Protoplanetary disks and dust dynamics



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Protoplanetary discs



- Two ALMA images of protoplanetary discs (ALMA partnership, 2015; Andrews et al., 2016)
- HL Tau is 140 pc away, 1 million years old
- TW Hya is 54 pc away, 10 million years old
- Emission comes mainly from mm-sized pebbles
- Dark rings are debated but may trace young planets

Planet formation in protoplanetary discs



- Planets form in protoplanetary discs around young stars as dust grains collide and grow to ever larger bodies
- Pebbles form as dust grains stick in collisions
- Pebbles spontaneously form dense clumps and clumps contract to form *planetesimals* – the building blocks of planets
- Planets grow by accretion of planetesimals and pebbles
- Gas giants like Jupiter form by contraction of gas from the protoplanetary disc onto a solid core of 10 Earth masses

Spectral energy distribution of young stars

The spectral energy distribution of young stars reveals two components: the stellar black body at short wavelengths and emission from warm circumstellar dust at long wavelengths



Irradiated dust



- Each ring radiates like a black body
- Temperature falls as r^{-q} (q = 1/2 in optically thin disc)
- Dust is the main opacity in protoplanetary discs
- ► Hydrogen molecules have very low opacity at low (10-100 K) temperatures ⇒ H₂ very difficult to detect
- Use instead dust mass to find mass of a protoplanetary disc $(M_{\rm disc} \approx M_{\rm dust}/0.01)$

Dust mass in protoplanetary discs

What is needed to determine the dust mass in a protoplanetary disc?

- The temperature of the dust
- The opacity of the dust
- The dust emission must be optically thin



- ⇒ Optically thin: emission proportional to emitting area of all particles dust mass known if opacity κ known
- \Rightarrow Optically thick: emission proportional to surface area of disc total dust mass unknown

Disc masses in Taurus-Auriga

- ► Taurus-Auriga complex is one of the nearest active star forming regions (d = 140 pc, M ~ 3.5 × 10⁴ M_☉)
- Andrews & Williams (2005) monitored 153 young stars for dust emission and found significant dust discs around 93 of them



Life-times of protoplanetary discs

- Stars in same star-forming region are pretty much the same age
- Compare instead disc fraction between regions of different age



Haisch et al. (2001)



 \Rightarrow Protoplanetary discs live for 1–5 Myr

Column density in the Minimum Mass Solar Nebula

- Spread rock and ice in the solar system planets evenly over the distance to the neighbouring planets
- \blacktriangleright Assume rock and ice represent ${\approx}1.8\%$ of total material \Rightarrow original gas contents

(Kusaka, Nakano, & Hayashi, 1970; Weidenschilling, 1977b; Hayashi, 1981)

$$\begin{split} \varSigma_{\rm r}(r) &= 7\,{\rm g\,cm^{-2}}\,\left(\frac{r}{{\rm AU}}\right)^{-3/2} & {\rm for} \quad 0.35 < r/{\rm AU} < 2.7\\ \varSigma_{\rm r+i}(r) &= 30\,{\rm g\,cm^{-2}}\,\left(\frac{r}{{\rm AU}}\right)^{-3/2} & {\rm for} \quad 2.7 < r/{\rm AU} < 36\\ \varSigma_{\rm g}(r) &= 1700\,{\rm g\,cm^{-2}}\,\left(\frac{r}{{\rm AU}}\right)^{-3/2} & {\rm for} \quad 0.35 < r/{\rm AU} < 36 \end{split}$$

Total mass of Minimum Mass Solar Nebula

$$M = \int_{r_0}^{r_1} 2\pi r \Sigma_{\mathrm{r+i+g}}(r) \mathrm{d}r \approx 0.013 M_{\odot}$$

