Topics in Effective Field Theory

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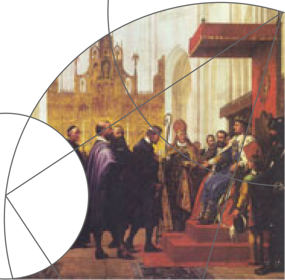

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Outline

Based on [arXiv:1711.07954] and [arXiv:1812.02991] with M. Trott and [arXiv:1803.08001] with M. Paraskevas and M. Trott.

1. Standard Model Effective Field Theory (SMEFT)
2. Gauge fixing
3. Renormalization of the vector current

Standard Model

The Standard Model is

$$\begin{aligned}\mathcal{L}_{\mathrm{SM}} =& -\frac{1}{4}G^A_{\mu\nu}G^{A\mu\nu} - \frac{1}{4}W^I_{\mu\nu}W^{I\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \overline{\psi}i\not{D}\psi \\ &+ (D_\mu H)^\dagger (D^\mu H) - \lambda\left(H^\dagger H - \frac{1}{2}v^2\right)^2 \\ &+ \left[H^{\dagger j}\overline{d}Y_d q_j + \tilde{H}^{\dagger j}\overline{u}Y_u q_j + H^{\dagger j}\overline{e}Y_e \ell_j + \mathrm{h.c.}\right] \end{aligned} \quad (1)$$

It works well, but it is not the final theory of Nature.
What should we do about physics Beyond the Standard Model?

To HEFT or to SMEFT, that is the question

We should state our assumptions clearly!
The effective field theory approach assumes that there are no new particles at the electroweak scale (sterile neutrinos?).
In addition, the two EFTs differ:

- The SMEFT assumes the Higgs boson is part of an SU(2) doublet.
- The HEFT does not assume that the Higgs boson is part of an SU(2) doublet.

SM $\subset$ SMEFT $\subset$ HEFT

Standard Model Effective Field Theory

Using the fields and symmetries of the Standard Model (SM), we add higher dimensional operators

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{SM} + \mathcal{L}^{(5)} + \mathcal{L}^{(6)} + \mathcal{L}^{(7)} + \dots \tag{2}$$

where

$$\mathcal{L}^{(d)} = \sum_{i=1}^{n_d} \frac{C_i^{(d)}}{\Lambda^{d-4}} Q_i^{(d)} \qquad \text{for } d > 4. \tag{3}$$

$C_i^{(d)}$: Wilson coefficient
$Q_i^{(d)}$: Operator with mass dimension d
Λ: Scale of New Physics

Dimension 5 operators - Weinberg operator

At dimension 5, we only have one operator [Weinberg, PRL **43**, 1566 (1979)]

$$\mathcal{L}^{(5)} = C^{(5)}_{\alpha\beta}(\overline{\ell}^{C}_{\alpha L}\tilde{H}^{*})(\tilde{H}^{\dagger}\ell_{\beta L}) + \text{h.c.} \tag{4}$$

It generates Majorana mass for left-handed neutrinos.
Not relevant for us.

Dimension 6 operators - Warsaw basis

Warsaw basis [1008.4884 Grzadkowski, Iskrzynski, Misiak, Rosiek]

$1: X^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$
Q_W	$\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$
$Q_{\widetilde{W}}$	$\epsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$

$2: H^6$	
Q_H	$(H^\dagger H)^3$

$3: H^4 D^2$	
$Q_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
Q_{HD}	$(H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$

$5: \psi^2 H^3 +$	
Q_{eH}	$(H^\dagger H)(\bar{l}_p e_r H)$
Q_{uH}	$(H^\dagger H)(\bar{q}_p u_r \widetilde{H})$
Q_{dH}	$(H^\dagger H)(\bar{q}_p d_r H)$

