Neutrino Quantum Kinetics in the Early Universe

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Neutrino Quantum Kinetics in Dense Environments

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Mission

A full Quantum Kinetic treatment of neutrinos in the Early Universe

Past Victories

1. Boltzmann Equation treatment of neutrino energy transport
2. 2-flavor "Mock-up" calculation of neutrino energy and flavor transport

Weak Decoupling: Overview

1. Initially: neutrinos at the same temperature as electrons and positrons
2. Electrons and positrons annihilate to produce photon pairs, slightly raising temperature of plasma
3. Two processes create heat flow between neutrinos and plasma

$$\nu_i + e^{\pm} \leftrightarrow \nu_i + e^{\pm}$$

$$\nu_i + \overline{\nu}_i \leftrightarrow e^- + e^+$$

4. Three processes redistribute energy within neutrino seas

$$\nu_i + \nu_j \leftrightarrow \nu_i + \nu_j$$

$$\nu_i + \overline{\nu}_j \leftrightarrow \nu_i + \overline{\nu}_j$$

$$\nu_i + \overline{\nu}_i \leftrightarrow \nu_j + \overline{\nu}_j$$

5. End result: neutrinos cooler than photons

Weak Decoupling: Collision Term

Reaction scheme: $1 + 2 \leftrightarrow 3 + 4$

$$\frac{df(q_1)}{dt} = \frac{1}{2q_1} \int \tilde{d}q_2 \tilde{d}q_3 \tilde{d}q_4 \langle |\mathcal{M}|^2 \rangle (2\pi)^4 \delta^4(q_1 + q_2 - q_3 - q_4)$$
$$\times [f_3 f_4 (1 - f_1)(1 - f_2) - f_1 f_2 (1 - f_3)(1 - f_4)]$$