Temperature in the Minimum Mass Solar Nebula

- Much more difficult to determine the temperature in the solar protoplanetary disc
- Several energy sources: solar irradiation, viscous heating, irradiation by nearby stars
- Simplest case: only solar irradiation in optically thin nebula

$$F_{\odot} = \frac{L_{\odot}}{4\pi r^2}$$

$$\begin{aligned} P_{\rm in} &= \pi \epsilon_{\rm in} R^2 F_\odot \\ P_{\rm out} &= 4\pi R^2 \epsilon_{\rm out} \sigma_{\rm SB} T_{\rm eff}^4 \end{aligned}$$

$$T_{\rm eff} = \left[\frac{F_{\odot}}{4\sigma_{\rm SB}}\right]^{1/4}$$
$$T = 280 \,\mathrm{K} \,\left(\frac{r}{\mathrm{AU}}\right)^{-1/2}$$



Vertical gravity Radial density structure of MMSN

$$\Sigma(r) = 1700 \,\mathrm{g \, cm^{-2}} r^{-1.5}$$

What about the vertical structure?

 \Rightarrow Hydrostatic equilibrium between gravity and pressure



► The distance triangle and the gravity triangle are similar triangles ⇒ g_z/g = z/d

$$g_z = g \frac{z}{d} = -\frac{GM_{\star}}{d^2} \frac{z}{d} \approx -\frac{GM_{\star}}{r^3} z = -\Omega_{\rm K}^2 z$$

Hydrostatic equilibrium structure



Equation of motion for fluid element at height z over the disc mid-plane:

$$\frac{\mathrm{d}\mathbf{v}_z}{\mathrm{d}t} = -\Omega_\mathrm{K}^2 z - \frac{1}{\rho} \frac{\mathrm{d}P}{\mathrm{d}z}$$

- For constant temperature T we can write P = c_s² ρ
 (isothermal equation of state with sound speed c_s=const)
- Look for hydrostatic equilibrium solution:

$$\mathbf{0} = -\Omega_{\mathrm{K}}^2 z - c_{\mathrm{s}}^2 \frac{1}{\rho} \frac{\mathrm{d}\rho}{\mathrm{d}z}$$

Scale height

Hydrostatic equilibrium condition:

$$0 = -\Omega_{\rm K}^2 z - c_{\rm s}^2 \frac{{\rm d}\ln\rho}{{\rm d}z}$$

• Rewrite slightly and introduce scale height $H = c_s / \Omega_K$:

$$\frac{\mathrm{d}\ln\rho}{\mathrm{d}z} = -\frac{\Omega_{\mathrm{K}}^2}{c_{\mathrm{s}}^2}z = -\frac{z}{H^2}$$

Solution in terms of ln ρ:

$$\ln \rho = \ln \rho_0 - \frac{z^2}{2H^2}$$

Solution in terms of ρ:

$$\rho(z) = \rho_0 \exp\left[-\frac{z^2}{2H^2}\right]$$

Mid-plane density

Vertical density structure of protoplanetary disc

$$\rho(z) = \rho_0 \exp\left[-\frac{z^2}{2H^2}\right]$$

• $\rho_0 = \rho(r, z = 0)$ is the mid-plane gas density

Problem: we only know the column density. Connection between Σ and ρ₀ comes from definite integral

$$\Sigma = \int_{-\infty}^{\infty} \rho(z) dz = \rho_0 \int_{-\infty}^{\infty} \exp[-z^2/(2H^2)] dz$$
$$= \sqrt{2}H\rho_0 \int_{-\infty}^{\infty} \exp[-\zeta^2] d\zeta = \sqrt{2\pi}H\rho_0$$

This yields the mid-plane density

$$\rho_0 = \frac{\Sigma}{\sqrt{2\pi}H}$$

Minimum Mass Solar Nebula overview

 As a starting point for planet formation models we can use the Minimum Mass Solar Nebula model of *Hayashi* (1981):

