Entanglement in Collective Neutrino Oscillations

A Many-body Approach

Michael J. Cervia

Department of Physics,
University of Wisconsin-Madison

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- Consider an initial many-body state, $|\Psi_0\rangle$
  - Example: in the (two-)flavor-basis, $|\nu_e \nu_x \nu_e\rangle$
- Adiabatically evolve with Schrödinger's Eq.

$$|\Psi\rangle \approx V e^{-i \int_0^t \Sigma(t') dt'} V_0^T \, |\Psi_0\rangle$$

  - V, V_0 real mapping of energy eigenstates to mass product states, parametrized by the 2^N solns of $\vec{\Lambda} \equiv (\Lambda_1, \ldots, \Lambda_N)$, at times $t, 0$
  - $\Sigma \equiv V H V^T$ real, diagonal; any energy degeneracies split by differing $\vec{\Lambda}$ parameters
  - Obtain both V, Σ efficiently using methods of Patwardhan *et al.*, 2019

Summary of Entanglement Measures

Density Matrix, Polarization Vector, & Entanglement Entropy

Consider a pure, many-body neutrino state $\rho = |\Psi\rangle\langle\Psi|$. Then, a single-neutrino reduced density matrix:

$$\rho_q \equiv \mathrm{Tr}_{1,\ldots,\widehat{q},\ldots,N}[\rho] = \sum_{i_1,\ldots,\widehat{i_q},\ldots,i_N=1}^{2} \langle \nu_{i_1}\ldots\widehat{\nu_{i_q}}\ldots\nu_{i_N}|\rho|\nu_{i_1}\ldots\widehat{\nu_{i_q}}\ldots\nu_{i_N}\rangle$$

- $\vec{P}(\omega_q)$, Polarization vector of neutrino q: $\rho_q = \frac{1}{2}\left(\mathbb{I} + \vec{P}(\omega_q)\cdot\vec{\sigma}\right)$
- $S(\omega_q)$, Entropy of entanglement between neutrino q and rest:

$$S(P) = -\frac{1-P}{2}\ln\left(\frac{1-P}{2}\right) - \frac{1+P}{2}\ln\left(\frac{1+P}{2}\right)$$

with $P = |\vec{P}(\omega_q)|$

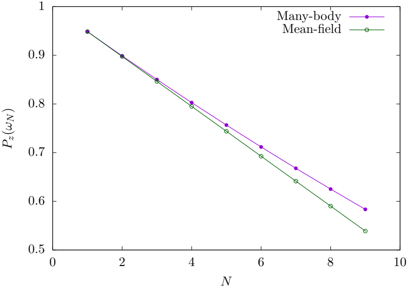
NB: S and $P \equiv |\vec{P}|$ are one-to-one, inversely related

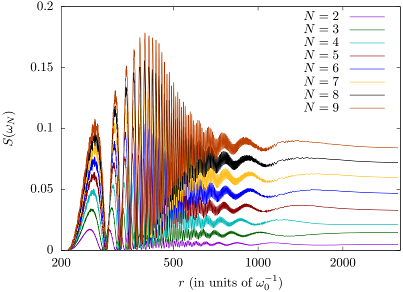
- $P = 1 \iff S = 0$ (Unentangled)
- $P = 0 \iff S = \ln(2)$ (*Maximally* Entangled)

Evolution of All-Electron Flavor Initial State

First Comparison of MF and MB

- Consider spectra of freq $\omega_1, \ldots, \omega_N$ where $\omega_p = p\omega_0$
- For $N = 2, \ldots, 9$, evolve from $|\Psi_0\rangle = |\nu_e \ldots \nu_e\rangle$
- As $\mu \sim 0$ $(r \gg R_\nu)$, H diag in mass-basis
  $\implies$ plot final spectra as $P_z = n_1 - n_2$



