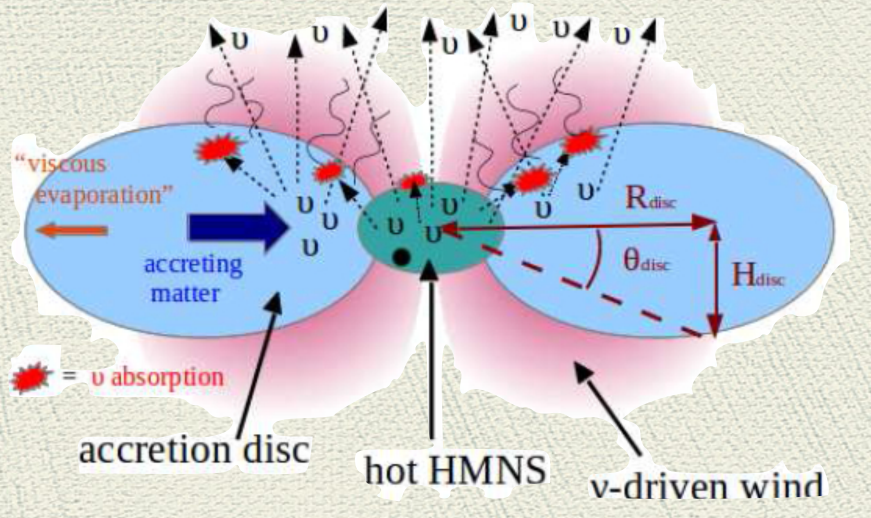
Theoretical aspects of neutrino propagation in dense environments

Maria Cristina Volpe

Laboratoire Astroparticule et Cosmologie (APC), Paris

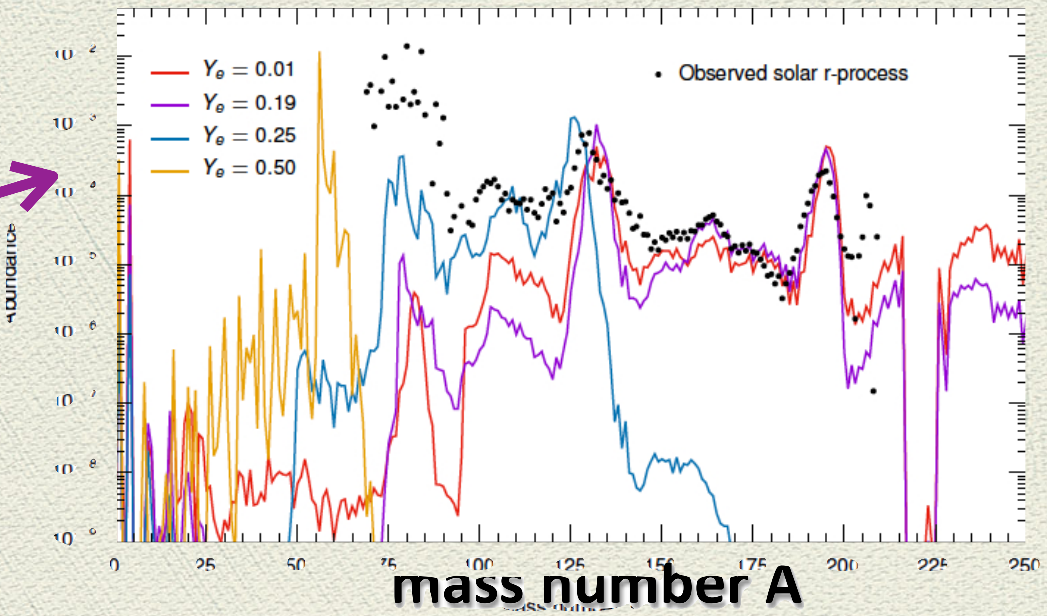


Observations

Predictions for future measurements :

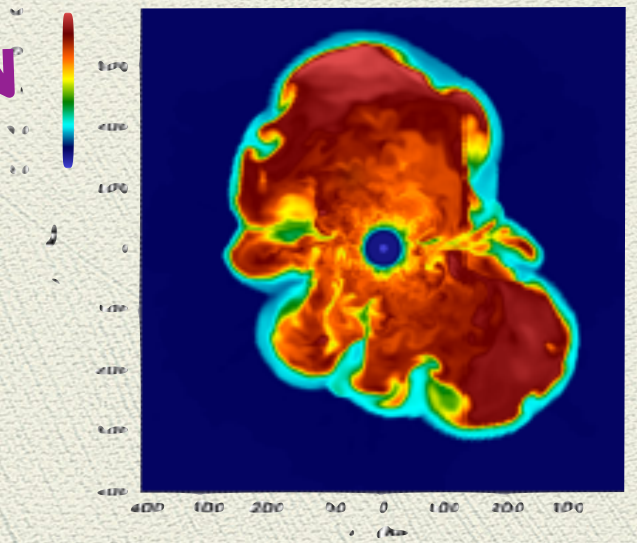
- diffuse supernova neutrino background, it starts - **Super-K + Gd**
- an (extra)galactic supernova - 10^4-10^6 events at 10 kpc - **SNEWS**

Heavy elements nucleosynthesis - **kilonovae**



Understanding the role of neutrinos and of flavor conversion in dense environments

