Spaatind 2020

Lecture 1
Gravitational Waves: Theoretical Background

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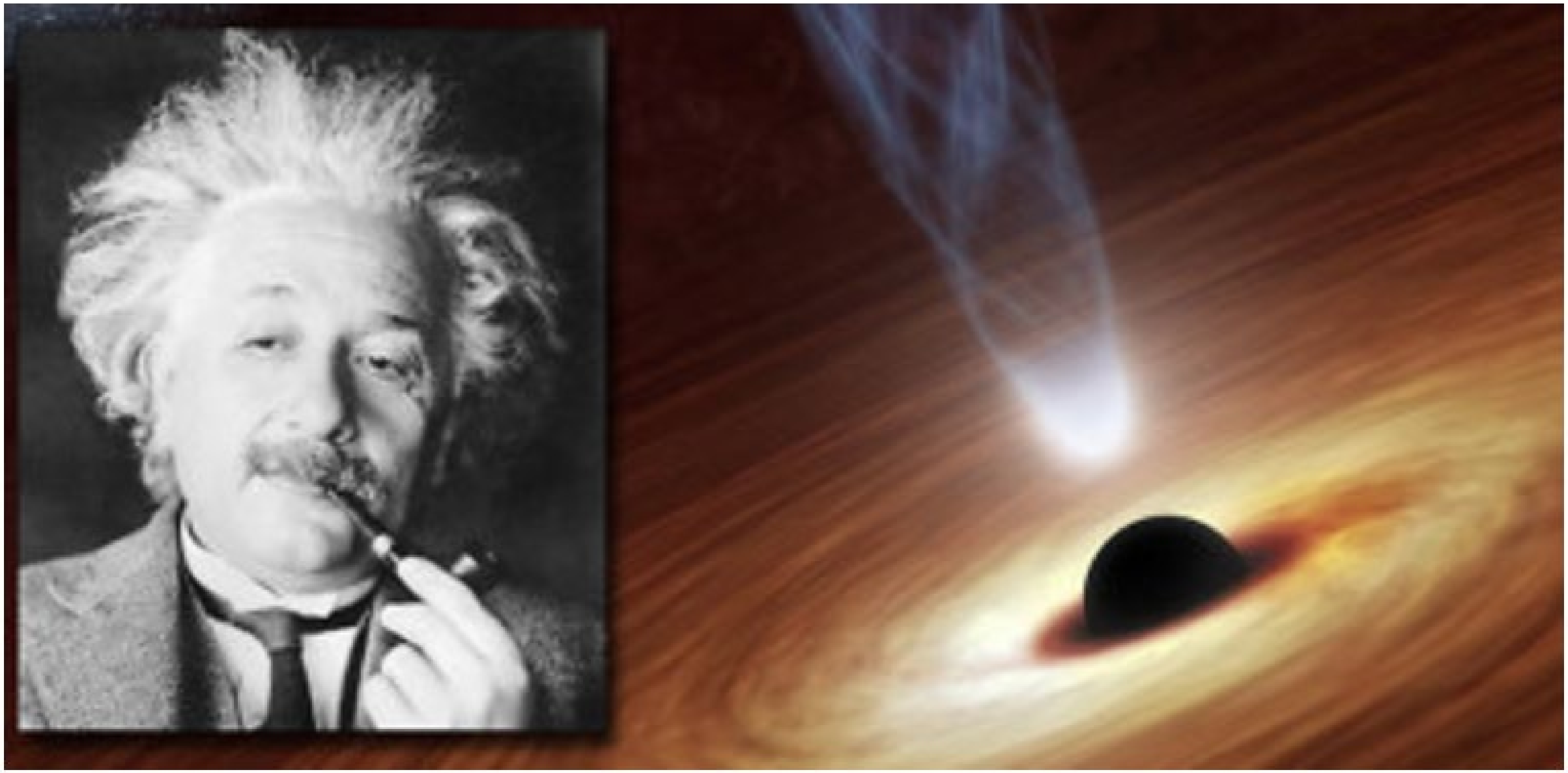


Forskere har funnet bevis for Einstein-teori:

- Det er vanskelig å finne store nok ord

Aftenposten, 11 Feb 2016

TOPICS

1) **What are gravitational waves?**
   Theoretical background

2) **How do we know we are seeing gravitational waves?**
   Detecting new signals

3) **What can we use gravitational waves for?**
   Astrophysics with gravitational waves
   Fundamental physics with gravitational waves

Einstein's changing attitude to gravitational waves

- **19 Feb 1916, letter to Schwarzshild:** *"Es gibt also keine Gravitationswellen, welche Lichtwellen analog wären"*
- **22 Jun 1916, article:** *"...so sieht man, daß A (die Ausstrahlung des Systems durch Gravitationswellen pro Zeiteneinheit) in allen nur denkbaren Fällen einen praktisch verschwindenden Wert haben muß." Nährungsweise Integration der Feldgleichungen, Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), 1916 688*
- **31 Jan 1918, article:** *"Da aber meine damalige Darstellung des Gegenstandes nicht genügend durchsichtig und außerdem durch einen bedauerlichen Rechenfehler verunstaltet ist, muß ich hier nochmals auf die Angelegenheit zurückkommen."Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften (Berlin), 1916 154*
- **1936 undated letter to Max Born:** *"Together with a young collaborator, I arrived at the interesting result that gravitational waves do not exist, though they have been assumed a certainty to the first approximation."*
- **1936 Princeton lecture:** *"If you ask me whether there are gravitational waves or not, I must answer that I do not know. But it is a highly interesting problem."*

Herrn John T. Tate
Editor The Physical Review
University of Minnesota
Minneapolis, Minn.

Sehr geehrter Herr:

Wir (Herr Rosen und ich) hatten Ihnen unser Manuskript zur Publikation gesandt und Sie nicht autorisiert, dasselbe Fachleuten zu zeigen, bevor es gedruckt ist. Auf die - übrigens irrtümlichen - Ausführungen Ihres anonymen Gewährsmannes einzugehen sehe ich keine Veranlassung. Auf Grund des Vorkommnisses ziehe ich es vor, die Arbeit anderweitig zu publizieren.

Mit vorzüglicher Hochachtung

P.S. Herr Rosen, der nach Sowjet-Russland abgereist ist, hat mich autorisiert, ihn in dieser Sache zu vertreten.

