QCD-like theories in strong magnetic fields

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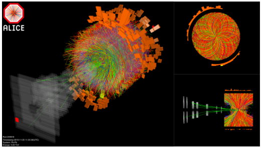
Joint work with Tomáš Brauner and Georgios Filios

QCD phase diagram



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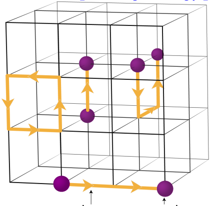
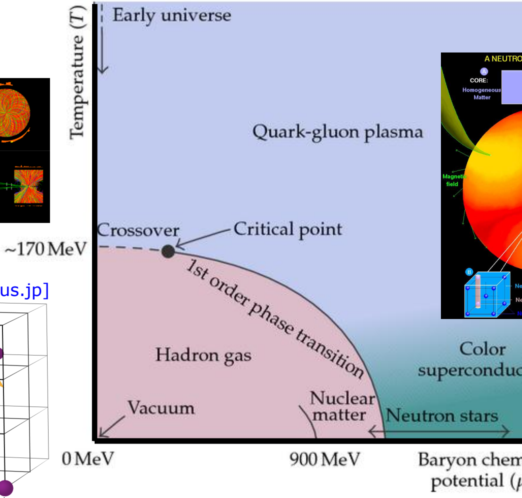
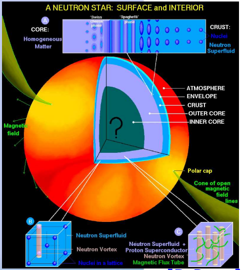


[Kim,Yi(2011)]





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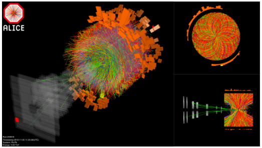

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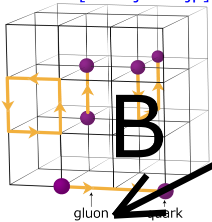

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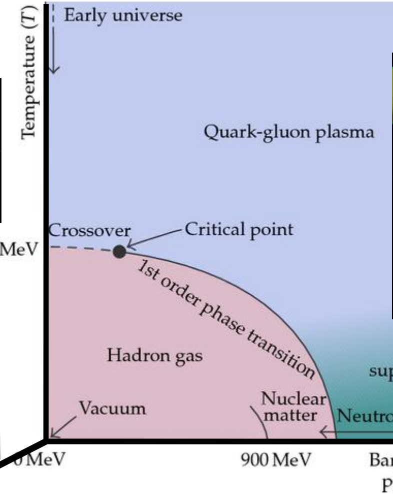

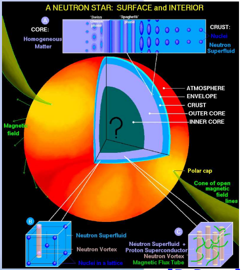

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Dense QCD matter in strong magnetic field

[Son,Stephanov(2008)][Brauner,Yamamoto(2017)]

- Method: chiral perturbation theory with $N_f = 2$

= Low energy effective field theory of Goldstone bosons arising from the spontaneous flavour symmetry breaking $SU(2)_L \times SU(2)_R \to SU(2)_V$

Topological Wess-Zumino-Witten term capturing the chiral anomaly

Standard LO χPT

matrix GB field

$$S = \frac{f_\pi^2}{4} \int \mathrm{d}^4x \, \mathrm{Tr}\left[D_\nu \Sigma D^\nu \Sigma^\dagger + m_\pi^2 \left(\Sigma + \Sigma^\dagger\right)\right] + S_{WZW}, \quad \Sigma = e^{i\frac{\pi^a \sigma^a}{f_\pi}}$$

$$D_\nu \Sigma = \partial_\nu \Sigma - i A_\nu \Sigma + i \Sigma A_\nu, \quad A_\nu = A_\nu^B \mathbb{1} + A_\nu^Q Q, \quad A_\nu^B = (\mu_B, 0, 0, 0)$$

minimal coupling to the gauge fields

(no effect of μ_B, only $\pi^\pm$ coupled to electromagnetic field)