Phase Space

$$\tilde{d}q_i = \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

Occupation number

$$n = \int \frac{d^3 p}{(2\pi)^3} f(p) = \frac{1}{2\pi^2} \int dp\, p^2 f(p)$$

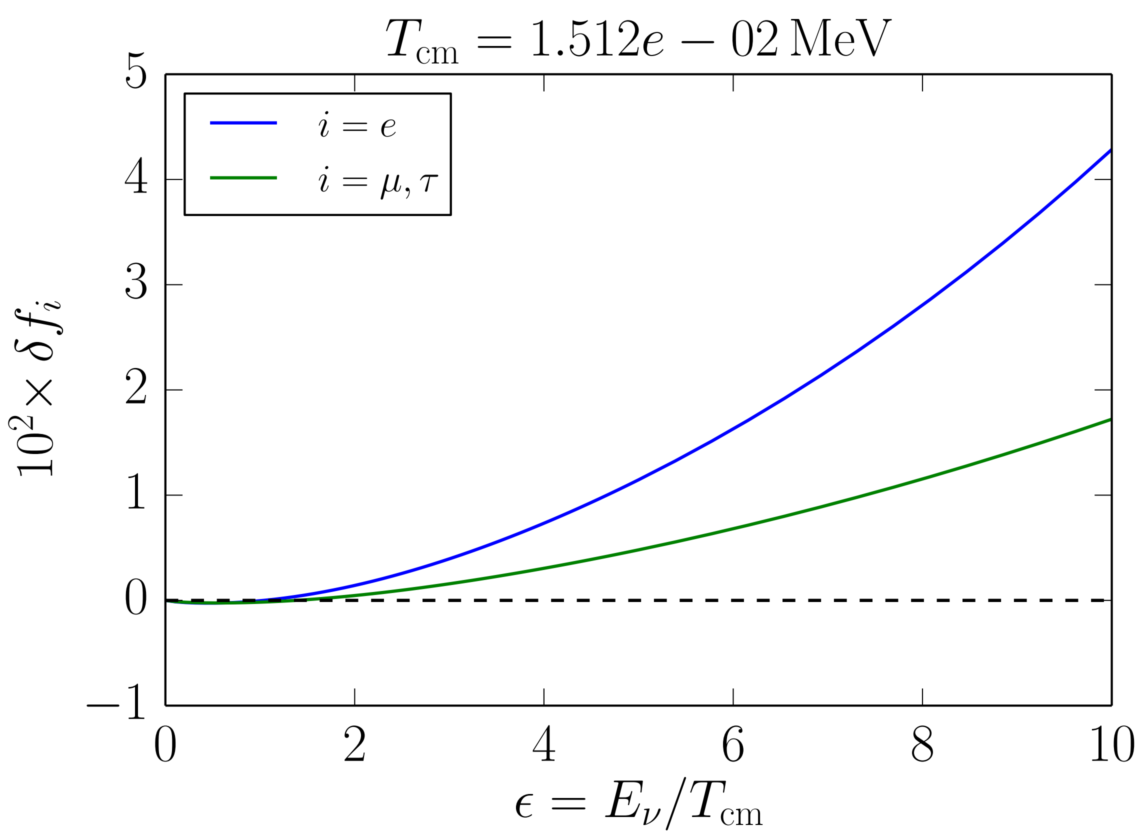
Matrix Element

$$\langle |\mathcal{M}|^2 \rangle \sim G_F^2 g(q_1, q_2, q_3, q_4)$$

4-momentum conservation

$$E_1 + E_2 = E_3 + E_4$$
$$\vec{p}_1 + \vec{p}_2 = \vec{p}_3 + \vec{p}_4$$

Out-of-Equilibrium Transport



Differential Visibility of Neutrino-Electron Scattering

Out-of-Equilibrium Neutrino Transport

$$\nu_i + \overline{\nu}_i \leftrightarrow e^- + e^+$$

$$\nu_i + e^\pm \leftrightarrow \nu_i + e^\pm$$

Red contours of constant differential visibility for electron flavor

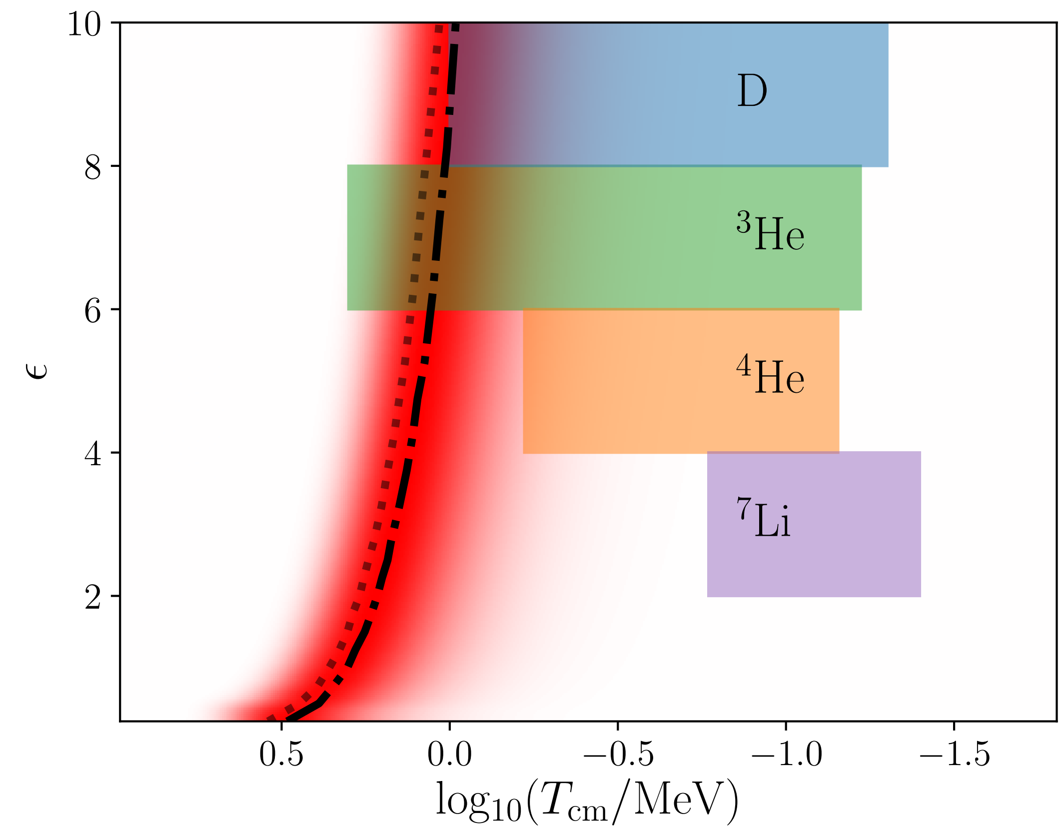
$$\frac{\Gamma'_{\nu_i}}{H} e^{-\tau_{\nu_i}}$$

