Beyond Isotropic Quantum Kinetics

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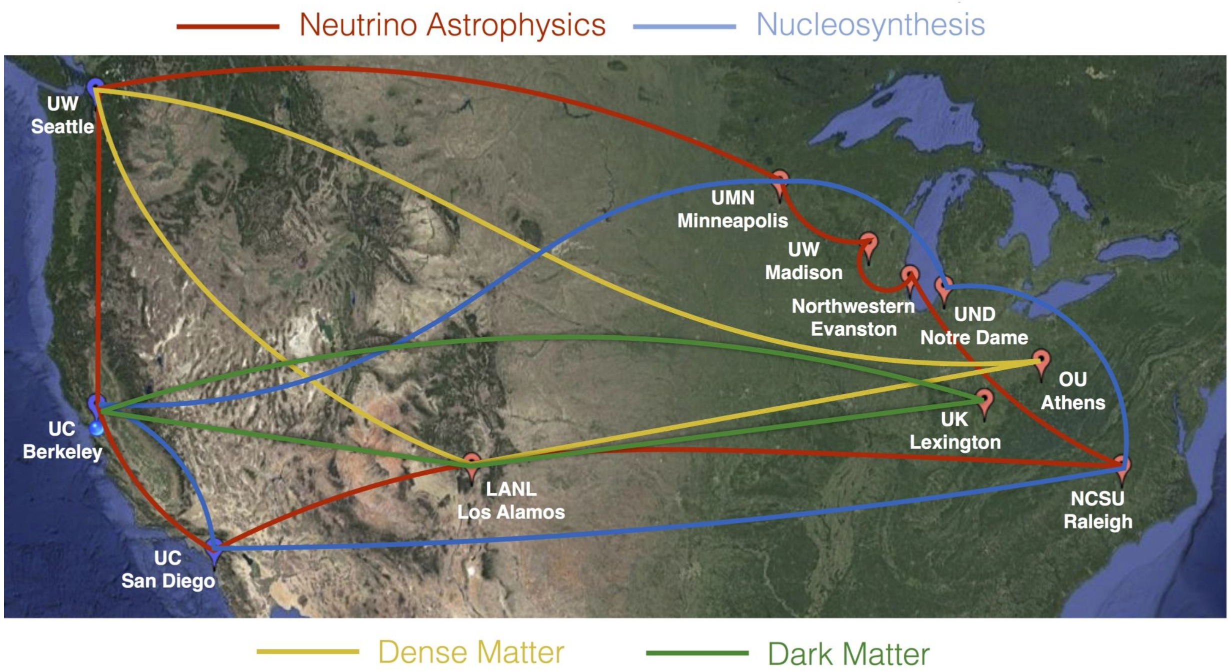
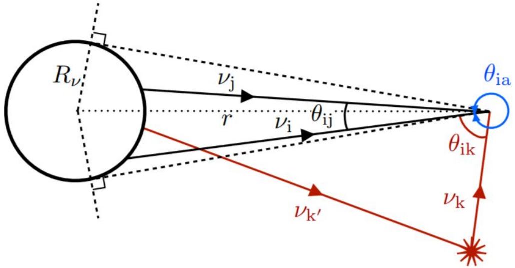
James Kneller
Alexey Vlasenko
Yonglin Zhu

“Network for Neutrinos, Nuclear Astrophysics, and Symmetries”



Neutrino Quantum Kinetics in Supernovae

Neutrino Halo Effect

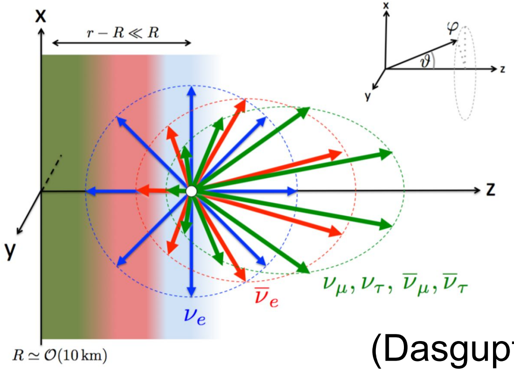


(Cherry+ 2012, 1203.1607)

Small number of reflected neutrinos can change neutrino flavors.

(Also, Cirigiliano+ 2018, 1807.07070, ...)

Fast Oscillations



(Dasgupta+ 2017)

Neutrino/anti-neutrino angular dependence cause instabilities

(orig. Sawyer 2005, also Izaguirre+ 2016, Capozzi+ 2017, Tamborra+2017, Dasgupta+ 2018, Capozzi+2019, Abbar+2019, Azari+2019, ...)

QKE Simulations in CCSNe have Begun!

	Dimensions	Time	Angularity	Interactions
Grohs+ (soon, cosmological**)**	0	Yes	Isotropic	Full
Shalgar & Tamborra (2019)	1	No	Multi-Angle	Simple
Capozzi+ (2019)	1	Yes	Two-Beam	Simple
Richers+ (2019)	0	Yes	Isotropic	Full

(+ much more in progress)

Neutrino Quantum Kinetic Equations

$$\frac{\partial f}{\partial t} = \{1 - f, \Pi^+\} - \{f, \Pi^-\} - i\,[\mathcal{H}, f]$$

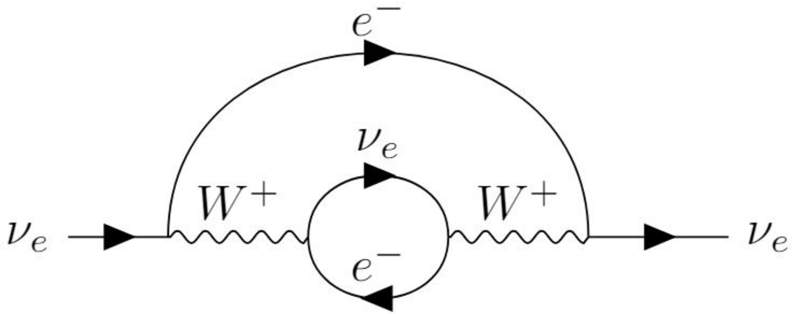
(See Vlasenko et al. 2014, Blaschke + Cirigliano 2016)

Evolve the (generalized) **occupation probability:**

$$f = \begin{bmatrix} f_{ee} & f_{e\mu} \\ f_{\mu e} & f_{\mu\mu} \end{bmatrix}$$

Oscillation Hamiltonian drives rotations

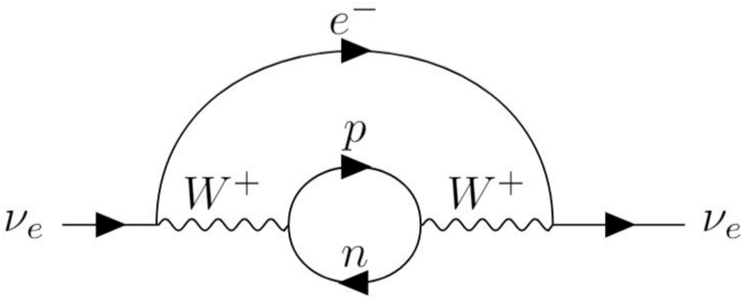
Self-energy drives rotations *and* changes number of neutrinos



$$\Pi^{\pm} \sim \int |\mathcal{M}|^2 d^3p' \; \{f' \, \mathrm{or} \, (1 - f')\}$$

Absorption and Emission

$$\frac{\partial f}{\partial t} = \left\{1 - f, \Pi^{+}\right\} - \left\{f, \Pi^{-}\right\} - i\left[\mathcal{H}, f\right]$$



No internal neutrino lines → Simple!

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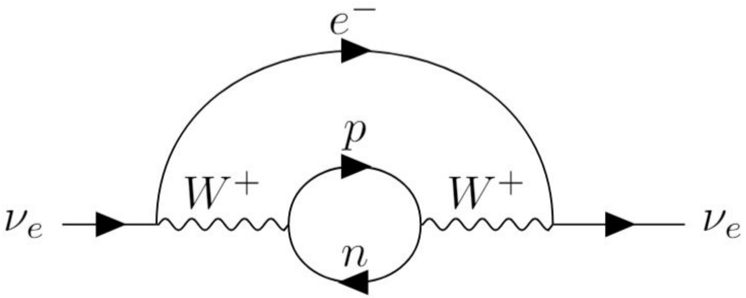

$$\frac{df}{dt} = A\begin{bmatrix}(1 - f_{ee}) & -f_{e\mu}/2 \\ -f_{\mu e}/2 & 0\end{bmatrix} - B\begin{bmatrix}f_{ee} & f_{e\mu}/2 \\ f_{\mu e}/2 & 0\end{bmatrix}$$

Absorption and Emission

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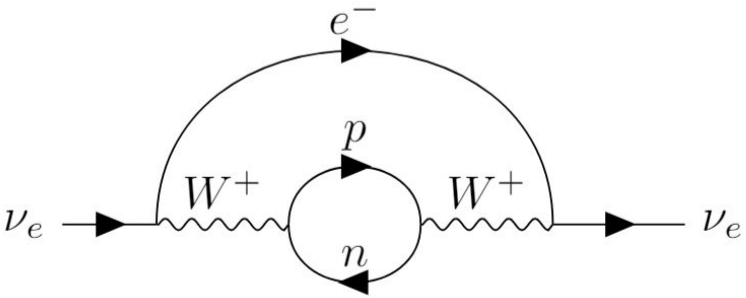

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But we already know this part!