Supernova explosion mechanism



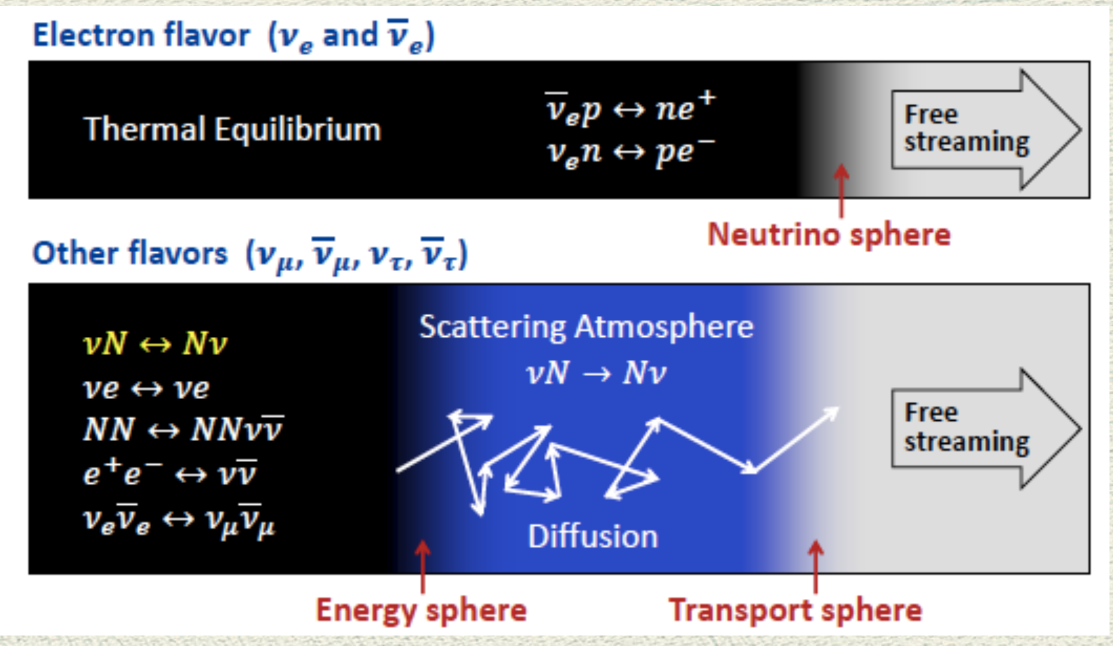
Mueller et al, 1705.00620

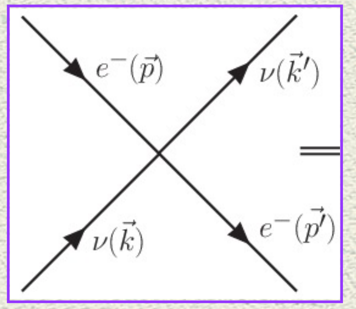
Outline

- Neutrino propagation in flat spacetime
  - Linearised equations
  - Extended mean-field equations
- Neutrino propagation in curved spacetime
  - Gravitational effects on neutrinos in dense environments
  - Decoherence on neutrino wave-packets

Neutrinos in dense environments

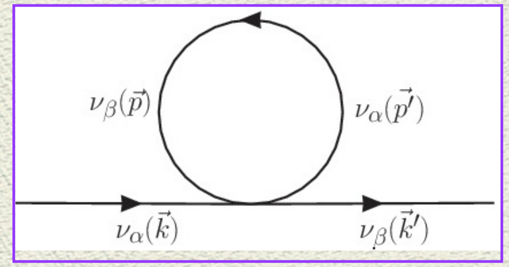
Raffelt (2012)





collisions dominated

Boltzmann approximation



flavor conversion

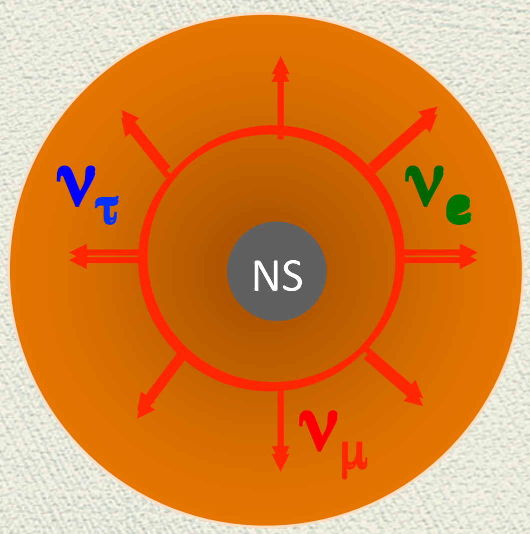
mean-field approximation

a non-linear many-body problem

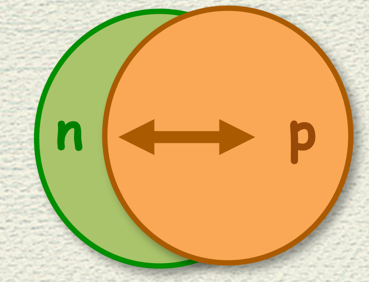
Pantaleone, PLB287 (1992)

the separation of scales…

Neutrino flavor evolution in dense environments : a many-body problem



ν in stars or accretion disks



atomic nucleus

10^{57}	**N**	200
weak	**interaction**	strong
unbound	**system**	bound
$\rho_{ji} = \langle a_i^{+} a_j \rangle$ neutrinos $\bar{\rho}_{ji} = \langle b_i^{+} b_j \rangle$ anti-neutrinos	**density**	$\rho = \langle a^{+} a \rangle$ neutrons protons

To determine the dynamics exactly:

$\rho_1 = \left\langle a^+ a \right\rangle$ one-body density

$\rho_{12} = \left\langle a^+ a^+ a a \right\rangle$ two-body

$\rho_{123} = \left\langle a^+ a^+ a^+ a a a \right\rangle$ three-body

... N-body

To determine the dynamics exactly:

$\rho_1 = \langle a^+ a \rangle$ one-body density $\quad \rho_{12} = \langle a^+ a^+ a a \rangle$ two-body $\quad \rho_{123} = \langle a^+ a^+ a^+ a a a \rangle$ three-body $\quad \dots$ N-body

$$i\dot{\rho}_1 = [t_1, \rho_1] + \mathrm{Tr}_{(2)} \{[v_{12}, \rho_{12}]\}$$

$$i\dot{\rho}_{12} = [t_1 + t_2 + v_{12}, \rho_{12}] + \mathrm{Tr}_{(3)} \{[v_{13} + v_{23}, \rho_{123}]\}$$

$$\vdots$$

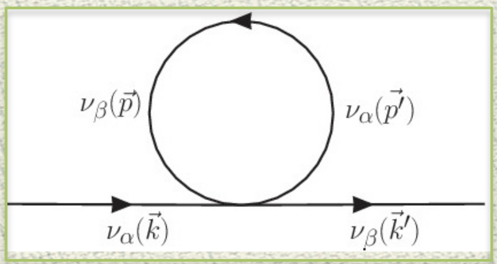
$$i\dot{\rho}_{1\cdots n} = \left[\sum_{i=1}^{n} t_i + \sum_{j>i=1}^{n} v_{ij}, \rho_{1\cdots n}\right] + \sum_{i=1}^{n} \mathrm{Tr}_{(n+1)} \left\{\left[v_{i(n+1)}, \rho_{1\cdots(n+1)}\right]\right\}$$

H = t + v
Hamiltonian

Born-Bogoliubov-Green-Kirkwood-Yvon (BBGKY) hierarchy

an infinite set of equations for a relativistic system

$$i\dot{\rho} = [h(\rho), \rho]$$



Liouville Von Neumann equations for neutrino and antineutrino one-body densities and beyond the usual mean-field.

