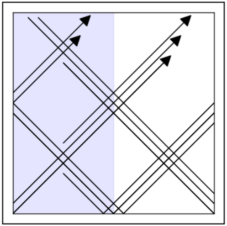

Effects of Extended Emission Regions on Neutrino Oscillations

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NBIA-LANL Neutrino Quantum Kinetics in Dense Environments, Copenhagen

August 27, 2019

Density matrix formalism

In the mean field approximation:

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{pmatrix} = \frac{1}{2} \begin{pmatrix} P_0 + P_z & P_x - iP_y \\ P_x + iP_y & P_0 - P_z \end{pmatrix} = \frac{1}{2}(P_0 + \vec{P} \cdot \vec{\sigma}).$$

Similarly for the Hamiltonian:

$$H = \frac{1}{2} \begin{pmatrix} V_z & V_x - iV_y \\ V_x + iV_y & -V_z \end{pmatrix} = \frac{1}{2} \vec{V} \cdot \vec{\sigma}.$$

Equation of motion (in absence of collisions):

$$i\dot{\rho} = [H, \rho] \qquad \Leftrightarrow \qquad \dot{\vec{P}} = \vec{V} \times \vec{P}$$

Homogeneous and isotropic environment

Include effect of damping, D:

$$\dot{\vec{P}} = \vec{V} \times \vec{P} - D\vec{P}_T.$$

For large V_z and D and $P_z \approx 1$:

$$P_x \approx \frac{V_x V_z}{V_z^2 + D^2}, \qquad P_y \approx \frac{-V_x D}{V_z^2 + D^2}.$$

Bell et al. 1998, Hannestad et al. 2012

Extended source Hansen and Smirnov 2019

Rough estimates:

- Neutrino sphere: $\sim 10\text{km}$.
- Width of neutrino sphere: $\sim 1\text{km}$.
- Oscillation length: $\sim \frac{1}{G_F n_e} \sim 10^{-8} - 10^{-7}\text{km}$.

Average over emission region suppresses oscillatory terms by $10^7 - 10^8$.

Changing background - only matter

Eigenstate basis ρ':

$$\rho'_{12}(r, r_e) = \rho'_{12,\mathrm{initial}} \exp\left(i \int_{r_e}^{r} \omega_m(r') dr' \right).$$

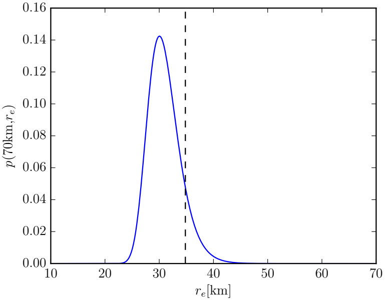
Average over emission point:

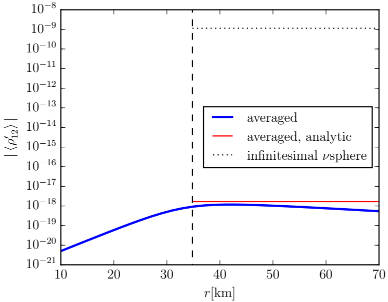
$$\left\langle \rho'_{12}(r) \right\rangle = \int_0^r p(r, r_e) \frac{1}{2} \sin 2\theta_m(r_e) \exp\left(i \int_{r_e}^{r} \omega_m(r') dr' \right) dr_e,$$

where

$$p(r, r_e) = \frac{F(r, r_e)}{F_0(r)}.$$

Changing background - only matter





$$\rho(r) = \rho_0 e^{-r/r_0}, \quad T = T_0 \frac{r_0}{r},$$

where $\rho_0 = 3 \times 10^{15} \mathrm{g/cm^3}$, $r_0 = 4\mathrm{km}$, $T_0 = 50\mathrm{MeV}$.

$$\langle \rho'_{12}(r) \rangle \approx -\frac{i}{2} \frac{\sin 2\theta_m}{r_0 V_e} \frac{r_0^2}{l_{\mathrm{abs}}^2},$$

neglecting Boltzmann suppression of n_e.