$$\begin{split} \Sigma(r) &= 1700 \,\mathrm{g \, cm^{-2}} \left(\frac{r}{\mathrm{AU}}\right)^{-3/2} \\ T(r) &= 280 \,\mathrm{K} \left(\frac{r}{\mathrm{AU}}\right)^{-1/2} \\ \rho(r,z) &= \frac{\Sigma(r)}{\sqrt{2\pi}H(r)} \exp\left[-\frac{z^2}{2H(r)^2}\right] \\ H(r) &= \frac{c_{\mathrm{s}}}{\Omega_{\mathrm{K}}} \qquad \Omega_{\mathrm{K}} = \sqrt{\frac{GM}{r^3}} \\ c_{\mathrm{s}} &= 9.9 \times 10^4 \,\mathrm{cm \, s^{-1}} \left(\frac{2.34}{\mu} \frac{T}{280 \,\mathrm{K}}\right)^{1/2} \\ H/r &= \frac{c_{\mathrm{s}}}{v_{\mathrm{K}}} = 0.033 \left(\frac{r}{\mathrm{AU}}\right)^{1/4} \end{split}$$

Minimum Mass Solar Nebula density

Density contours in Minimum Mass Solar Nebula:



- Mid-plane gas density varies from 10⁻⁹ g/cm³ in the terrestrial planet formation region down to 10⁻¹³ g/cm³ in the outer nebula
- Blue line shows location of z = H
- ► Aspect ratio increases with r, so solar nebula is slightly *flaring*

Conditions for planet formation



 Young stars are orbited by dusty protoplanetary discs

• Disc masses of
$$10^{-4}$$
– 10^{-1} M_{\odot}

Disc life-times of 1–5 million years



Observed dust growth in protoplanetary discs



• Dust opacity as a function of frequency $\nu = c/\lambda$:

- $\kappa_{\nu} \propto \nu^2$ for $\lambda \gg a$
- $\kappa_{\nu} \propto \nu^0$ for $\lambda \ll a$
- $F_{\nu} \propto \nu^{\alpha} \propto \kappa_{\nu} B_{\nu} \propto \kappa_{\nu} \nu^2 \propto \nu^{\beta} \nu^2$
- By measuring α from SED, one can determine β from $\beta = \alpha 2$
- Knowledge of β gives knowledge of dust size

Pebbles in protoplanetary disks



- ► Many nearby protoplanetary disks observed in mm-cm wavelengths show opacity indices below β = 2 (κ_ν ∝ ν^β)
- Typical pebble sizes of mm in outer disk and cm in inner disk
- Protoplanetary disks are filled with pebbles

Drag force

Gas accelerates solid particles through drag force:

(Whipple, 1972; Weidenschilling, 1977)

$$\frac{\partial \boldsymbol{v}}{\partial t} = \dots - \frac{1}{\tau_{\rm f}} (\boldsymbol{v} - \boldsymbol{u})$$
Particle velocity
Gas velocity

In the Epstein drag force regime, when the particle is much smaller than the mean free path of the gas molecules, the friction time is

$$\tau_{\rm f} = \frac{R\rho_{\bullet}}{c_{\rm s}\rho_{\rm g}} \qquad \begin{array}{c} R: \mbox{ Particle radius} \\ \rho_{\bullet}: \mbox{ Material density} \\ c_{\rm s}: \mbox{ Sound speed} \\ \rho_{\rm g}: \mbox{ Gas density} \end{array}$$

Important nondimensional parameter in protoplanetary discs:

$$St = \Omega \tau_{f}$$
 (Stokes number)

 \varOmega is the Keplerian frequency

Particle sizes



(Johansen et al., 2014, Protostars & Planets VI)

- In the Epstein regime $St = \frac{\sqrt{2\pi}R\rho_{\bullet}}{\Sigma_{\sigma}}$
- Other drag force regimes close to the star yield different scalings with the gas temperature and density (Whipple, 1972)

Radial drift



- Disc is hotter and denser close to the star
- ► Radial pressure gradient force mimics decreased gravity ⇒ gas orbits slower than Keplerian
- Particles do not feel the pressure gradient force and would orbit at Keplerian speed in absence of gas
- Headwind from sub-Keplerian gas drains angular momentum from particles, so they spiral in through the disc