Volpe et al., PRD 87 (2013)

Neutrino evolution equations in flat spacetime

mean-field approximation

Samuel, PRD48 (1993); Sawyer PRD72 (2005),
Pehlivan and Balantekin, J. Phys. G 34 (2007)

linearised mean-field equations

Banerjee, Dighe, Raffelt, PRD84 (2011)
Väänänen and Volpe PRD88 (2013)

A dispersion relation approach

Izaguirre, Raffelt, Tamborra, PRL 118 (2017)

mean-field and extended mean-field

Volpe, Väänänen Espinoza, PRD87 (2013)
Serreau and Volpe PRD90 (2014)

towards the many-body solution

Birol, Pehlivan, Balantekin, Kajino, PRD98 (2018)
Cervia, Patwardhan, Balantekin, Coppersmith, Johnson, 1908.03511

quantum kinetic equations

Sigl and Raffelt, Nucl. Phys. B 406 (1993)
Vlasenko, Fuller, Cirigliano, PRD89 (2014)

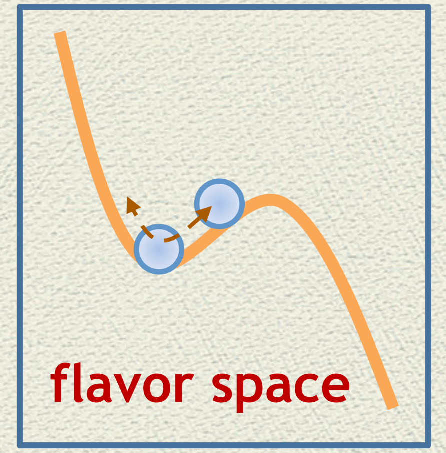
see Capozzi, Dasgupta, Mirizzi, Sen, Sigl, PRL 122 (2019), Richters, McLaughlin, Kneller, Vlasenko, PRD99 (2019)

Evolution equations derived with numerous theoretical frameworks

Volpe, Int. J. Mod. Phys. E24 (2015), a review

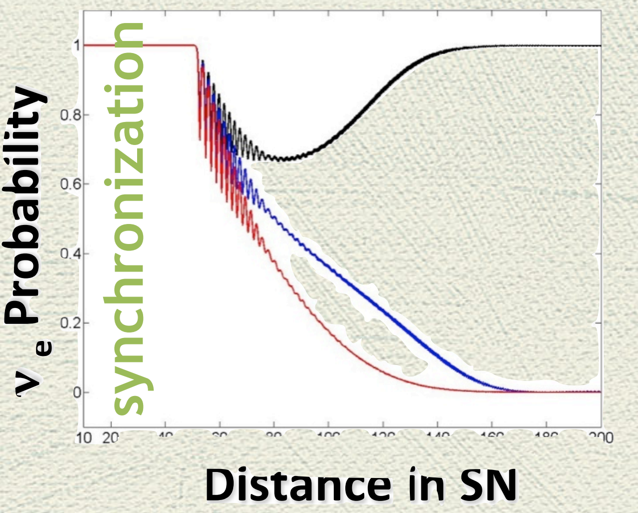
Linearised evolution equations

Banerjee, Dighe, Raffelt, PRD84 (2011)
Väänänen and Volpe PRD88 (2013)



$$i\dot{\rho} = [h(\rho), \rho] \qquad [h^0, \rho^0] = 0$$

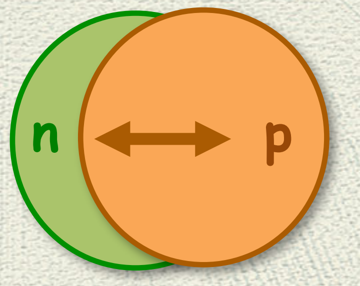
$$\delta\rho = \rho_0 + \delta\rho(t) = \rho^0 + \rho' e^{-i\omega t} + \rho'^{\dagger} e^{i\omega^* t}.$$



$$\begin{pmatrix} A & B \\ \bar{B} & \bar{A} \end{pmatrix} \begin{pmatrix} \rho' \\ \bar{\rho}' \end{pmatrix} = \omega \begin{pmatrix} \rho' \\ \bar{\rho}' \end{pmatrix}$$

S eigenvalues :
-> real : stable collective
-> imaginary : instabilities

The Random-Phase-Approximation (RPA) in the study of atomic nuclei.
Small amplitudes variations around a stationary solution.



nuclear resonances

The diagonalisation of the RPA matrix gives both collective and non-collective solutions - the Giant Resonances (particle-hole excitations).

Some of the solutions are spurious, i.e. unphysical…
(e.g. the displacement of the atomic nucleus as a whole).

Extended mean-field equations - spin/helicity coherence

- The most general mean-field equations include supplementary contributions from wrong helicity contributions because of the neutrino mass. Need anisotropy.

$\zeta = \langle a_+^+ a_- \rangle$ correlators with helicity change

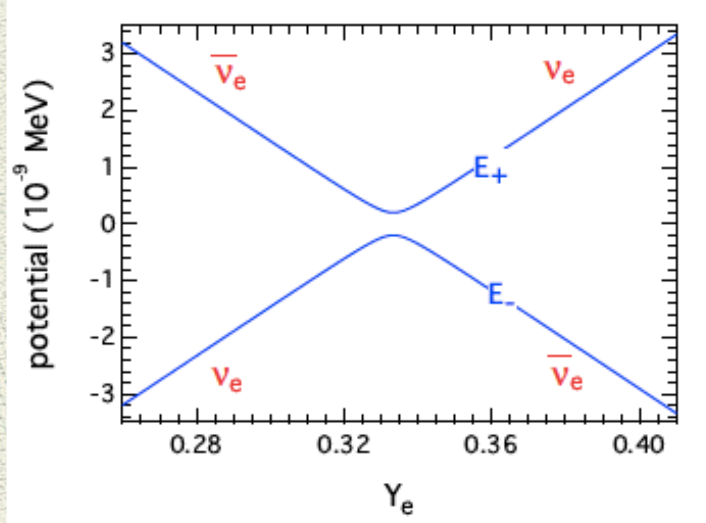
$\mathcal{R} = \begin{pmatrix} \rho & \zeta \\ \zeta^* & \bar{\rho} \end{pmatrix}$ $\quad \mathcal{H} = \begin{pmatrix} h & \Phi \\ \Phi^* & \bar{h} \end{pmatrix}$ ← Φ couples ν with $\bar{\nu}$

helicity (or spin) coherence

$\Phi \sim (h_{mat}^{perp} + h_{\nu\nu}^{perp}) \times m/2E$

$\mathcal{R}$ and $\mathcal{H}$ have helicity and flavor structure ($2\mathcal{N}_f \times 2\mathcal{N}_f$).