Source: APS/Kennefick, Traveling at the Speed of Thought: Einstein and the Quest for Gravitational Waves

Relativistic argument

perturbation propagation in Newtonian gravity → elliptical equation (Poisson's equation) → instantaneous

No gravitational waves in Newtonian gravity

perturbation propagation in Einstein gravity → hyperbolic equation (wave equation) → at speed of light

Gravitational waves are a generic prediction of relativity plus gravity, not just of Einstein's theory

Speed of gravity

. 1693 Edmund Halley, looking at ancient Babylonian and Arabic eclipse observations, noticed the period of the moon's orbit seemed to be getting shorter.

. 1776, Laplace examined possible explanations, in *Traité de mécanique céleste,* Volume IV, Book X, Chapter VII

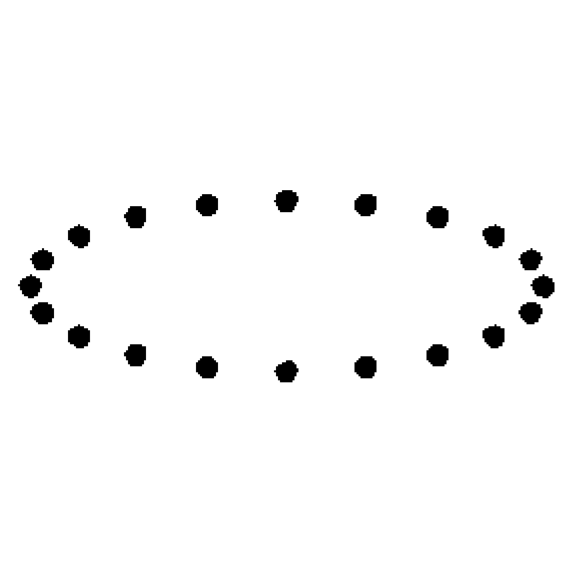
Newtonian finite-speed gravity: $v_g > 7x10^6c$

What are gravitational waves?

$G_{ab} = 8\pi G T_{ab}$ Einstein equations

Small linear perturbation: $g_{ab} = \eta_{ab} + h_{ab}$

Wave equation: $\nabla^2 \bar{h}_{ab} = 0$



$$Q_{ij} \equiv \int d^3 x \rho \left(x_i x_j - \frac{1}{3} r^2 \delta_{ij} \right)$$

Einstein quadrupole formula $h_{ij} = \frac{2G}{d_L} \frac{d^2 Q_{ij}}{dt^2}$

Tidal forces

Effect of gravitational field can be measured by tidal forces on freely falling particles

Tidal force due to ordinary Newtonian potential: $\nabla\nabla\phi \sim r^{-3}$

Tidal force due to gravitational wave perturbation: $\nabla\nabla h \sim r^{-1}\lambda^{-2}$

Newtonian force dominates in the near zone, r < λ , gravitational waves dominate far away r >> λ

Exact gravitational wave solutions

pp-waves in Brinkmann coordinates (u, v, x, y)

$$ds^2 = H(u,x,y)du^2 + 2dudv + dx^2 + dy^2$$

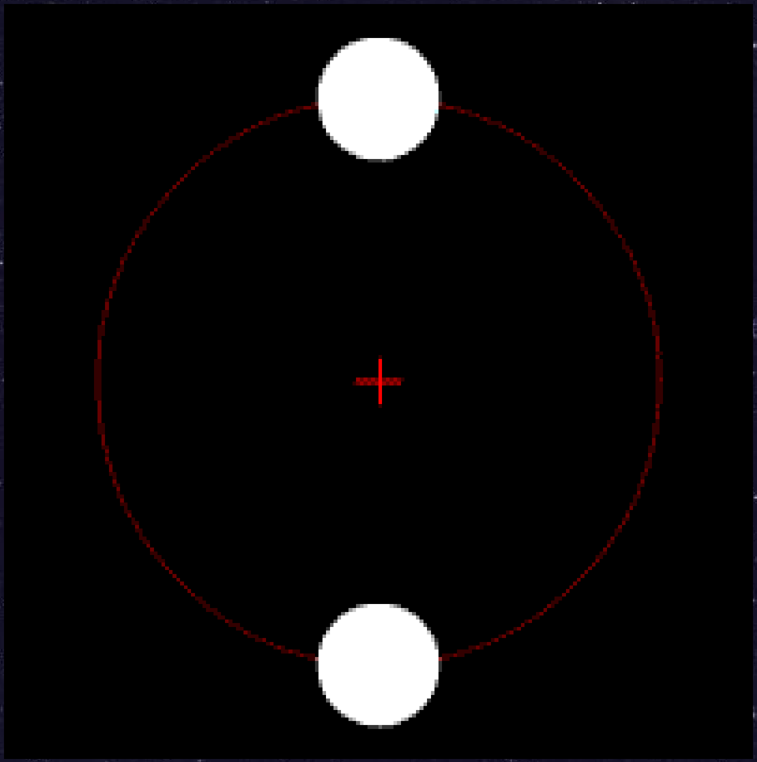
Vacuum solution of Einstein equations if

$$H(u,x,y) = a(u)(x^2 - y^2) + 2b(u)xy$$

Solution is not flat but

$$0 = R_{abcd}R^{abcd} = R_{abcd}R^{cdef}R_{ef}{}^{ab} = \ldots$$

Why binaries for Gravitational Waves?



File source: Zhatt

Einstein quadrupole formula:

$$h_{ij} = \frac{2G}{r}\frac{d^2}{dt^2}Q_{ij}$$

Power output of binaries

$$\text{Inspiral power} \quad \sim 6 \times 10^{40} \left(\frac{M}{M_{sun}}\right)^{\frac{10}{3}} \left(\frac{f}{10\text{Hz}}\right)^{\frac{10}{3}} W$$

Binary black hole at 100Hz	$\sim 10^{47}\ W$
EM luminosity of Sun	$\sim 10^{26}\ W$
EM luminosity of Milky Way	$\sim 10^{37}\ W$
EM luminosity of all visible galaxies	$\sim 10^{49}\ W$

Total inspiral	$E_{tot} \sim 5\%$	of rest mass energy
Ringdown	$E_{tot} \sim 1\%$	of rest mass energy
Supernova	$E_{tot} \sim 10^{-5}\%$	of rest mass energy

Quantization and gravitons

"the primary motivation for the study of [gravitational radiation] theory is to prepare for quantization of the gravitational field." *Felix Pirani*

Energy at ~ 1Hz is ~ 6×10^{-34}J

Power output (non-inertial) ~ 10^{-54}W

One graviton every ~10^{21} seconds >> $L_{universe}$

Energy of gravitational waves

Landau-Lifshitz energy pseudotensor (1951):

$$t^{ab} = \frac{c^4}{16\pi G(-g)} \partial_c \partial_d \left((-g)\left(g^{ab} g^{cd} - g^{ac} g^{bd}\right) \right)$$

Isaacson tensor (1968):

$$T_{ab}{}^{GW} = \frac{1}{32\pi} \langle h_{cd;a} h^{cd}{}_{;b} \rangle$$

Gravitational waves carry energy-momentum, but unlike other fields, this cannot be localised to a point (cf equivalence principle) and must be averaged over a region of size > λ.