Radial drift speed

Balance between drag force and head wind gives radial drift speed (Adachi et al. 1976; Weidenschilling 1977)

$$v_{\mathrm{drift}} = -rac{2\Delta v}{arOmega_{\mathrm{K}} au_{\mathrm{f}} + (arOmega_{\mathrm{K}} au_{\mathrm{f}})^{-1}}$$

for Epstein drag law $\tau_{\rm f}=a\rho_{\bullet}/(c_{\rm s}\rho_{\rm g})$



 MMSN Δν ~ 50...100 m/s

 Drift time-scale of 100 years for particles of 30 cm in radius at 5 AU

Drift-limited pebble growth



(Birnstiel et al., 2015)

- Particles in the outer disc grow to a characteristic size where the growth time-scale equals the radial drift time-scale (*Birnstiel et al.*, 2012)
- Growth time-scale $t_{\rm gr} = R/\dot{R}$, drift time-scale $t_{\rm dr} = r/\dot{r}$
- Yields dominant particle size that increases as pebble drifts inwards
- Pebble sizes agree well with observations
- The drift-limited solution shows a fundamental limit to particle growth
- Bouncing and fragmentation would result in even smaller particle sizes

Particle concentration



(Johansen et al., Protostars and Planets VI, 2014)

Three categories of particle concentration mechanisms:

Between small-scale low-pressure eddies

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(Cuzzi et al., 2001, 2008; Pan et al., 2011)
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In pressure bumps and vortices

(Whipple, 1972; Barge & Sommeria, 1995; Klahr & Bodenheimer, 2003; Johansen et al., 2009a)

By streaming instabilities

(Youdin & Goodman, 2005; Johansen & Youdin, 2007; Johansen et al., 2009b; Bai & Stone, 2010a,b,c)

Streaming instability

- Gas orbits slightly slower than Keplerian
- Particles lose angular momentum due to headwind
- Particle clumps locally reduce headwind and are fed by isolated particles



 \Rightarrow Youdin & Goodman (2005): "Streaming instability"

Linear analysis



- The streaming feeds off the velocity difference between gas and particles
- Particles move faster than the gas and drift inwards, pushing the gas outwards
- In total there are 8 linear modes (density waves modified by drag)
- One of the modes is unstable (Youdin & Goodman, 2005; Jacquet, Balbus, & Latter, 2011)
- Requires both radial and vertical displacements
- Fastest growth for large particles and local dust-to-gas ratio above unity

Streaming instability

Evolution of the flow of cm-sized pebbles embedded in gas:



High particle concentrations driven by the streaming instability (Youdin & Johansen, 2007; Johansen & Youdin, 2007; Johansen et al., 2007; 2009; 2012; Bai & Stone, 2010a,b,c)

Stratified simulations

- Johansen, Youdin, & Mac Low (2009) presented stratified simulations of streaming instabilities
- Pebble sizes $\Omega \tau_{\rm f} = 0.1, 0.2, 0.3, 0.4$ (3–12 cm at 5 AU, 1–4 cm at 10 AU)
- Metallicity $Z = \Sigma_p / \Sigma_g$ is a free parameter



Convergence tests

- Criterion for gravitational collapse: $\rho_{\rm p} \gtrsim \Omega^2/G \sim 100 \rho_{\rm g}$
- Maximum density increases with increasing resolution
- Particle density up to 10,000 times local gas density





(Johansen, Youdin, & Lithwick, 2012)

Gravitational collapse



- Particle concentration by streaming instabilities reach at least 10,000 times the gas density
- ► Filaments fragment to planetesimals with contracted radii 25-200 km (Johansen, Mac Low, Lacerda, & Bizzarro, 2015)
- \Rightarrow Initial Mass Function of planetesimals at up to 512³ resolution (through European PRACE supercomputing grant)

Planetesimal birth sizes



- \blacktriangleright Differential size distribution is well fitted by a power law with ${\rm d}N/{\rm d}M \propto M^{-1.6}$
- Results with Pencil Code and Athena code are very similar
- Most of the mass resides in the largest planetesimals
- Small planetesimals dominate in number
- Size of largest planetesimal decreases with decreasing particle column density, down to 100 km at MMSN-like density at 2.5 AU

