Neutrino Quantum Kinetic Equations: The Collision Term

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*based mostly on DNB & V. Cirigliano, PRD **94** (2016) 033009*

current collaborators on this topic:

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Motivation

Why study neutrino collisions in detail and
why use quantum kinetic equations (QKE)?

Good description of neutrino interactions

- Neutrino interactions impact abundance of heavy elements in neutrino driven winds in supernovae, BH accretion-discs, neutron-star mergers

- Needed for complete description of neutrino transport in early universe, core collapse supernovae and compact mergers

SN 1994D, NGC 4526 galaxy

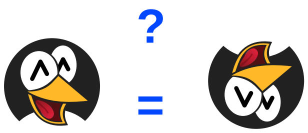
- Quantum Kinetic Equations (QKE): evolution of ensemble of neutrinos in hot dense media

- Closed Time Path formalism for non-equilibrium QFT

Why QKE?

- Account for **kinetic**, **flavor** and **spin** degrees of freedom: study interaction of all flavors of neutrinos with electrons, protons, neutrons
- More detailed than mean field approach
- Anisotropic regions: spin-flip yields
  - Neutrino-antineutrino transformation for Majorana neutrinos
  - Active-sterile transformation for Dirac neutrino
  - QKEs depend on absolute mass scale

→ *Possible path to distinguishing between Majorana vs Dirac neutrinos*



Further details: Cirigliano, Fuller, Vlasenko, *Phys.Lett.* **B747** (2015) 27; Serreau, Volpe, *PRD* **90** (2014) 125040

Background

Review setup and notation:

QKEs in the closed time path formalism, approximations for neutrinos in hot, dense medium

Effective interactions

Assume neutrino energy below electroweak scale (<< 100 GeV), effective Lagrangians after integrating out *W, Z*:

$$\mathcal{L}_{\nu\nu} = -\frac{G_F}{\sqrt{2}}\, \bar{\nu}\gamma_\mu P_L \nu\ \bar{\nu}\gamma^\mu P_L \nu,$$

$$\mathcal{L}_{\nu e} = -2\sqrt{2}G_F \left(\bar{\nu}\gamma_\mu P_L \underline{Y_L} \nu\ \bar{e}\gamma^\mu P_L e + \bar{\nu}\gamma_\mu P_L \underline{Y_R} \nu\ \bar{e}\gamma^\mu P_R e\right)$$

$$\mathcal{L}_{\nu N} = -\sqrt{2}G_F \sum_{N=p,n} \bar{\nu}\gamma_\mu P_L \nu\ \bar{N}\gamma^\mu \underline{\left(C_V^{(N)} - C_A^{(N)}\gamma_5\right)} N\,,$$

$$\mathcal{L}_{CC} = -\sqrt{2}G_F\, \bar{e}\gamma_\mu P_L \nu_e\, \bar{p}\,\gamma^\mu \left(1 - \underline{g_A}\gamma_5\right) n + \text{h.c.}$$

$$P_{L,R} = (\mathbb{1} \mp \gamma_5)/2\,, \qquad \nu = \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}.$$

Neutrinos in hot/dense medium

ensemble of neutrinos described by incoherent mixture of states

neutrinos $\langle a^{\dagger}_{j,h'}(\vec{k}')\, a_{i,h}(\vec{k})\rangle \propto \delta^{(3)}(\vec{k}-\vec{k}')f^{ij}_{hh'}(\vec{k})$

antineutrinos $\langle b^{\dagger}_{j,h'}(\vec{k}')\, b_{i,h}(\vec{k})\rangle \propto \delta^{(3)}(\vec{k}-\vec{k}')\bar{f}^{ij}_{hh'}(\vec{k})$

$2n_f \times 2n_f$ matrix structure: Dirac case, need F and $\bar{F}$

$$F = \begin{pmatrix} f_{LL} & f_{L,R} \\ f_{R,L} & f_{RR} \end{pmatrix}$$

active-sterile coherence

$$\bar{F} = \begin{pmatrix} \bar{f}_{RR} & \bar{f}_{R,L} \\ \bar{f}_{L,R} & \bar{f}_{LL} \end{pmatrix}$$

Majorana case: $F = \begin{pmatrix} f & \phi \\ \phi^{\dagger} & \bar{f}^{T} \end{pmatrix}$

neutrino-antineutrino coherence

Power counting / approximations

Assume neutrino masses, mass-splitting, matter potentials (induced by forward scattering), and external gradients are much smaller than neutrino energy:

$$m_\nu / E \sim \Delta m_\nu / E \sim \Sigma_{\mathrm{forward}} / E \sim \partial_X / E \sim O(\epsilon)$$

$$\Sigma_{\mathrm{inelastic}} / E \sim O(\epsilon^2)$$

i.e. assume physical quantities vary slowly on the scale of the neutrino de Broglie wavelength

QKEs include second order effects $\mathrm{O}(\epsilon^2)$

details: Vlasenko, Fuller, Cirigliano, *PRD* **89** (2014) 105004

Quantum Kinetic Equations (QKE)

$$i\,DF = [H, F] + i\,C$$

generalized "Vlasov" term

coherent evolution, generalizes MSW

collision term

$$H = \begin{pmatrix} H_R & H_m \\ H_m^\dagger & H_L \end{pmatrix}, \quad H_m$$ depends ***linearly*** on the neutrino mass