- $\pi^\pm = 0$:

$$S_{WZW} = \frac{1}{4\pi^2 f_\pi} \int \mathrm{d}^4x \, \mu_B \, \mathbf{B} \cdot \boldsymbol{\nabla} \pi^0$$

Dense QCD matter in strong magnetic field

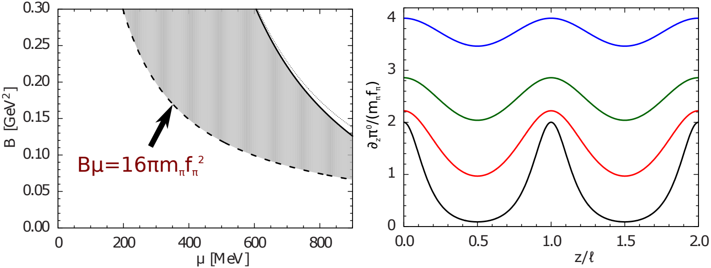
[Son,Stephanov(2008)][Brauner,Yamamoto(2017)]

- Ground state for $B\mu \geq 16\pi m_\pi f_\pi^2$: inhomogeneous condensate of neutral pions carrying baryon charge and magnetic moment

$$n_B(z) = \frac{B}{4\pi^2 f_\pi}\partial_z \pi^0(z), \quad m(z) = \frac{\mu}{4\pi^2 f_\pi}\partial_z \pi^0(z)$$

- parity and translations in z direction broken!
- named "chiral soliton lattice" in the analogy with chiral magnets
- for $B\mu^2 \geq 16\pi^4 f_\pi^4$ BEC of charged pions

[Brauner, Yamamoto(2017)]

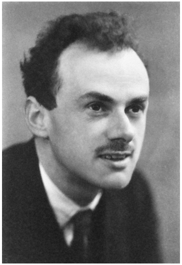


Dense QCD matter on lattice?

$$\mathcal{L}^E_{QCD} = \bar{\psi} M(A) \psi + \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}$$

$M(A)$: Dirac operator

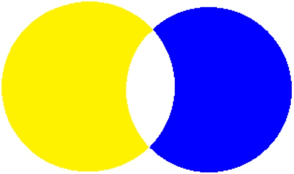
$$\int \mathcal{D}A\, \mathcal{D}\psi\, \mathcal{D}\bar{\psi}\, e^{-S^E_{QCD}} = \int \mathcal{D}A \,\det M(A)\, e^{-S_{YM}}$$



Standard lattice Monte Carlo methods work only if $\det M(A) > 0$**!**

theory	$M(A)$	properties	sign problem
QCD at $\mu_B = 0$	$\not{D} + m$	$M^\dagger = \gamma_5 M \gamma_5$	absent
QCD at $\mu_B \neq 0$	$\not{D} + m - \mu_B \gamma_0$	-	present

QCD with two colors?



- Gauge group $= SU(2)$, quarks in fundamental representation
- Quarks in pseudoreal representation of the gauge group: $\sigma_i^* = -\sigma_2 \sigma_i \sigma_2$

NB: In general quarks in (pseudo)real representation $\Leftrightarrow$ $T_i^* = -P^{-1} T_i P$
(3-color QCD with adjoint quarks, G_2 as a gauge group...)

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QCD at $\mu_B = 0$	$\not{D} + m$	$M^\dagger = \gamma_5 M \gamma_5$	absent
QCD at $\mu_B \neq 0$	$\not{D} + m - \mu_B \gamma_0$	-	present
QCD-like	$\not{D} + m - \mu_B \gamma_0$	$M^* = (C\gamma_5 P) M (C\gamma_5 P)^{-1}$	absent for $2N_f$

QCD with two colors?

- Gauge group $= SU(2)$, quarks in fundamental representation
- Quarks in pseudoreal representation of the gauge group: $\sigma_i^* = -\sigma_2\sigma_i\sigma_2$
- Enlarged flavour symmetry! $SU(2N_f)$ spontaneously broken to $Sp(2N_f)$
- $N_f = 2$: $SU(4)/Sp(4) \simeq SO(6)/SO(5) \Rightarrow$ 5 Goldstone bosons

pseudo-GB field	baryon number	isospin
d	$+1$	0
$\bar{d}$	-1	0
π^+	0	$+1$
π^-	0	-1
π^0	0	0

QCD with two colors?