High $T_{\rm cm}$

$\tau_{\nu_i} >> 1$

Low $T_{\rm cm}$

$\Gamma'_{\nu_i} << H$



c/o Matthew J. Wilson

QKEs in the Early Universe

Change array dimensions (Majorana or Dirac):

$$\{f_i(\epsilon)\}, \{\overline{f}_i(\epsilon)\} \rightarrow f_{ij}(\epsilon), \overline{f}_{ij}(\epsilon)$$

2 Generalized 3 × 3 density matrices (ϕ=0)

Equations of motion for neutrinos:

$$\frac{df}{dt} = -i[H, f] + \hat{C}(f, \overline{f})$$

H: Hamiltonian-like potential (coherent)

$\hat{C}$: Collision term from Blaschke & Cirigliano (2016)

Nonlinear coupled ODEs

Coherent Term in the Early Universe

$$H = H_V + H_A + H_S$$

$$H_V = \frac{1}{2\epsilon T_{\mathrm{cm}}} U M^2 U^\dagger$$

Vacuum Oscillations

$$H_A = \sqrt{2} G_F (L + \widetilde{L})$$

Asymmetric Term (proportional to number difference)

$$H_S = -\frac{8\sqrt{2} G_F \epsilon T_{\mathrm{cm}}}{3 m_W^2} (E + \cos^2\theta_W \widetilde{E})$$

Symmetric term (proportional to energy density)

Collisions

Positron-Electron Annihilation

$$\nu(k)\bar{\nu}(q_3) \rightarrow e^+(q_2)e^-(q_1)$$

Loss Potential (Blaschke & Cirigliano 2016)

$$\Pi_R^+(k) = \frac{-32G_F^2}{|\vec{k}|}\int \tilde{d}q_1\tilde{d}q_2\tilde{d}q_3(2\pi)^4 \sum_{I=L,R}\Big[(1-f_{e,1})(1-\bar{f}_{e,2}) \times Y_I\bar{f}_3\Big(2Y_I\mathcal{M}_I^R(q_1,-q_2,-q_3,k) - Y_{J\neq I}\mathcal{M}_m(q_1,-q_2,-q_3,k)\Big)\Big]$$

Amplitudes

$$\mathcal{M}_I^R(q_1,q_2,q_3,k) = \Big(\delta_I^L(q_3q_1)(kq_2) + \delta_I^R(q_3q_2)(kq_1)\Big)\delta^{(4)}(k-q_3-q_1+q_2)$$

$$\mathcal{M}_m(q_1,q_2,q_3,k) = m_e^2(kq_3)\delta^{(4)}(k-q_3-q_1+q_2)$$

Collisions (Cont.)

Matrices in Weak eigenbasis

$$Y_L = \begin{bmatrix} \frac{1}{2} + \sin^2\theta_W & 0 & 0 \\ 0 & -\frac{1}{2} + \sin^2\theta_W & 0 \\ 0 & 0 & -\frac{1}{2} + \sin^2\theta_W \end{bmatrix}$$

$$Y_R = \sin^2\theta_W \times \mathbb{1}$$

Collision Term

$$C[f(\epsilon)] = \frac{1}{2}\{\Pi_R^+, f(\epsilon)\} - \frac{1}{2}\{\Pi_R^-, \mathbb{1} - f(\epsilon)\}$$