Spin flip sensitive to absolute mass scale!

details: Vlasenko, Fuller, Cirigliano, *PRD* **89** (2014) 105004;
Cirigliano, Fuller, Vlasenko, *Phys.Lett.* **B747** (2015) 27;
Vlasenko, Fuller, Cirigliano, arXiv:1406.6724

Collision term

$$C = \frac{1}{2}\left\{\Pi^{+}, F\right\} - \frac{1}{2}\left\{\Pi^{-}, \mathbb{1} - F\right\}$$

$$\Pi^{\pm} = \begin{pmatrix} \Pi_R^{\kappa\pm} & 2P^{\pm} \\ 2P^{\pm\dagger} & \Pi_L^{\kappa\pm} \end{pmatrix}$$

($2n_f \times 2n_f$ matrix)

$$F = \begin{pmatrix} f & \phi \\ \phi^{\dagger} & \bar{f}^T \end{pmatrix}$$

(occupation # in diagonal,
coherence in off-diagonal)

The collision term has a non-diagonal matrix structure in both flavor and spin space.

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Results

DNB, V. Cirigliano, *Phys. Rev.* **D94** (2016) 033009, (Editors' suggestion)

Contributions to the collision term

Π :

- Neutrino-nucleon scattering processes
- Neutrino absorption and emission (charged-current processes)
- Neutrino-electron processes
- Neutrino-neutrino processes

} only left topology

DNB, V. Cirigliano, *Phys. Rev.* **D94** (2016) 033009, (Editors' suggestion)

Example: NN-scattering

$$\Pi^{\pm}_{ab}(k) = -2G_F^2 \int \frac{d^4q_1\, d^4q_2\, d^4q_3}{(2\pi)^8} \delta^{(4)}(k - q_3 - q_1 + q_2)$$

$$\times \sum_{N=n,p} \Big\{ \gamma_\mu (P_L - P_R) G^{(\nu)\pm}_{ab}(q_3) \gamma_\nu (P_L - P_R)$$

$$\times \mathrm{Tr}\Big[\Gamma^\nu_N\, G^{(N)\mp}(q_2)\, \Gamma^\mu_N\, G^{(N)\pm}(q_1) \Big] \Big\}$$

$$G^{(N)+}(p) = 2\pi\delta(p^2 - m_N^2)(\not{p} + m_N)\Big[\theta(p^0)(1 - f(\vec{p})) - \theta(-p^0)\bar{f}(-\vec{p})\Big]$$

Neglect neutrino mass in these expressions because the collision term is already second order $\mathrm{O}(\epsilon^2)$

12 integrals and 7 delta functions

Approximations for the early universe

- isotropy and homogeneity
- Assume all statistical functions depend only on the absolute values of the momenta (not their angles), spin coherence disappears.
- Therefore can explicitly integrate over all angles (e.g. following techniques of Dolgov, Hansen & Semikoz 1997).
- Initially, have a total number of 9 integrals and 4 delta functions; integrating all angles (in k-space) leaves us with 3 integrals and 1 delta function.

→ ***2-dim. Integrals can be solved numerically***

Example: neutrino-neutrino processes

spin coherence disappears in early universe, therefore:

$$C = -\frac{G_F^2}{E_k^2}\int \frac{dE_1 dE_2 dE_3}{2\pi^3}\Big(\Big(E_1E_3D_2(E_1,E_3;E_2,E_k) + \underline{D_3(E_1,E_2,E_3,E_k)}$$

$$+ E_2E_k\underline{D_2(E_2,E_k;E_1,E_3)} + E_1E_2E_3E_k\underline{D_1(E_1,E_2,E_3,E_k)}\Big)\times$$

$$\times\Big(\delta(E_k - E_3 - E_1 + E_2)\Big\{\Big(\mathrm{tr}\big((1-f_1)f_2\big) + (1-f_1)f_2\Big)(1-f_3), f\Big\}$$

$$+ \delta(E_k - E_3 + E_1 - E_2)\Big\{\Big(\mathrm{tr}\big(\bar f_1(1-\bar f_2)\big) + \bar f_1(1-\bar f_2)\Big)(1-f_3), f\Big\}$$

$$+ \delta(E_k + E_3 - E_1 - E_2)\Big\{\Big(\mathrm{tr}\big((1-f_1)(1-\bar f_2)\big) + (1-f_1)(1-\bar f_2)\Big)\bar f_3, f\Big\}\Big)$$

$$- f \leftrightarrow (1-f)$$

where D_i are polynomials in E_i (Dolgov, Hansen, Semikoz 1997)

→ *multi-flavor generalization*

Further reading:

1. D. N. Blaschke, V. Cirigliano, *PRD **94** (2016) 033009, (Editors' suggestion)*

2. V. Cirigliano, G. M. Fuller, A. Vlasenko, *Phys. Lett.* **B747** (2015) 27

3. A. Vlasenko, G. M. Fuller, V. Cirigliano, *PRD* **89** (2014) 105004 and arXiv:1406.6724

4. E. Grohs, G. M. Fuller, C. T. Kishimoto, M. W. Paris, A. Vlasenko, *PRD* **93** (2016) 083522

5. L. Johns, G. M. Fuller, *PRD* **95** (2017) 043003

6. V. Cirigliano, M. W. Paris, S. Shalgar, *Phys. Lett.* **B774** (2017) 258

Thank you for your attention!