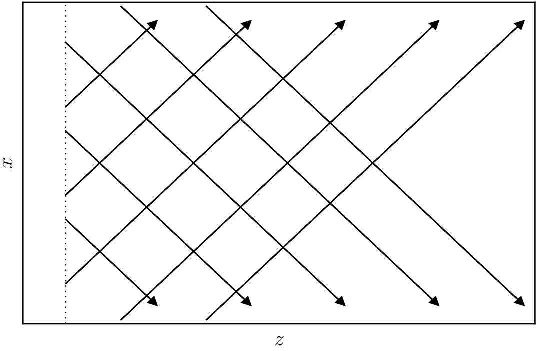
Changing background - only matter

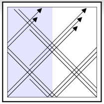
- A suppression of $\sim 10^{-9}$ is seen when neglecting neutrino-neutrino interactions.

Changing background - only matter

- A suppression of $\sim 10^{-9}$ is seen when neglecting neutrino-neutrino interactions.
- Does this hold when collective oscillations are introduced?

A very simple model with an extended emission region





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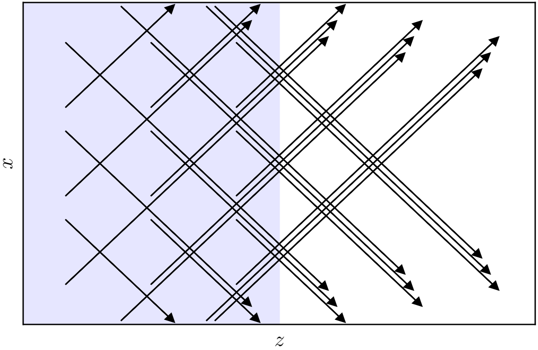
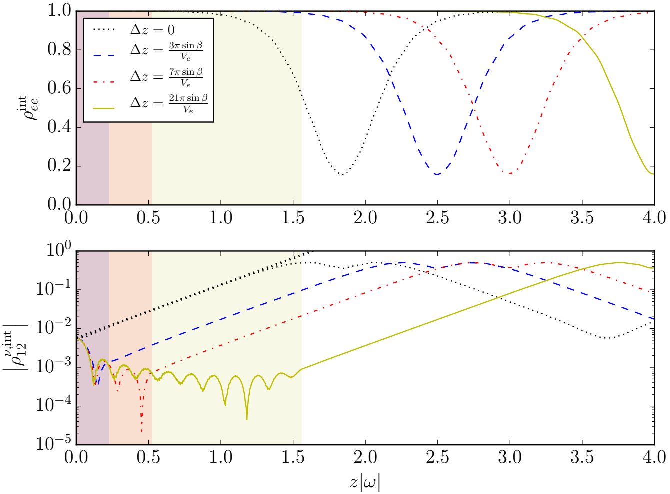


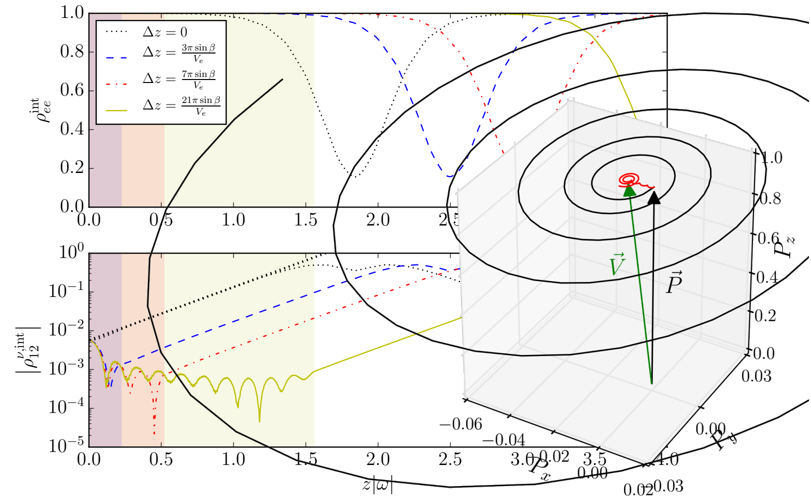
- Single energy.
- Single angle.
- Multiple emission points in z.
- Solve

$$i\dot{\rho} = [H, \rho].$$

Extended emission region



Extended emission region



Linear stability analysis Banerjee, Dighe, Raffelt 2011

Linear analysis demonstrate stability or instability.