Vlasenko, Fuller, Cirigliano, PRD89 (2014)
Serreau,Volpe, PRD90 (2014)

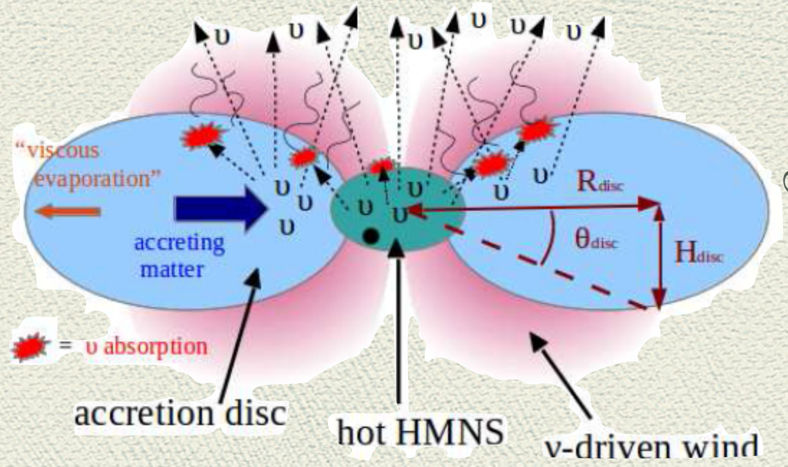
- 

<u>One flavor schematic model</u>
Resonance conditions fulfilled for helicity coherence.
Impact on neutrino fluxes found.

Vlasenko, Fuller, Cirigliano, arXiv: 1406.6724

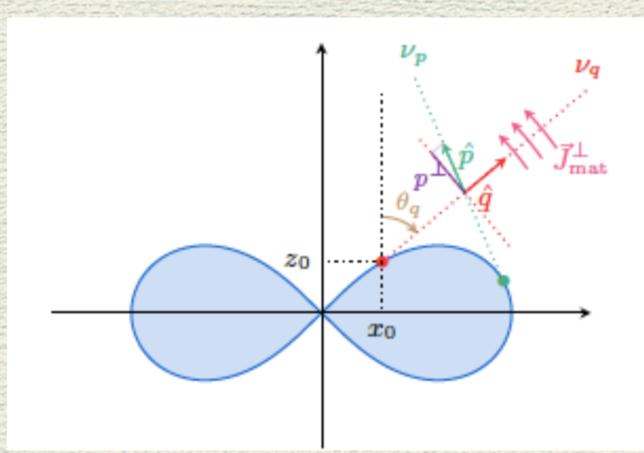
Flavor evolution and binary neutron star merger remnants

- In binary neutron star merger remnants, an electron antineutrino excess.



Perego et al, 2014

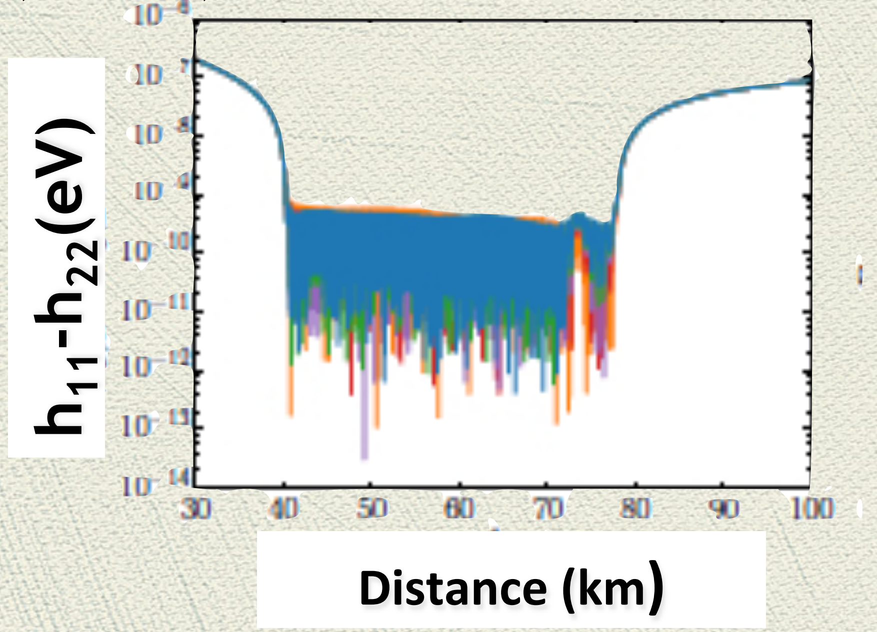
	$\langle E_\nu \rangle$	L_ν	R_ν (km)
ν_e	10.6	15	84
$\bar{\nu}_e$	15.3	30	60
ν_x	17.3	8	58
	MeV	10^{51} erg/s	

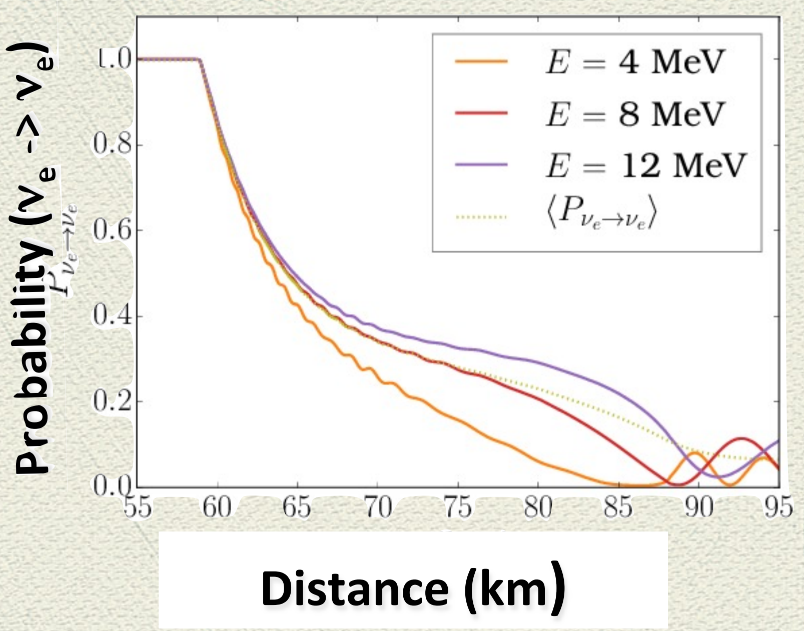


- Matter and self-interaction potentials can cancel : Matter-Neutrino Resonance.