$4: X^2 H^2$	
Q_{HG}	$H^\dagger H\, G_{\mu\nu}^A G^{A\mu\nu}$
$Q_{H\widetilde{G}}$	$H^\dagger H\, \widetilde{G}_{\mu\nu}^A G^{A\mu\nu}$
Q_{HW}	$H^\dagger H\, W_{\mu\nu}^I W^{I\mu\nu}$
$Q_{H\widetilde{W}}$	$H^\dagger H\, \widetilde{W}_{\mu\nu}^I W^{I\mu\nu}$
Q_{HB}	$H^\dagger H\, B_{\mu\nu} B^{\mu\nu}$
$Q_{H\widetilde{B}}$	$H^\dagger H\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$
Q_{HWB}	$H^\dagger \tau^I H\, W_{\mu\nu}^I B^{\mu\nu}$
$Q_{H\widetilde{W}B}$	$H^\dagger \tau^I H\, \widetilde{W}_{\mu\nu}^I B^{\mu\nu}$

$6: \psi^2 X H +$	
Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r)\tau^I H W_{\mu\nu}^I$
Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) H B_{\mu\nu}$
Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r)\widetilde{H}\, G_{\mu\nu}^A$
Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\tau^I \widetilde{H}\, W_{\mu\nu}^I$
Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r)\widetilde{H}\, B_{\mu\nu}$
Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) H\, G_{\mu\nu}^A$
Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r)\tau^I H\, W_{\mu\nu}^I$
Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) H\, B_{\mu\nu}$

$7: \psi^2 H^2 D$	
$Q_{Hl}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{l}_p \gamma^\mu l_r)$
$Q_{Hl}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$
Q_{He}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$
$Q_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$
$Q_{Hq}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$
Q_{Hu}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$
Q_{Hd}	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
Q_{Hud} + h.c.	$i(\widetilde{H}^\dagger D_\mu H)(\bar{u}_p \gamma^\mu d_r)$

Dimension 6 operators - Warsaw basis

Four fermion operators [1008.4884 Grzadkowski, Iskrzynski, Misiak, Rosiek]

$8:(\bar{L}L)(\bar{L}L)$	
Q_{ll}	$(\bar{l}_p\gamma^\mu l_r)(\bar{l}_s\gamma_\mu l_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{q}_s\gamma_\mu q_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma^\mu\tau^I q_r)(\bar{q}_s\gamma_\mu\tau^I q_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma^\mu l_r)(\bar{q}_s\gamma_\mu q_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma^\mu\tau^I l_r)(\bar{q}_s\gamma_\mu\tau^I q_t)$

$8:(\bar{R}R)(\bar{R}R)$	
Q_{ee}	$(\bar{e}_p\gamma^\mu e_r)(\bar{e}_s\gamma_\mu e_t)$
Q_{uu}	$(\bar{u}_p\gamma^\mu u_r)(\bar{u}_s\gamma_\mu u_t)$
Q_{dd}	$(\bar{d}_p\gamma^\mu d_r)(\bar{d}_s\gamma_\mu d_t)$
Q_{eu}	$(\bar{e}_p\gamma^\mu e_r)(\bar{u}_s\gamma_\mu u_t)$
Q_{ed}	$(\bar{e}_p\gamma^\mu e_r)(\bar{d}_s\gamma_\mu d_t)$
$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma^\mu u_r)(\bar{d}_s\gamma_\mu d_t)$
$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma^\mu T^A u_r)(\bar{d}_s\gamma_\mu T^A d_t)$

$8:(\bar{L}L)(\bar{R}R)$	
Q_{le}	$(\bar{l}_p\gamma^\mu l_r)(\bar{e}_s\gamma_\mu e_t)$
Q_{lu}	$(\bar{l}_p\gamma^\mu l_r)(\bar{u}_s\gamma_\mu u_t)$
Q_{ld}	$(\bar{l}_p\gamma^\mu l_r)(\bar{d}_s\gamma_\mu d_t)$
Q_{qe}	$(\bar{q}_p\gamma^\mu q_r)(\bar{e}_s\gamma_\mu e_t)$
$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{u}_s\gamma_\mu u_t)$
$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{u}_s\gamma_\mu T^A u_t)$
$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma^\mu q_r)(\bar{d}_s\gamma_\mu d_t)$
$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma^\mu T^A q_r)(\bar{d}_s\gamma_\mu T^A d_t)$

$8:(\bar{L}R)(\bar{R}L)+$	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_{tj})$

$8:(\bar{L}R)(\bar{L}R)+$	
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\epsilon_{jk}(\bar{q}_s^k d_t)$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\epsilon_{jk}(\bar{q}_s^k T^A d_t)$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\epsilon_{jk}(\bar{q}_s^k u_t)$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$