- Lattice simulations addressing the phase diagram in $\mu - T$ plane
  [Kogut,Toublan,Sinclair(2001,2002)][(Boz),(Cotter),(Fister),(Giudice),Hands,(Kim),(Mehta),Skullerud(2006,2010,2013)],
  [Braguta,(Ilgenfritz),Kotov,(Molochkov),Nikolaev,(Vlagushev)(2015,2016)]
  agree with the predictions of χPT (in the range of it's validity)
  [Splittorf,Toublan,Verbaarschot(2002)]

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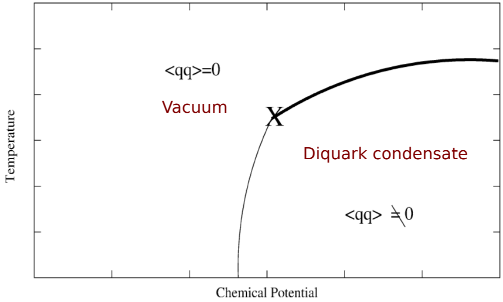

Fig. 1. Schematic phase diagram of diquark condensation in the T–μ plane. The thin (thick) line consists of second (first) order transitions. X labels the tricritical point.

QCD with two colors in strong magnetic field?

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EFT with different coset space

$\Rightarrow$ different shape of the Wess-Zumino-Witten term!

Wess-Zumino-Witten term

- Standard machinery for constructing EFT for Goldstone bosons = coset construction [Callan,Coleman,Wess,Zumino(1969)]
- But $\pi^0 \to \gamma\gamma$ was missing in the χPT!
- The term in χPT capturing the chiral anomaly first identified in [Wess,Zumino(1971)], its geometrical meaning given by [Weinberg (1983)]

$$S_{WZW} = \int_Q \omega_5$$

(spacetime compactified to 4-sphere M; $U : M \to SU(3)$, the 4-sphere in $SU(3)$ defined by $U(x)$ = boundary of 5-dimensional disc Q; ω_5: closed $SU(3)$-invariant 5-form)

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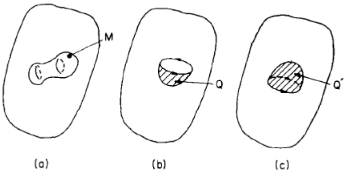


Fig. 2. Space-time, a four-sphere, is mapped into the SU(3) manifold. In part (a), space-time is symbolically denoted as a two sphere. In parts (b) and (c), space-time is reduced to a circle that bounds the discs Q and Q′. The SU(3) manifold is symbolized in these sketches by the interior of the oblong.

Gauged Wess-Zumino-Witten term for general coset space

[H.K., Tomáš Brauner; arXiv:1809.05310]

$$S_{WZW} = \int_Q \omega_5 = \int_Q \mathrm{d}\omega_4 \overset{\text{Stokes}}{=} \int_{\partial Q} \omega_4 = \int_{U(M)} \omega_4 = \int_M U^* \omega_4$$

(spacetime compactified to 4-sphere M; $U : M \to SU(3)$, the 4-sphere in $SU(3)$ defined by $U(x)$ = boundary of 5-dimensional disc Q; ω_5: closed $SU(3)$-invariant 5-form)

- Using methods based on theory of cohomology [D'Hoker(1995)] we got

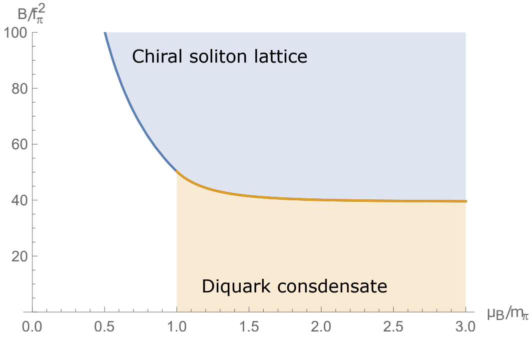
$$\omega_5 = \mathrm{Tr}\left[\frac{1}{10}\bar{\phi}^5 - \frac{1}{2}(\bar{W} + \bar{F})\bar{\phi}^3 + (\bar{W}^2 + \bar{F}^2)\bar{\phi} + \frac{1}{2}(\bar{W}\bar{F} + \bar{F}\bar{W})\bar{\phi}\right]$$