$$A = j_{(\nu_e)} \qquad B = \kappa_{\mathrm{abs},(\nu_e)}$$

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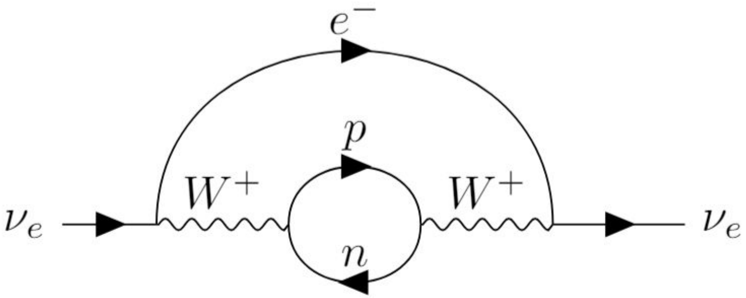

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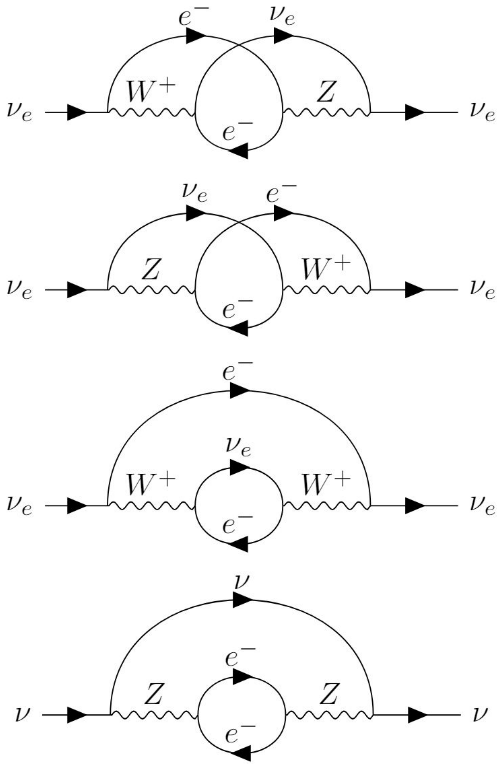
But we already know this part!

$$A = j_{(\nu_e)} \qquad B = \kappa_{\mathrm{abs},(\nu_e)}$$

The full QKE source term is **easy** to write and implement!

Electron Scattering (inelastic)



$$\mathcal{C}^{+}_{ab} = \int d^3\nu' \left[R^{+}_{ab} f'_{ab} - \varsigma^{+}_{ab}\right]$$

$$\mathcal{C}^{-}_{ab} = \int d^3\nu' \left[\langle R\rangle^{-}_{ab} f_{ab} - \varsigma^{-}_{ab}\right]$$

Straightforward combination of
R, f, f’

$$\varsigma^{\pm}_{ab} = \frac{1}{2}\sum_{c}\left(R^{\pm}_{cb} f_{ac} f'_{cb} + R^{\pm}_{ac} f'_{ac} f_{cb}\right)$$

Neutrino Quantum Kinetic Equations

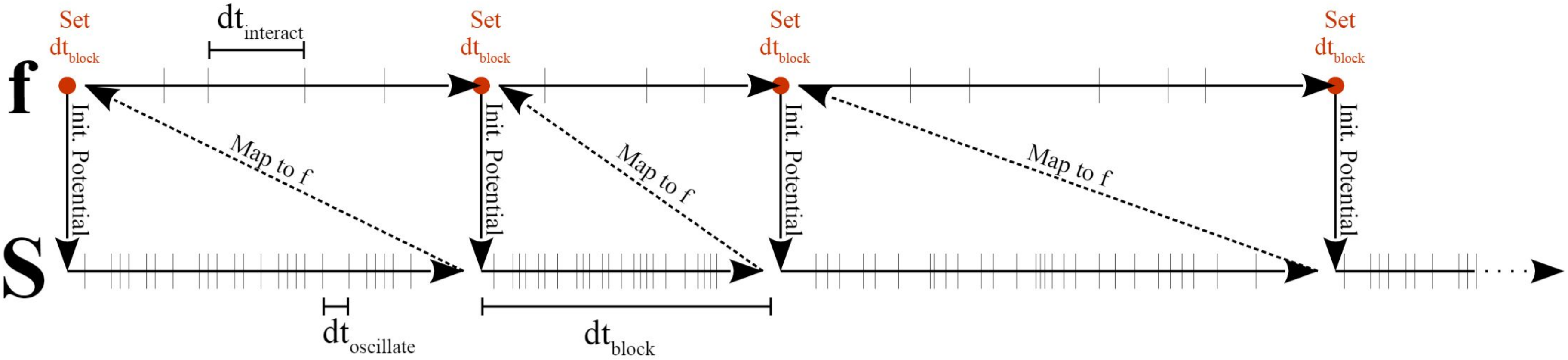
$$\frac{\partial f}{\partial t} = \left\{1 - f, \Pi^{+}\right\} - \left\{f, \Pi^{-}\right\} - i\left[\mathcal{H}, f\right]$$

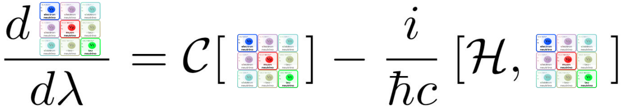
(See Vlasenko et al. 2014
Blaschke + Cirigliano 2016)

It is straightforward to extend **known** interactions to a **full quantum kinetic treatment**!

IsotropicSQA

github.com/srichers/IsotropicSQA





1) Evolve **oscillations only** for some “block” time
2) Evolve **interactions only** for same amount of “block” time
3) Check the impact of the interactions and adjust “block” time

Details:

- Must randomize block time to accurately sample distribution
- Must evolve oscillations with high accuracy (many many timesteps)

Moment-Based Transport

Neutrino Transport

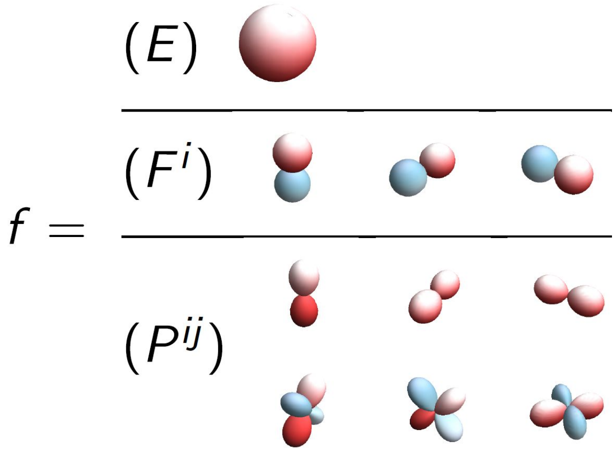
$f(x^\mu, p^\mu)$ is (# of neutrinos) per (volume) per (energy) per (solid angle)

Classical Boltzmann Equation

$$\frac{1}{c}\frac{\mathrm{d}f}{\mathrm{d}\tau} = \mathcal{C}$$

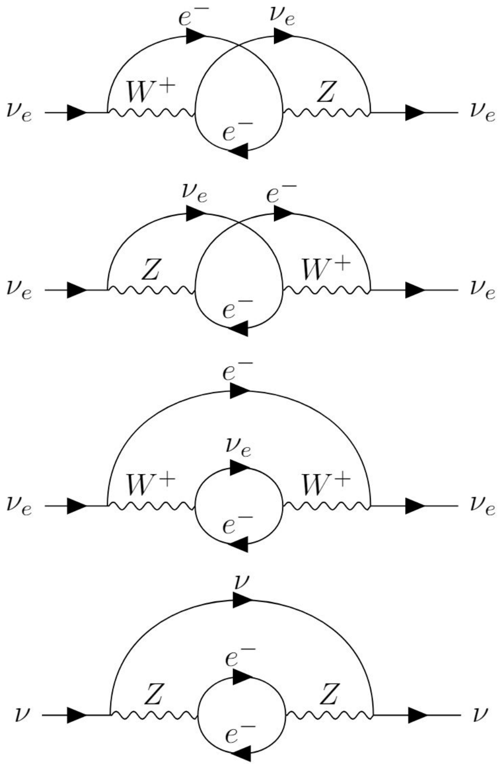
Separate moment for each component

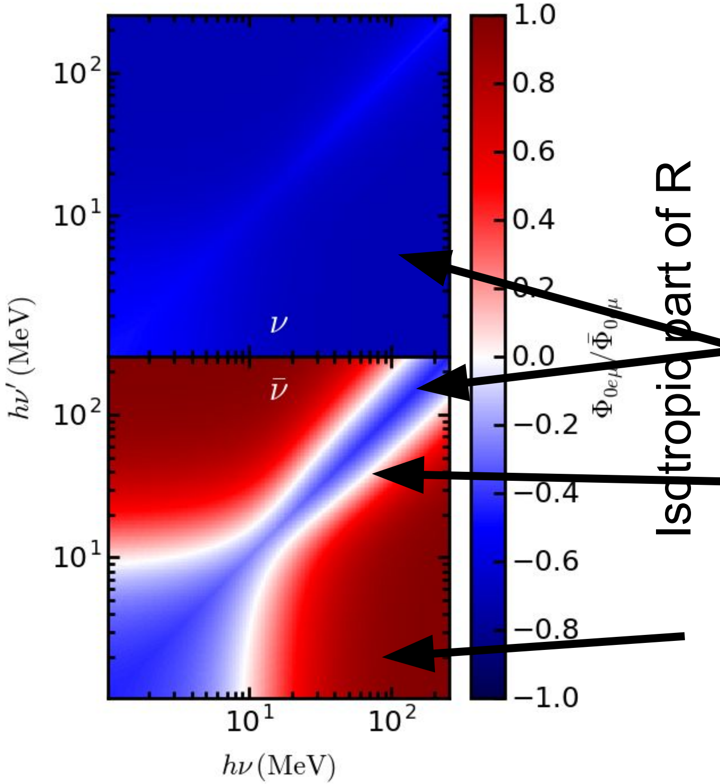
The full EOMs are also known!