$\Delta B = \Delta L = 1+$	
Q_{duql}	$\epsilon^{\alpha\beta\gamma}\epsilon^{ij}(d_{\alpha p}^T C u_{\beta r})(q_{\gamma is}^T C l_{jt})$
Q_{qque}	$\epsilon^{\alpha\beta\gamma}\epsilon^{ij}(q_{\alpha ip}^T C q_{\beta jr})(u_{\gamma s}^T C e_t)$
Q_{qqql}	$\epsilon^{\alpha\beta\gamma}\epsilon^{i\ell}\epsilon^{jk}(q_{\alpha ip}^T C q_{\beta jr})(q_{\gamma ks}^T C l_{\ell t})$
Q_{duue}	$\epsilon^{\alpha\beta\gamma}(d_{\alpha p}^T C u_{\beta r})(u_{\gamma s}^T C e_t)$

Problems with gauge fixing the SMEFT

How should one gauge fix the Standard Model Effective Field Theory?
We have the same number of degrees of freedom as in the Standard Model. Thus, one could impose a similar gauge fixing presciption as in the Standard Model.
We want to

- cancel Goldstone-gauge boson bilinear mixing
- cancel A-Z mixing present at tree level
- use the background field method

However, using the normal Standard Model gauge fixing procedure leads to [1505.02646 Hartmann, Trott]

$$\frac{c_W s_W}{\xi_B \xi_W}(\xi_B - \xi_W)(\partial^\mu A_\mu \partial^\nu Z_\nu) - \frac{C_{HWB} v^2 (s_w^2 - c_W^2)(s_W^2 \xi_B + c_W^2 \xi_W)}{\xi_B \xi_W}(\partial^\mu A_\mu \partial^\nu Z_\nu) + \ldots \quad (5)$$

The A-Z mixing doesn't cancel for $\xi_W = \xi_B$.

Geometry of scalar field space

The bilinear field interactions can be thought of in terms of connections on the field space manifold [1511.00724 1605.03602 Alonso, Jenkins, Manohar].
Consider [1803.08001 AH, Paraskevas, Trott]

$$\begin{aligned}\mathcal{L}_{\text{scalar,kin}} =& \left(D_\mu H\right)^\dagger \left(D^\mu H\right) + \frac{C_{H\Box}}{\Lambda^2}\left(H^\dagger H\right)\Box\left(H^\dagger H\right) \\ &+ \frac{C_{HD}}{\Lambda^2}\left(H^\dagger D_\mu H\right)^*\left(H^\dagger D^\mu H\right) \\ =& \frac{1}{2} h_{IJ}(\phi)\left(D_\mu \phi\right)^I \left(D^\mu \phi\right)^J\end{aligned} \tag{6}$$

where $I, J \in \{1, \ldots, 4\}$ and

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix}\phi_2 + i\phi_1 \\ \phi_4 - i\phi_3\end{bmatrix}. \tag{7}$$

Metric of the scalar field manifold

The metric is non-trivial

$$h_{IJ}(\phi) = \delta_{IJ} - 2\frac{C_{H\Box}}{\Lambda^2}\phi_I\phi_J + \frac{1}{2}\frac{C_{HD}}{\Lambda^2}f_{IJ}(\phi), \tag{8}$$

where

$$f_{IJ}(\phi) = \begin{bmatrix} a & 0 & d & c \\ 0 & a & c & -d \\ d & c & b & 0 \\ c & -d & 0 & b \end{bmatrix}, \quad \begin{aligned} a &= \phi_1^2 + \phi_2^2 \\ b &= \phi_3^2 + \phi_4^2 \\ c &= \phi_1\phi_4 + \phi_2\phi_3, \\ d &= \phi_1\phi_3 - \phi_2\phi_4. \end{aligned} \tag{9}$$

- The Riemann curvature tensor calculated from the scalar field metric is non-vanishing. The scalar manifold is curved due to the power counting expansion.
- Field redefinitions cannot turn the metric into a trivial form.
- Physical quantities depend on field redefinition invariant quantities.