$$\rho = \frac{f_{\nu_e} + f_{\nu_x}}{2} + \frac{f_{\nu_e} - f_{\nu_x}}{2} \begin{pmatrix} s & S \\ S^* & -s \end{pmatrix}$$

$$|S| = |P_x + iP_y| \ll 1, \quad s^2 + S^2 = 1 \Rightarrow s = P_z \approx 1.$$

Linearised equation:

$$i\dot{S} = (\omega + \lambda + \mu)S - \mu \int d\Gamma'(1 - v \cdot v')S',$$

Linear stability analysis

In Fourier space: $S = e^{-i\Omega t} Q$

$$\Omega Q = (\omega + \lambda + \mu) Q - \mu \int d\Gamma' (1 - v \cdot v') Q'$$

- Unstable if $\mathrm{Im}(\Omega) \neq 0$. (See also Capozzi et al. 2017)
- Discrete modes: solve matrix equation.
- Continious modes: Decompose in independent functions.
- Can also be formulated as a dispersion relation. (Izaguirre, Raffelt and Tamborra, 2016) ect.
- The basis where the equation is derived should be such that $S = 0$ is a "fixed point".

A comment on the 3-neutrino case

Döring, Hansen and Lindner, 2019

$$H'_{\mathrm{diag}} \approx \begin{pmatrix} \lambda + \omega_i(s_{12}^2 c_{13}^2 + \eta s_{13}^2) & 0 & 0 \\ 0 & \omega_i c_{12}^2 & 0 \\ 0 & 0 & \omega_i(s_{12}^2 s_{13}^2 + \eta c_{13}^2) \end{pmatrix},$$

where $\eta = \Delta m_{31}^2 / \Delta m_{21}^2$.

The vacuum part of the linearized equations become:

$$\Omega \begin{pmatrix} Q_{12} \\ Q_{13} \\ Q_{23} \end{pmatrix} = \omega_i \begin{pmatrix} -c_{12}^2 + c_{13}^2 s_{12}^2 + \eta s_{13}^2 & 0 & 0 \\ 0 & C_{13}(s_{12}^2 - \eta) & 0 \\ 0 & 0 & c_{12}^2 - s_{12}^2 s_{13}^2 - \eta c_{13}^2 \end{pmatrix} \begin{pmatrix} Q_{12} \\ Q_{13} \\ Q_{23} \end{pmatrix}$$

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$-c_{12}^2 + c_{13}^2 s_{12}^2 + \eta s_{13}^2 > 0$ for the measured mixing parameters.

This corresponds to an effective inverted hierarchy!

Extended emission region

- Take into account that μ changes inside the emission region.
- $\tilde{\mu}(z) = \mu \frac{z}{\Delta z}$ is so small for small z that no instability exists.

The fastest possible growth that one can expect is:

$$|S^{\mathrm{int}}(z)| \approx \frac{\sin\beta \sin 2\theta_m}{\Delta z V_e} \exp\left[\frac{\sqrt{|\omega|(2\mu - |\omega|)}}{\sin\beta}(z - \Delta z)\right].$$

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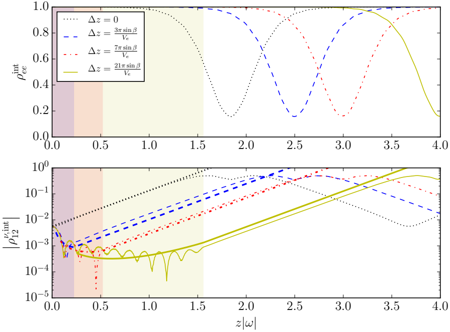
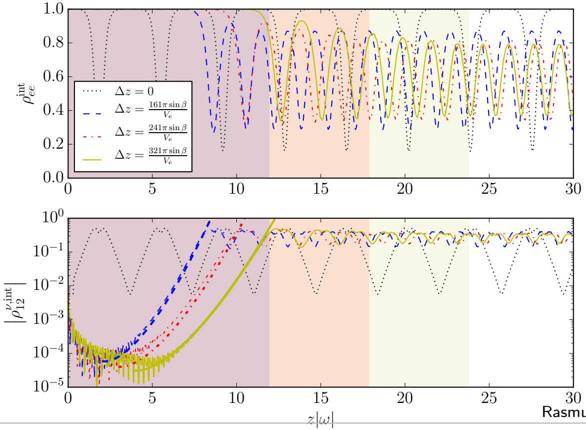