2-flavor "Mock-up" Calculation

1. Ensure calculation is CP-symmetric

$$H_A = \sqrt{2} G_F \widetilde{L}$$

2. Reduce collision terms

$$\nu + \overline{\nu} \leftrightarrow e^- + e^+$$

3. Change mixing angles

$$\theta_{13} = \theta_{23} = 0$$

4. Change couplings in $e^{\pm}$ annihilation collision term

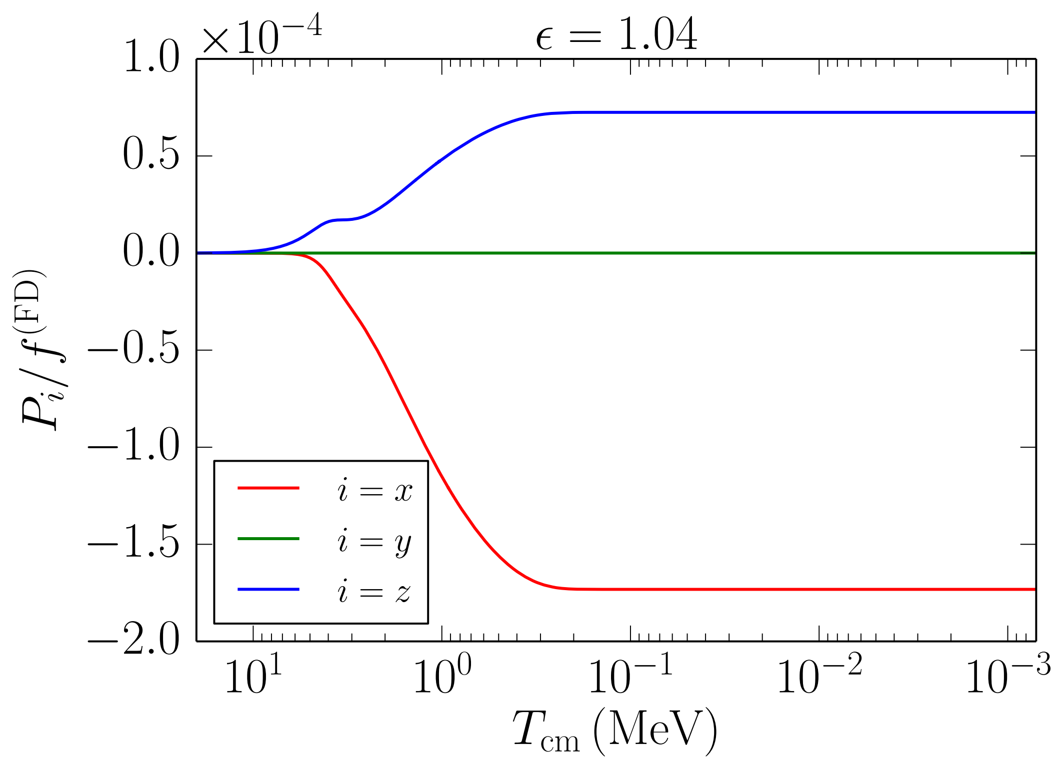
$$Y_{L,\tau\tau} = Y_{R,\tau\tau} = 0$$

5. End result: τ-flavor inert

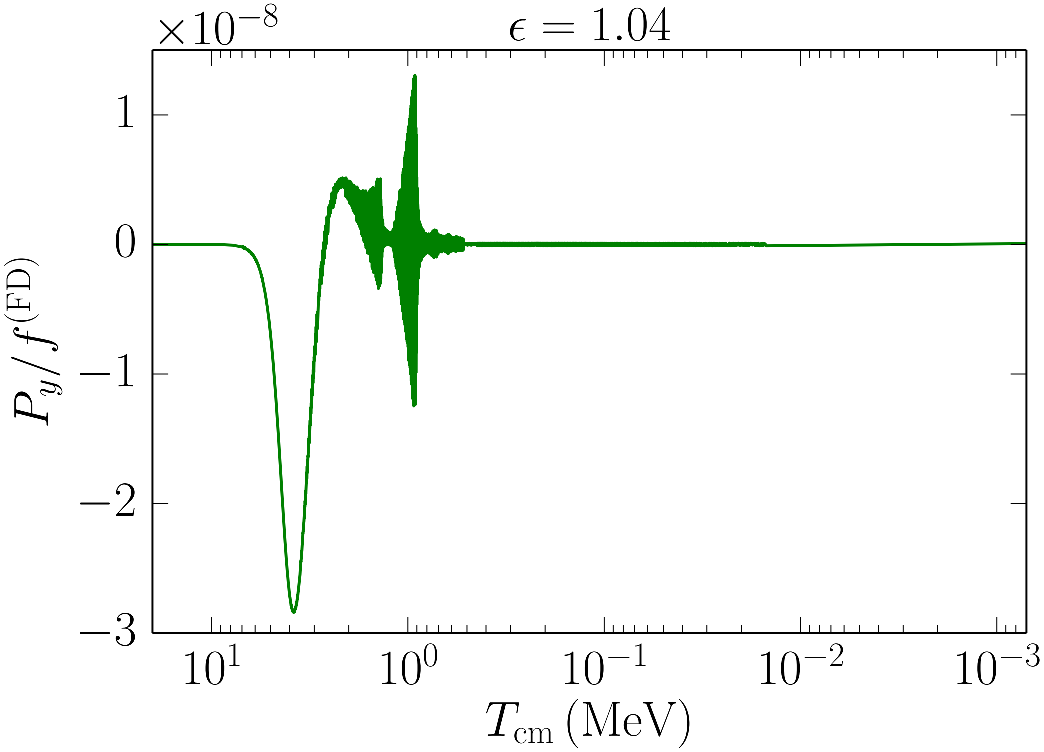
$\theta_{12} = 33.6^\circ$

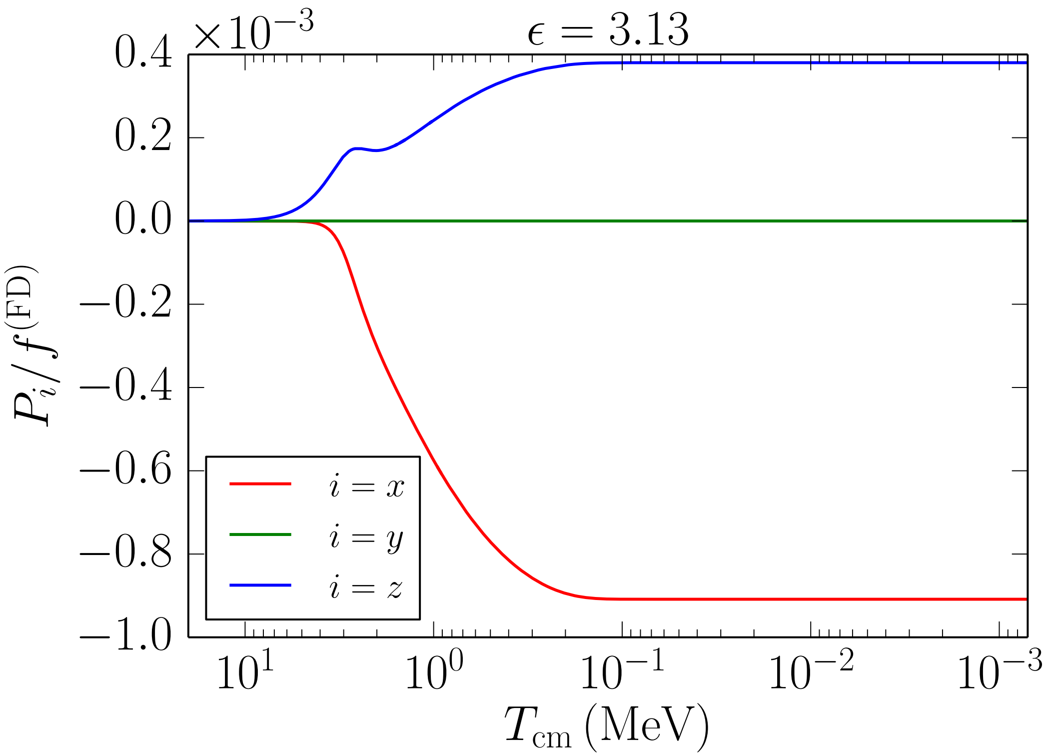
$\delta m_\odot^2 = 7.53 \times 10^{-5}\,\mathrm{eV}^2$