Geometry of gauge field space

Analogously, we can describe the kinetic part of the gauge fields in terms of connections on the field space manifold

$$\begin{aligned}\mathcal{L}_{\text{gauge,kin}} =& -\frac{1}{4}W^a_{\mu\nu}W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \frac{C_{HB}}{\Lambda^2}H^\dagger H B_{\mu\nu}B^{\mu\nu} \\ &+ \frac{C_{HW}}{\Lambda^2}H^\dagger H W^a_{\mu\nu}W^{a,\mu\nu} + \frac{C_{HWB}}{\Lambda^2}H^\dagger\sigma^a H W^a_{\mu\nu}B^{\mu\nu} \\ =& -\frac{1}{4}g_{AB}(H)W^A_{\mu\nu}W^{B,\mu\nu}, \qquad A, B = 1, \dots, 4,\end{aligned} \tag{10}$$

where

$$\begin{aligned} g_{ab} &= \left(1 - 4\frac{C_{HW}}{\Lambda^2}H^\dagger H\right)\delta_{ab}, \qquad g_{44} = 1 - 4\frac{C_{HB}}{\Lambda^2}H^\dagger H, \\ g_{a4} &= g_{4a} = -2\frac{C_{HWB}}{\Lambda^2}H^\dagger\sigma_a H, \qquad a = 1, 2, 3. \end{aligned} \tag{11}$$

The gauge field manifold is curved.

Background field method

- The background field method splits fields into background and quantum fields $F \to \hat{F} + F$.
  $\hat{F}$: background field
  F: quantum field
- The background field method provides technical simplifications due to the background field gauge invariance being preserved and the resulting Ward identities.
- The Standard Model was formulated using the background field method [9410338 Denner, Dittmaier, Weiglein]

Real representation of the scalar field

We cannot use the Pauli matrix representation when we have the ϕ^I fields. We use the real representation

$$\gamma^I_{1,J} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \quad \gamma^I_{2,J} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$\gamma^I_{3,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \gamma^I_{4,J} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}. \tag{12}$$

We have that

$$[\gamma_a, \gamma_b] = 2\epsilon^c_{ab}\gamma_c, \qquad \tilde{\gamma}_A = \begin{cases} g_2\gamma_A & \text{for } A = 1,2,3 \\ g_1\gamma_A & \text{for } A = 4, \end{cases}$$

$$[\gamma_a, \gamma_4] = 0, \qquad \tilde{\epsilon}^A_{BC} = g_2\epsilon^A_{BC}. \tag{13}$$

Gauge fixing the Standard Model Effective Field Theory

A gauge fixing choice which preserves the geometric structure of the theory is

$$\mathcal{L}_{\mathrm{GF}} = -\frac{\hat{g}_{AB}}{2\xi}\mathcal{G}^A\mathcal{G}^B,$$
$$\mathcal{G}^X \equiv \partial_\mu W^{X,\mu} - \tilde{\epsilon}^X_{CD}\hat{W}^C_\mu W^{D,\mu} + \frac{\xi}{2}\hat{g}^{XC}\phi^I\hat{h}_{IK}\tilde{\gamma}^K_{C,J}\hat{\phi}^J. \quad (14)$$

Background field gauge invariance is preserved.

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Background field gauge invariance is preserved.

Background field gauge transformations

It is useful to note the following background field gauge transformations ($\delta\hat{F}$), with infinitesimal local gauge parameters $\delta\hat{\alpha}_A(x)$ when verifying the explicitly the background field gauge invariance of this expression

$$\begin{aligned}
\delta\,\hat{\phi}^I &= -\delta\hat{\alpha}^A\,\frac{\tilde{\gamma}^I_{A,J}}{2}\hat{\phi}^J,\\
\delta\,(D^\mu\hat{\phi})^I &= -\delta\hat{\alpha}^A\,\frac{\tilde{\gamma}^I_{A,J}}{2}(D^\mu\hat{\phi})^J,\\
\delta\hat{\mathcal{W}}^{A,\mu} &= -\partial^\mu(\delta\hat{\alpha}^A) - \tilde{\epsilon}^A{}_{BC}\,\delta\hat{\alpha}^B\,\hat{\mathcal{W}}^{C,\mu},\\
\delta\hat{h}_{IJ} &= \hat{h}_{KJ}\,\frac{\delta\hat{\alpha}^A\,\tilde{\gamma}^K_{A,I}}{2} + \hat{h}_{IK}\,\frac{\delta\hat{\alpha}^A\,\tilde{\gamma}^K_{A,J}}{2},\\
\delta\hat{\mathcal{W}}^A_{\mu\nu} &= -\tilde{\epsilon}^A{}_{BC}\,\delta\hat{\alpha}^B\,\hat{\mathcal{W}}^C_{\mu\nu},\\
\delta\hat{g}_{AB} &= \hat{g}_{CB}\,\tilde{\epsilon}^C{}_{DA}\,\delta\hat{\alpha}^D + \hat{g}_{AC}\,\tilde{\epsilon}^C{}_{DB}\,\delta\hat{\alpha}^D.
\end{aligned} \tag{15}$$