Larger Δz

$$|S^{\rm int}(z)| \approx \frac{\sin\beta \sin 2\theta_m}{z V_e} \exp\left[\frac{2|\omega| z_0}{3\sin\beta}\left(\frac{z}{z_0}-1\right)^{3/2}\right].$$



Summary

- The non-negligible width of the neutrino sphere affects the neutrino state due to averaging over different emission points.
- The angle between the neutrino state and the Hamiltonian in polarization space is reduced by a factor $\sim 10^8$ at the neutrino sphere by the averaging.
- A "fixed point" has to be used for the linear stability analysis. For three-neutrino oscillations this reveals an effective inverted hierarchy.
- The onset of neutrino conversion can be analysed using linear stability analysis, and for a given model, it can be calculated if conversion has the potentially to occur.

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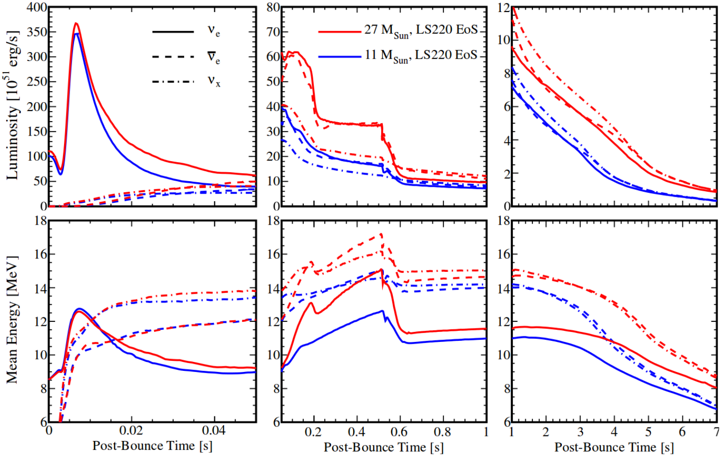
Thank you for your attention!

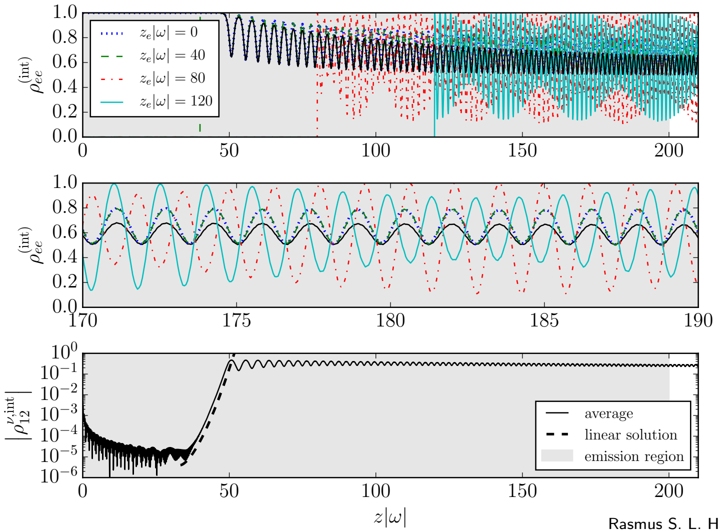
Neutrino emission

Deleptonisation Accretion Cooling



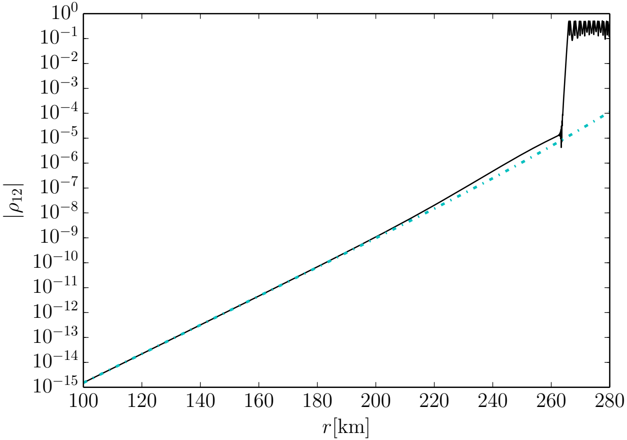
Lang et al. 2016

Very large Δz



Collective oscillations

Can collective oscillations still occur? YES!