Bondi-Metzner-Sachs (BMS) metric (1962)

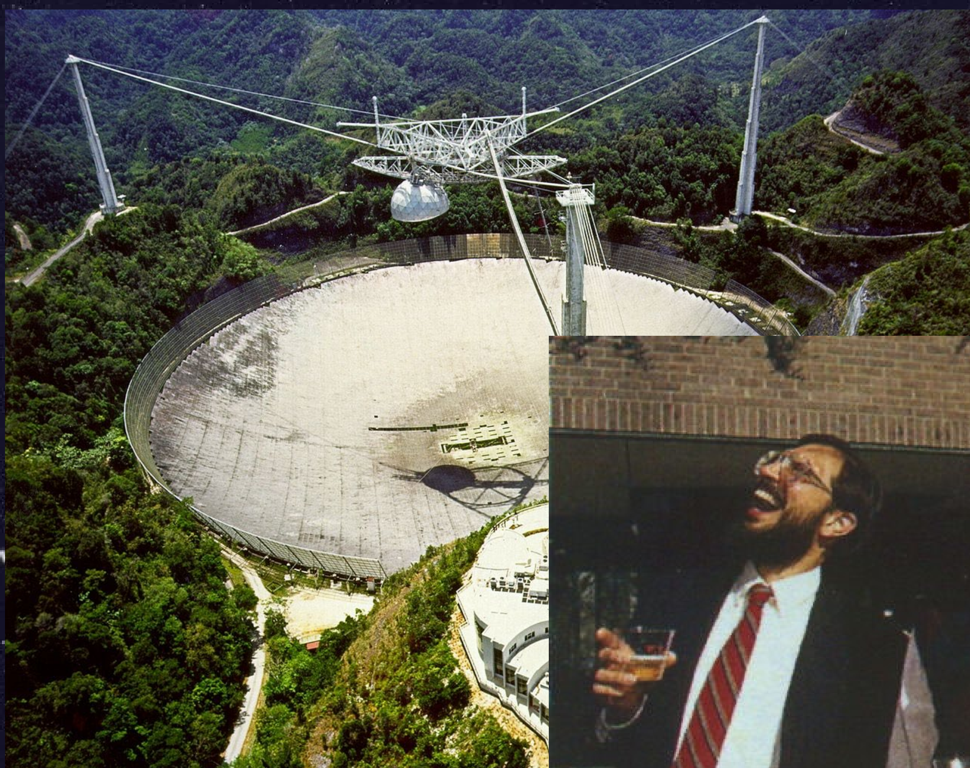
$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$
$$+\frac{2m_B}{r} + rC_{zz}dz^2 + rC_{\bar{z}\bar{z}}d\bar{z}^2 + D^zC_{zz}dudz + D^{\bar{z}}C_{\bar{z}\bar{z}}dud\bar{z}$$
$$+\frac{1}{r}\left(\frac{4}{3}(N_z + u\partial_z m_B) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c.$$

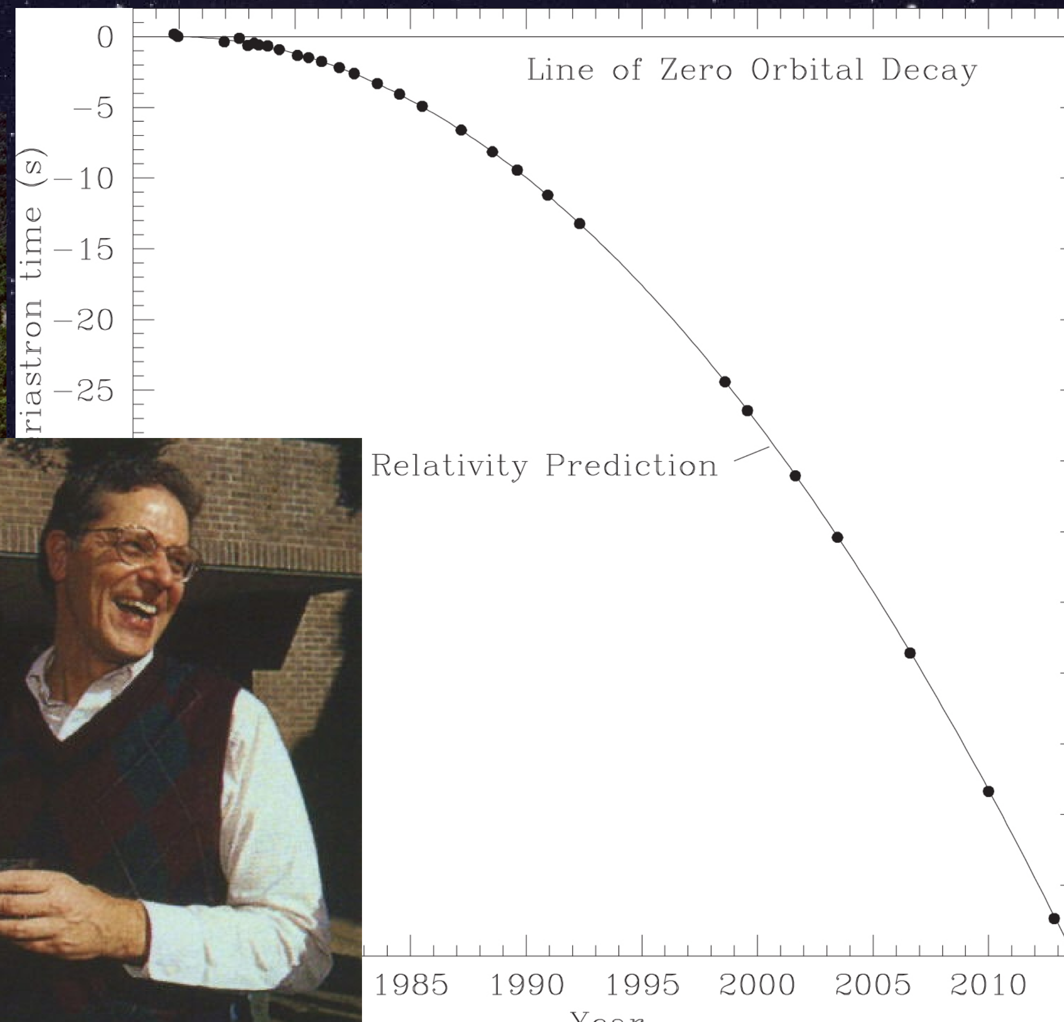
Bondi News function $N_{zz} = \partial_u C_{zz}$

Energy flux at infinity $\sim {N_{zz}}^2$

Hulse–Taylor Radio Pulsar PSR1913+16 (1974)

Source : NSF





Weisberg and Huang ApJ 829 (2016) 55

Nobel Prize 1993

Mathematical doubts persist...

