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Scalar Effective Field Theories and Soft Recursion Amplitude Methods

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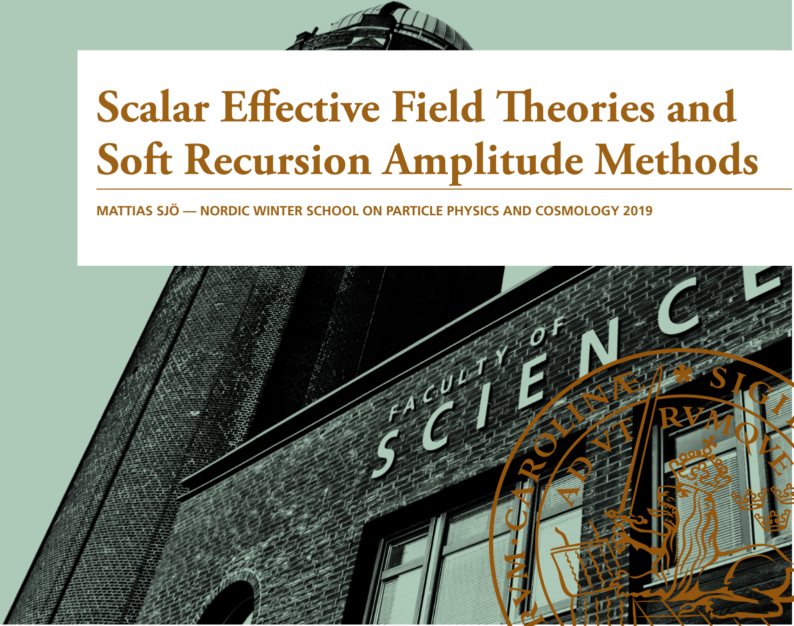


Background of the topic

This talk is connected to my my master's thesis project, supervised by Karol Kampf and Johan (Hans) Bijnens.

- I analytically calculate high-order scalar particle scattering at tree level.
- Straightforward but extremely cumbersome (we're talking billions of terms).
- Shortcuts are needed.
- This outlines one major shortcut.

Stripped amplitudes

- An n-particle amplitude can be written as

$$\mathcal{M}_n^{a_1 \dots a_n}(p_1, \dots, p_n) = \sum_{\sigma \in \mathcal{S}_n} \varphi_n^\sigma(a_1 \dots a_n) \mathcal{M}_n^\sigma(p_1, \dots, p_n)$$

  where a_i are colour/flavour indices, φ a colour/flavour structure,
  σ permutations of $1, \dots, n$.
- It can be shown that

$$\mathcal{M}_n^\sigma(p_1, \dots, p_n) = \mathcal{M}_n(p_{\sigma(1)}, \dots, p_{\sigma(n)})$$

  where $\mathcal{M}_n = \mathcal{M}_n^{\mathrm{id}}$ and $\mathrm{id}(k) = k$ (same for φ).
- $\mathcal{M}_n$ is called the “stripped amplitude”, and contains all information of the full amplitude.
- Much easier to caclulate (still tough to do diagrammatically).

Recursive Amplitude Methods

The general idea of amplitude methods

- Take an n-particle stripped amplitude $\mathcal{M}_n(p_1, \dots, p_n)$
- Shift each momentum into an analytic function

$$p_i \to p_i(z) \quad \text{with} \quad p_i(0) = p_i \quad \text{for each } i$$

preserving

$$p_i(z)^2 = 0, \qquad \sum_i p_i(z) = 0.$$

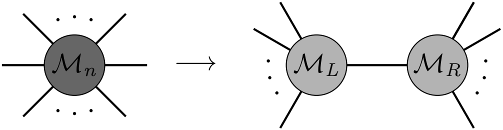
- Get an analytic function $\mathcal{M}_n(z)$; $\mathcal{M}_n(0)$ is our amplitude.
- Apply Cauchy's theorem:

$$0 = \oint \frac{\mathcal{M}_n(z)}{z} dz = \mathcal{M}_n(0) + \sum_{\text{poles } z_k} \operatorname*{Res}_{z=z_k} \frac{\mathcal{M}_n(z)}{z}$$

- With cleverly chosen shifts, the residues are much simpler to compute than the amplitude itself.

BCFW Recursion

- The original recursive amplitude method is due to BCFW: Britto, Cachazo, Feng & Witten (2005).
- Tree-level amplitudes can only have simple poles like



  with propagator on-shell — Factorisation!
- Each half is a physical amplitude with fewer particles — Recursion!
- Using the BCFW choice of momentum shift,

$$\operatorname*{Res}_{z=z_k} \mathcal{M}_n(z) = \mathcal{M}_L(z_k) \frac{i}{P^2(0)} \mathcal{M}_R(z_k)$$

  where $P(z)$ is the propagator momentum.
- The base case depends on the theory.

Soft Recursion

Failure of BCFW recursion

- BCFW is excellent for e.g. gluons.
- It would be nice for our EFT, but

$$0 = \oint \frac{\mathcal{M}_n(z)}{z} dz = \mathcal{M}_n(0) + \sum_{\text{poles } z_k} \operatorname*{Res}_{z=z_k} \frac{\mathcal{M}_n(z)}{z}$$

  assumes that $\mathcal{M}(z)$ falls off at infinity.

- $\mathcal{M}_n(z) \sim z^k$ with $k > 0$ as $z \to \infty$, so this fails!

Soft behaviour

- The hard limit is awful, but what about the soft limit?
- EFTs (e.g. the NLSM) generally have "Adler zeroes":

$$\mathcal{M}_n(p_1, \ldots, p_n) \sim p_i^\sigma \quad \text{as } p_i \to 0$$

- $\sigma > 0$ is the "soft degree"; the NLSM has $\sigma = 1$.
- We can captialise on this using soft recursion, due to Cheung, Kampf, Novotny, Shen & Trnka (2016).

Amplitudes using the soft limit

- Define an Adler-zero-preserving shift

$$p_i \to p_i(z) = p_i(1 - z a_i)$$

with suitable a_i that preserve $\sum_i p_i = 0$

- Define

$$F_n(z) = \prod_i (1 - a_i z)^\sigma, \qquad F_n(0) = 1.$$

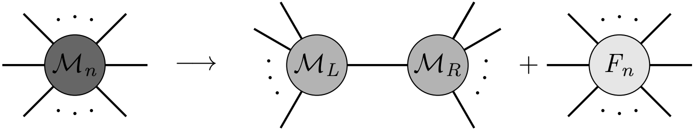
- Then $\mathcal{M}_n(z)/F_n(z)$ has the same poles as $\mathcal{M}_n(z)$, but falls off faster.
- This gives the modified expression

$$0 = \oint \frac{\mathcal{M}_n(z)}{zF_n(z)} dz = \mathcal{M}_n(0) + \sum_{\text{poles } z_k} \operatorname*{Res}_{z=z_k} \frac{\mathcal{M}_n(z)}{zF_n(z)}$$

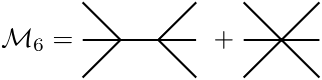
valid if $\mathcal{M}_n(z) \sim z^k$ as $z \to \infty$, with $k < n\sigma$.

Soft recursion

- Amplitude factorises just as for BCFW.
- Factorised amplitude does not have Adler zeroes, so $F_n(z)$ gives additional poles.



- Simplest nontrivial example: 6-particle NLSM amplitude at leading order



- First diagram factorises into two 4-particle base cases.
- Second diagram comes from the additional poles — no need to derive it from the Lagrangian.

Thank you for listening! Questions?