Malkus et al, PRD86 (2012), 96 (2016)

$$h_{\mathcal{G},11} - h_{\mathcal{G},22} = -2\omega c_{2\theta} + \sqrt{2} G_F n_B Y_e + h^{ee}_{\nu\nu} - h^{xx}_{\nu\nu} \simeq 0.$$





Evolution for the matter-neutrino resonance (mostly) adiabatic.

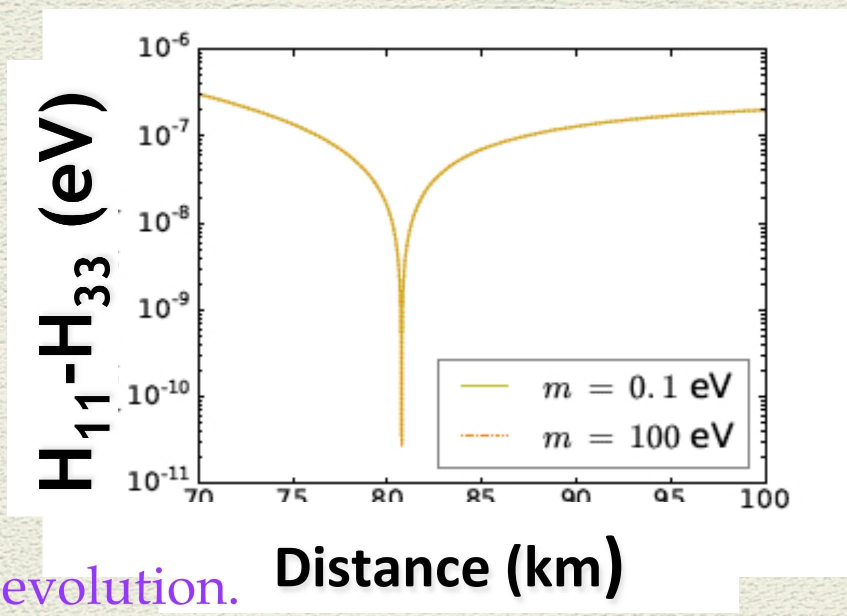
Does it survive if the full angular dependence in the self-interaction hamiltonian retained ?

Shalgar, JCAP 1802 (2018) ; Vlasenko and McLaughlin, PRD97 (2018)

Helicity coherence and non-linear feedback

- Two flavors : 4 conditions possible. Resonance conditions for helicity coherence similar to the matter-neutrino resonance.

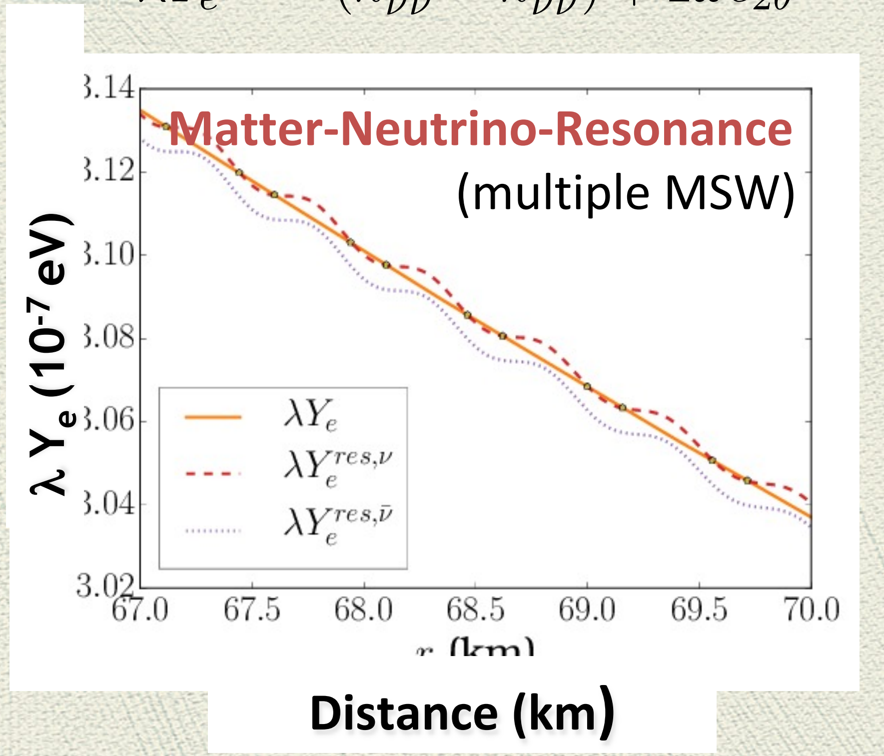
$$h_{\mathcal{G},11} - h_{\mathcal{G},33} = \sqrt{2} G_F n_B (3Y_e - 1) + 2h^{ee}_{\nu\nu} \simeq 0$$



- Performed detailed investigation on numerous trajectories, based on detailed simulations of binary neutron star merger remnants.

Non-linear feedback not sufficient for adiabatic evolution.

$$\lambda Y_e \simeq -(h^{ee}_{\nu\nu} - h^{xx}_{\nu\nu}) + 2\omega c_{2\theta} \qquad n_B(3Y_e - 1) \simeq -\frac{2}{\sqrt{2}G_F} h^{ee}_{\nu\nu}.$$



Perturbative analysis shows matching conditions between matter and self-interaction terms require peculiar matter densities.

Results hold for core-collapse supernovae.