Flux-Limited Diffusion
Two-Moment (M1)

Electron Scattering (inelastic)





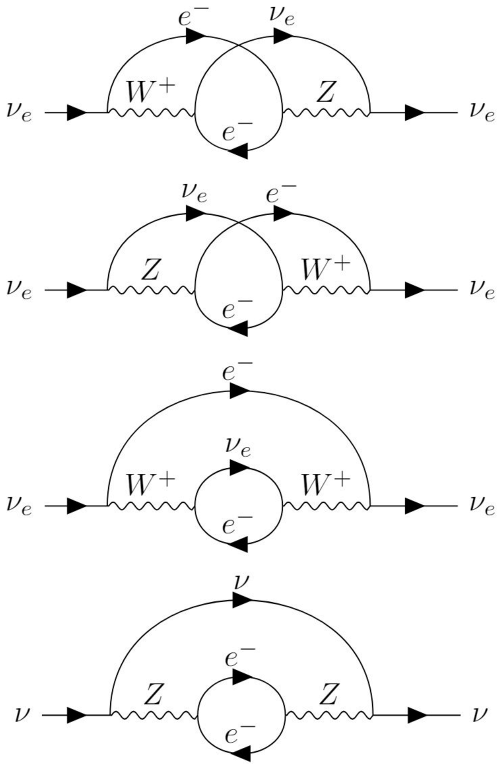
CC scattering helps suppress flavor coherence

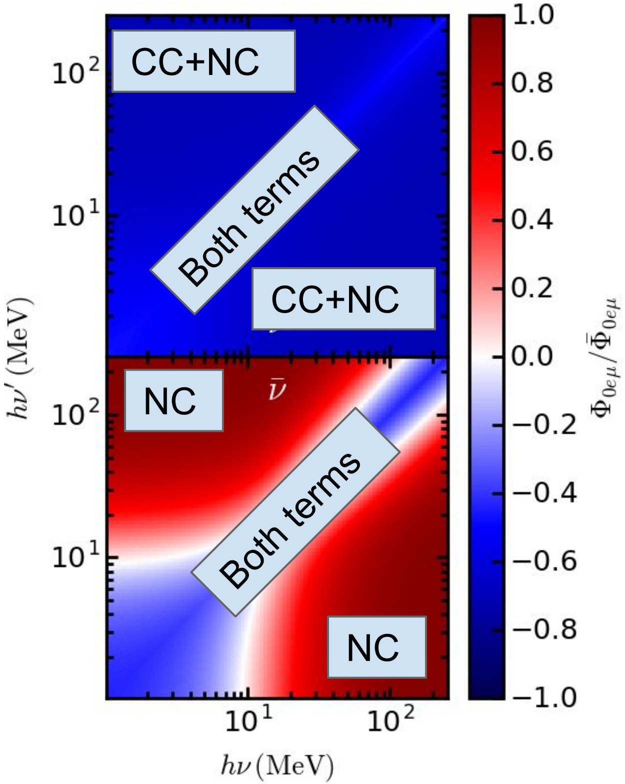
Scattering flips quantum state

Scattering collapses quantum state

Scattering preserves quantum state

Electron Scattering (inelastic)





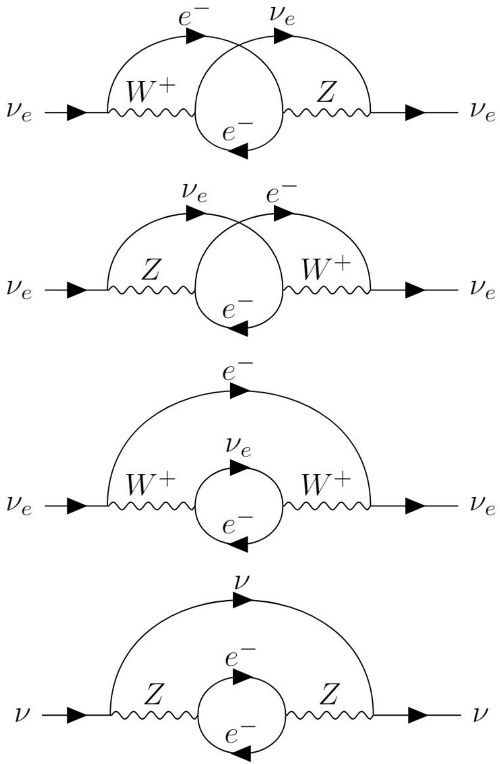
Isotropic part of R

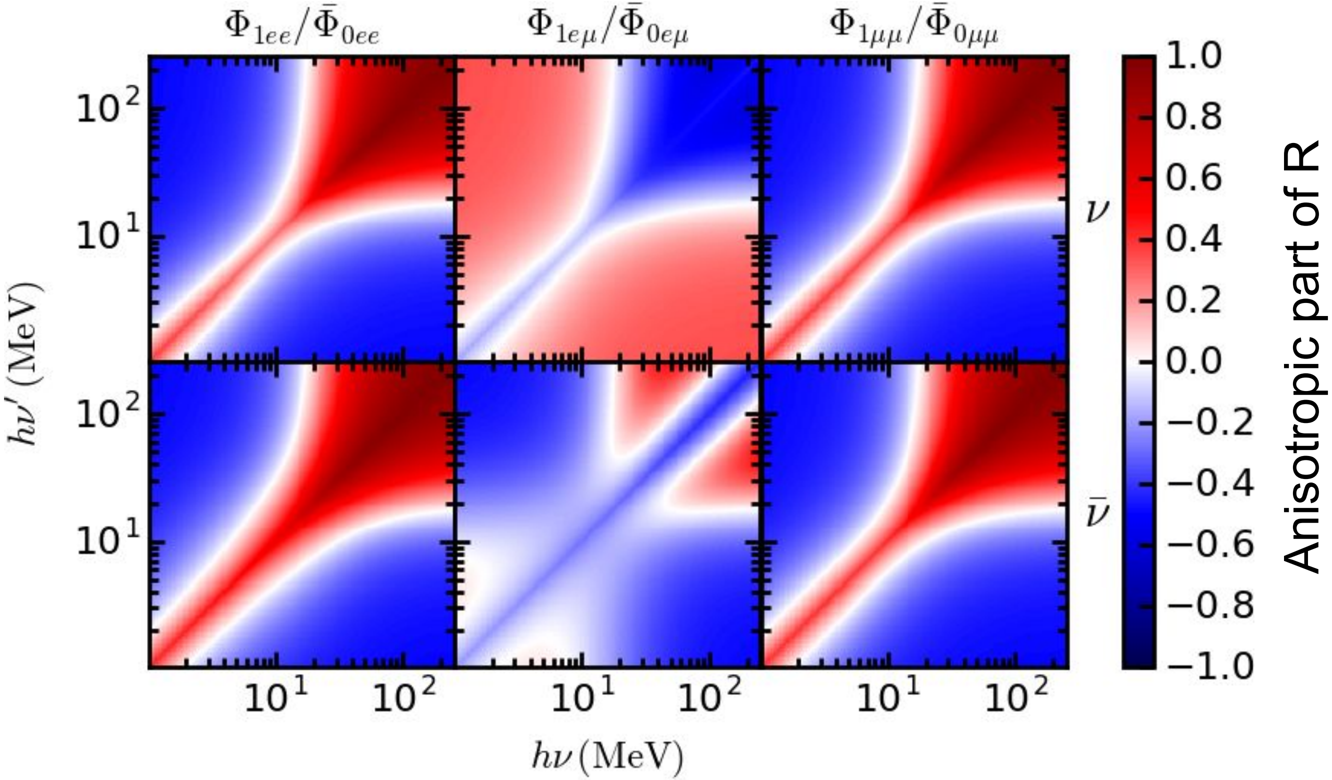
CC + NC

$$R \sim \int d^3p_e[(C_V + C_A)^2(p_e \cdot p_\nu)(p'_e \cdot p'_\nu)$$
$$+(C_V - C_A)^2(p'_e \cdot p_\nu)(p_e \cdot p'_\nu)]$$

NC only

Electron Scattering (inelastic)





Neutrino Transport

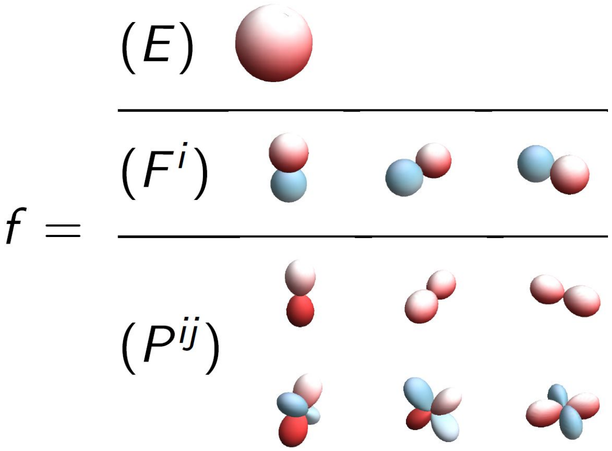
$f(x^\mu, p^\mu)$ is (# of neutrinos) per (volume) per (energy) per (solid angle)