- $\omega_5 - \omega_5^{A=0} = \mathrm{d}\omega_{4A}$

$$\begin{aligned}\omega_{4A} = \mathrm{Tr}\Big\{ & \tfrac{1}{2}\phi^3(\bar{A} + \bar{A}_\parallel) + \tfrac{1}{4}\phi\bar{A}_\perp\phi(\bar{A} + \bar{A}_\parallel) + \tfrac{1}{2}\phi^2[\bar{A}_\perp, \bar{A}_\parallel] + \tfrac{1}{2}\bar{A}_\perp\bar{A}_\parallel^3 - \tfrac{1}{2}\bar{A}_\parallel\bar{A}_\perp^3 - \tfrac{1}{4}\bar{A}_\parallel\bar{A}_\perp\bar{A}_\parallel\bar{A}_\perp \\ & + \phi(\tfrac{1}{2}\bar{A}_\perp^3 + \tfrac{3}{4}\bar{A}_\perp^2\bar{A}_\parallel + \tfrac{3}{4}\bar{A}_\parallel\bar{A}_\perp^2 + \tfrac{1}{2}\bar{A}_\parallel^2\bar{A}_\perp + \tfrac{1}{2}\bar{A}_\parallel\bar{A}_\perp\bar{A}_\parallel + \tfrac{1}{2}\bar{A}_\perp\bar{A}_\parallel^2 + \bar{A}_\parallel^3) \\ & + \tfrac{1}{2}\bar{F}[\bar{A} + \tfrac{1}{2}\bar{A}_\parallel, \phi] + \tfrac{1}{2}(\bar{W} + W)[\tfrac{1}{2}\bar{A} + \bar{A}_\parallel, \phi] + (\tfrac{1}{2}\bar{F} + \tfrac{1}{2}\bar{W} + \tfrac{1}{4}W)[\bar{A}_\parallel, \bar{A}_\perp]\Big\}\end{aligned}$$

Preliminary study of the 2-color QCD phase diagram

- We have derived the WZW term in case of $SU(4)/Sp(4)$ coset space
- For $\pi^0 = 0$, the WZW term is absent $\Rightarrow$ diquark condensate phase expected
- For $d, \bar{d} = 0$, the Lagrangian is identical with the QCD case $\Rightarrow$ chiral soliton lattice phase expected
- Comparison of the two ground state energies $\Rightarrow$



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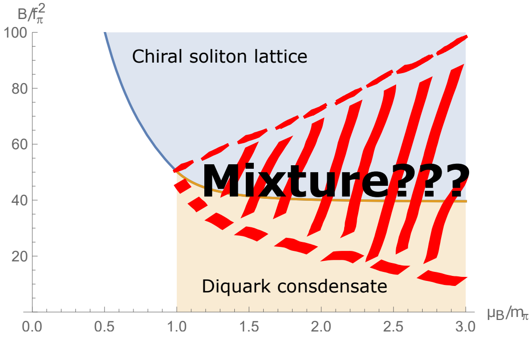

Conclusions

- Chiral Soliton Lattice phase found recently to be the ground state of dense QCD matter in strong magnetic field
- Such a phase could be seen on lattice if present for QCD-like theories
- EFT study: construction of the WZW term for general G/H necessary
- Side product: formula for gauged WZW term which could have application in different fields of physics (composite Higgs models, composite dark matter models, solid state physics...)
- Preliminary results suggest the presence of CSL phase in case of 2-color QCD
- Inhomogeneous phase present in the theory without the sign problem! (not expected in the $B = 0$ case [Splittorff,Son,Stephanov(2001)])

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Thank you for your attention!