Chatelain, Volpe, PRD 95, (2017)

Gravitational effects on neutrinos in dense environments

Strong gravitational fields nearby compact objects impacts
neutrino propagation, or flavor evolution through

- the vacuum oscillation phase, if matter outside the compact object neglected

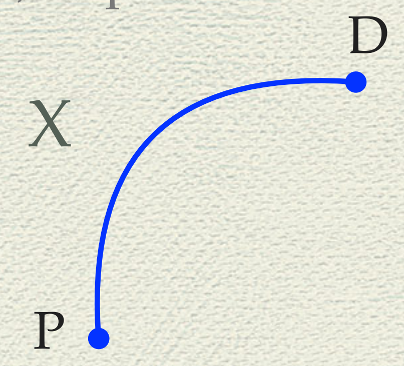
- the bending of neutrino trajectories

- the energy redshift

- a modification of wave packet decoherence

Neutrino propagation in curved spacetime

- The covariant phase acquired by a particle propagating from P to D, in presence of strong gravitational field, nearby a compact object



$$\phi_X(P,D) = \int m_X ds = \int_P^D p_\mu dx^\mu$$

with the canonical conjugate momentum to the coordinate x^μ
(ds - line element along the particle trajectory)

$$p_\mu = m_X g_{\mu\nu} \frac{dx^\nu}{ds}$$

with $g_{\mu\nu}$ the metric tensor and m particle mass.

L. Stodolsky, «Matter and light wave interferometry in gravitational fields», 11(1979)391

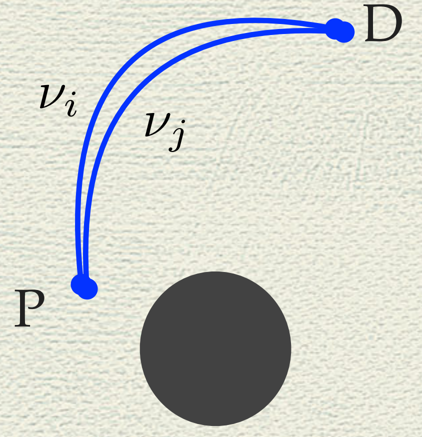
- The covariant phase acquired by the j-th mass eigenstate of mass m_j

$$\phi_j(P,D_j) = \int_P^{D_j} p_\mu^{(j)} \mathrm{d}x^\mu$$

discussed in a series of papers with different metrics - Schwarzschild, Kerr, …(often in the weak field limit)

Ahluwalia et al, Gen. Relativ. Gravit 29 (1997), Bhattacharya et al, PRD59 (1999), Lambiase et al, PRD71 (2005), Visinelli, Gen. Relativ. Gravit. 47 (2015), …

Vacuum oscillation phase in presence of gravitational fields



- For a stationary gravitational field from a compact object with spherical symmetry, the corresponding Schwarzschild metric is

$$\mathrm{d}s^2 = -B(r)\mathrm{d}t^2 + \frac{1}{B(r)}\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta d\varphi^2 \quad \text{with} \;\; B(r) = 1 - \frac{r_s}{r} \qquad r_s = 2M$$

- Since it is isotropic, the trajectories are confined to a plane. $(\theta = \pi/2, d\theta = 0)$

- The canonical momenta

$$p_t^{(j)} = -m_j B(\vec{r})\frac{\mathrm{d}t}{\mathrm{d}s},$$
$$p_\varphi^{(j)} = m_j r^2 \frac{\mathrm{d}\varphi}{\mathrm{d}s}$$
$$p_r^{(j)} = \frac{m_j}{B(\vec{r})}\frac{\mathrm{d}r}{\mathrm{d}s},$$

are related by the mass on-shell relation

$$m_j^2 = g^{\mu\nu}p_\mu^{(j)}p_\nu^{(j)} = \frac{1}{B(r)}(p_t^{(j)})^2 - B(r)(p_r^{(j)})^2 - \frac{(p_\phi^{(j)})^2}{r^2}$$

- Interference phase between the kth and jth mass eigenstates, in the radial case $d\phi = 0$ along the light-ray trajectory from P to D, in the approximation $m_j^2 << E_j^2$ at infinity :

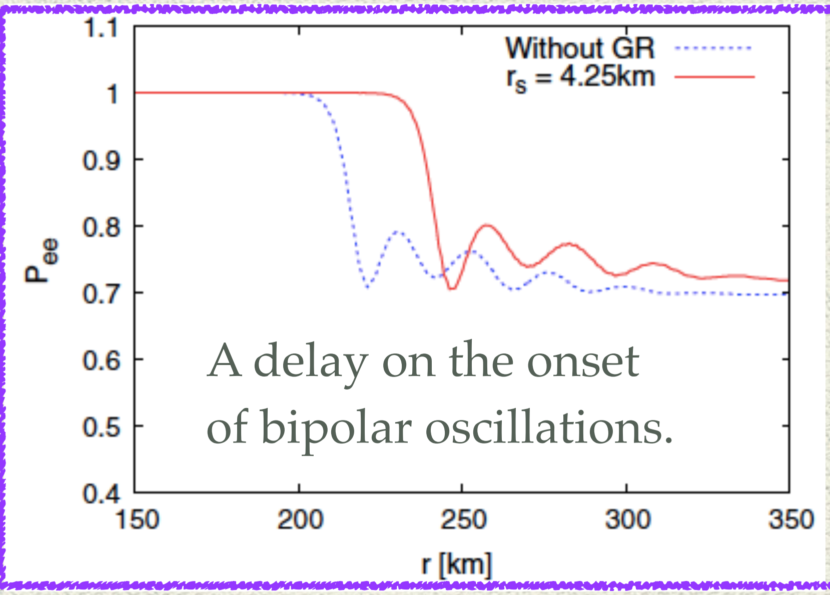
$$\boxed{\Phi_{kj} \simeq \frac{\Delta m_{kj}^2}{2E_0}|r_D - r_P|}$$

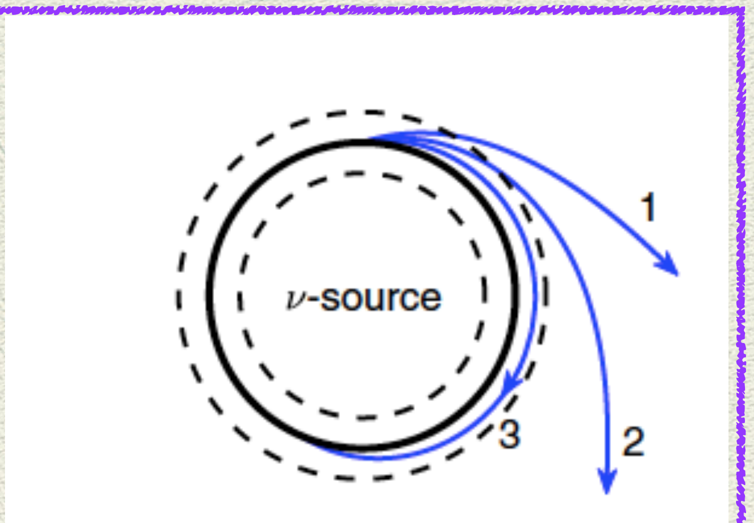
E_0 constant energy at infinity for a massless particle, r is the local radial coordinate (not a physical distance)

- Gravity affects location and adiabaticity of the Mikheev-Smirnov-Wolfenstein (MSW) effect (energy redshift).