THE ASTROPHYSICAL JOURNAL, **208**:L77–L81, 1976 September 1

COMMENTS ON GRAVITATIONAL RADIATION DAMPING AND ENERGY LOSS IN BINARY SYSTEMS

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ABSTRACT

It is argued that a formula for the energy loss due to gravitational radiation of bound systems such as binaries has not yet been derived either exactly or by means of a consistent approximation method within general relativity, a view which contradicts some widely accepted claims in the literature. The main approaches used to obtain such a formula are critically reviewed, and it is pointed out that the derivations presented so far either contain inconsistencies or are incomplete.

Subject headings: gravitation — relativity — stars: binaries

The determination of radiation reaction forces is a fundamental problem of any field theory of gravitation. More specifically, the discovery of the binary pulsar PSR 1913+16 and systems such as Ariel 1118−61 has concerned here not with a field theory of gravitation in the abstract, but with a particular one: Einstein's general theory of relativity. While in some respects Einstein's theory indeed has the same structure and

Propagation of gravitational waves

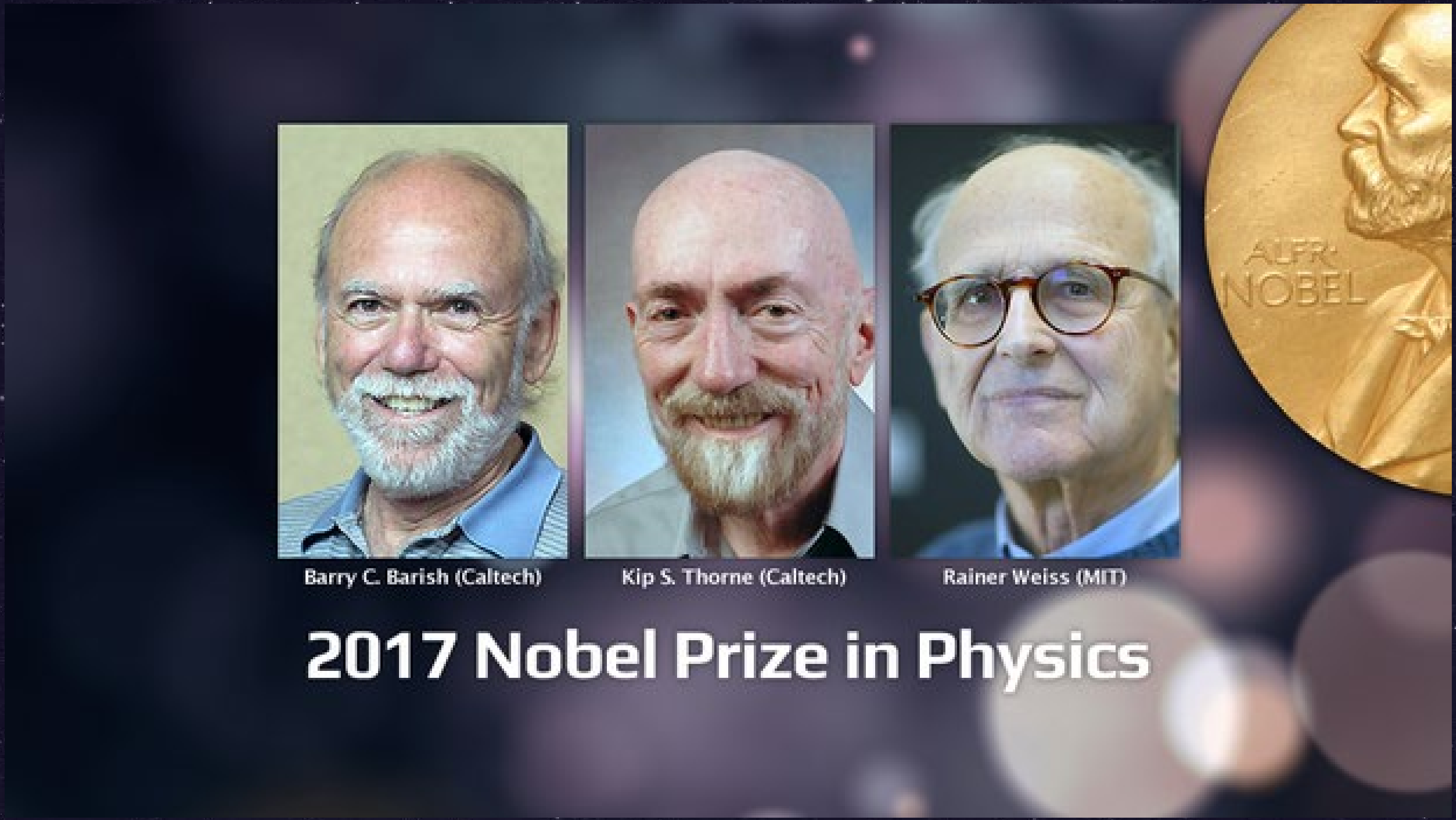
Plasma mean free path: $$L = \frac{1}{n\sigma} \approx \frac{c^4}{nG^2 m_p^2} \approx (10^{110})m$$

"...any reasonable regularization of the vacuum expectation value of the energy momentum tensor of the field must vanish. This means that a gravitational wave far from its source will propagate without hindrance by quantum effects."