Classical Boltzmann Equation

$$\frac{1}{c}\frac{\mathrm{d}f}{\mathrm{d}\tau} = \mathcal{C}$$

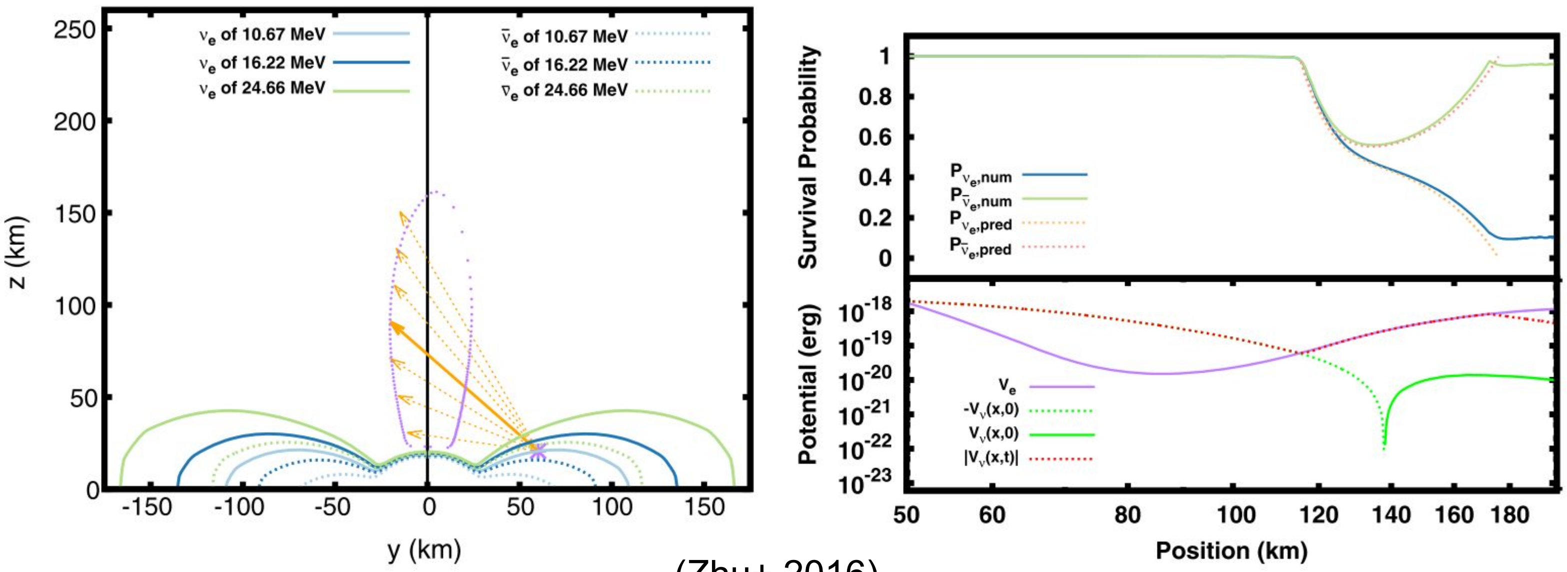
Potential Issues:

- Distribution gets only 2(4) phase angles in 1(3)D.
- Required resolution much higher than regular transport.