Fornengo, Giunti, Kim, Song, « Gravitational effects on neutrino oscillations », PRD56 (1997) ; Cardall and Fuller, « Neutrino oscillations in curved spacetime : a heuristic treatment », PRD55(1997)7960

- The « bulb » model with matter and ν self-interactions outside the compact object. Self-interactions enhanced. A neutrino «halo» can be formed.



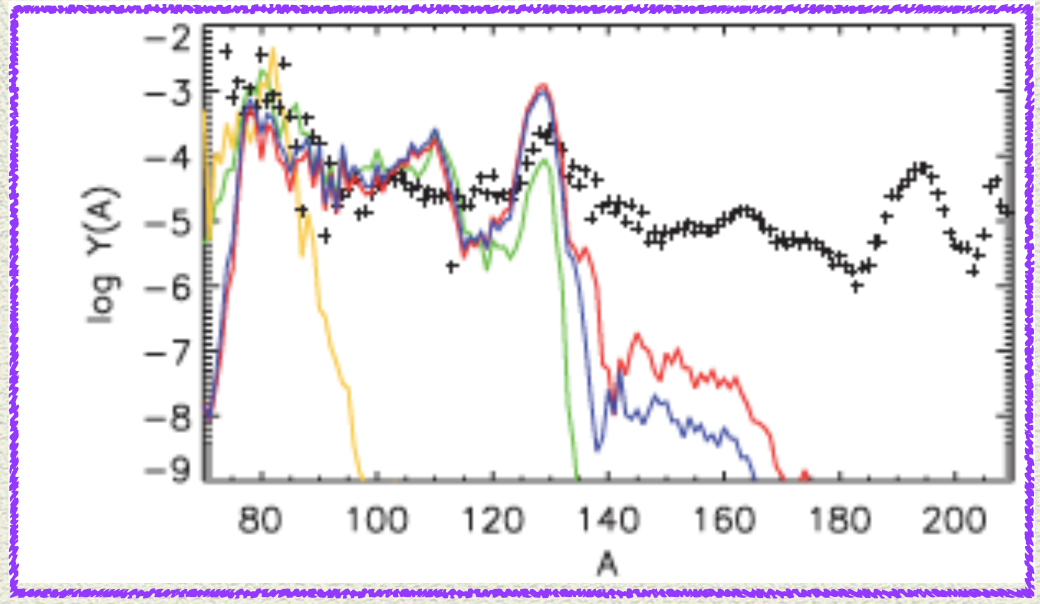


A neutrino « halo »

Yang and Kneller, PRD96 (2017)

Gravitational effects on nucleosynthesis

- The inclusion of trajectory bending on neutrino propagation and energy redshifts can impact the neutrino spectra and r-process nucleosynthetic outcomes. An example for an accretion-disk black hole model with Schwarzschild and Kerr metrics (no flavor transformation included).

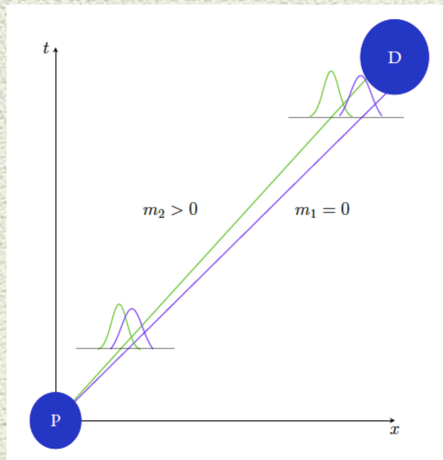


Caballero, McLaughlin and Surman,Astr. Journ. 745 (2012).

Works in curved spacetime consider neutrinos as plane waves.
Wave-packets account for neutrinos, as localized particles. Then one can have

Neutrino decoherence by wave packet propagation

- Neutrino mass eigenstates wave packets (WP) can loose coherence during propagation. In flat spacetime, the neutrino coherence length L_{coh} quantifies the distance at which coherence is lost.



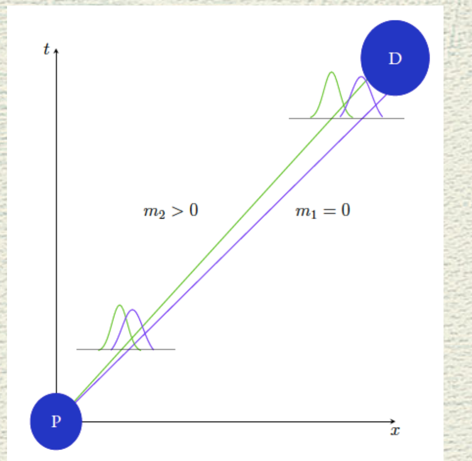
$$L = L_{coh} \quad \Delta x = \sigma_x$$

σ_x- WP width

$$L_{\mathrm{coh}}^{jk} = \frac{4\sqrt{2}E^2}{\left|\Delta m_{jk}^2\right|}\sigma_x$$

see e.g. Giunti, Found. Phys. Lett. 17 (2004);
Kersten and Smirnov, Eur. Phys. J. C76 (2016) ;
Akhmedov, Kopp, Lindner, JCAP 1709 (2017)

Wave packet decoherence in flat spacetime



- The coordinate-space wave function is

$$\psi_j(t,\vec{x}) = \int_{\vec{p}} e^{i\vec{p}\cdot\vec{x}}\psi_j(t,\vec{p}) \qquad \int_{\vec{p}} \equiv \int \frac{d^3p}{(2\pi)^3}$$

whose Fourier components evolve according to

$$\psi_j(t,\vec{p}) = f_{\vec{p}_j}(\vec{p})e^{-iE_j(\vec{p}))t}$$

with $f_{\vec{p}_j}(\vec{p})$ the momentum distribution functions centered in $\vec{p}_j$, describing the wave-packet associated to the **j-th** mass eigenstate.