Metallicity threshold



- The streaming instability makes dense filaments above a threshold metallicity (Carrera et al., 2015)
- Lowest around a sweetspot at St \sim 0.1 (1 mm at 30 AU)
- Increases to smaller and larger St
- ▶ The threshold also depends on the radial pressure support (Bai & Stone, 2010)

Achieving the conditions for the streaming instability



- Possible to form pebble sizes needed for streaming instability outside of the ice line (Drazkowska & Dullemond, 2014)
- But bouncing stalls silicate particles at mm sizes inside of the ice line
- About half of the solid mass remains in tiny grains unable to participate in the streaming instability
- Photoevaporation can increase the dust-to-gas ratio towards the end of the disc life-time (Gorti et al., 2015; Carrera, Gorti, & Johansen, 2017)
- Raising the metallicity to trigger the streaming instability is a very active research area (e.g., Drazkowska et al., 2016; Gonzalez et al., 2017)

Forming planetesimals by photoevaporation



(Carrera, Gorti, Johansen, & Davies, 2017)

- Photoevaporation models including X-rays, EUV and FUV show evolution in gas-to-dust ratio (Gorti et al., 2015)
- Typically 50–100 M_E of dust remains after gas disc gone
- Pebbles turn into planetesimals when including prescription for streaming instability (Carrera et al., 2017)
- $\Rightarrow\,$ Efficient delivery of planetesimals to debris disc phase
 - ? How to form planetesimals that grow to gas-giant cores?

Classical core accretion scenario



- 1. Dust grains and ice particles collide to form km-scale planetesimals
- 2. Large protoplanet grows by run-away accretion of planetesimals
- 3. Protoplanet attracts hydrostatic gas envelope
- 4. Run-away gas accretion as $M_{
 m env} pprox M_{
 m core}$
- 5. Form gas giant with $M_{
 m core} pprox 10 M_{\oplus}$ and $M_{
 m atm} \sim M_{
 m Jup}$

(Safronov, 1969; Mizuno, 1980; Pollack et al., 1996)

All steps must happen within 1–3 Myr while there is gas orbiting the star

Core formation time-scales

The size of the protoplanet relative to the Hill sphere:

$$\frac{R_{\rm p}}{R_{\rm H}} \equiv \alpha \approx 0.001 \left(\frac{r}{5\,{\rm AU}}\right)^{-1}$$

 Maximal growth rate by gravitational focussing

$$\dot{M} = \pi R_{\rm p}^2 v \rho_{\rm s} \alpha^{-1}$$
$$= \alpha R_{\rm H}^2 F_{\rm H}$$

- $\Rightarrow \text{ Only 0.1\% (0.01\%) of planetesimals} \\ \text{entering the Hill sphere are accreted at 5} \\ \text{AU (50 AU)} \end{cases}$
- $\begin{array}{rl} \Rightarrow & \mbox{Time to grow to 10} \ M_\oplus \mbox{ is } \\ & \sim 10 & \mbox{Myr at 5} \ \ AU \\ & \sim 50 & \mbox{Myr at 10} \ AU \\ & \sim 5,000 \ \ \mbox{Myr at 50} \ \ AU \end{array}$



Directly imaged exoplanets



(Marois et al., 2008; 2010)

(Kalas et al., 2008)

- HR 8799 (4 planets at 14.5, 24, 38, 68 AU)
- Fomalhaut (1 controversial planet at 113 AU)
- ⇒ No way to form the cores of these planets within the life-time of the protoplanetary gas disc by standard core accretion