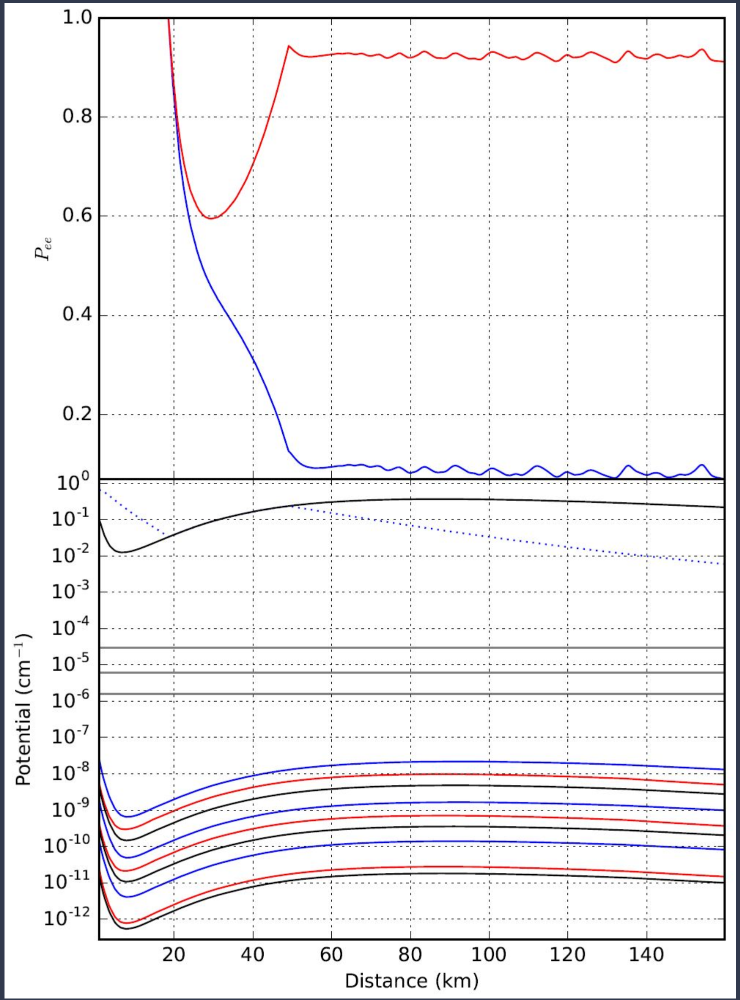
Ray-Based Transport

Pure oscillations in neutron star mergers

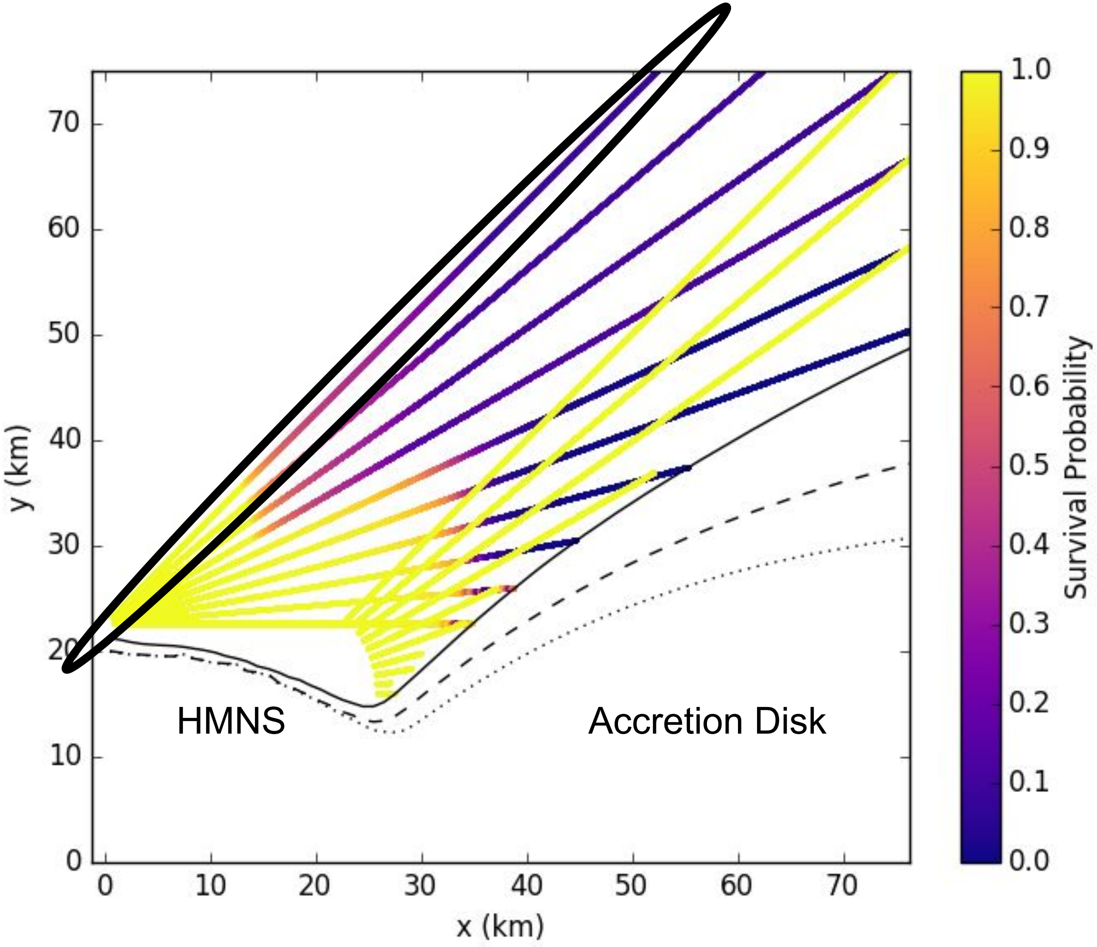


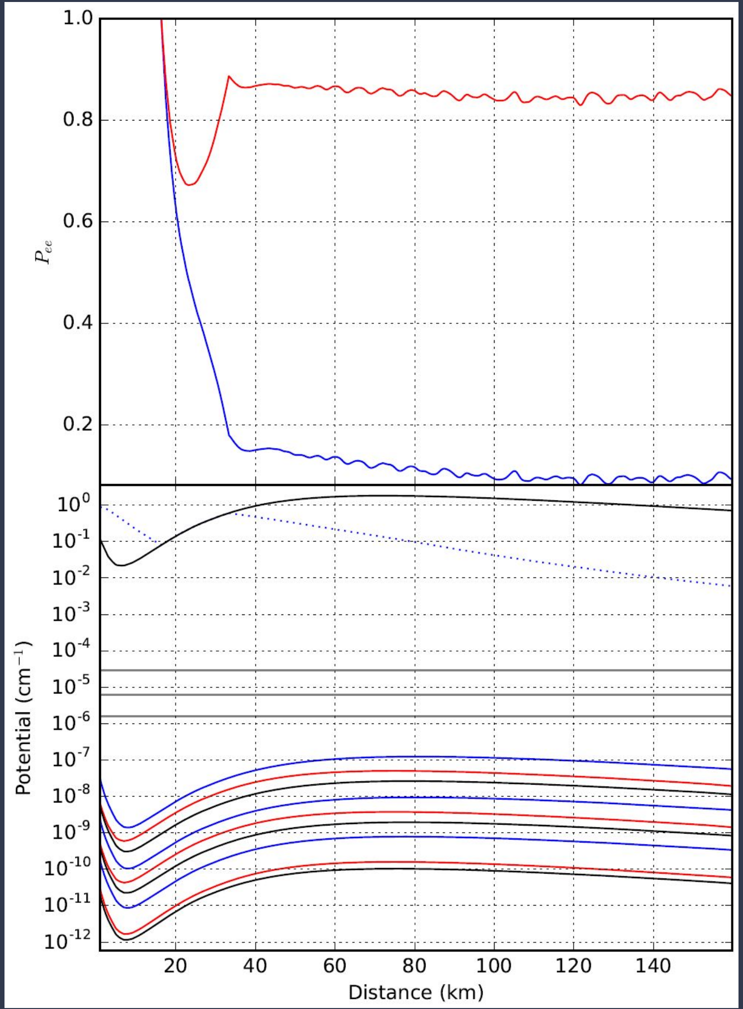
(Zhu+ 2016)

Neutrino-matter resonance efficiently transforms neutrinos.