- The one-body density matrix is $\rho_{jk}(t,\vec{x}) = U^*_{\alpha j}U_{\alpha k}\psi_j(t,\vec{x})\psi^*_k(t,\vec{x})$.

- Assuming Gaussian wave packets

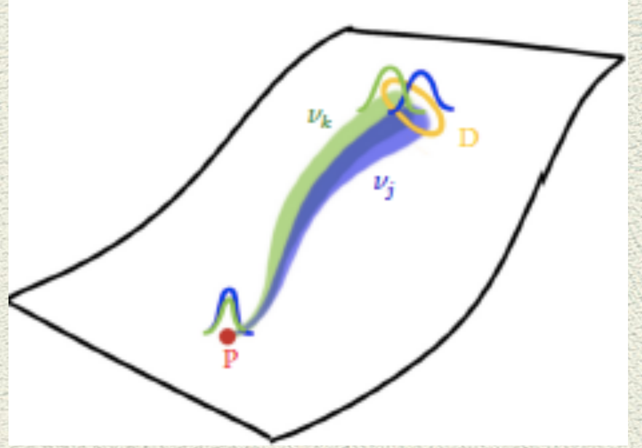
$$\rho_{jk}(t,\vec{x}) = N^{\alpha}_{jk}\int_{\vec{p},\vec{q}} \exp\left[-i\left[E_j(\vec{p}) - E_k(\vec{p})\right]t\right\}\exp\left[i(\vec{p}-\vec{q})\vec{x} - \frac{(\vec{p}-\vec{p}_j)^2}{4\sigma_p^2} - \frac{(\vec{q}-\vec{p}_k)^2}{4\sigma_p^2}\right],$$

- In order to calculate the coherence length, we expand the energies around the peak momenta $E_j(\vec{p}) = E_j + (\vec{p}-\vec{p}_j)\vec{v}_j + \mathcal{O}\left[(\vec{p}-\vec{p}_j)^2\right]$.

- By performing Gaussian integrals one gets $\rho_{jk}(\vec{x}) = A^{\alpha}_{jk}\ \rho^{osc}_{jk}(\vec{x})\ \rho^{damp}_{jk}(\vec{x})$

$$\rho^{damp}_{jk}(\vec{x}) = \exp\left[-\frac{(\vec{v}_j - \vec{v}_k)^2x^2}{4\sigma_x^2(v_j^2+v_k^2)}\right] \qquad L^{jk}_{\text{coh}} = \frac{4\sqrt{2}E^2}{\left|\Delta m^2_{jk}\right|}\sigma_x$$

Wave packet decoherence in curved spacetime



- A neutrino flavor state produced in a spacetime point P

$$|\nu_\alpha(P)\rangle = U^*_{\alpha j}|\nu_j(P)\rangle$$

evolves to a « detection » point D

$$|\nu_j(P, D_j)\rangle = e^{-i\phi_j(P,D_j)}|\nu_j(P)\rangle,$$

with the covariant form of the quantum mechanical phase

$$\phi_j(P, D_j) = \int_P^{D_j} p^{(j)}_\mu \mathrm{d}x^\mu \qquad p^{(j)}_\mu = m_j g_{\mu\nu}\frac{\mathrm{d}x^\nu}{\mathrm{d}s}$$

- By performing Gaussian integrals one gets

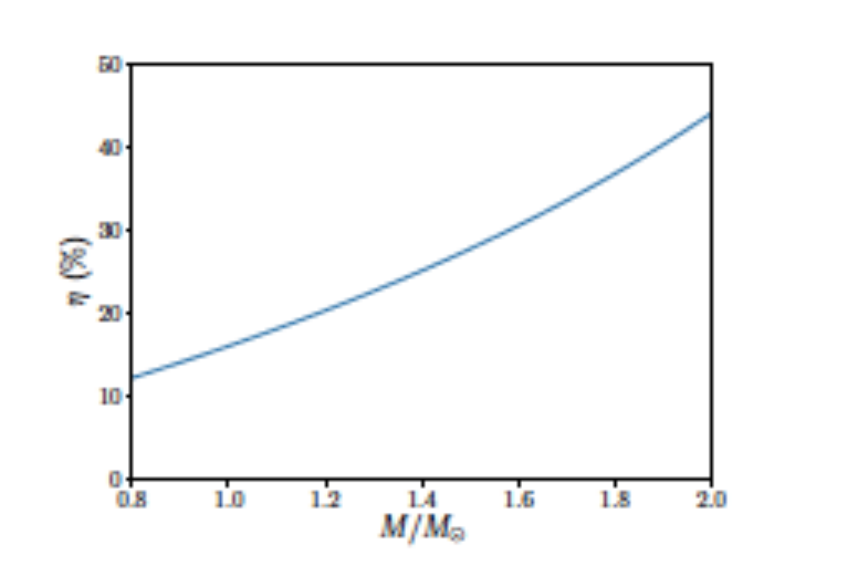
$$\rho_{jk}(r_P, r_D) = A^\alpha_{jk}\ \rho^{osc}_{jk}(r_P, r_D)\ \rho^{damp}_{jk}(r_P, r_D)$$

$$\rho^{damp}_{jk}(r_P, r_D) = \exp\left[-\frac{\Delta m^4_{jk} r^2_{PD}}{32\sigma^2_x E^4}\right],$$

$$r^{coh}_{PD} = \frac{4\sqrt{2}\sigma_x E^2}{\Delta m^2_{jk}}$$

$$\tau_{coh} = \sqrt{B(r^{coh}_D)}\left[r^{coh}_{PD} + r_s \ln\left(1 + \frac{r^{coh}_{PD}}{r_P - r_s}\right)\right]$$

$$\eta = \frac{\tau_{coh} - L_{coh}}{L_{coh}}(\%)$$



$E = 11\ MeV\ \ r_P = 10\ km\ \ \sigma_x = 4\times 10^{-12} cm$

Chatelain and Volpe, arXiv:1906.12152

Conclusions and perspectives

- Investigations of the neutrino evolution equations in dense media have brought numerous interesting developments - linearised, dispersion relation, QKE and connections to other many body systems, opening new possibilities.

Many body problems means complexity…

- Most general mean-field equations include helicity coherence. A two-flavor model based on detailed simulations does not produce significant flavor modification.

Non-linear feedback does not appear to operate in dense environments (perturbative argument).

- Strong gravitational fields influence neutrino propagation, flavor evolution, nucleosynthesis and neutrino decoherence by wave packet separation…