Gibbons, Commun. Math. Phys 45 (1975) 191

$$\langle 0|T_{ab}|0\rangle = 0$$

2017 Nobel Prize in Physics

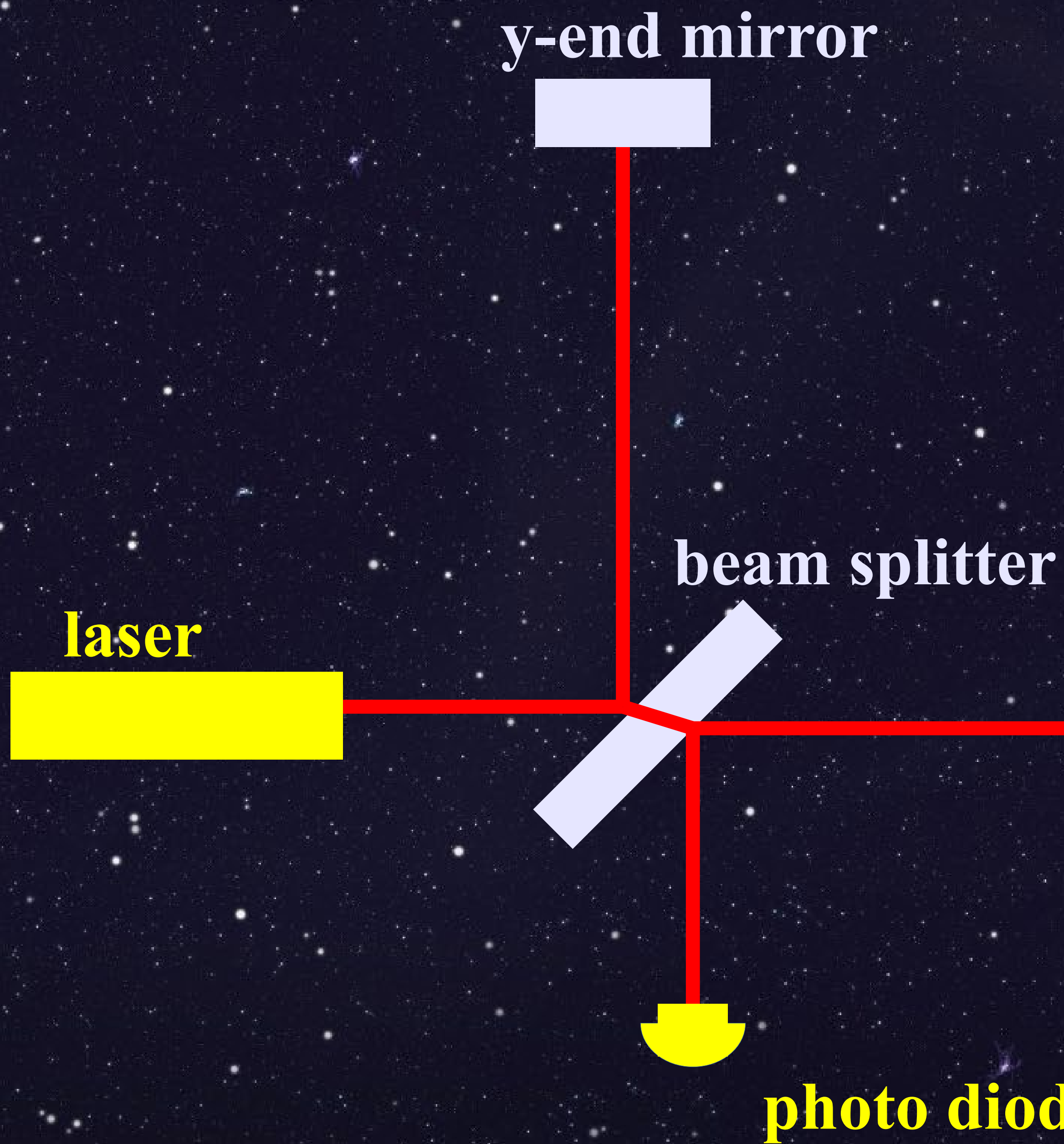


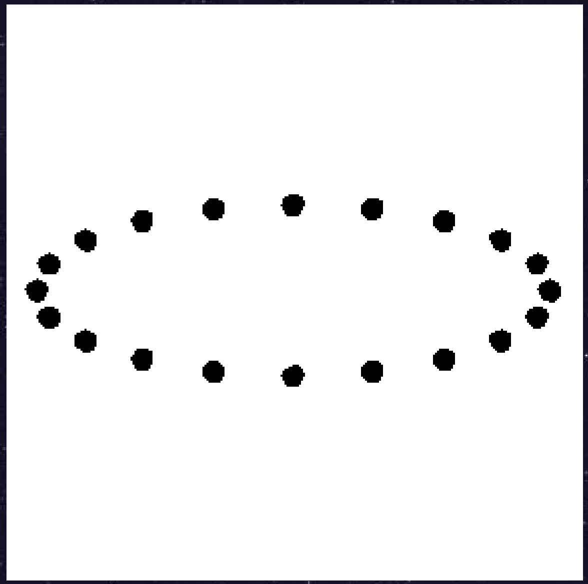
“"for decisive contributions to the LIGO detector and the observation of gravitational waves"”