Matter-Neutrino Resonance!





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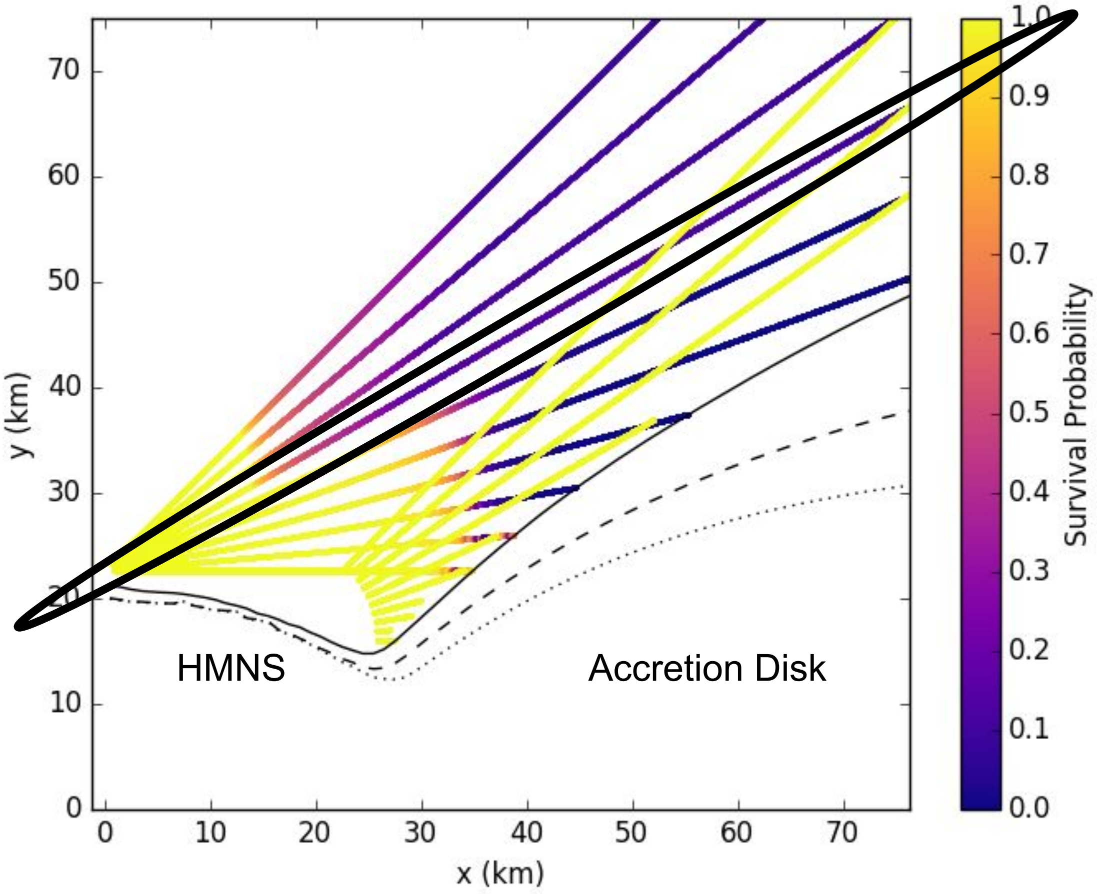
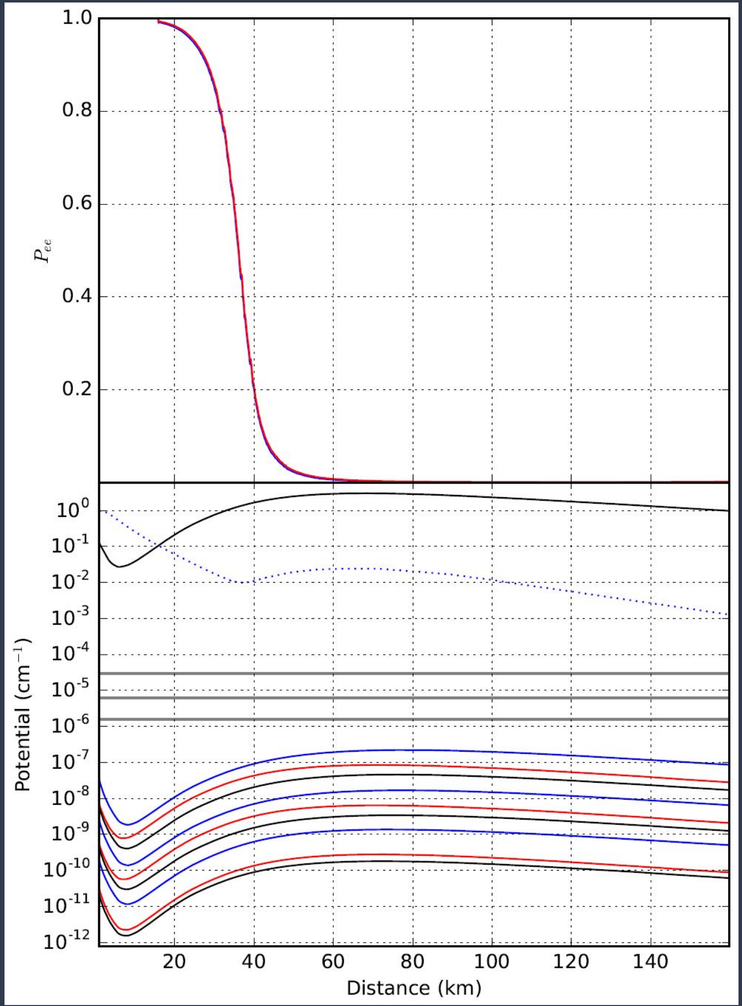
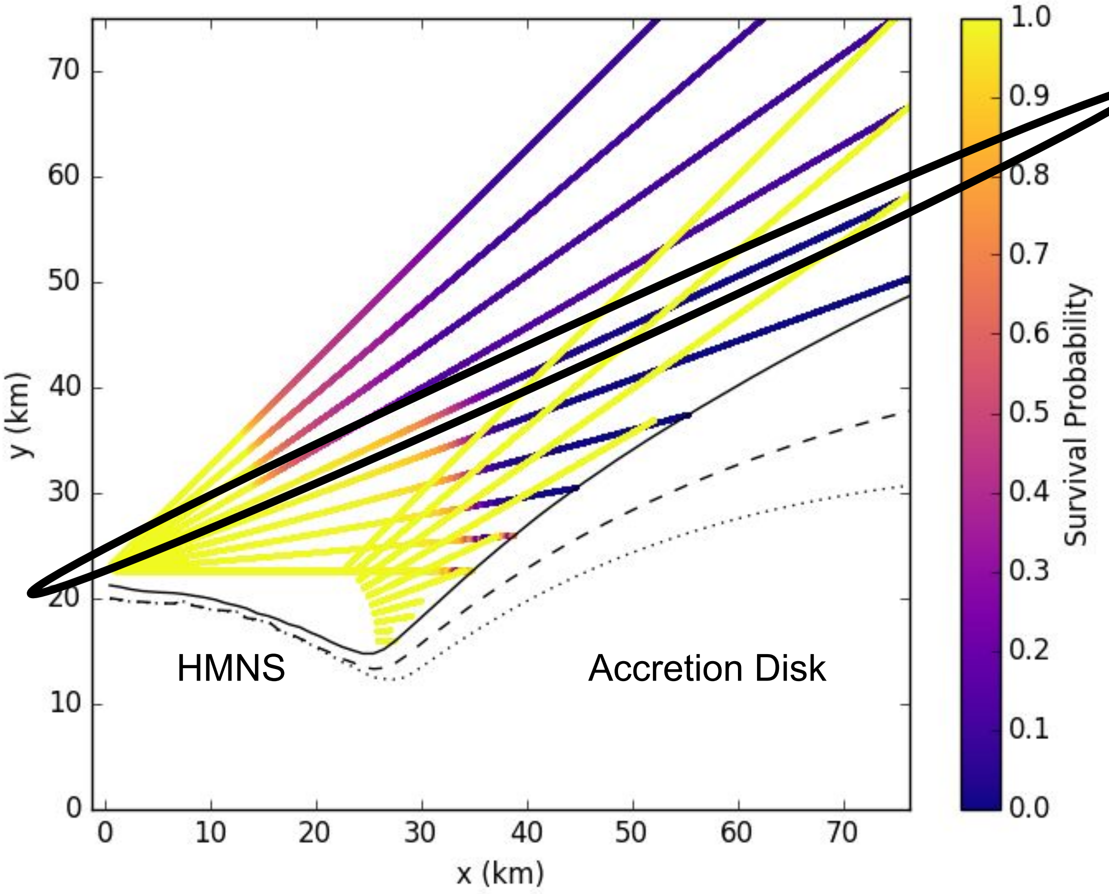
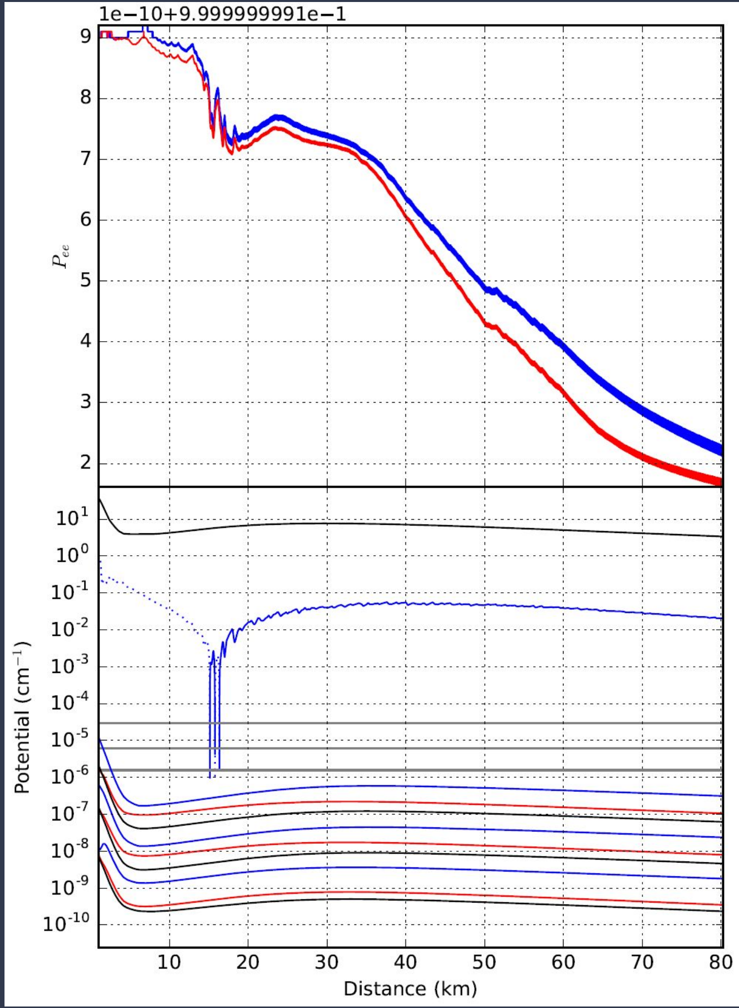
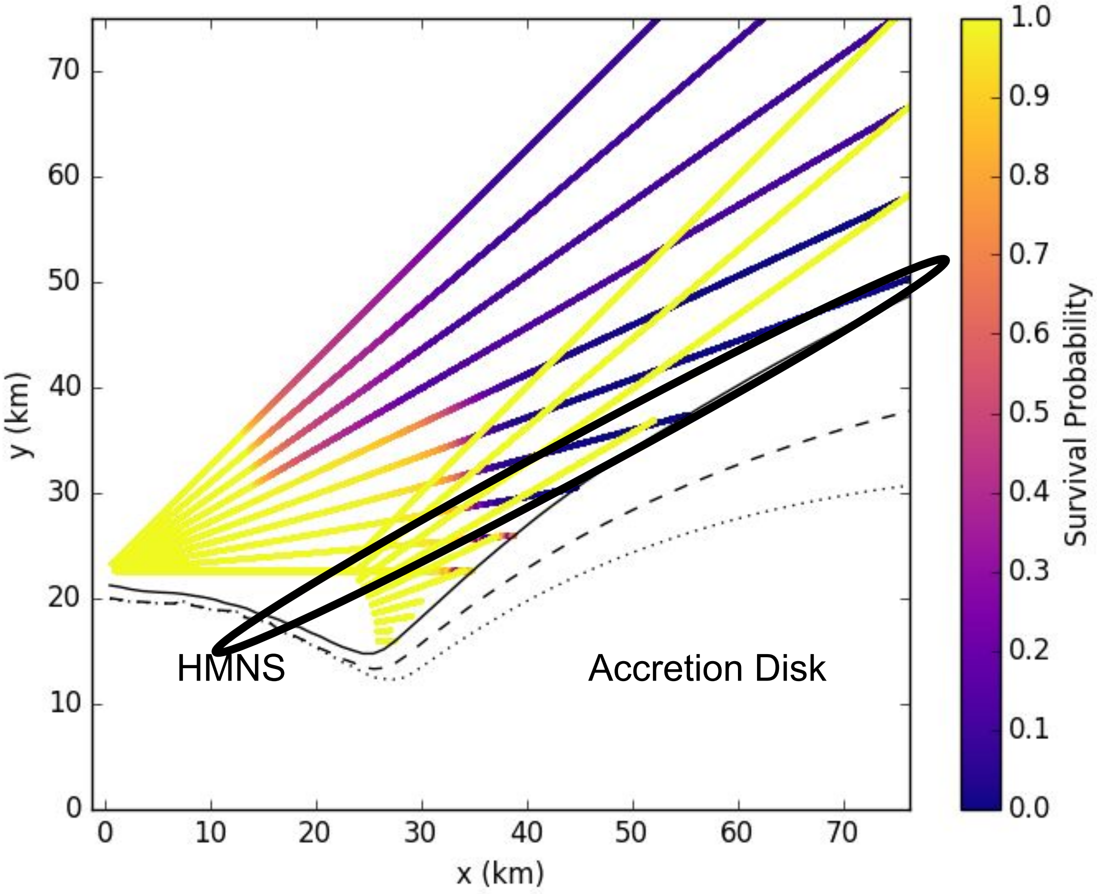