Normal mass hierarchy

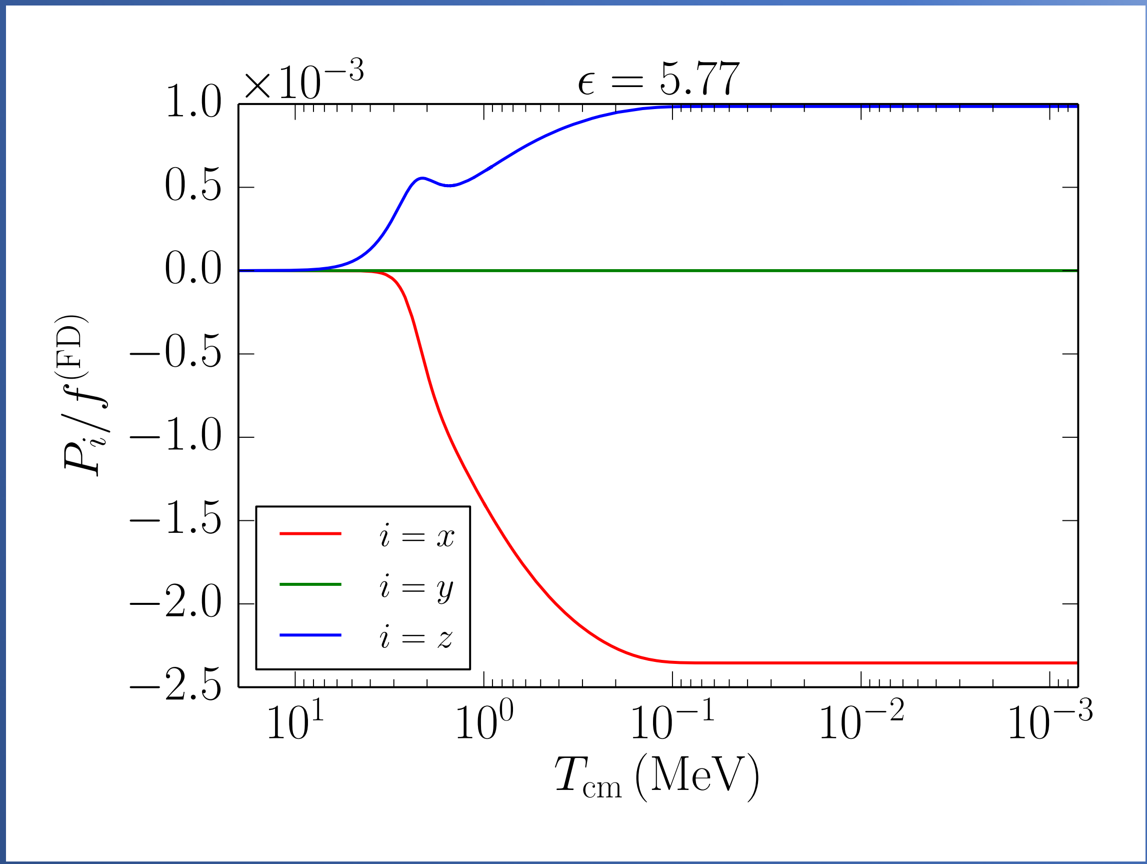


$\epsilon = 1.04$
$\times 10^{-4}$
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
$P_i/f^{(\mathrm{FD})}$
$i = x$
$i = y$
$i = z$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
T_{cm} (MeV)