USD 620 million*

Image source: otherhand.org
* Nature, 16 July 2014

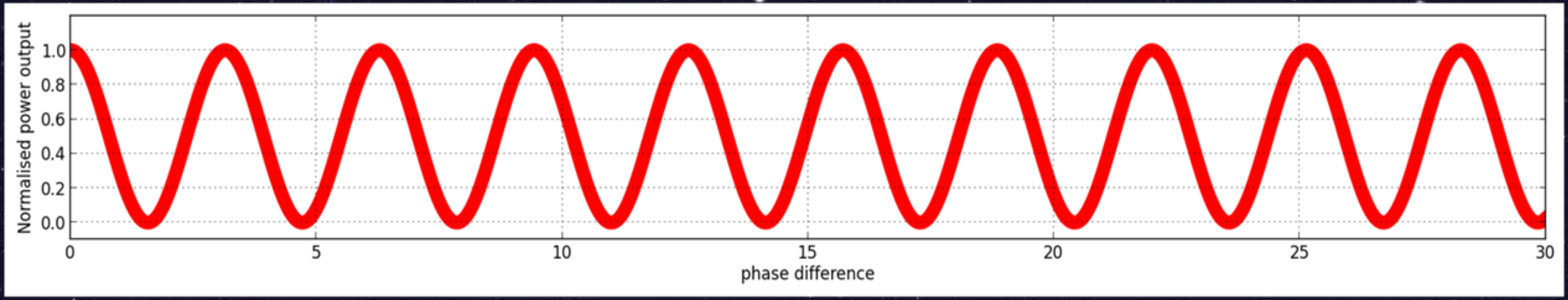
Interferometers





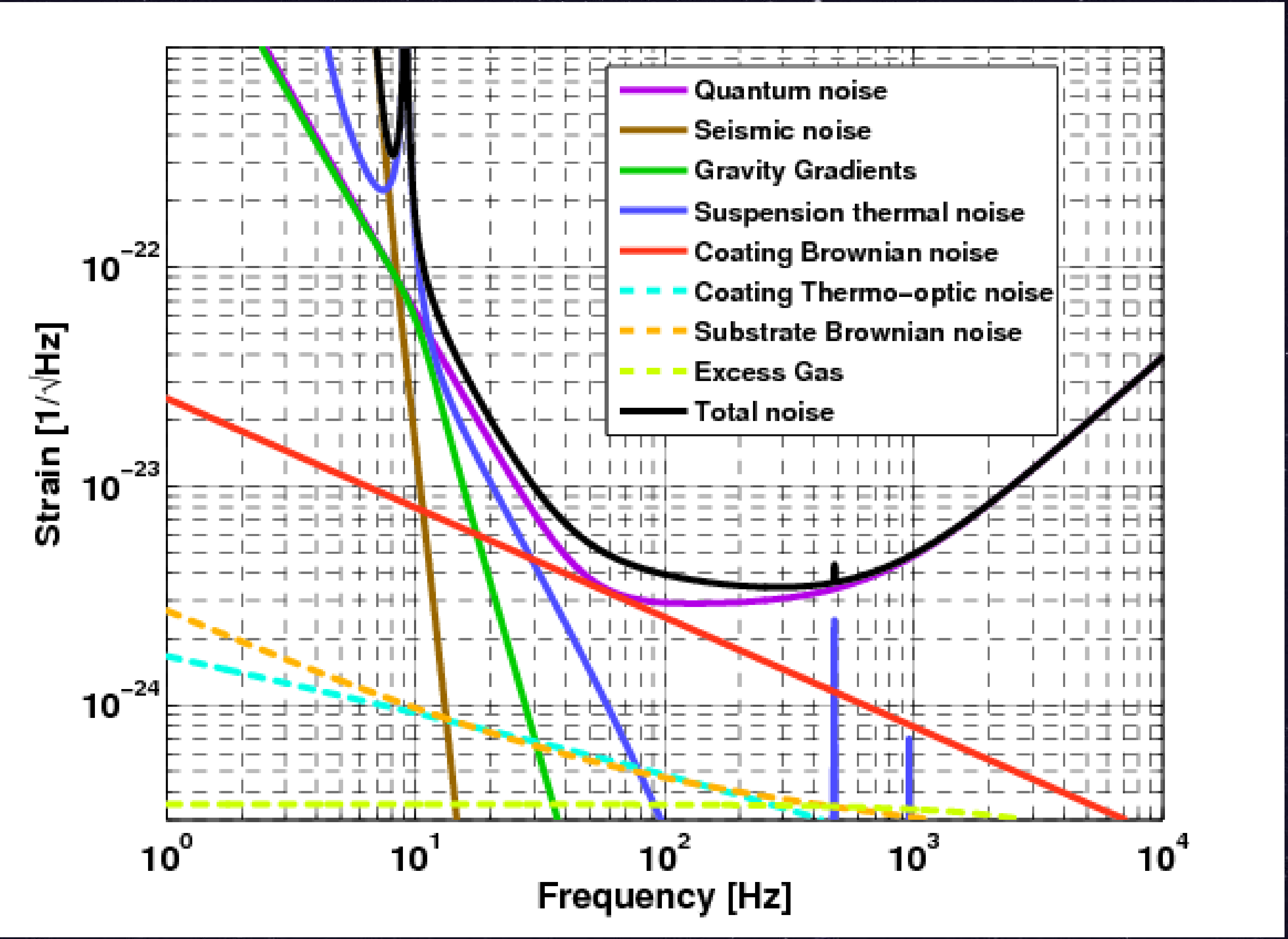
Reading between the lines

Interference pattern: $$P_{out} = P_{max}\cos^2\Delta\phi$$



$$\Delta\phi = \frac{\pi}{2} + B\frac{c\Delta T}{\lambda}$$:Accumulated phase difference

Displacement sensitivity: $$\Delta L = \frac{\lambda}{B}\sqrt{\frac{P_{out}}{P_{max}}}$$