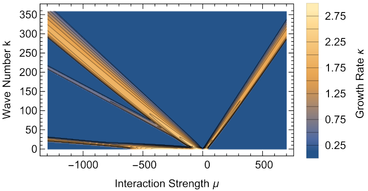


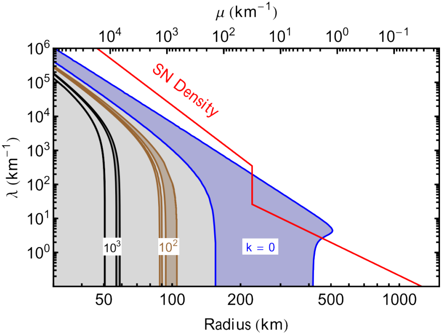
Multiple angles, in-homogeneous

Linear stability analysis of a more realistic model:

$$(\Omega + \mathbf{v}\cdot\mathbf{k})Q = (\omega + \lambda + \mu(\epsilon - \mathbf{v}\cdot\phi))Q - \mu \int d\Gamma'(1 - \mathbf{v}\cdot\mathbf{v}')g'Q'$$

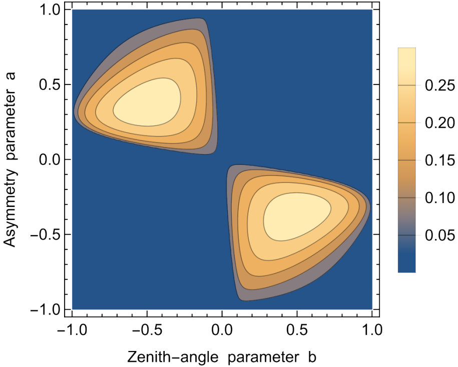
- μ and λ functions of r.
- Multi angle matter effect.
- Homogeneous mode: $k = 0$

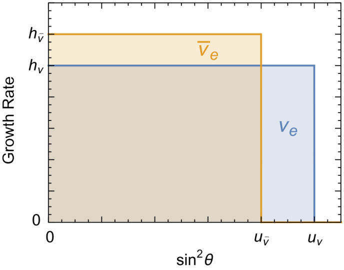




Chakraborty, RSLH, Izaguirre and Raffelt, 2015

Very fast flavour conversion R. F. Sawyer



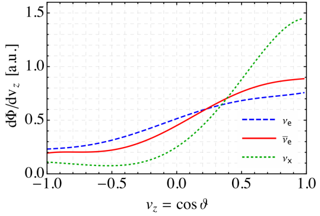


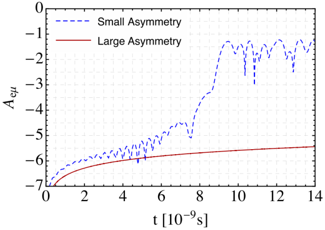
Chakraborty, RSLH, Izaguirre and Raffelt, 2016

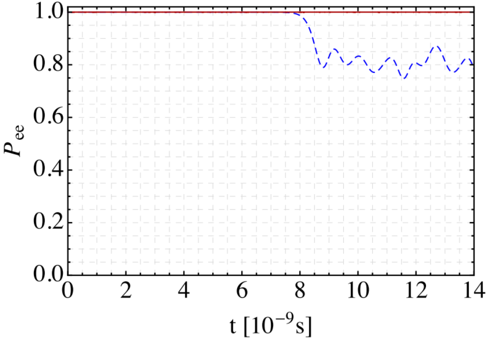
Conversion on meter-scale.

Can also occur in a supernova. Dasgupta, Mirizzi and Sen, 2016

Very fast flavour conversion







Dasgupta, Mirizzi and Sen, 2016