}|_{\phi^{\text{min}}}=\left[V_0+\hbar V_1+\hbar^2\left(V_2+\phi_1\partial V_1+\frac{\phi_1^2}{2}\partial^2 V_0\right)+\dots\right]_{\phi=\phi_0}$$

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IR divergences

IR divergences

$$G(\phi)|_{\phi=\phi_0} = 0,$$
$$V_3(\phi) \sim \log G(\phi),$$
$$V_4(\phi) \sim \frac{1}{G(\phi)}$$

At the L^{th} loop level

$$V_L(\phi) \sim G^{3-L}(\phi) + \dots$$

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Effective potential

Gauge Dependence

IR divergences

IR Divergence Cancellation

L^{th} order

$$V|_{\phi=\phi^{min}} = \dots \hbar^L \left(V_L + \phi_1 \partial V_{L-1} + \frac{\phi_1^2}{2!} \partial^2 V_{L-2} + \dots \right) + \dots$$

$$V_L|_{\phi\approx\phi_0} \sim G^{3-L}(\phi)$$

$$\partial^n V_L|_{\phi\approx\phi_0} \sim G^{3-L-n}(\phi)$$

$V(\phi,\xi)|_{\phi=\phi^{min}}$ is Finite & Gauge invariant order by order