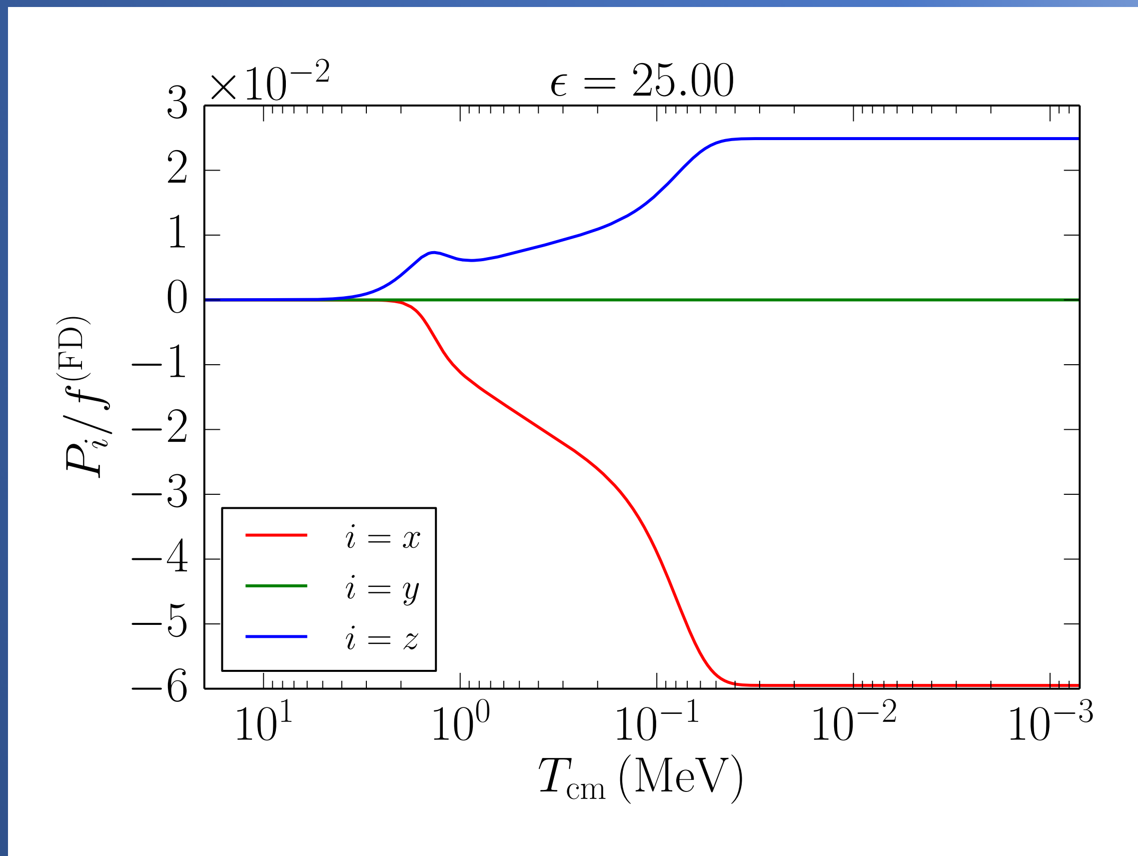

$\times 10^{-8}$
$\epsilon = 1.04$
$P_y / f^{(\mathrm{FD})}$
1
0
−1
−2
−3
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
T_{cm} (MeV)



$\epsilon = 3.13$
$\times 10^{-3}$
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
$P_i/f^{(\mathrm{FD})}$
$i = x$
$i = y$
$i = z$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
T_{cm} (MeV)



×10^−3
ϵ = 5.77
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−2.5
P_i/f^(FD)
i = x
i = y
i = z
10^1
10^0
10^−1
10^−2
10^−3
T_cm (MeV)



$\times 10^{-2}$
$\epsilon = 25.00$
3
2
1
0
−1
−2
−3
−4
−5
−6
$P_i/f^{(\mathrm{FD})}$
$i = x$
$i = y$
$i = z$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
T_{cm} (MeV)

The Path Forward

1. Full collision and coherent 2-flavor Mock-up calculation

2. Atmospheric mass in 2-flavor

3. Three-flavor calculation with entire coherent term and reduced collision term

4. Full QKE calculation

Early Universe Code

BURST

BBN

- ➔ Predict primordial nuclear abundances

UNITARY

- ➔ Preserve unitarity in nuclear reaction network
- ➔ Quantify errors

RECOMBINATION

- ➔ Treat recombination with three-level atom similar to recfast
- ➔ Isolate neutrino signatures in cosmological power spectra

SELF-CONSISTENT

- ➔ Maintain self-consistency over large range of epochs

TRANSPORT

- ➔ Follow evolution of neutrino spectra

Helium vs. Neutron lifetime

Bottle expt.
Steyerl et al (2016)
$\tau_n = 882.5 \pm 2.1\,\mathrm{s}$

Beam expt.
(1309.2623)
$\tau_n = 887.7 \pm 3.1\,\mathrm{s}$

UCNτ
(1707.01817)
$\tau_n = 877.7 \pm 1.1\,\mathrm{s}$