Synchronized MSW





Too many electrons!

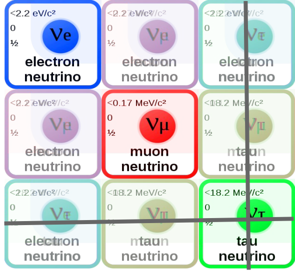
Conclusions

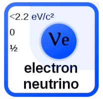
- Existing neutrino interaction rates can be extended to **full QKE source terms!**

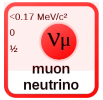
- **Anisotropic numerical quantum kinetics** is finally an active field.

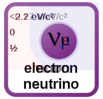
- **Needs:**
  - **Multiple numerical techniques**
  - **HPC** code infrastructure
  - Numerical techniques for **multi-rate** equations (error accumulation and cost)
  - Force/drift terms? Multi-flavor? Majorana/Dirac? Sterile? Non-standard interactions? Spin coherence? Magnetic fields? ...

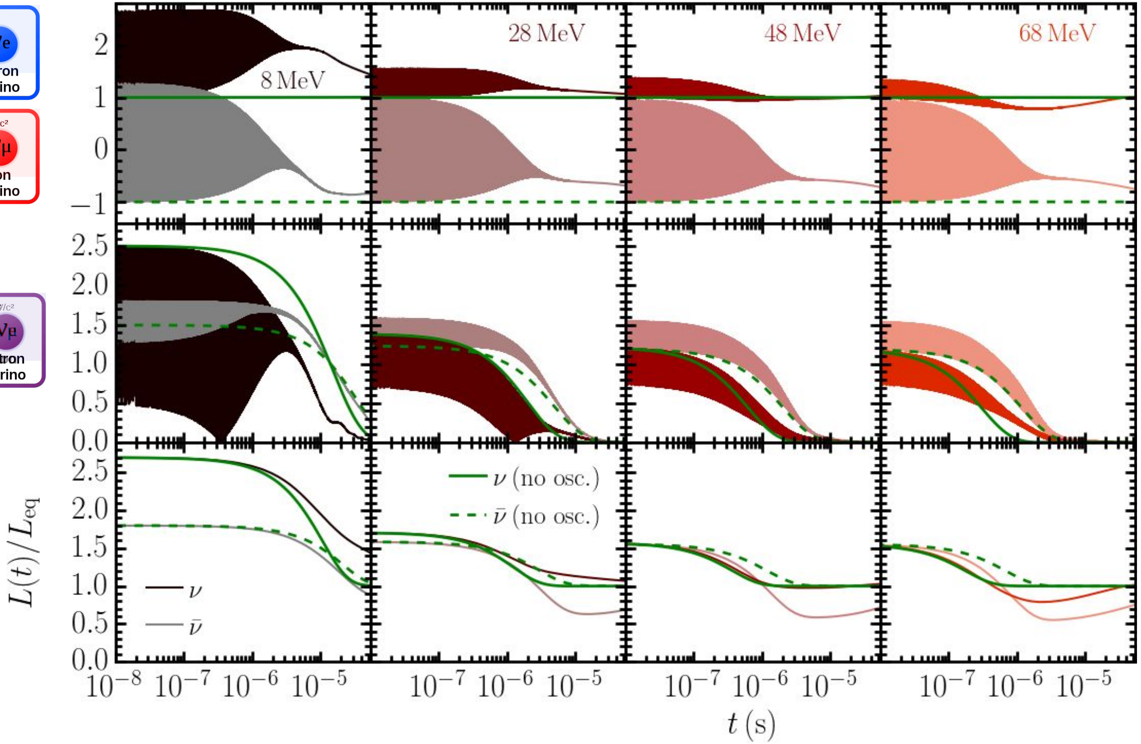
Everything Together









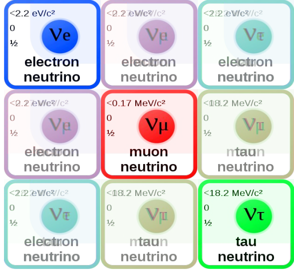


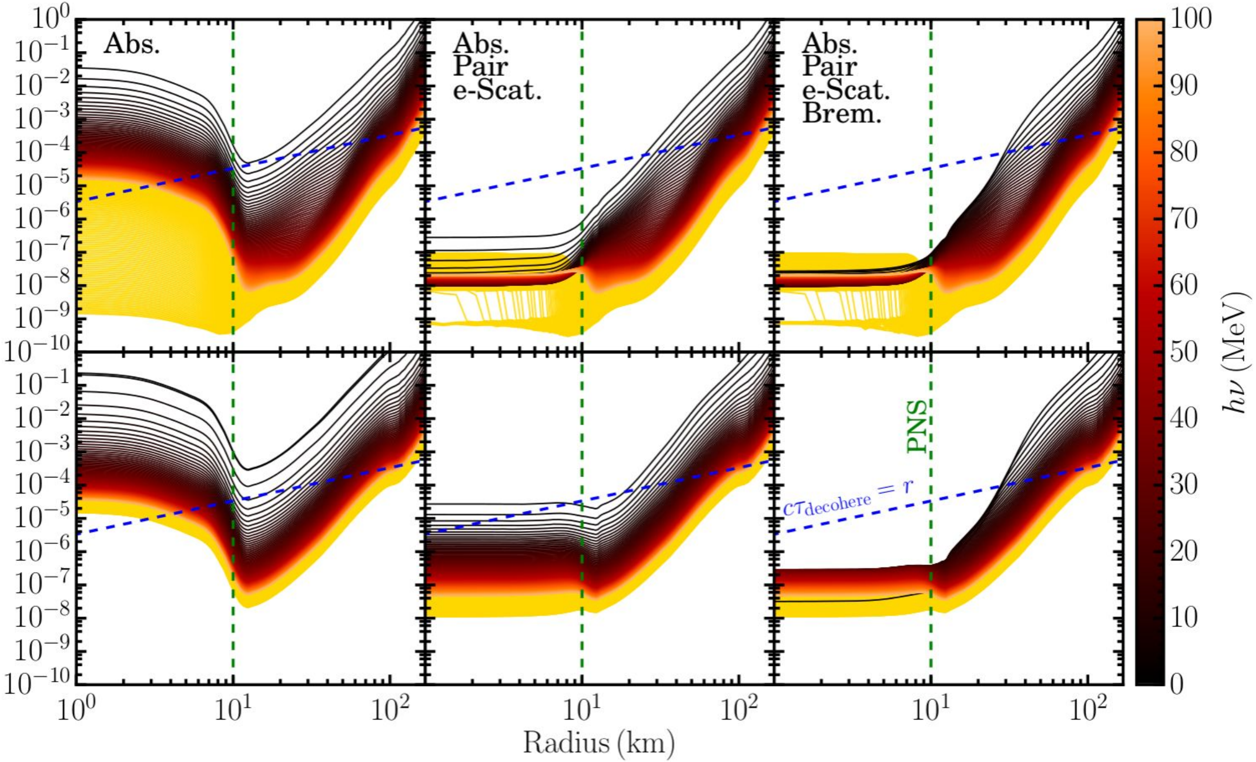
Decoherence with/without oscillations is **similar**

Separate timescales:

1) Damping oscillations
2) Relaxing to equilibrium

Many collision processes are important!



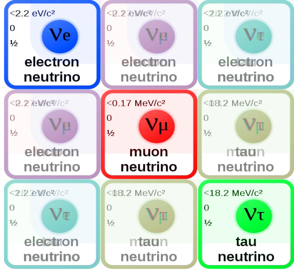


Core:

- Absorption/Emission
- Pair Processes
- Scattering
- Bremsstrahlung
- Neutrino-Neutrino

Heating Region:

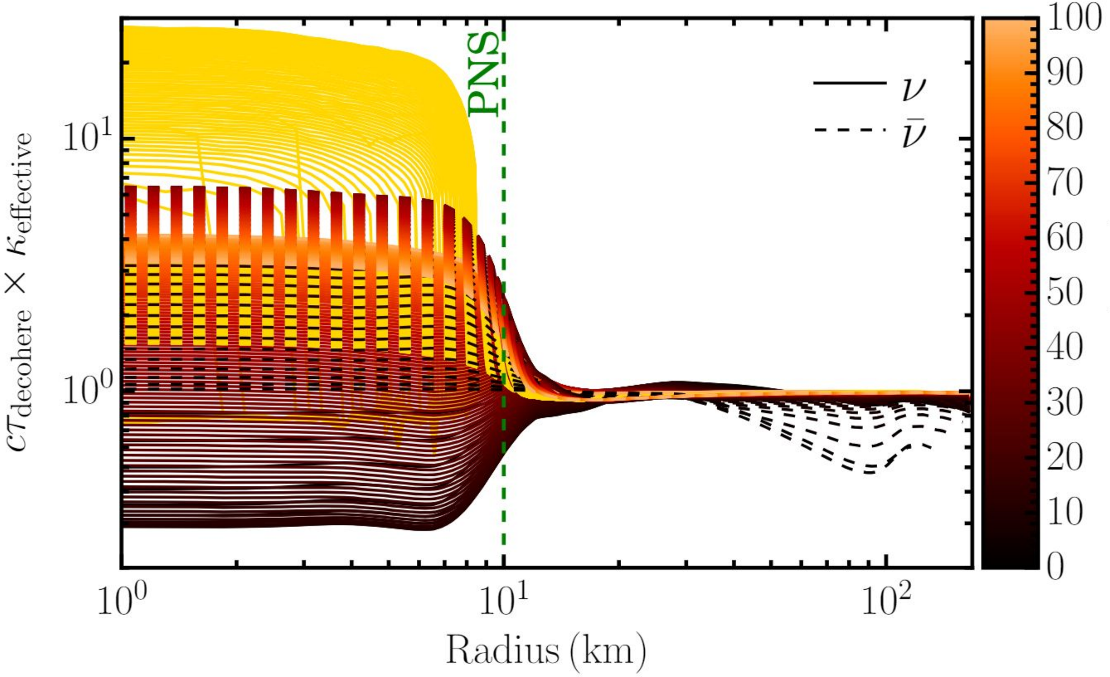
- Absorption/Emission



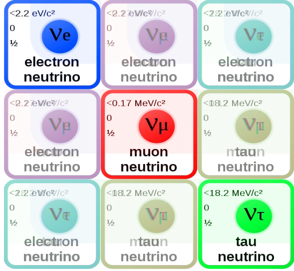
Simple "effective opacity"

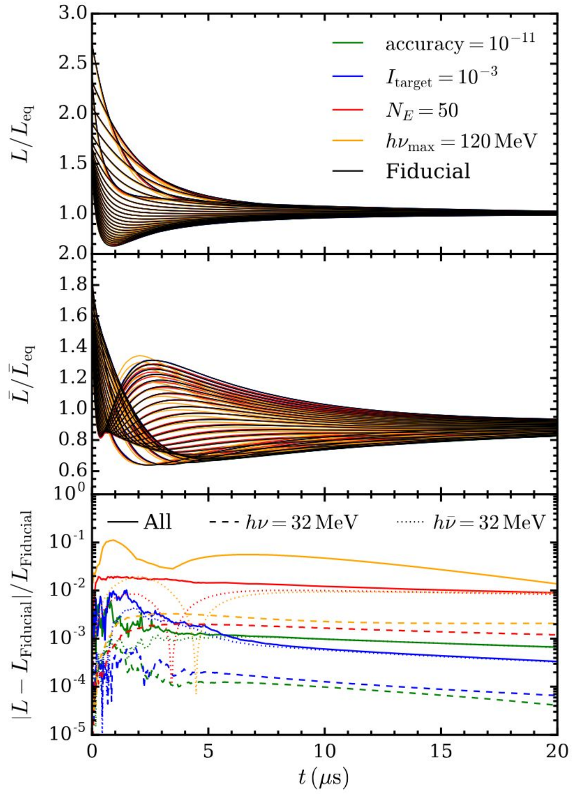
$$\kappa_{\text{effective}} := \kappa_{\text{abs},e\mu} + \frac{1}{2}\widetilde{\kappa}_{\text{scat},e\mu}$$

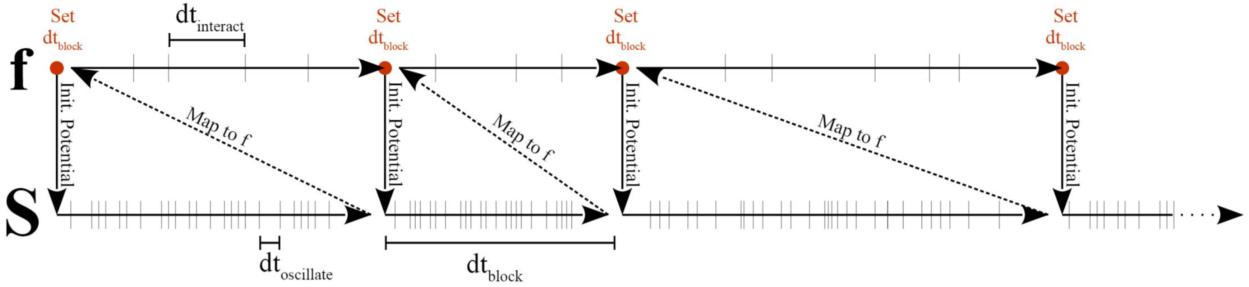
More effective than **mean free path** or **absorption opacity**