Source: *S. Hild (2012)*

Spectral lines

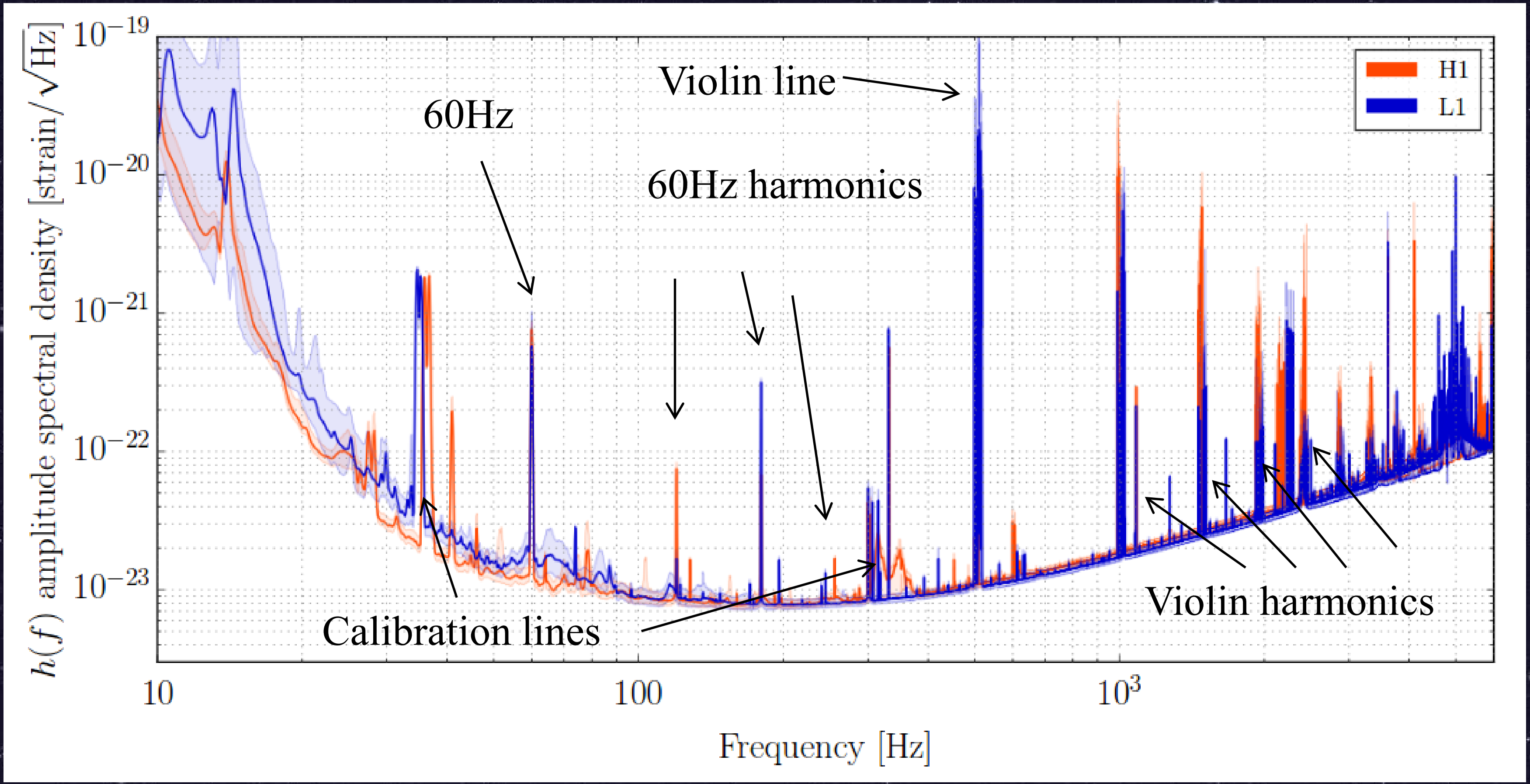
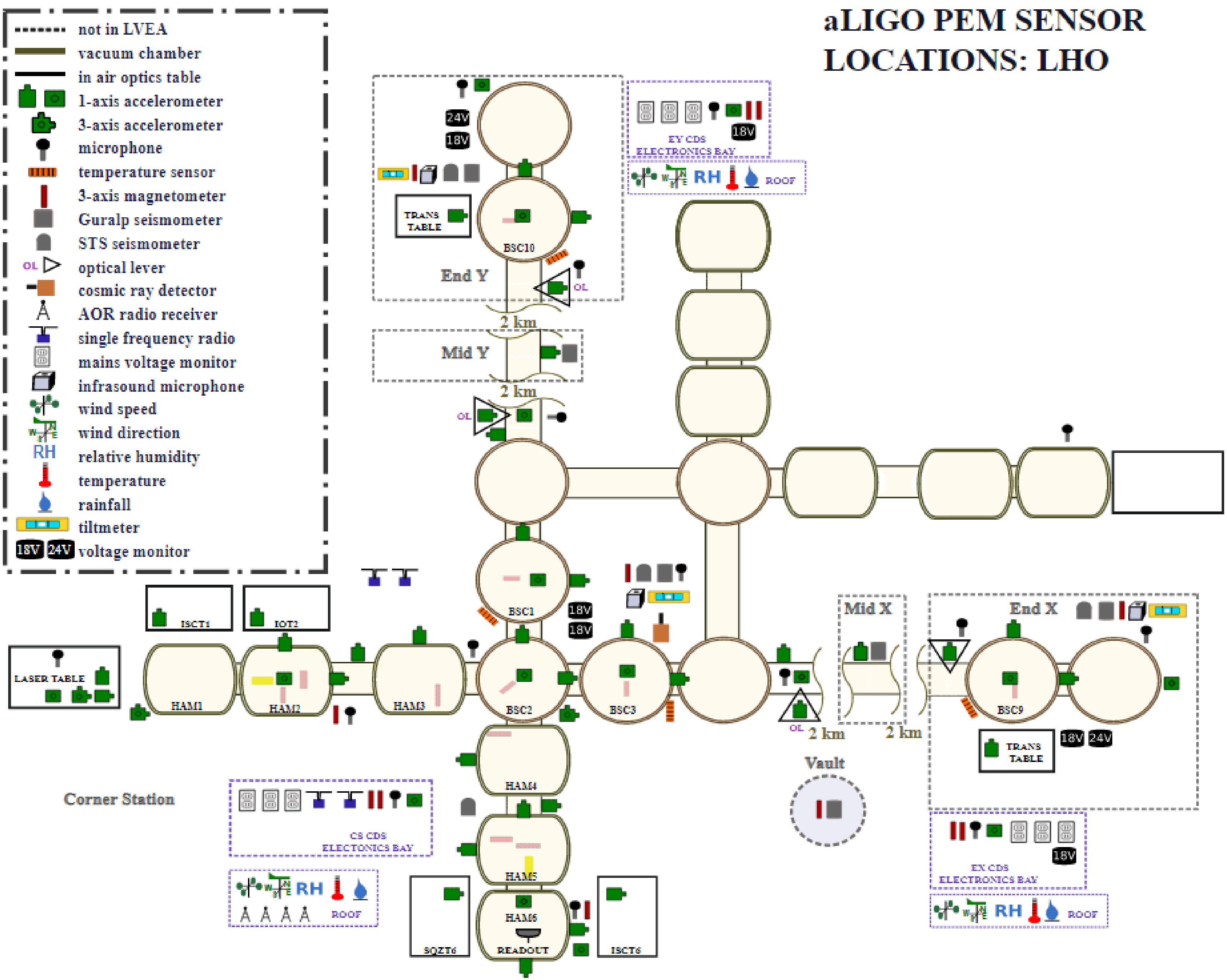