Pebble accretion



- Most planetesimals are simply scattered by the protoplanet
- Pebbles spiral in towards the protoplanet due to gas friction
- ⇒ Pebbles are accreted from the entire Hill sphere
- Growth rate by planetesimal accretion is

$$\dot{M} = \alpha R_{\rm H}^2 F_{\rm H}$$

 Growth rate by pebble accretion is

$$\dot{M}=R_{\rm H}^2F_{\rm H}$$

Relevant parameters for pebble accretion

- Hill radius R_H = [GM_p/(3Ω²)]^{1/3}
 Distance over which the gravity of the protoplanet dominates over the the tidal force of the central star
- Bondi radius R_B = GM/(Δν)²
 Distance over which a particle with approach speed Δν is significantly deflected by the protoplanet (in absence of drag)
- Sub-Keplerian speed Δν
 Orbital speed of gas and pebbles relative to Keplerian speed
- ▶ Hill speed $v_{\rm H} = \Omega R_{\rm H}$ Approach speed of gas and pebbles at the edge of the Hill sphere

Pebble accretion regimes



Two main pebble accretion regimes: (Lambrechts & Johansen, 2012)

- 1. Bondi regime (when $\Delta v \gg v_{\rm H}$) Particles pass the protoplanet with speed Δv , so $\dot{M} \propto R_{\rm B}^2 \propto M^2$
- 2. Hill regime (when $\Delta v \ll v_{\rm H}$) Particles enter protoplanet's Hill sphere with speed $v_{\rm H} \approx \Omega R_{\rm H}$, so $\dot{M} \propto R_{\rm H}^2 \propto M^{2/3}$

Time-scale of pebble accretion



- ⇒ Pebble accretion speeds up core formation by a factor 1,000 at 5 AU and a factor 10,000 at 50 AU (Ormel & Klahr, 2010; Lambrechts & Johansen, 2012; Nesvorny & Morbidelli, 2012)
- ⇒ Cores form well within the life-time of the protoplanetary gas disc, even at large orbital distances
- Requires large planetesimal seeds to accrete in Hill regime, consistent with planetesimal formation by gravitational collapse

Halting pebble accretion



- Pebble accretion is stopped when the protoplanet grows massive enough to carve a gap in the pebble distribution
- ► Gap formation known for Jupiter-mass planets (Paardekooper & Mellema, 2006)
- Lambrechts et al. (2014) demonstrate that pebble accretion is stopped already at 20 M_{\oplus} at 5 AU, with isolation mass scaling as

$$M_{\rm iso} = 20 \left(\frac{r}{5{
m AU}}
ight)^{3/4} M_{\oplus}$$

 \Rightarrow Collapse of the gas envelope

Growth tracks of giant planets



- Pebble accretion combined with planetary migration (Johansen & Lambrechts, 2017, Annual Review of Earth and Planetary Sciences)
- Giant planets undergo substantial migration
- Embryo at 6 AU forms hot Jupiter; Jupiter-analogue starts at 16 AU
- Ice giants are stranded by photoevaporation

Growth tracks of wide-orbit planets



- Many wide-orbit exoplanet systems now, including HR 8799 (Marois et al., 2008; 2010)
- Migration is very severe in wide orbits
- Three inner planets start at 50 100 AU
- The outer planet is challenging, even for pebble accretion

Growth tracks of super-Earths



 The pebble isolation mass is around 5 M_E in the inner disc (Lambrechts et al., 2014)

- ► Gas contraction is slow on small cores (Piso & Youdin, 2014; Lee et al., 2014)
- Super-Earths are stuck in the slow gas contraction phase (Ikoma & Hori, 2012)
- Planetesimal accretion dominates growth to embryo sizes, then pebble accretion takes over

Emergence regions of planetary classes



(Bitsch et al., 2015)

Summary



- Protoplanetary discs are really good pebble factories
- Radial drift limits pebble growth to mm-cm sizes
- Pebbles are concentrated to very high densities by the streaming instability
- Rapid pebble accretion leads to the formation a wide range of planetary classes despite planetary migration
- Details depend on protoplanetary disc structure, pebble sizes, planetesimal birth masses, ...
- Some plots from "Forming Planets via Pebble Accretion" (Johansen & Lambrechts, Annual Review of Earth and Planetary Sciences, 2017)