The background field gauge invariance is established by using these transformations in conjuction with a linear change of variables on the quantum fields

$$\mathcal{W}^{A,\mu} \to \mathcal{W}^{A,\mu} - \tilde{\epsilon}^{A}_{\ BC}\, \delta\hat{\alpha}^{B}\, \mathcal{W}^{C,\mu},$$
$$\phi^{I} \to \phi^{I} - \frac{\delta\hat{\alpha}^{B}\, \tilde{\gamma}^{I}_{B,K}}{2}\phi^{K}. \quad (16)$$

The transformation of the gauge fixing term is

$$\delta\mathcal{G}^{X} = -\tilde{\epsilon}^{X}_{\ AB}\, \delta\hat{\alpha}^{A}\mathcal{G}^{B}. \quad (17)$$

With these transformations, the background field gauge invariance of the gauge fixing term is directly established.

Ghost term

The quantum fields gauge transformations are

$$\Delta\mathcal{W}^A_\mu = -\partial_\mu \Delta\alpha^A - \tilde{\epsilon}^A{}_{BC}\,\Delta\alpha^B\,(\mathcal{W}^C_\mu + \hat{\mathcal{W}}^C_\mu),$$
$$\Delta\phi^I = -\Delta\alpha^A\,\frac{\tilde{\gamma}^I_{A,J}}{2}\,(\phi^J + \hat{\phi}^J). \qquad (18)$$

As the hatted field metrics depend only on the background fields and do not transform under quantum field gauge transformations, the Faddeev-Popov ghost term still follows directly; we find

$$\begin{aligned}\mathcal{L}_{\mathrm{FP}} = &- \hat{g}_{AB}\bar{u}^B \Big[-\partial^2\delta^A_C - \overleftarrow{\partial}_\mu \tilde{\epsilon}^A{}_{DC}(\mathcal{W}^{D,\mu} + \hat{\mathcal{W}}^{D,\mu}) \\ &+ \tilde{\epsilon}^A{}_{DC}\hat{\mathcal{W}}^D_\mu \overrightarrow{\partial}^\mu - \tilde{\epsilon}^A{}_{DE}\tilde{\epsilon}^E{}_{FC}\hat{\mathcal{W}}^D_\mu(\mathcal{W}^{F,\mu} + \hat{\mathcal{W}}^{F,\mu}) \\ &- \frac{\xi}{4}\hat{g}^{AD}(\phi^J + \hat{\phi}^J)\tilde{\gamma}^I_{C,J}\,\hat{h}_{IK}\,\tilde{\gamma}^K_{D,L}\,\hat{\phi}^L)\Big]\,u^C. \qquad (19)\end{aligned}$$

Summary

- We have showed how to gauge fix the Standard Model Effective Field Theory preserving the background field gauge invariance.
- This approach can be directly generalized to higher orders in the power counting expansion.

The key point

We gauge fix the fields on the curved field space due to the power counting expansion.

Electromagnetic vector current is renormalized

Gauss's law relates the time component of the electromagnetic vector current $J^\mu = \overline{\psi}_e \gamma^\mu \psi_e$ to

$$J^0 = \frac{\nabla \cdot \mathbf{E}}{-e_{phys}}. \quad (20)$$

Here $e_{phys} = 1.6021766208(98) \times 10^{-19}\ C$ is the electron charge in the usual SI units.