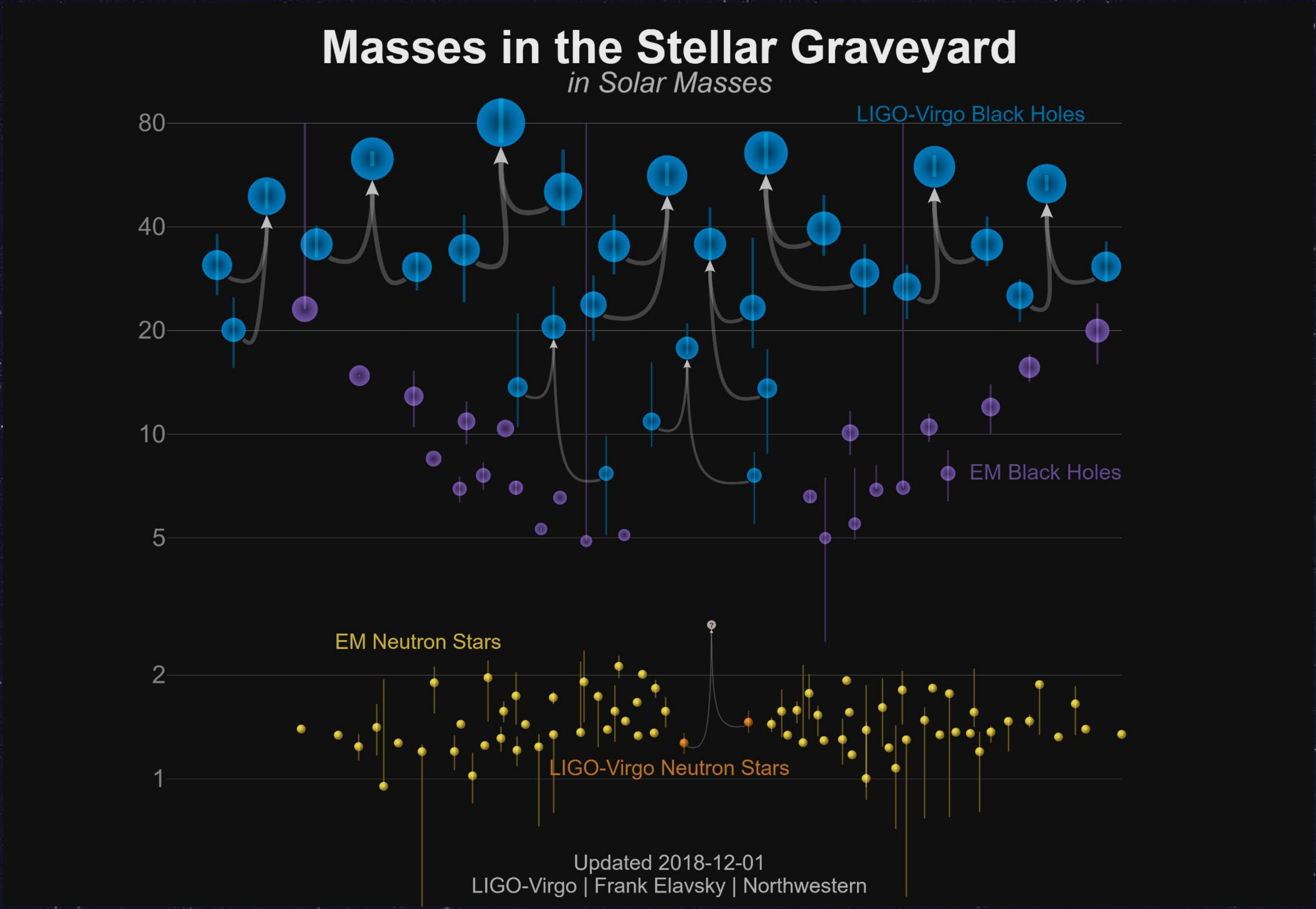
Fig. 1 Abbott et al. CQG33 (2016) 134001

aLIGO PEM SENSOR LOCATIONS: LHO



Source: ligo.pem.org

End of Lecture 1



[Image credit: LIGO-Virgo/Frank Elavsky/Northwestern University]

Technology development for (Advanced) LIGO

- **Long arms:** Earth's curvature over 4km is ~1m
- **High vacuum:** One trillionth atm, 10^{-9} torr in 10,000m^3
- **High power laser:** 20W 1064 nm Nd:YAG (neodymium-doped yttrium aluminium garnet) (will be up to 180W)
- **Higher power beams:** Fabry-Perot cavities, 100kW, power and signal recycling
- **Low-loss mirror coatings:** Titanium-doped tantalum pentoxide
- **Heavy test masses:** 40kg suspended by fused-silica wires 0.4mm thick, Heraeus Suprasil 3001
- **Near-dark photo diodes:** 50 mW, homodyne detection
- **Active seismic isolation:** at ~10^{-13} m, isolated reaction masses
- **Passive suspension:** four stage down to ~10^{-19} m

Einsteinturm

Astronomical-messenger firsts

- Optical: 32,000 BCE, Aurignacian, bones, moon's phases
- Meteorites: Jean-Baptiste Biot 1803, L'Aigle
- Cosmic Ray: Victor Hess 1912, balloon flights
- Radio: Karl Jansky 1931, from Milky Way
- X-ray: US Navy V-2 1949, from the Sun
- Gamma-ray: Explorer 11, 1961, <100 isotropic
- Neutrinos: Raymond Davis Jr and John Bahcall, 1968, Homestake Mine, from the Sun
- Gravitational waves: LVC 2015, from BBH GW150914

National Archeology Museum, Paris

Gravitationally bound orbits

- Earth-Moon system, period ~ 1 month !
- Earth-Sun system, period ~ 1 year !
- Sun-Galaxy, period ~ 250 million years
- Equal Mass Black Hole Binary, up to 100 times a second

Gravitationally bound orbits

- Earth-Moon system, period ~ 1 month!

measured +4cm out-spiral per year

- Earth-Sun system, period ~ 1 year!

measured to ~3metres, expected +2cm per year

- Sun-Galaxy, period ~ 250 million years

measured to ~0.5% of value

- Equal Mass Black Hole Binary, up to 100 times a second

measured to 1000km in 0.1 seconds!

Source: LIGO Lab

Emission of gravitational waves

Mass quadrupole $$Q_{ij} \equiv \int \rho\, x_i x_j d^3x$$

Mass octupole $$O_{ijk} \equiv \int \rho\, x_i x_j x_k d^3x$$

Einstein quadrupole formula $$h_{ij} = \frac{2}{r}\frac{d^2}{dt^2} Q_{ij}$$

Octupole emission formula $$h_{ij} = \frac{2}{3r} n^k \frac{d^2}{dt^2} O_{ijk}$$

… and so on …

Sticky bead argument

Sources / The Role of Gravitation in Phy… / An Expanded Version of the Rem… Support for autho

26 Summary of Conference

27 An Expanded Version of the Remarks by R.P. Feynman on the Reality of Gravitational Waves

I think it is easy to see that if gravitational waves can be created they can carry energy and can do work. Suppose we have a transverse-transverse wave generated by impinging on two masses close together. Let one mass A carry a stick which runs past touching the other B. I think I can show that the second in accelerating up and down will rub the stick, and therefore by friction make heat. I use coordinates

The Role of Gravitation in Physics

Report from the 1957 Chapel Hill Conference, DeWitt, Cecile M. et al.

Energy emitted

Einstein quadrupole formula: $$\frac{dE}{dt} = \frac{c^3 r^2}{32\pi G}\int d\,\Omega\langle \dot{h}_{ij}\dot{h}_{ij}\rangle$$

Equal mass circular motion: $$\frac{dE}{dt} = \frac{2}{5}\frac{G}{c^5}\Omega^6 M^2 D^4$$

Gravitationally bound motion: $$\frac{dE}{dt} = \frac{2}{5}\frac{c^5}{G}\left(\frac{R_S}{D}\right)^5$$

“...so sieht man, daß A (die Ausstrahlung des Systems durch Gravitationswellen pro Zeiteneinheit) in allen nur denkbaren Fällen einen praktisch verschwindenden Wert haben muß.”

Einstein, Preussische Akademie der Wissenschaften, Sitzungsberichte, 1916 688