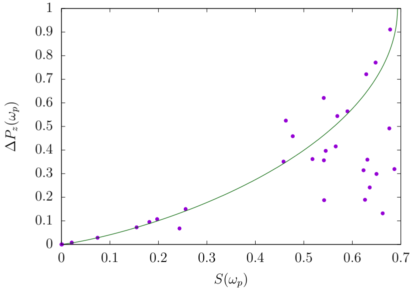


NB: In the MF case, $S = 0$ *always*

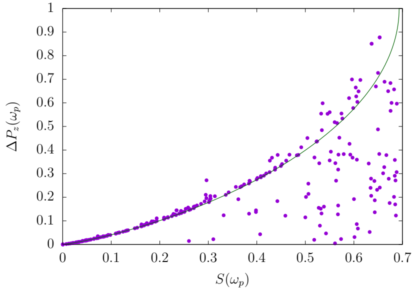
Calculate $\Delta P_z(\omega) \equiv |P_z^{\mathrm{MF}}(\omega) - P_z^{\mathrm{MB}}(\omega)|$ at $r \gg R_\nu$

- For $N = 4$: all ICs with definite flavor ν_e, ν_x (e.g., $|\nu_e, \nu_x, \nu_x, \nu_x\rangle$)
- For $N = 8$: same ICs, but plus four ν_e to top/bottom of spectrum

$(N = 4)$



$(N = 8)$

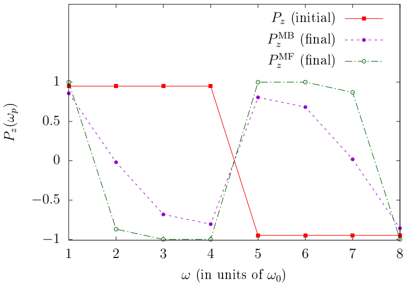


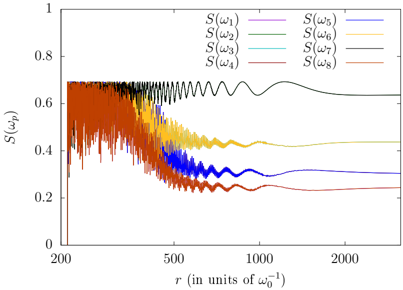
Trendline: $y(S) \equiv P^{\mathrm{MF}}(S) - P^{\mathrm{MB}}(S) = 1 - P(S)$

Comparison of P_z Spectra

Weakening the Spectral Swap

- Evolve $|\Psi_0\rangle = |\nu_e\nu_e\nu_e\nu_e\nu_x\nu_x\nu_x\nu_x\rangle$ until $r \gg R_\nu$
- Pairs (p, q) with $P_z(\omega_p) = -P_z(\omega_q)$ also share $S(\omega_p) = S(\omega_q)$



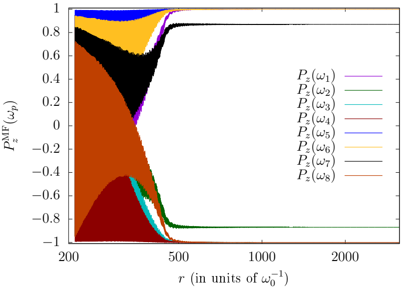


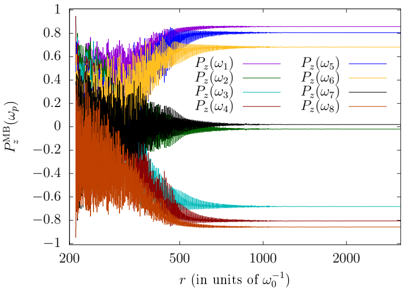
Cervia, Patwardhan, Balantekin, Coppersmith, Johnson
arXiv e-print:1908.03511

Comparison of Intermediate P_z Spectra

While $r \gtrsim R_\nu$

- Same IC, but observe P_z before $r \gg R_\nu$





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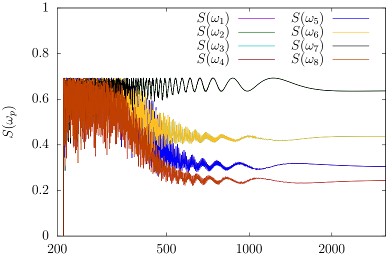
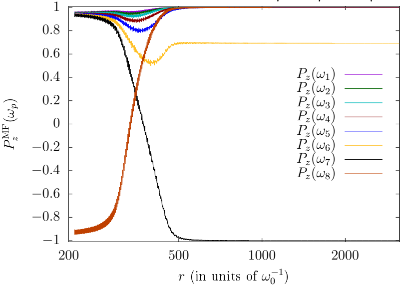
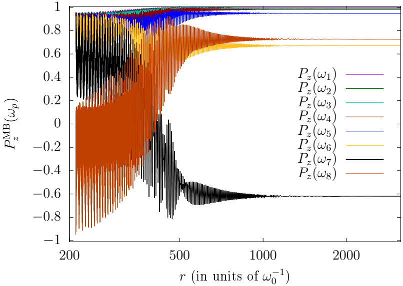