Divergence counter-term with higher dimensional operator

The QED case was discussed in [0512187 Collins, Manohar, Wise]. The first higher-dimensional operator we can add to QED is a dipole operator [1812.02991 AH, Trott]

$$\mathcal{L}_5 = C_{e\gamma} \left(\overline{e}_L \sigma^{\nu\mu} e_R\right) F_{\mu\nu} + \mathrm{h.c.} \tag{21}$$

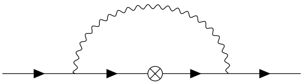
We include this operator in the discussion of the renormalization of the vector current.

Divergence counter-term

Usual textbook arguments conclude that

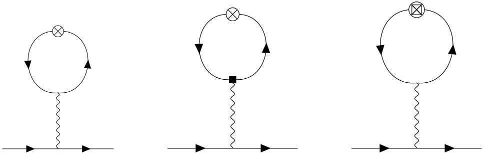
$$\mu \frac{d}{d\mu} J_N^{\nu} = 0, \tag{22}$$

consistent with the current being conserved. This corresponds to the diagram not giving any new divergence structures

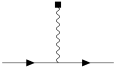


Divergence counter-term

However, the following diagrams are divergent



These divergences are cancelled by a counter-term of the form $\partial^\nu F_{\nu\mu}$, shown in the following diagram



The four-divergence of the operator vanishes identically.

Derivation using the equation of motion

Vary the Lagrangian with respect to the gauge field

$$
\begin{aligned}
0 =& \frac{\delta S}{\delta A_\mu} = e\mu^\epsilon J^\mu_N + Z_3 \partial_\nu F^{\nu\mu} + \frac{1}{\xi}\partial^\mu \partial \cdot A \\
&+ \sqrt{Z_3} Z_2 \partial_\nu \left(Z_C C_{e\gamma} \left(\overline{e}_L \sigma^{\nu\mu} e_R\right) + Z_C^\star C^*_{e\gamma} \left(\overline{e}_R \sigma^{\nu\mu} e_L\right)\right) + \ldots
\end{aligned} \tag{23}
$$

We define a $\overline{\mathrm{MS}}$-renormalized current

$$
\begin{aligned}
J^\mu_{\overline{\mathrm{MS}}} =& J^\mu_N + \frac{Z_3 - 1}{e\mu^\epsilon} \partial_\nu F^{\nu\mu} \\
&+ \frac{\sqrt{Z_3} Z_2}{e\mu^\epsilon} \partial_\nu \left(Z_C C_{e\gamma} \left(\overline{e}_L \sigma^{\nu\mu} e_R\right) + Z_C^* C^*_{e\gamma} \left(\overline{e}_R \sigma^{\nu\mu} e_L\right)\right) + \ldots
\end{aligned} \tag{24}
$$

The renormalization group flow of the current is

$$
\mu \frac{d}{d\mu} J^\mu_{\overline{\mathrm{MS}}} = 2\gamma_A \frac{1}{e_0 Z_3} \partial_\nu F^{(0),\nu\mu}. \tag{25}
$$

Physical current

We define the physical current

$$J^{\mu}_{\text{phys}} = J^{\mu}_{\overline{\text{MS}}} - \frac{\Pi(0)}{e\mu^{\epsilon}} \partial_{\nu} F^{\nu\mu}, \tag{26}$$

where $\Pi(0)$ is the electron vacuum polarization. It follows that

$$F^{\nu\mu}_{\text{phys}} = \left[1 + \Pi(0)\right]^{1/2} F^{\nu\mu}, \qquad e_{\text{phys}} = \left[1 + \Pi(0)\right]^{-1/2} e\mu^{\epsilon}. \tag{27}$$

In the $\overline{\text{MS}}$ scheme

$$\Pi(0) = -\frac{e^2}{12\pi^2} \log \frac{m_e^2}{\mu^2} + \frac{e\, q_e}{2\,\pi^2} \left(C_{\underset{11}{e\gamma}} [M_e]_{11} + C^{\star}_{\underset{11}{e\gamma}} [M_e^{\dagger}]_{11} \right) \log \frac{m_e^2}{\mu^2} + \dots$$

From these results one directly defines the time component of the physical current as

$$j^{0}_{\text{LEFTphys}} = \frac{\nabla \cdot \mathbf{E}_{\text{LEFT,phys}}}{-e_{\text{LEFT,phys}}}, \tag{28}$$

which is the appropriate generalization of the source in Gauss's law into the LEFT.

Thank You!