Numerical Challenges

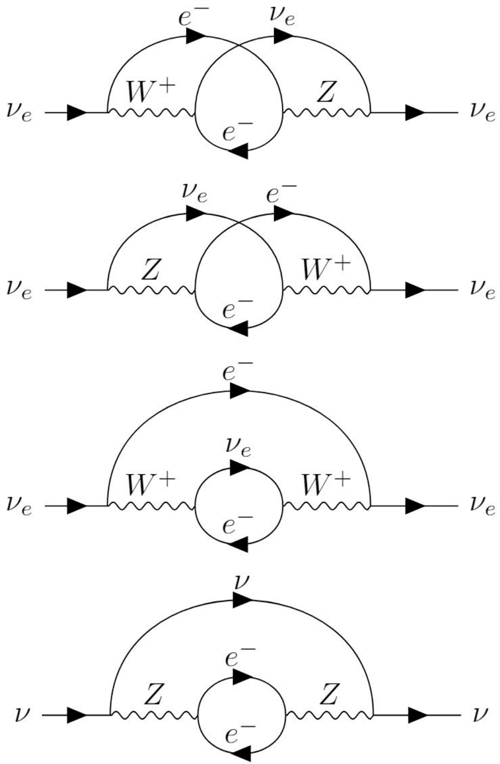






- Few evolution variables are **difficult to parallelize**
- **Conversion between S and f** induces error
- **Neutrino-neutrino interactions** scale with N_E^4 and require evenly-spaced energy grid
- Sensitive to size of the energy domain

Electron Scattering (inelastic)

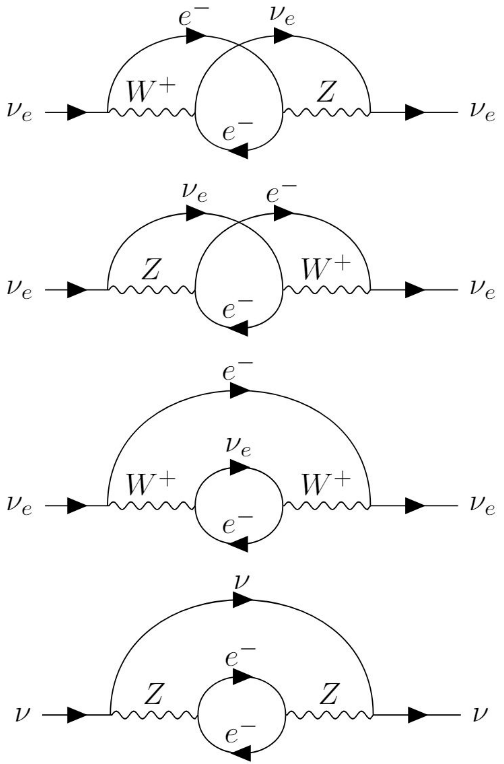


$$\frac{\partial f}{\partial t} = \{1 - f, \Pi^+\} - \{f, \Pi^-\} - i\,[\mathcal{H}, f]$$

$$\Pi^- = \int d^3p' \{AY_L(1-f')Y_L + BY_R(1-f')Y_R + C[Y_L(1-f')Y_R + Y_R(1-f')Y_L]\}$$

$$\Pi^+ = \int d^3p' \{AY_L f' Y_L + BY_R f' Y_R + C[Y_L f' Y_R + Y_R f' Y_L]\}$$

Electron Scattering (inelastic)



$$\frac{\partial f}{\partial t} = \left\{1 - f, \Pi^+\right\} - \left\{f, \Pi^-\right\} - i\left[\mathcal{H}, f\right]$$

$$\Pi^- = \int d^3p' \{A Y_L (1-f') Y_L + B Y_R (1-f') Y_R + \cancel{C[Y_L(1-f')Y_R + Y_R(1-f')Y_L]}\}$$

$$\Pi^+ = \int d^3p' \{A Y_L f' Y_L + B Y_R f' Y_R + \cancel{C[Y_L f' Y_R + Y_R f' Y_L]}\}$$

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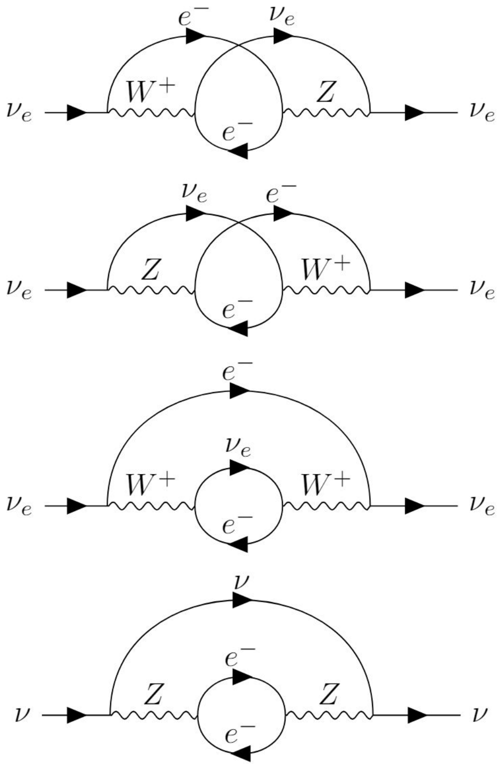

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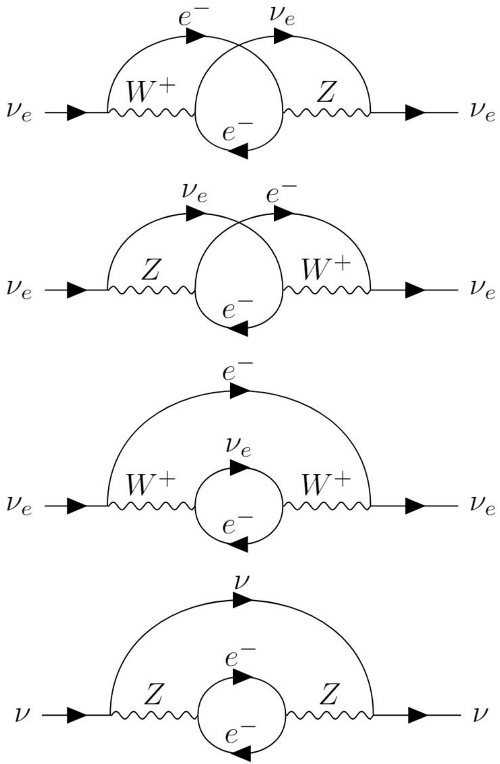
$$\cancel{C[Y_L(1 - f')Y_R + Y_R(1 - f')Y_L]}\}$$

$$\Pi^+ = \int d^3p' \{ A Y_L f' Y_L + B Y_R f' Y_R +$$
$$\cancel{C[Y_L f' Y_R + Y_R f' Y_L]}\}$$

Once again, we know the diagonals. **Just solve for A and B!**

(the hard part: multiplying matrices)

Electron Scattering (inelastic)



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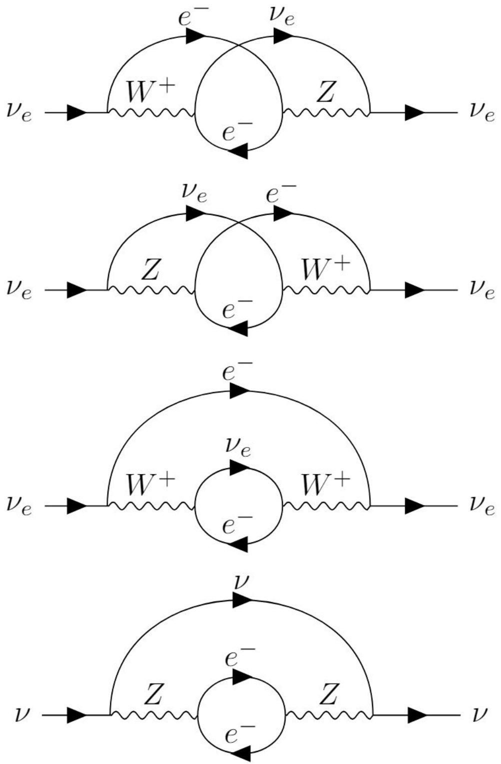
Electron Scattering (inelastic)



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$$\mathcal{C}^{-}_{ab} = \int d^3\nu' \left[\langle R\rangle^{-}_{ab} f_{ab} - \varsigma^{-}_{ab}\right]$$

Simple combination of
known scattering rates

$$\langle R\rangle = \begin{bmatrix} R_{(\nu_e)} & \frac{R_{(\nu_e)}+R_{(\nu_\mu)}}{2} \\ \frac{R_{(\nu_e)}+R_{(\nu_\mu)}}{2} & R_{(\nu_\mu)} \end{bmatrix}$$

$$R = \langle R\rangle - \begin{bmatrix} 0 & \frac{R_{(\nu_e)}-R_{(\nu_\mu)}}{4\sin^2\theta_W} \\ \frac{R_{(\nu_e)}-R_{(\nu_\mu)}}{4\sin^2\theta_W} & 0 \end{bmatrix}$$