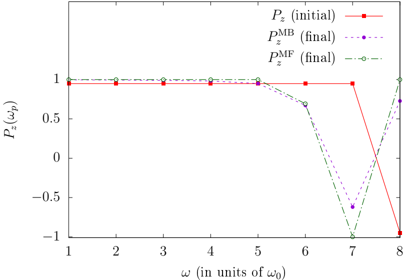
Comparison of Intermediate P_z Spectra

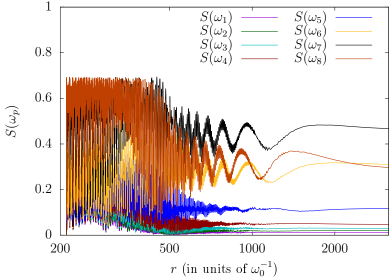
While $r \gtrsim R_\nu$, $N = 8$ alternative IC

$$|\Psi_0\rangle = |\nu_e\nu_e\nu_e\nu_e\nu_e\nu_e\nu_e\nu_x\rangle$$



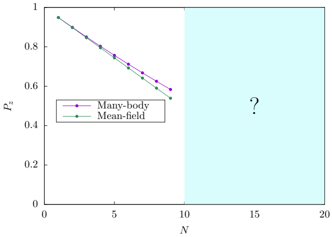






Future Work

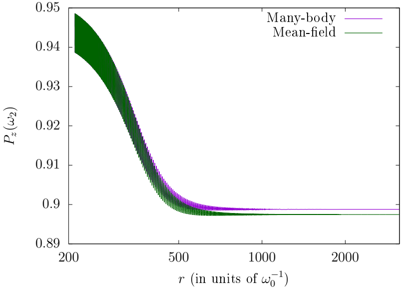
- Y_e Predictions $\rightarrow$ Nucleosynthesis yields
- Next steps in calculations
  - Larger N
  - Inclusion of $\bar{\nu} \iff \omega < 0$, here
  - Inclusion of frequency degeneracies
  - Beyond single-angle approximation $\mu \rightarrow \mu_{\mathbf{pq}}$

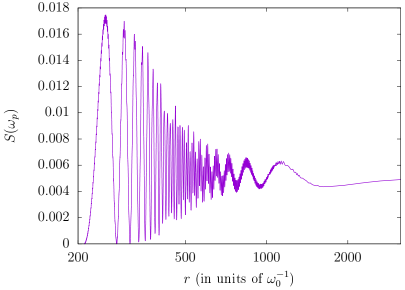


Comparison of Intermediate P_z Spectra

While $r \gtrsim R_\nu$, $N = 2$ mono-flavor initially

- $|\Psi_0\rangle = |\nu_e \nu_e\rangle$, and observe P_z before $r \gg R_\nu$

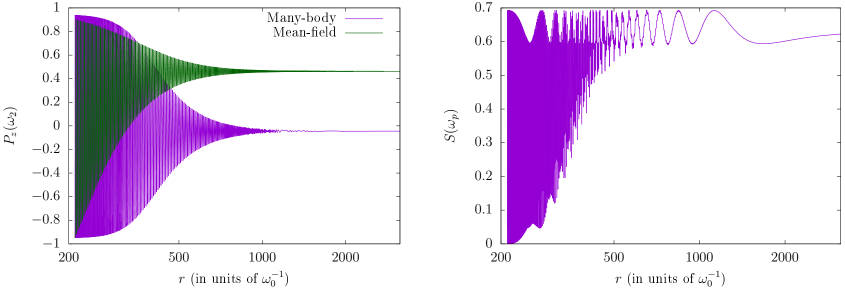




Comparison of Intermediate P_z Spectra

While $r \gtrsim R_\nu$, $N = 2$ different-flavor initially

- $|\Psi_0\rangle = |\nu_e \nu_x\rangle$, and observe P_z before $r \gg R_\nu$



Entanglement in Individual Eigenstates

Highest frequency ν to the rest, $N = 5$

- $|\Psi\rangle$ eigenstates of H, for $N = 5$
- hightest/lowest-weight states are trivial